

Imprecision plateaus in quantum steering

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We study tests of quantum steering in which the trusted party does not have perfect control of their measurements. We report on steering inequalities that remain unaffected when introducing up to a threshold magnitude of measurement imprecision. This phenomenon, which we call an imprecision plateau, thus permits a departure from the standard assumption of idealized measurements without incurring any cost to the detection power of steering experiments. We provide an explanation for why imprecision plateaus are possible, a simple criterion for their existence, and tools for analyzing their properties. We also demonstrate that these plateaus have natural applications when the assumption of perfect measurements is relaxed: They allow for maintaining both the noise robustness and loss robustness of standard steering tests and the performance rate of idealized one-sided device-independent random number generators.

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Introduction. Quantum steering is the impossibility for a hidden variable to explain how quantum measurements on one particle can alter the local state of another particle. The scenario can be understood as a black-box party, Alice, influencing the laboratory of another party, Bob, in a way that proves shared entanglement. The last two decades have seen much interest in studying quantum steering in theory and experiments [1,2], and developing its applications within e.g., one-sided device-independent quantum key distribution [3,4], randomness certification [5,6], and quantum teleportation [7].

In the steering scenario, Alice selects an input, x , and performs a corresponding measurement, $\{A_{a|x}\}_a$, on her share of a bipartite state ρ_{AB} . The (subnormalized) states created remotely for Bob become $\sigma_{a|x} = \text{tr}_A(A_{a|x} \otimes \mathbb{1} \rho_{AB})$. The set $\{\sigma_{a|x}\}$ is called the steering assemblage. Bob must now determine whether the assemblage can be modeled via a local hidden state (LHS), i.e., whether it admits the form $\sigma_{a|x} = \sum_{\lambda} p(\lambda) p(a|x, \lambda) \sigma_{\lambda}$, where λ is a hidden variable and σ_{λ} is the hidden quantum state [8]. If the answer is negative, the assemblage is steerable, implying that ρ_{AB} is entangled. To detect steerability, Bob must select an input y and perform an associated measurement $\{B_{b|y}^{\text{targ}}\}_b$. The assemblage demonstrates steering if and only if the input/output statistics can violate a suitable inequality that is respected by all local hidden state (LHS) models but not by quantum theory [8]. Such a steering inequality takes the form

$$\mathcal{W}_0 \equiv \sum_{a,b,x,y} c_{abxy} \text{tr}(\sigma_{a|x} B_{b|y}^{\text{targ}}) \stackrel{\text{LHS}}{\leq} \beta_0, \quad (1)$$

where c_{abxy} are arbitrary real coefficients and β_0 is the tight bound for LHS models.

However, standard steering inequalities (1) require a strong assumption: that Bob is able to *perfectly* implement his targeted measurements. This is an idealization to which experiments can only aspire. A series of works, focused on experiments for tomography [9], communication [10,11], entanglement [9,12,13], and steering [14–16], have shown that if Bob's operations only nearly correspond to those targeted, it can lead to a high risk of false positives. For example, the perhaps most well-known test of two-qubit steering involves Alice and Bob measuring the three Pauli observables $\{X, Y, Z\}$. Specifically, in an LHS model, this “Pauli witness” is

$$\mathcal{W}_0^{\text{Pauli}} = \frac{1}{\sqrt{3}} (\langle A_1 \otimes X \rangle - \langle A_2 \otimes Y \rangle + \langle A_3 \otimes Z \rangle) \leq 1 \quad (2)$$

for arbitrary observables A_i with eigenvalues ± 1 , where $\langle \cdot \rangle$ denotes the expectation value [17,18]. Quantum theory achieves violations up to $\mathcal{W}_0^{\text{Pauli}} = \sqrt{3}$. However, if Bob's laboratory observables turn out to have only a nearly perfect fidelity of, say, 99.5%, over 25% of the possible quantum violation range can be explained by false positives stemming from LHS models [15]. The high sensitivity to small measurement imprecisions is illustrated for the Pauli witness by the steep increase around the origin of the blue curve in Fig. 1. Similar effects arise for other common steering inequalities [15], and only some refined methods are known that can detect steering without this drawback [14].

