

Fate of the Mollow triplet in strongly coupled atomic arraysOrazio Scarlatella¹ and Nigel R. Cooper¹*TCM Group, Cavendish Laboratory, University of Cambridge, J. J. Thomson Avenue, Cambridge CB3 0HE, United Kingdom*

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Subwavelength arrays of quantum two-level emitters have emerged as an interesting platform displaying prominent collective effects that can be harnessed for applications. Here, we study such arrays under strong coherent driving, realizing an open quantum many-body problem in a strongly nonlinear regime. For this we introduce an approach to this problem in terms of a dynamical mean-field theory, paving the way for further studies. We show that the spectrum of scattered light, characterized by the famous Mollow triplet for a single atom, develops a characteristic line shape with flat sidebands determined by dipolar interactions and relevant for experiments. Remarkably, this is to some extent independent of the specific geometry, but is sensitive to the ordered arrangement of the atoms. This line shape therefore characterizes atomic arrays and distinguishes them from disordered ensembles and noninteracting emitters.

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The collective behavior of ensembles of quantum emitters, resulting from photons scattering between them, has long attracted attention [1–3]. Considering two-level atoms, when the average interatomic distance becomes comparable to the wavelength of the two-level transition, the resulting dipolar interactions become strong and can lead to collective effects, such as superradiance and subradiance [2,4] and Lamb shifts [5,6].

Cold atom systems are natural places where strong dipolar interactions can be studied and exploited. There, the emitters can be arranged in ordered configurations, allowing one to enhance and control collective effects [7]. Such arrays of quantum emitters have recently been investigated for several applications in quantum information and optics [8–10], including realizing single atomic layer mirrors [11–13] with coherent control [10,14], as well as for photon storage [15–17]. Several effects have been studied including the shift of collective resonances [18,19], electromagnetically induced transparency [14,20,21], and superradiance and subradiance [16,22–25].

Nevertheless, most works so far focused on weak drive or single excitation regimes, in which the emitters are described by noninteracting theories [16,26,27]. Less is known in nonlinear regimes, where the system constitutes a strongly interacting far-from-equilibrium quantum many-body problem.

A typical nonlinear effect in quantum optics characterizes the spectrum of light scattered by a single two-level atom that is strongly driven on resonance. This has a three-peak structure known as the Mollow triplet [28], due to a dressing of the atomic levels with drive photons [29]. It was observed in several platforms including atomic beams [30], ions [31],

molecules [32], quantum dots [33], and cold atoms [34]. A number of works investigated how this is affected by dipolar interactions in the case of few atoms [35–40] and more recently for disordered clouds [41–43]. However, its fate for large arrays of strongly coupled atoms is still unclear, and was only investigated in small systems [43,44].

Here, we show that for large arrays of strongly coupled atoms the Mollow triplet acquires a characteristic line shape with flat sidebands determined by dipolar interactions. Remarkably, we find that this structure of the line shape is to some extent independent of the specific lattice geometry and detection and drive directions, but yet is sensitive to the ordered arrangement of atoms. The triplet line shape therefore characterizes atomic arrays and sets them apart from noninteracting emitters and interacting emitters in disordered configurations [43]. It could be measured in experiments [13,14,45,46], where it would provide a clear experimental signature of strong coupling in nonlinear regimes, beyond current demonstrations that are limited to linear phenomena [13,14,18].

These predictions are made introducing a dynamical mean-field theory (DMFT) approach to the steady states of atomic arrays, and comparing with exact diagonalization calculations. DMFT has been very successful in the context of strongly correlated electrons [47,48], and describes the limit of large lattice dimensionality. Here, we leverage a DMFT approach to Markovian open quantum systems [49] and to spin systems in equilibrium [50], to formulate a DMFT approach for atomic arrays. This paves the way to investigate a broad range of phenomena in these systems beyond a Gutzwiller mean-field approximation, including steady-state superradiance [22], lasing, and interaction-induced transparency [51]. It complements other methods such as generalized mean-field approaches based on a factorization of spatial cumulants [52,53], that break down when these describe important correlations [52,54], and a recent truncated Wigner approximation [55], inaccurate when subradiant states are significant.

