

Scaling laws governing the collapse of a Bose-Einstein condensate

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We study the collapse of an attractive Bose-Einstein condensate, where an unstable system evolves towards a singularity, by numerically solving the underlying cubic-quintic nonlinear Schrödinger equation. We find good agreement between our simulations and the atom-loss measurements with a ^{39}K condensate. Our simulations reveal an interplay of weak collapse and the propensity of the system to form a hotspot and we uncover scaling laws that govern this behavior. We also identify promising signatures of the theoretically predicted, but so far experimentally elusive, elastic three-body interactions.

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Collapse of nonlinear systems is ubiquitous in nature, from the breaking of ocean waves [1,2] to the gravitational collapse of stars [3]. When attractive forces overwhelm dispersive ones, systems evolve towards a singularity in finite time. In practice, dissipative processes typically become relevant as the singularity is approached and can stabilize the system.

The same cubic nonlinear Schrödinger equation is at the heart of the theoretical description of a variety of collapse phenomena [4,5], including self-focusing of light [6], collapse of Langmuir waves in plasmas [7–9], and white-caps on choppy water [1]. A key role is played by the dimensionality: lower-dimensional systems tend to feature soliton solutions [10], including the celebrated Townes soliton in two dimensions [11–13], whereas three-dimensional (3D) systems predominantly exhibit collapse. For the 3D collapse, there is a rich variety of self-similar solutions that predict how a singularity is approached, including nonpeaked solutions [5] and the counterintuitive weak collapse [14–17], where a stronger attractive nonlinearity leads to less dissipation. In comparison, little is understood about the dynamics after the collapse [4,5], with possibilities including further collapse events and the formation of a hotspot [17–24], where an emergent effective potential sustains a wave-action flux into a dissipation region in which the wave remains localized for a prolonged time.

Ultracold atoms provide powerful analog simulators of this nonlinear Schrödinger equation; the sign and the strength of the cubic nonlinearity can be tuned using Feshbach resonances [25] and inelastic three-body recombination naturally acts as a quintic dissipation mechanism. Collapse experiments have been performed with atomic Bose-Einstein condensates in harmonic [26–32] and box traps [33] (see also [34]) and

both the criterion and the time for the collapse to occur have been understood (see also [42–47]). Moreover, by isolating single-collapse events, the weak nature of the 3D collapse was observed [33]. However, the scaling of the atom loss with the strengths of the cubic and quintic nonlinearities did not follow Zakharov’s analytic prediction [15] and was never fully understood.

In this Letter, we perform numerical simulations of collapse in the cubic-quintic nonlinear Schrödinger equation. Focusing on single-collapse events, the extracted total loss agrees with the measurements with a box-trapped ^{39}K Bose-Einstein condensate in Ref. [33] and the additional measurements we took in the same system for this study. The time-resolved loss dynamics feature an interplay of weak collapse and the propensity of the system to form a hotspot and we uncover scaling laws that govern this behavior. We also study the effects of the theoretically predicted, but experimentally elusive, elastic three-body interactions [48–53]. At their predicted strength, these interactions modify the single-collapse loss only slightly, at a level that is not resolved in the experiments. However, we find promising signatures in the time-resolved loss dynamics, including significantly longer dissipation times and suppression of the hotspot behavior.

In dimensionless form, the nonlinear Schrödinger equation with an elastic cubic and dissipative quintic term reads

$$i\frac{\partial\psi}{\partial t} = -\nabla^2\psi + \alpha|\psi|^2\psi - i\eta|\psi|^4\psi, \quad (1)$$

where α and $\eta > 0$ are real. We initially ($t = 0$) normalize the wave function $\psi(r, t)$ to unity inside a spherical box of unit diameter and enforce the boundary condition $\psi = 0$ at the trap walls. To convert from dimensionless units to the experimental ones for the collapse of a box-trapped condensate [33], one rescales $r \rightarrow r/L$, $\psi \rightarrow L^{-3/2}\psi$, and $t \rightarrow t/\tau$, where L is the characteristic box size, $\tau = 2mL^2/\hbar$ the characteristic timescale, and m the atom mass, and uses

$$\alpha = \frac{8\pi a N_0}{L}, \quad \eta = \frac{N_0^2 m \mathcal{L}_3}{\hbar L^4}, \quad (2)$$