It has been proposed to eliminate the perfect-measurement assumption by replacing it with a quantitative limitation of Bob's lack of control. References [9,13,15,19] consider the case where Bob aims to perform a target measurement corresponding to a basis $\{|e_b\rangle\}$, but instead implements a different laboratory measurement $\{E_b\}$. The laboratory measurement can be subject to arbitrary imperfection; it is only assumed to

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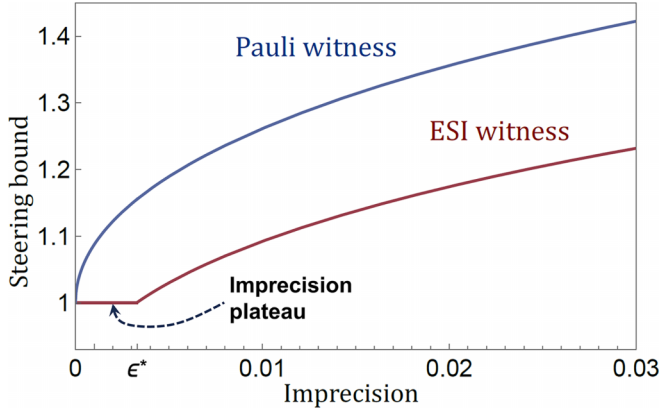


FIG. 1. Imprecise measurements for the trusted party typically leads to stronger LHS correlations, i.e., a higher steering bound (blue curve). However, some steering witnesses remain unchanged, up to a critical value, even though the measurements are increasingly uncontrolled. This leads to an imprecision plateau (red curve).

be a good approximation of the target measurement, fulfilling the constraint

$$\langle e_b | E_b | e_b \rangle \geq 1 - \epsilon_b, \quad (3)$$

for some small imprecision parameter $\epsilon_b \in [0, 1]$. This definition has the advantages of being platform independent and operationally meaningful. Indeed, ϵ_b can be estimated with small resources in the laboratory. However, when permitting imprecise measurements in our model, we reduce our assumed prior knowledge of the experiment. This means that the LHS bound in (1) in general must increase to account for all measurements compatible with the condition (3), leading to an ϵ -dependent criterion, $\mathcal{W}_\epsilon \leq \beta_\epsilon$ (e.g., the blue curve in Fig. 1).

Here, we reveal that it is possible to conduct steering tests without assuming idealized measurements that nevertheless cost no additional detection power. Specifically, we show that there exist steering inequalities of the form (1) where the LHS bound does not change when relaxing the assumption of perfectly controlled measurements up to a critical imprecision magnitude ϵ^* . That is, when the imprecision is below the threshold, i.e., $\epsilon \leq \epsilon^*$, the steering inequality remains $\mathcal{W}_\epsilon \leq \beta_0$. This corresponds to an *imprecision plateau* for steering inequalities, in which the ability to detect steering remains as good as if the measurements had been assumed perfect (see the red curve in Fig. 1). We give an intuitive explanation for the existence of imprecision plateaus in terms of properties of well-chosen steering witnesses. For our main example, we analytically characterize the plateau's length (see ϵ^* in Fig. 1), while for generic steering witnesses we develop both analytical and numerical relaxation methods to estimate it. We then proceed to showcase the utility of these plateaus in tests of steering and its quantum information applications.

Imprecision plateaus. To show the existence of imprecision plateaus, we consider a steering witness based on the so-called elegant Bell inequality [20,21], which we, in analogy, will refer to as the elegant steering inequality (ESI). In the ESI scenario, Alice and Bob have four and three inputs, respectively, and binary outputs. Alice's measurements are

unknown, whereas Bob's three target measurements correspond to the Paulis, i.e., $B_1^{\text{targ}} = X$, $B_2^{\text{targ}} = Y$, and $B_3^{\text{targ}} = Z$. The ESI reads

$$\begin{aligned} \mathcal{W}_0^{\text{ESI}} = & \frac{1}{4} (\langle (A_1 + A_2 - A_3 - A_4) \otimes B_1^{\text{targ}} \rangle \\ & + \langle (A_1 - A_2 + A_3 - A_4) \otimes B_2^{\text{targ}} \rangle \\ & + \langle (A_1 - A_2 - A_3 + A_4) \otimes B_3^{\text{targ}} \rangle) \stackrel{\text{LHS}}{\leq} 1. \end{aligned} \quad (4)$$