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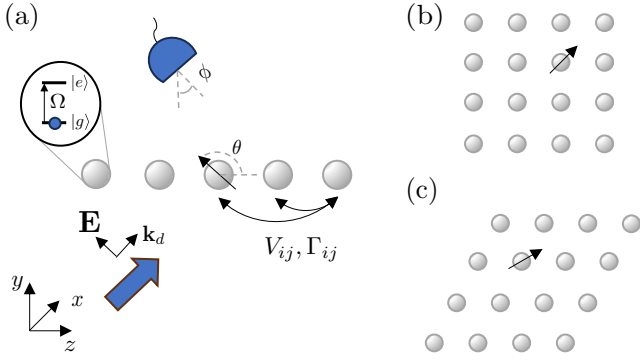


FIG. 1. An array of two-level atoms driven by an electric field. The atoms are subject to coherent dipole-dipole interactions and collective dissipation. The photons emitted are collected by a detector. A one-dimensional (1D) geometry is shown in (a), a 2D square lattice in (b), and a triangular lattice in (c). The arrows indicate the orientation of the induced dipoles.

Model. We consider a large number N of two-level atoms ordered in a periodic array. The atoms are illuminated with a uniform plane wave and are coupled to the free-space electromagnetic field, giving rise both to coherent dipole-dipole interactions and to collective dissipation. These can be described by means of a Markovian master equation [3]:

$$\dot{\rho} = -\frac{i}{\hbar}[H_0 + H_{\text{drive}}, \rho] + \mathcal{D}[\rho], \quad (1)$$

$$H_0 = \frac{\hbar\Delta}{2} \sum_i \sigma_i^z + \sum_{ij; i \neq j} \hbar V_{ij} \sigma_i^+ \sigma_j^-, \quad (2)$$

$$H_{\text{drive}} = \frac{\hbar\Omega}{2} \sum_i (\sigma_i^+ e^{i\mathbf{k}_d \cdot \mathbf{r}_i} + \sigma_i^- e^{-i\mathbf{k}_d \cdot \mathbf{r}_i}), \quad (3)$$

$$\mathcal{D}[\rho] = \frac{1}{2} \sum_{i,j} \Gamma_{ij} (2\sigma_i^- \rho \sigma_j^+ - \sigma_i^+ \sigma_j^- \rho - \rho \sigma_i^+ \sigma_j^-). \quad (4)$$

Here, σ_l^α with $\alpha = x, y, \text{ or } z$ are the Pauli matrices on site l and $\sigma_l^\pm = \sigma_l^x \pm i\sigma_l^y$ the raising and lowering operators; $\Delta = \omega_d - \omega_0$ is the detuning of the drive from the two-level transition energy ω_0 , $\Omega = 2\mathbf{d} \cdot \mathbf{E}/\hbar$ the Rabi coupling given by the vector of transition dipole moments $\mathbf{d}$ and the electric field vector $\mathbf{E}$, and $\mathbf{k}_d$ is the drive momentum (see Fig. 1).

The dipolar interactions and collective decay terms are given by [16,52]

$$V_{il} = \frac{3\Gamma}{4} \left\{ -\alpha \frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})} + \beta \left[\frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})^2} + \frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})^3} \right] \right\}, \quad (5)$$

$$\Gamma_{il} = \frac{3\Gamma}{2} \left\{ \alpha \frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})} + \beta \left[\frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})^2} - \frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})^3} \right] \right\}, \quad (6)$$

where the single-atom decay rate is $\Gamma = |\mathbf{d}|^2 k_0^3 / 3\pi\epsilon_0 \hbar$ with $r_{il} = |\mathbf{r}_i - \mathbf{r}_l| = a|i - l|$, $\mathbf{r}_i$ is the position of atom i on the lattice and a the lattice spacing. $k_0 = 2\pi/\lambda_0$ is the wave vector of the two-level transition, with wavelength λ_0 . $\alpha = 1 - \cos^2 \theta$ and $\beta = 1 - 3\cos^2 \theta$, where θ represents the angle between the common atomic dipole orientation and the line connecting atoms i and j .