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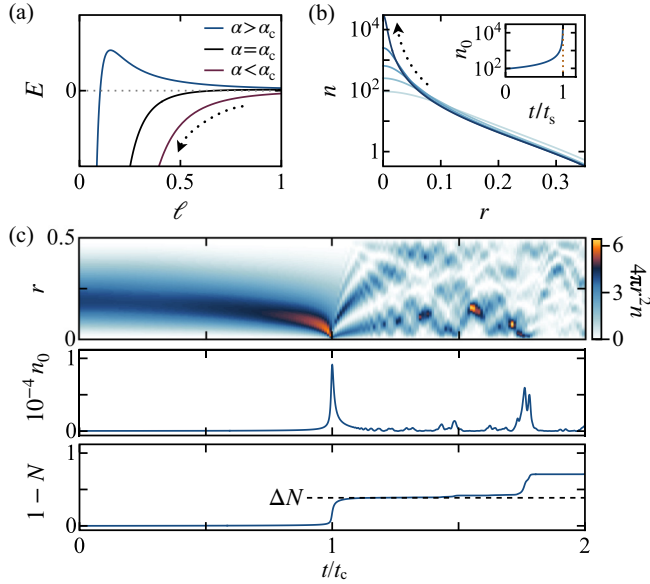


FIG. 1. Weak collapse. (a) Sketch of the effective energy landscape for a wave function of extent ℓ . The interaction energy is $\propto \alpha/\ell^3 < 0$ and collapse occurs (arrow) for $\alpha < \alpha_c$. (b) Typical evolution of the density profile $n(r, t)$ after a quench of α from above to below α_c . As the system approaches the singularity (light to dark colors), the central density diverges at a time t_s (see inset) and a diminishing fraction of the wave approaches the singularity, with stationary tails left behind. (c) Simulation results for a spherical box with $\alpha_c = -3.89(1)$, $\alpha = -4.2$, and nonzero dissipation strength $\eta = 3 \times 10^{-4}$ [see Eq. (1)]. We show the dynamics of the radial population density $4\pi r^2 n$ (top), the central density n_0 (middle), and the fractional loss $1 - N$ (bottom), where $N = \int 4\pi r^2 n dr \leq 1$; t_c is the time when n_0 first peaks (note that $t_c \rightarrow t_s$ for $\eta \rightarrow 0$ [57]). The dashed line indicates ΔN , the total single-collapse loss.

where a is the s -wave scattering length, N_0 the initial condensate atom number, and $\mathcal{L}_3$ the three-body loss coefficient.

Assuming $\eta = 0$ and a characteristic wave function extent ℓ in Eq. (1), the kinetic energy is $\propto 1/\ell^2$ and the interaction energy is $\propto \alpha/\ell^3$. As sketched in Fig. 1(a), for slightly negative α collapse is prevented by a kinetic-energy barrier, which disappears at a negative critical value α_c .

As illustrated in Fig. 1(b), after α is quenched (at $t = 0$) from above to below α_c , the system approaches a singularity in time t_s . The density profile, $n(r, t) = |\psi(r, t)|^2$, is predicted [15], for $t \rightarrow t_s$, to evolve self-similarly with a peaked scaling solution χ :

$$n(r, t) = n_0(t)\chi[r/r_0(t)], \quad (3)$$

with $\chi(0) = 1$, $\chi \propto (r_0/r)^2$ for $r \gg r_0$, $r_0 \propto \sqrt{t_s - t}$, and $n_0(t) = n(0, t) \propto 1/[|\alpha|(t_s - t)]$. The collapse is termed *weak* because the fraction of the wave that is collapsing towards the origin, $n_0 r_0^3 \propto \sqrt{t_s - t}/|\alpha| \propto 1/\sqrt{n_0|\alpha|^3}$, diminishes as n_0 diverges, with stationary tails, $n(r) \propto 1/(|\alpha|r^2)$, left behind (see also [54]).