Here, Alice's observables are defined as $A_x = A_{0|x} - A_{1|x}$ and can be conveniently written as $A_x = \vec{n}_x \cdot (X, Y, Z)$ for each input $x = \{1, 2, 3, 4\}$, where $\vec{n}_x$ are Bloch vectors. The LHS bound can be computed as a largest-eigenvalue problem. The maximal quantum violation of the ESI is $\mathcal{W}_0 = \sqrt{3}$ and it is reached by sharing a maximally entangled state $|\phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ and selecting $\{\vec{n}_x\}_{x=1}^4$ as $\{(1, -1, 1), (1, 1, -1), (-1, -1, -1), (-1, 1, 1)\}/\sqrt{3}$, forming a regular tetrahedron on the Bloch sphere. This choice gives $\mathcal{W}_0^{\text{ESI}} = (\langle X \otimes X \rangle - \langle Y \otimes Y \rangle + \langle Z \otimes Z \rangle)/\sqrt{3}$ which is identical to the optimal way to violate the Pauli witness (2).

Now, we replace the target measurements in (4) with ϵ -imprecise measurements, $\{B_y^\epsilon\}$, and set out to compute the new, ϵ -dependent, LHS bound. From Eq. (3), one finds that this means considering all qubit observables such that $\text{tr}(B_y^{\text{targ}} B_y^\epsilon) \geq 2 - 4\epsilon$. Since the witness is linear, we need only to consider all deterministic input-output strategies of Alice. Labeling them by $(a_1, a_2, a_3, a_4) \in \{0, 1\}^4$, the values of $\mathcal{W}_\epsilon$ become [22]

$$\begin{aligned} \bigcup & (0, 0, 0, 0) \Rightarrow \mathcal{W}_\epsilon = 0 \\ \bigcup & \begin{cases} (0, 0, 0, 1) \Rightarrow \mathcal{W}_\epsilon = \langle B_1^\epsilon + B_2^\epsilon - B_3^\epsilon \rangle_\psi / 2 \\ (0, 0, 1, 0) \Rightarrow \mathcal{W}_\epsilon = \langle B_1^\epsilon - B_2^\epsilon + B_3^\epsilon \rangle_\psi / 2 \\ (1, 0, 1, 1) \Rightarrow \mathcal{W}_\epsilon = \langle B_1^\epsilon - B_2^\epsilon - B_3^\epsilon \rangle_\psi / 2 \\ (0, 1, 1, 1) \Rightarrow \mathcal{W}_\epsilon = \langle B_1^\epsilon + B_2^\epsilon + B_3^\epsilon \rangle_\psi / 2 \end{cases} = f^*(\epsilon), \\ \bigcup & \begin{cases} (0, 0, 1, 1) \Rightarrow \mathcal{W}_\epsilon = \langle B_1^\epsilon \rangle_\psi \\ (0, 1, 0, 1) \Rightarrow \mathcal{W}_\epsilon = \langle B_2^\epsilon \rangle_\psi \\ (0, 1, 1, 0) \Rightarrow \mathcal{W}_\epsilon = \langle B_3^\epsilon \rangle_\psi \end{cases} = g^*(\epsilon). \end{aligned} \quad (5)$$

Due to the Bloch sphere symmetry and the unbiasedness of the three target measurements, we see that these eight cases reduce to just three qualitatively different classes, as indicated above. Class 1 is trivial. For class 2, the steering bound is $f(\epsilon) \equiv \max f^*(\epsilon) = \frac{\sqrt{3}}{2} [1 + 2\sqrt{2\epsilon(1-\epsilon)} - 2\epsilon]$, which is proven in the Supplemental Material (SM) I [23]. Note that for $\epsilon = 0$ we have $f = \frac{\sqrt{3}}{2}$. Next, we make the key observation: The value for class 3 is *independent* of ϵ , because trivially $g(\epsilon) \equiv \max g^*(\epsilon) = 1$. Since $g(0) > f(0)$, this is the LHS bound appearing in (4). Crucially, the bound $\mathcal{W}_\epsilon^{\text{ESI}} \leq 1$ remains valid until ϵ is large enough for $f(\epsilon)$ to overtake $g(\epsilon) = 1$. This happens at

$$\epsilon^* = \frac{1}{2} - \frac{1}{3\sqrt{3}} - \frac{1}{3}\sqrt{\frac{5}{6}} \approx 3.3 \times 10^{-3}. \quad (6)$$

Thus, the ESI witness under imprecise measurements becomes $\mathcal{W}_\epsilon^{\text{ESI}} \leq 1$ for $\epsilon \leq \epsilon^*$ and $\mathcal{W}_\epsilon^{\text{ESI}} \leq f(\epsilon)$ for $\epsilon^* < \epsilon \leq (3 - \sqrt{3})/6$, where the upper demarcation corresponds to the

maximal value of f [26]. This threshold implies the imprecision plateau (see the red curve in Fig. 1) and ϵ^* can therefore be interpreted as the plateau's length.