Throughout this Letter, we will assume units of $\Gamma = 1$ and $\hbar = 1$. The interactions and losses are controlled by the parameter $k_0 a$: In the extreme limit $k_0 a \rightarrow \infty$ interactions vanish and dissipation becomes local with $\Gamma_{il} = \Gamma \delta_{il}$, recovering the independent atoms case, while for $k_0 a \rightarrow 0$ interactions diverge while dissipation becomes negligible, going to a finite value while also acquiring an all-to-all character $\Gamma_{il} = \Gamma$. Here, we concentrate on intermediate values [see Supplemental Material (SM) [56] for a plot].

DMFT for atomic arrays. We formulate a DMFT approach to investigate the steady state of the model in the large atom number limit $N \rightarrow \infty$ (see SM [56] for details). To apply DMFT to the present case of a spin Markovian master equation, we first map this onto a Keldysh action using spin-coherent state path integrals. Then, we decouple the dipolar interactions and collective dissipation terms by introducing auxiliary bosonic fields, and apply a bosonic DMFT approach [49,57–61], on the lines of Ref. [50] and of an extended DMFT approach [62]. Due to the Markovian nature of the problem, we need to define a discretization of Green's functions at $t = 0$ such that the Keldysh causality structure [63] is preserved.

Bosonic DMFT maps a lattice model onto an effective impurity model, describing a single site of the lattice coupled to an effective field, as in a standard Gutzwiller mean-field approach, and to an effective environment. Assuming a local self-energy in space, both can be determined such as to satisfy a set of self-consistent equations. The effective environment introduces corrections of order $1/z$, where z is the lattice coordination number, to a Gutzwiller mean field corresponding to $1/z \rightarrow 0$ [61]. In the present case in which interactions are not limited to nearest neighbors, the accuracy of DMFT is expected to improve. We later confirm our results by comparing with exact diagonalization (ED).

In the present case, the impurity model is a generalized spin-boson model, that in the steady state is described by the time-translation invariant Keldysh action

$$S^{\text{imp}} = S_0^i + \frac{1}{2} \int_{-\infty}^{\infty} dt [\varsigma_i^\dagger(t) \tau_3 b + \text{H.c.}] - \frac{1}{2} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \varsigma_i^\dagger(t) \tau_3 \mathcal{W}(t-t') \tau_3 \varsigma_i(t'). \quad (7)$$

Here, $\varsigma_i = (d_{i+}, \bar{d}_{i+}, d_{i-}, \bar{d}_{i-})^T$ is a vector in Nambu and Keldysh formalisms, where $d = \langle \sigma^- \rangle$ and $\bar{d} = \langle \sigma^+ \rangle$ are average values on spin-coherent states [64], and the $+$ and $-$ indices indicate the branch of the Keldysh double contour to which the fields belong. b and $\mathcal{W}$ represent the effective field and effective environment. These are taken site independent assuming a homogeneous steady state, as verified by a linear stability analysis. τ_3 is a diagonal matrix with entries $(1, 1, -1, -1)$ arising in the Keldysh-Nambu formalism. Note that here a Nambu formalism is needed to allow for finite values of the anomalous correlators $\langle \sigma^\pm \sigma^\pm \rangle$, that arise due to the drive term, breaking the $U(1)$ symmetry of the undriven problem.

The self-consistent conditions correspond to a set of matrix relations in frequency and momentum space: $\Pi_{\text{loc}} = [\tau_3 \chi_{ii}^{-1} \tau_3 + \mathcal{W}]^{-1}$, $U_{ii} = \frac{1}{N} \sum_k [W_k^{-1} - \Pi_{\text{loc}}]^{-1}$, and $\mathcal{W}^{-1} = \Pi_{\text{loc}} + U_{ii}^{-1}$, and $b_\pm = [\mathcal{W}^R(\omega = 0) - W_{k=0}^R(\omega = 0)] \langle \varsigma_{i\pm} \rangle$.

Here, the superscript R indicates the retarded component and $b_{\pm}$ and $\langle \zeta_{i\pm} \rangle$ the $+$ or $-$ components of the corresponding vectors. W_k is a known matrix with entries defined by the Fourier transforms of (5) and (6), reported in SM [56]. Assuming one can compute from the impurity action (7) the local connected Green's function $\chi_{ii}(\tau) = -i \lim_{t \rightarrow \infty} \langle \zeta_i(t + \tau) \zeta_i^\dagger(t) \rangle_{\text{con}}^{\text{imp}}$ and expectation value $\langle \zeta_i \rangle$, then these equations determine $\mathcal{W}$, b , Π_{loc} , and U_{ii} , such that the impurity problem represents the original lattice problem.