Introducing weak dissipation, $\eta \ll |\alpha|$, the quintic term in Eq. (1) significantly affects the dynamics once it becomes comparable to the cubic one, which happens for $n_0 \propto |\alpha|/\eta$ and corresponding $r_0 \propto \eta^{1/2}/|\alpha|$. At this point in time, the

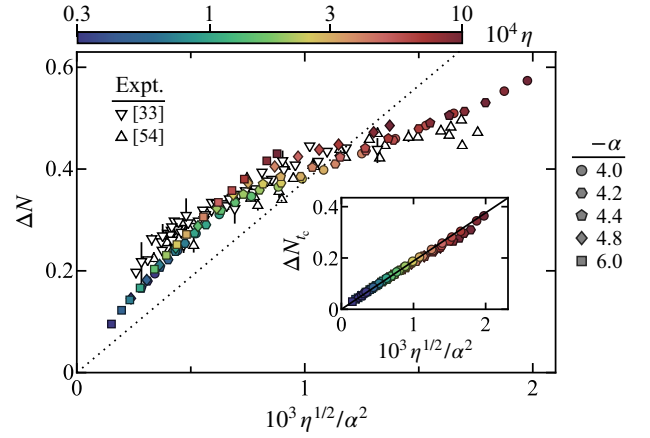


FIG. 2. Single-collapse atom-loss scaling laws. Plot of ΔN versus $\eta^{1/2}/\alpha^2$, comparing the experimental data (open triangles) to our simulations with different α (colored symbols) and η (color bar). The inset shows the simulated ΔN_c , the loss up to t_c , which follows the prediction in Eq. (4) (solid line), and suggests that the different scaling of ΔN arises due to dynamics in the aftermath of the singularity; the dotted line shows $\Delta N = 2\Delta N_c$. Note that the systematic uncertainty in experimentally determining $\eta^{1/2}/\alpha^2$ is $\pm 20\%$.

proportion of the wave that is still collapsing towards the origin is $\propto \eta^{1/2}/\alpha^2$. Assuming that this part is lost due to dissipation, while the tails are unaffected, gives the Zakharov scaling law for the fractional loss ΔN in a single-collapse event [7]:

$$\Delta N \propto \eta^{1/2}/\alpha^2, \quad (4)$$

which, counterintuitively, is smaller for a more unstable system (larger $|\alpha|$). These scaling arguments provide some intuition, but a complete understanding of collapse in Eq. (1) is lacking [5] and it is in general difficult to simulate due to the necessary spatiotemporal resolution [23,58,59].

Here, we assume spherical symmetry [60] and numerically solve the radial part of Eq. (1) in 1D (see [54] for details). We mimic the experimental protocol from Ref. [33] by starting from the metastable state for $\alpha = -3.6$, above $\alpha_c \approx -3.9$. We then quench to $\alpha < \alpha_c$ and record the evolution of ψ for different values of η .

In Fig. 1(c) we show a typical example of the simulated dynamics, which reveals an isolated first collapse event, followed by rich dynamics, including refocusing and subsequent collapses (see also [5,24,64–66]). We plot the evolution of the radial population density $4\pi r^2 n(r, t)$ (top), the central density n_0 (middle), and the loss $1 - N(t)$ (bottom), where $N(t) = \int 4\pi r^2 n(r, t) dr \leq 1$. Here t_c is the time when n_0 first peaks [57] and the dashed line shows the extracted ΔN ; note that throughout this Letter we focus on the first collapse event.

In Fig. 2, we compare our results for $\alpha \in [-4, -6]$ and $\eta \in [0.3, 10] \times 10^{-4}$ (colored symbols) to the measurements of ΔN with a box-trapped ^{39}K condensate (open triangles), finding good agreement; note that here we include both the data from Ref. [33] and new data taken for this study (see [54] for details). In both simulation and experiment, the data do not follow the prediction $\Delta N \propto \eta^{1/2}/\alpha^2$ [Eq. (4)]. However, as shown in the inset, the loss up to t_c , denoted ΔN_c , does