Importantly, imprecision plateaus as a phenomenon are *independent* of the specific quantifier of measurement imprecision. For instance, we could replace the fidelity measure (3) with a limitation on the norm of the anti-commutators $\| \{B_y, B_{y'}\} \|_\infty \leq \epsilon$ [27]. We still have $g(\epsilon) = 1$ for class 3 and via the Cauchy-Schwarz inequality we obtain for class 2, $f^*(\epsilon) \leq \sqrt{3 + \sum_{j < k} \langle \{B_j^\epsilon, B_k^\epsilon\} \rangle_\psi} / 2 \leq \sqrt{3(1 + \epsilon)/2} = f(\epsilon)$. For $\epsilon \leq \frac{1}{3}$, $f(\epsilon)$ does not exceed $g(\epsilon) = 1$ and hence we have a plateau in ϵ .

Construction of plateau witnesses. In general, a steering witness supports an imprecision plateau if and only if all deterministic strategies associated with the tight LHS bound ($\epsilon = 0$) correspond to witness expressions [as in Eq. (6)] that depend only on a *single* input of Bob. In an LHS model, this permits Bob's local state to align itself with the relevant observable independently of ϵ . In contrast, this is impossible when the witness expression depends on more than a single observable, because the metric distance of two imprecise observables will be ϵ dependent. This allows us to determine whether a given witness has an imprecision plateau by solving a set of largest-eigenvalue problems.

However, how does one construct witnesses that have this property? A simple systematic way is to take $\mathcal{W}_0 = \sum_{x,y} c_{xy} (A_x \otimes B_y^{\text{targ}})$ and select the coefficient matrix, c , with the following properties: (i) The first column is $(1, \dots, 1)^T$ and (ii) all other columns individually sum to zero [28]. By construction, if Alice always outputs $A_x = 1$, then $\mathcal{W}_0 \propto B_1^{\text{targ}}$. Other deterministic strategies may correspond to expectation values involving several observables. Since the LHS value associated with the first type of strategy is independent of the choice of B^{targ} , whereas the other are not, one can then select $\{B_y^{\text{targ}}\}$ so that the former corresponds to the LHS bound. This implies the existence of an imprecision plateau.

We now construct a relevant family of plateau witnesses that naturally generalize the ESI. The family is parametrized by an integer $n \geq 3$. Let Alice have 2^{n-1} possible inputs represented as a string $x = (0, x_2, \dots, x_{2^{n-1}})$ with $x_i \in \{0, 1\}$. Let Bob have n target observables, selected so that they pairwise anticommute, i.e., $\{B_j^{\text{targ}}, B_k^{\text{targ}}\} = 0$ for $j \neq k$. Next, define witness coefficients $c_{xy} = (-1)^{x_y} / 2^{n-1}$ and note that properties (i) and (ii) are satisfied. Note also that for $n = 3$ this is equivalent to the ESI witness. In SM II we first prove that the LHS bound for this witness is $\beta_0 = 1 \forall n$ and then we prove that all the deterministic strategies that saturate it correspond to witness expressions that depend only on one target observable [23]. Thus, this witness has a plateau for every n . These witnesses have two noteworthy properties. (I) The steering inequality is valid in any Hilbert space dimension. In particular, a set of n anticommuting target observables can always be identified in dimension $d = 2^{\lceil n/2 \rceil}$. Thus, by selecting n , this witness family is tailored for steering in arbitrary high-dimensional Hilbert spaces. (II) A quantum violation exists achieving $\mathcal{W}_0 = \sqrt{n}$ by using a d -dimensional maximally entangled state (see SM II) [23]. Consequently, even though the noise tolerance improves with n the imprecision plateau always remains possible.