The connected propagator of the lattice $\chi_{ij}(\tau) = -i \lim_{t \rightarrow \infty} \langle \zeta_i(t + \tau) \zeta_j^\dagger(t) \rangle_{\text{con}}$ can then be computed by $\chi^{-1}(k, \omega) = \tau_3 [\Pi_{\text{loc}}^{-1}(\omega) - W_k] \tau_3$.

Solving the impurity model is a challenge by itself and a viable numerically exact approach is still lacking. Here, we use a method based on a noncrossing approximation (NCA) [49,65,66]. More details on DMFT/NCA are found in SM [56].

Mollow triplet in DMFT. The spectrum of the light emitted from a single two-level atom which is strongly and continuously driven on resonance, $\Omega \gg \Gamma$ and $\Delta = 0$, shows a central peak at the two-level transition energy $\omega = \omega_0$, accompanied by two side peaks at $\omega = \omega_0 \pm \Omega$, known as the Mollow triplet [28,67,68]. This is understood by a picture in which the atom is dressed by the laser field [29,67]. In a frame rotating at the atomic frequency ω_0 , where energy differences are shifted by this quantity, the three resonances correspond to incoherent transitions induced by the jump operator $L = \sqrt{\Gamma} \sigma^-$ between the eigenstates $|+\rangle, |-\rangle$ of the single-atom Hamiltonian $H = \Omega \sigma^x / 2$, with transition energies $\omega = 0$ and $\omega = \pm \Omega$. These are represented graphically in Fig. 2(a).

In the case of several atoms and for $V_{ij} \ll \Omega$, single-particle transitions can be understood as taking place between two bands of width set by V_{ij} , separated by the drive intensity Ω , as depicted in Fig. 2. The width of the bands corresponds to a broadening of the triplet lines, where interband transitions correspond to the central peak and intraband ones to the sidebands. Predicting how the triplet broadens is nevertheless nontrivial. In fact, no broadening is predicted by a Gutzwiller mean-field approach, as well as by approximating the lattice self-energy with that of a single site, that instead allows to analytically estimate the Hubbard bands of a Mott insulator [47,69]. This motivates a DMFT approach.

The spectrum of emitted light in the case of atomic arrays is related to the atomic correlation functions $g_{ij}(\tau) = \lim_{t \rightarrow \infty} \langle \sigma_i^+(t + \tau) \sigma_j^-(t) \rangle$ [3]. In DMFT, a key role is played by the local Green's function $g_{ii}(\tau)$. This could be measured by near-field microscopy [70,71] and also characterizes the far-field emission, as discussed later. It is shown in Fig. 2(b) for different values of $k_0 a$ and different geometries, namely a 1D chain with dipoles along the chain ($\theta = 0$) and 2D square and triangular lattices with dipoles oriented as in Fig. 1. The drive direction is chosen along x .

We see that far from the limit of large $k_0 a$, where the single-atom triplet composed of three Lorentzian peaks is recovered, the resonances acquire a characteristic line shape with the sidebands becoming flat, while the central resonance remains peaked. They eventually merge when $\Omega \lesssim V_{ij}$. Figure 2(c) shows that the resonance widths are set by the dipolar interactions, scaling as the nearest-neighbor contribu-