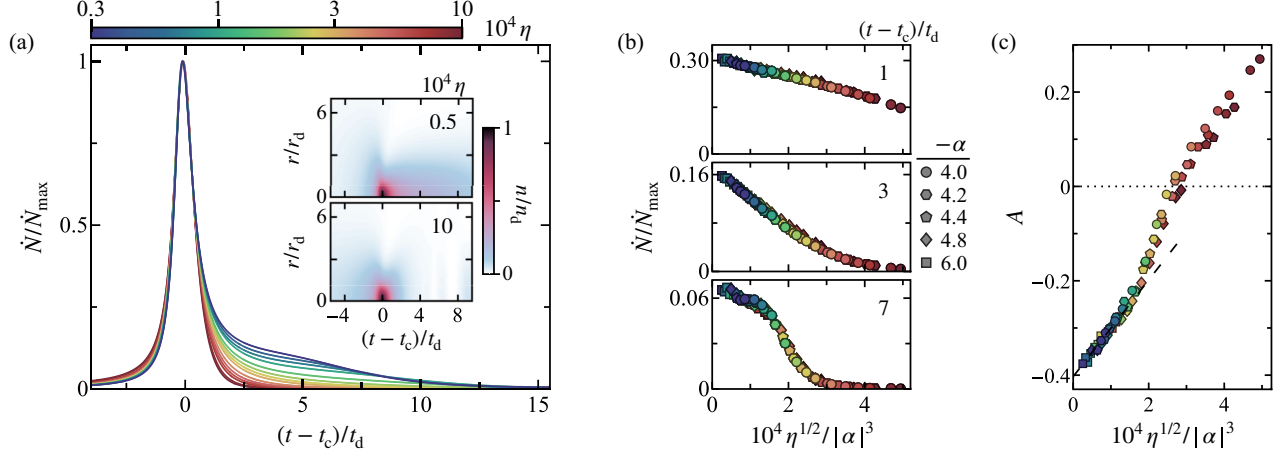


FIG. 3. Time-resolved loss dynamics revealing the scaling laws that govern the propensity to form a hotspot. (a) Normalized loss rate $\dot{N}/\dot{N}_{\max}$ versus $(t - t_c)/t_d$ for $\alpha = -4.0$ and varying η ; here t_d is the full width at half maximum of $|\dot{N}(t)|$. For smaller η , the universal weak-collapse peak is accompanied by a growing “shoulder” (for $t > t_c$). The insets show $n(r, t)$ for $\eta = 5 \times 10^{-5}$ (top) and $\eta = 10^{-3}$ (bottom), where for the smaller η the density remains high and localized for a prolonged time, a key signature of hotspot behavior; here $n_d = n(0, t_c)$ and r_d is defined so that $n(r_d, t_c) = n_d/2$. (b) Plotting $\dot{N}/\dot{N}_{\max}$ at three characteristic $(t - t_c)/t_d > 0$ (panels) versus $\eta^{1/2}/|\alpha|^3$ for different α (legend) and η [colors, as in (a)] reveals remarkably universal behavior. (c) The loss asymmetry $A = 2\Delta N_c/\Delta N - 1$ also shows universal behavior in $\eta^{1/2}/|\alpha|^3$. For $\eta^{1/2}/|\alpha|^3 \rightarrow 0$ we find that A approaches ≈ -0.4 ; the dashed line shows a linear fit for $\eta^{1/2}/|\alpha|^3 < 1 \times 10^{-4}$.

show this scaling (solid line), so the difference must occur after t_c (cf. dotted line, which shows $\Delta N = 2\Delta N_c$).

To elucidate the collapse dynamics, we look at the time-resolved loss rate $\dot{N} = dN/dt = -8\pi\eta \int n^3 r^2 dr$. In Fig. 3(a), for $\alpha = -4.0$ and various η , we plot $\dot{N}/\dot{N}_{\max}$ versus $(t - t_c)/t_d$, where $\dot{N}_{\max}$ is the maximal instantaneous loss rate and t_d the dissipation time, defined as the full width at half maximum of $|\dot{N}(t)|$.

Before and near t_c , the curves coincide, meaning that $\Delta N_c \propto t_d |\dot{N}_{\max}|$. This is consistent with the Zakharov weak-collapse picture [23], with the characteristic size of the dissipation region $r_d \propto r_0(t_c) \propto \eta^{1/2}/|\alpha|$, density $n_d = n_0(t_c) \propto |\alpha|/\eta$, and time $t_d \propto \eta/\alpha^2$, such that $t_d |\dot{N}_{\max}| \propto t_d \eta n_d^3 r_d^3 \propto \eta^{1/2}/\alpha^2$, as in Eq. (4). We define r_d so that $n(r_d, t_c) = n_d/2$ and have verified the scaling laws for n_d , r_d , t_d , and $\dot{N}_{\max}$ (see [54]).

However, for the smaller η values the loss extends to longer times, with a prominent “shoulder” at $t > t_c$. As illustrated in the inset, the corresponding density profiles remain localized for much longer ($\gtrsim 5t_d$), which is a key signature of hotspot formation [17–24] (see also [67]).

In Fig. 3(b) we extend our analysis to different α values and show that the hotspot shoulder exhibits remarkably universal behavior: at a fixed $(t - t_c)/t_d > 0$, we empirically find that $\dot{N}/\dot{N}_{\max}$ is a function of $\eta^{1/2}/|\alpha|^3$ [68]. To our knowledge, this scaling has not been predicted and invites further study [69].