Semidefinite programming methods. While the ESI is simple enough for analytically determining the plateau's length, this is in general rarely the case. Consider that we are given an arbitrary steering witness and we find that it supports a plateau. How can we bound the plateau's length? Already for the more experimentally relevant variant of the ESI, in which each of Bob's three measurements has a different imprecision parameter, $\vec{\epsilon} = (\epsilon_x, \epsilon_y, \epsilon_z)$, we have been unable to solve the problem exactly. One possibility is to apply the main theorem of Ref. [15] to each of the deterministic strategies of Alice. This theorem gives a bound on the operators in terms of the imprecision parameter and allows one to compute ϵ -dependent upper bounds on any steering witness $\mathcal{W}_\epsilon$ as perturbations to the standard steering inequality (1), but it does not give a tight bound (see SM III) [23]. Another, less rigorous option, also discussed in SM III, is to use a Taylor approximation in the relevant regime of small imprecision [23]. This gives better bounds, but still not tight. To overcome this, we instead address the problem of bounding the plateau length using semidefinite programming (SDP) relaxations based on moment matrices [29].

The method, discussed in detail in SM IV, is based on three main ideas [23]. First, contrasting the common approach to Bell-type scenarios [30,31], we define a tracial moment matrix, $\Gamma_{u,v} = \text{tr}(u^\dagger v)$, where u and v are monomials of observables in the selected relaxation. Second, we use the idea of Ref. [32] to add dimensionality constraints on Γ by constructing a basis of moment matrices via random sampling. Third, we follow the idea of Ref. [10] and explicitly insert the target measurements into the monomial set to incorporate the fidelity constraints into the SDP relaxation. Combined, this lets us express the steering witness as a linear combination over Γ and realize (3) as linear constraints. Given that Γ is positive semidefinite by construction, it can be solved as an SDP. To showcase the usefulness of this method, we have applied it to the ESI witness with the three independent imprecision parameters. Already at first level relaxation, corresponding to an SDP matrix of dimension 14 with 19 variables, it systematically returns bounds on the plateau length that match our brute-force numerical search values up to at worst five decimals. The plateau length in ϵ_z as a function of ϵ_x and ϵ_y is given in SM IV [23].

Application to noise and loss. Imprecision plateaus have natural applications in experiments that recognize the lack of perfect measurement control. Typically, as we have exemplified in Fig. 1 for the Pauli witness (2), the quantum violation gap rapidly shrinks when accounting for imperfect measurements. This is equivalent to the decrease of the corresponding noise tolerance of steering. However, the plateau permits experiments to account for measurement imperfections while maintaining the noise tolerance associated with idealized measurements. For example, for perfect measurements ($\epsilon = 0$), both the Pauli witness and the ESI witness detect steering for the isotropic state $\rho_v = v|\phi^+\rangle\langle\phi^+| + (1-v)\frac{1}{4}$ when $v > \frac{1}{\sqrt{3}} \approx 0.577$. However, when permitting measurement imprecision corresponding to the plateau length ϵ^* , the equivalence is far gone. The former now requires $v \gtrsim 0.667$ whereas the latter still has $v \gtrsim 0.577$. Note that a nonlinear test for steering exists, where Bob implements Pauli measure-

ments on qubits without any assumption on the precision [14]. This works for any $v \gtrsim 0.577$ too, but is not straightforward to generalize to arbitrary systems.

A similar advantage applies for scenarios with detection losses, i.e., when Alice's measurement fails with probability $\eta \in [0, 1]$. Since in our picture the measurement errors of Bob are uncontrolled, we do not allow him to actively use the ϵ cone around his target measurement toward closing the detection loophole. Instead, we assume that he performs the target measurements. It is well known that the critical efficiency, η_{crit} , needed for demonstrating steering is much lower than the efficiency needed for a Bell inequality violation, where Bob's measurements are entirely uncharacterized [33–35]. Therefore, with the gradual loss of measurement control, one expects η_{crit} to increase towards the Bell inequality limit. However, the imprecision plateau permits us to circumvent this problem for imprecision magnitudes within the plateau. For perfect measurement control, the critical detection efficiency for both the Pauli witness and the ESI witness is $\eta_{\text{crit}} = \frac{1}{3}$. However, at imprecision $\epsilon = \epsilon^*$, the advantage becomes large: The ESI witness still has $\eta_{\text{crit}} = \frac{1}{3}$ whereas the Pauli witness now requires $\eta_{\text{crit}} \approx 0.606$. In SM V, we compute the optimal witness values at fixed detection efficiency [23].

Application: Random number generation. Imprecision plateaus also enable advantages in one-sided device-independent protocols. We showcase this for a quantum random number generation scenario, in which we restrict ourselves to individual eavesdropper attacks (see, e.g., Ref. [36] for more general types of attacks). Using the ESI witness as a testbed, we show that for imprecisions within the plateau, one can both certify randomness whenever the original steering inequality is violated and obtain the same rate of randomness as when assuming perfect measurements. Thus, the performance of the protocol is fully maintained while eliminating idealizations that otherwise may be used to compromise the security.