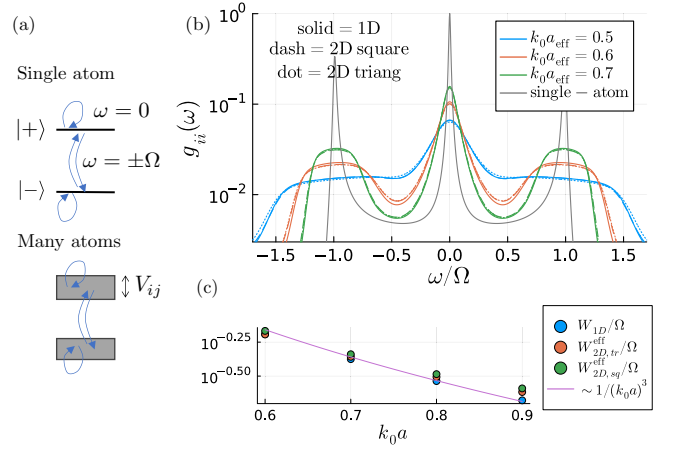


FIG. 2. (a) Sketch of the single-particle transitions determining the Mollow triplet. In the case of many atoms (bottom) they take place between two bands determined by the dipolar interactions. (b) The local atomic correlation function for a 1D chain (solid lines) and 2D square and triangular lattices as defined in the body (dashed and dotted lines) as a function of lattice spacing and in DMFT, for $N \rightarrow \infty$. (c) The width of the sidebands W , extracted by the extrema of the derivatives, scales as the dipolar interaction (5) with a leading order $1/(k_0 a)^3$ behavior. The widths for the different geometries collapse on the 1D curve once rescaled as $W^{\text{eff}} = W \|V_{1D}\|_2 / \|V\|_2$. Similarly, in (b) the 2D curves are plotted for a lattice spacing a such that $\|V(a)\|_2 = \|V_{1D}(a_{\text{eff}})\|_2$. Here, $\Omega = 60\Gamma$, dipoles point as in Fig. 1 (with $\theta = 0$ in 1D), and drive is along x (orthogonal to the array).

tion with a power of $1/(k_0 a)^3$ at small $k_0 a$. This is in contrast with the case of a single atom, where the peak widths are determined by the decay rate [67].

Remarkably, the line shape remains unchanged for all geometries within our DMFT approach. We find that it depends on the effective interaction parameter $\|V\|_2 = \sqrt{\sum_j V_{ij}^2}$ (see SM [56]). Curves for different geometries superimpose when plotted for a lattice spacing a such that $\|V(a)\|_2 = \|V_{1D}(a_{\text{eff}})\|_2$, where a_{eff} is a 1D-effective lattice spacing, as shown in Fig. 2(b). Similarly, in Fig. 2(c) we show that the sideband widths fall on the same curve when rescaled by this factor. Similar results are also found as a function of dipole orientation in SM [56]. While the specific geometry is not important, a regular arrangement still is, as the characteristic flat sideband structure we find differs markedly from the structureless broadening expected in disordered ensembles [43].

A similar spectrum also characterizes the light emitted in the far field. This is proportional to the momentum-resolved propagator $g(\mathbf{k}, \omega) = \int_{-\infty}^{\infty} d\tau \sum_{ij} e^{i(\omega\tau + \mathbf{k} \cdot \mathbf{r}_{ij})} \lim_{t \rightarrow \infty} \langle \sigma_i^+(t + \tau) \sigma_j^-(t) \rangle$, computed at $\mathbf{k} = k_0 \mathbf{n}$ where $\mathbf{n}$ is a unit vector indicating the emission direction [3]. Note that only momenta $|\mathbf{k}| \leq k_0$ are probed by the far-field emission, where the largest momentum is attained for an emission direction $\mathbf{n}$ parallel to the array. In DMFT, and for the same lattice geometries as in Fig. 2, we find that this propagator becomes effectively momentum independent in the regime of large drive of the Mollow triplet and coincides with the local propagator $g(\mathbf{k}, \omega) \approx g_{ii}(\omega)$. The predicted line shape therefore also

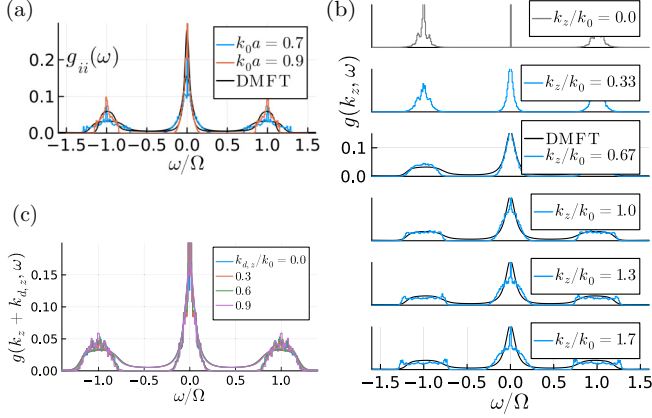


FIG. 3. A comparison of the DMFT Green's functions for $N \rightarrow \infty$ (smooth curves), with an approximation from exact diagonalization (ED) of the Hamiltonian for a chain of $N = 10$ sites, calculated as a histogram (noisy curves). In all panels $\Omega = 60\Gamma$ and the drive is along x . (a) shows the local Green's function. (b) shows the momentum-resolved Green's function $g(k_z, \omega)$ for $k_0a = 0.7$. At $k_z = 0$ the gray curve indicates that the ED-based approximation breaks down. In these panels dipoles point along z ($\theta = 0$). (c) shows the momentum-resolved Green's function for different drive directions labeled by $k_{d,z}$. Here, $k_z a = k_0 a = 0.5$ and $\theta = \pi/2$.