In Fig. 3(c) we introduce the loss asymmetry $A \equiv 2\Delta N_c/\Delta N - 1$, which is also a universal function of $\eta^{1/2}/|\alpha|^3$. Interestingly, in the regime where the hotspot behavior is most pronounced ($\eta^{1/2}/|\alpha|^3 \rightarrow 0$), A approaches ≈ -0.4 (rather than -1), so that $\Delta N \approx 3.3 \Delta N_c$. Note that this recovers the Zakharov scaling $\Delta N \propto \eta^{1/2}/\alpha^2$ and, since A grows linearly for small $\eta^{1/2}/|\alpha|^3$ (dashed line), the next-order correction to ΔN is $\propto \eta/|\alpha|^5$. For large $\eta^{1/2}/|\alpha|^3$, A exceeds 0

because a significant fraction of the wave is lost already before t_c .

In experimental units, our simulations predict a size of the dissipation region $r_d L \approx 1.9 \times (m\mathcal{L}_3/\hbar)^{1/2}/(8\pi|a|)$, a peak density $n_d N_0/L^3 \approx 0.72 \times 8\pi\hbar|a|/(m\mathcal{L}_3)$, and a dissipation time $t_d \tau \approx 25 \times m^2 \mathcal{L}_3/(32\pi^2 \hbar^2 a^2)$ [54]; note that these do not depend on either N_0 or L . For ^{39}K with $\mathcal{L}_3 = 1.3(5) \times 10^{-41} \text{ m}^6 \text{ s}^{-1}$ [70] at $a = -0.3a_0$ [71] (where a_0 is the Bohr radius), $r_d L \approx 0.4 \mu\text{m}$, $n_d N_0/L^3 \approx 4 \times 10^4 \mu\text{m}^{-3}$, and $t_d \tau \approx 2 \text{ ms}$. By leveraging nondestructive imaging techniques [74–76] to monitor the same cloud as it collapses, or by preparing samples with a small N_0 uncertainty [77] (to minimize shot-to-shot variations in t_c), it should be possible to experimentally resolve the collapse dynamics.

Note that beyond-mean-field effects [80], which are not captured by Eq. (1), are expected to be negligible in the current ^{39}K experiments, since $\sqrt{(n_d N_0/L^3)}|a|^3 \approx 10^{-4}(a/a_0)^2 \lesssim 3 \times 10^{-4}$. However, by increasing $|a|$ while keeping $\alpha \approx \alpha_c$, it should be possible to also probe these effects; e.g., using $a = -30a_0$ would offer a sizable $\sqrt{(n_d N_0/L^3)}|a|^3 \approx 0.1$ at t_c .

In the final part of this Letter, we extend our simulations to include elastic three-body interactions [48–53], predicted to accompany the inelastic ones in atomic gases [52,53], by adding a real quintic term $\kappa|\psi|^4\psi$ to Eq. (1), i.e., $-i\eta \rightarrow -i\eta + \kappa$. The strengths of the elastic and inelastic three-body interactions are, respectively, $\kappa = \text{Re}[D]N_0^2/L^4$ and $\eta = -\text{Im}[D]N_0^2/L^4$, where D is the complex three-body scattering hypervolume [51]. Near the zero crossing of a Feshbach resonance, $\text{Im}[D] = -\mathcal{L}_3 m/\hbar$ is nonuniversal (as it depends on details of the short-range interactions) but it can readily be measured, whereas the elastic three-body interactions have been predicted to be universally set by the van der Waals interaction length r_{vdW} , with $\text{Re}[D] \approx 100r_{\text{vdW}}^4$ [52,53].