We certify randomness from Alice's output averaged over her four possible inputs. The randomness is given by $R = -\log_2(P_g)$ where $P_g = \max \frac{1}{4} \sum_{a,x} p(a|x)$ is the average probability of an eavesdropper guessing her outcome [37], maximized over all strategies. Thus, we must compute the largest possible P_g compatible with the observed value of the ESI witness. This can be achieved by evaluating P_g separately for each of Alice's deterministic strategies. When $\epsilon = 0$, each of these optimizations can be straightforwardly cast as an SDP. When $\epsilon > 0$, we can bound P_g from above (corresponding to a lower bound on R) by using the previously discussed SDP relaxation method. This is applied separately to each of the deterministic strategies [38]. To this end, we have selected $\epsilon = 3 \times 10^{-3} < \epsilon^*$ and employed a moment matrix of size 296 featuring 1371 variables. The randomness rates are illustrated in Fig. 2. Our certified randomness bound is essentially equal to the amount of randomness obtained for $\epsilon = 0$, with a worst-case difference of $\Delta R = 0.0016$. The small discrepancy is likely due to our choice of relaxation level, which is a compromise between accuracy and practicality. This illustrates the maintained protocol performance in face of measurement imprecisions. In contrast, when operating outside the plateau, e.g., at $\epsilon = 5 \times 10^{-5} > \epsilon^*$, we cannot maintain either the

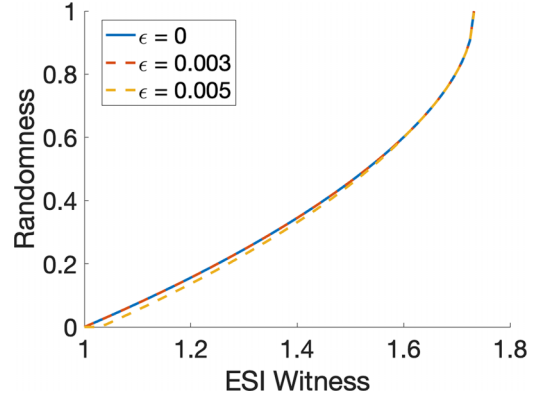


FIG. 2. The performance of the randomness generation remains unchanged when introducing measurement imprecisions below the plateau length (blue and red curve). However, imprecisions above the plateau length lead to the expected decrease in noise tolerance and generation rate (yellow curve).

randomness rate or the noise tolerance. This is illustrated by the yellow curve in Fig. 2, which is an upper bound on R obtained from numerical search. It implies a clear reduction in the performance when not having access to the imprecision plateau.

Discussion. Quantum steering experiments are commonly performed under the assumption that the trusted party has perfectly controlled measurements. This is an unrealistic assumption, whose near validity is also rarely evidenced in practice. For steering scenarios, the problem of how to make witness tests valid under arbitrary imprecision was addressed in Ref. [15]. However, the main drawback is that most of the well-known steering witnesses rapidly lose their detection power even as tiny imprecision is introduced. In this Letter, we have shown a sharp contrast to this observation, namely that well-chosen tests of steering can remain unchanged even when perfect control of the trusted measurement is relaxed. We have seen how this is made possible in steering because of the phenomenon of imprecision plateaus. These plateaus lead to several natural questions. How does one construct steering witnesses that both have a long imprecision plateau and detect entanglement that is far from Bell nonlocal? For a given steerable state, what is the longest imprecision plateau possible? It seems likely that imprecision plateaus also should appear more broadly, for example in multipartite steering or steering with quantum inputs [39,40].

We have also shown that imprecision plateaus have natural applications. These can be divided into two types. First, the plateaus pave the way for tests of steering that remove the undesirable assumption of perfect measurements, thus becoming more resilient to false positives, but without any cost to either the noise or loss requirements of the experiment. We have exemplified these advantages in our analysis of the ESI. Importantly, the plateau lengths that we have found are significantly larger than the imprecision parameters reported in a recent experiment [19]. Second, they can be used to make one-sided device-independent quantum information protocols more faithful while maintaining the performance of the original, idealized protocol. We illustrated this possibility in a proof-of-principle analysis of a quantum random number

generator. This suggests that similar advantages may be more widely possible.

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