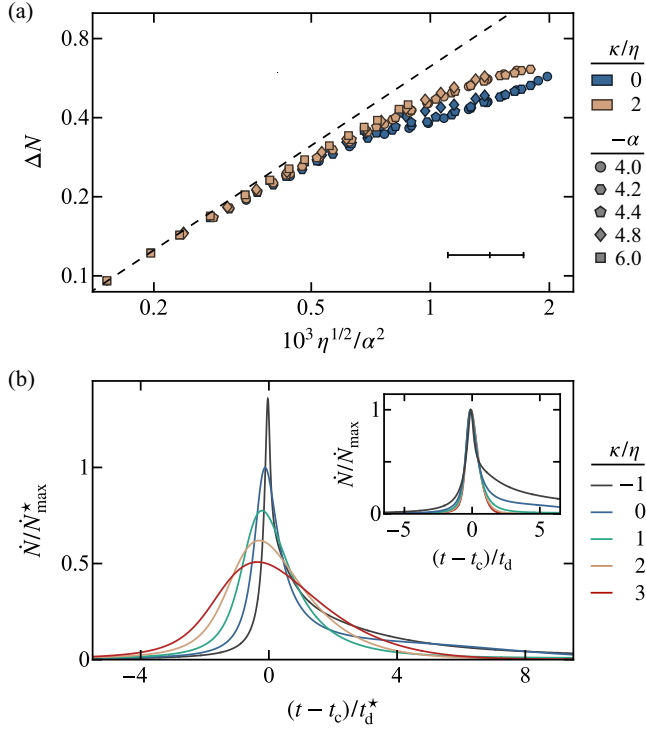


FIG. 4. Effects of elastic three-body interactions. (a) Plot of ΔN versus $\eta^{1/2}/\alpha^2$ for $\kappa/\eta = 2$ and $\kappa = 0$ (colors) on log-log scale, also varying both α (symbols) and $\eta \in [0.3, 10] \times 10^{-4}$ as in Fig. 2. For reference, the dashed line shows $\propto \eta^{1/2}/\alpha^2$ and the x error bar in the bottom right corner indicates the $\pm 20\%$ systematic uncertainty in experimentally determining $\eta^{1/2}/\alpha^2$ [78]. (b) Time-resolved $\dot{N}/\dot{N}_{\max}^*$ for different κ/η (legend) and fixed $\alpha = -4.4$ and $\eta = 10^{-4}$, highlighting that the dissipation time t_d and hotspot behavior are highly sensitive to κ/η , even though the differences in ΔN are small. The main panel shows $\dot{N}/\dot{N}_{\max}^*$ versus $(t - t_c)/t_d^*$, where the stars (*) indicate $\kappa = 0$ values, while the inset shows individually normalized curves.

Combining this prediction with the independently measured $-\text{Im}[D] = 60(20)r_{\text{vdW}}^4$ for ^{39}K [70] suggests $\kappa/\eta \approx 2$.

In Fig. 4(a) we show ΔN versus $\eta^{1/2}/\alpha^2$ for both $\kappa/\eta = 2$ and $\kappa = 0$ on log-log scale, with the same α and η values as in Fig. 2. The differences in ΔN between the two are

relatively small ($\lesssim 20\%$) and, moreover, cannot be resolved within the systematic uncertainty of $\pm 20\%$ in experimentally determining $\eta^{1/2}/\alpha^2$.

However, as illustrated in Fig. 4(b) for $\eta^{1/2}/\alpha^2 \approx 5 \times 10^{-4}$ ($\alpha = -4.4$ and $\eta = 10^{-4}$), the time-resolved loss dynamics are very sensitive to κ . For increasing κ , the time t_d increases, $\dot{N}_{\max}$ decreases, and the hotspot behavior is suppressed. Note that, when $\kappa > 0$, the elastic three-body term plays a similar role to the dissipative one in slowing down the approach to the singularity (see [54] for additional details).

Experimental studies of the collapse dynamics in ^{39}K could thus offer a setting for observing elastic three-body interactions. To study the hotspot behavior, one could instead use other atoms with access to smaller predicted κ/η , e.g., for ^7Li with $-\text{Im}[D] \approx 5000r_{\text{vdW}}^4$ [81] one has $\kappa/\eta \approx 0.02$.

In conclusion, we numerically investigated collapse in the 3D cubic-quintic nonlinear Schrödinger equation, finding good agreement between our simulations and experiments performed with attractive box-trapped Bose-Einstein condensates. We demonstrated the propensity of the system to form a hotspot, uncovering scaling laws that extend the Zakharov picture of weak collapse, and showed that collapse dynamics can be a sensitive probe of elastic three-body interactions. In the future, it would be interesting to further study the interplay between elastic and inelastic three-body interactions, for instance in the formation of quantum droplets [52,53,82–84]. It would also be interesting to study how the presence of a harmonic trap modifies the collapse dynamics, where soliton formation can play a role [29] and delayed collapses have been predicted [85]. More generally, studying collapse dynamics of other quantum fluids, including quantum mixtures [86] and dipolar condensates [87,88], could offer more insights into the interactions governing their behavior (see, e.g., Refs. [89–92]).

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Data availability. The data and code that support the findings of this Letter are openly available [93].

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