characterizes the far-field emission in a generic emission direction, as we further discuss below.

Validation with exact diagonalization. A more complex dependence on geometry and momentum is expected beyond DMFT, which assumes a momentum-independent self-energy. Nevertheless, here we confirm that DMFT still captures correct qualitative features of the Mollow triplet broadening, comparing with exact diagonalization (ED) in the worst case scenario of a 1D chain, for which the largest deviations from DMFT (describing the large dimensional limit) are expected.

For ED calculations we assume open boundary conditions. To access larger system sizes, we disregard the dissipator as the triplet broadening is mostly determined by dipolar interactions and these dominate at small k_0a (see also SM [56]). Also, in this large-drive regime the stationary state is found to be featureless and to approach an infinite-temperature state.

The local Green's function can then be approximated by weighting the density of states (DOS) $\sum_{n,m} \delta(\omega - E_n - E_m)$ with the matrix elements $|\langle \sigma_i^- \rangle_{nm}|^2 = |\langle \psi_n | \sigma_i^- | \psi_m \rangle|^2$ —see, e.g., Ref. [72], namely $g_{ii}(\omega) \approx 2\pi/2^N \sum_{n,m} |\langle \sigma_i^- \rangle_{nm}|^2 \delta(\omega - E_n - E_m)$, where $|\psi_n\rangle$ and E_n are the eigenstates and eigenvalues of the Hamiltonian. This is compared with the DMFT result in Fig. 3(a), displaying very good agreement. We checked that the different broadening of the three peaks results from the matrix elements, probing the many-body eigenstates, while is not reflected in the DOS, as expected from the two-band picture sketched in Fig. 2(a).

Similarly, we approximate the momentum-resolved Green's function with a weighted sum of the DOS with matrix elements $|\langle \sigma_k^- \rangle_{nm}|^2/N = |\langle \psi_n | \sigma_k^- | \psi_m \rangle|^2/N$ in Fig. 3(b). Remarkably, this compares well with the momentum-independent DMFT prediction for most values of k_z , but also reveals some expected momentum dependence beyond DMFT, and beyond finite-size effects [56]. Comparison with

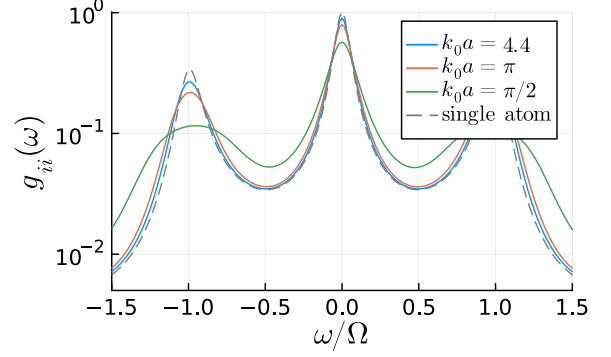


FIG. 4. The local atomic correlation function for experimentally relevant parameters for the realization in Ref. [13]. This is obtained for a 2D square lattice geometry and within DMFT for $N \rightarrow \infty$. Here, $\Omega/\Gamma = 10$, dipoles point along z , and the drive is along x (orthogonal to the array).

DMFT deteriorates close to $k_z = 0$, where the resonances narrow down up to a delta-function-like central peak due to a total spin conservation at $k_z = 0$ (see SM [56]), and the ED approximation of neglecting the dissipator is not justified there. We remark that the special case $k_z = 0$ was previously studied in Ref. [43] for a small 1D chain, while the characteristic line shape for other emission directions was not predicted there.

The success of DMFT in capturing the line-shape broadening of the Mollow triplet can be understood from the physics being effectively short ranged, despite the long-range character of interactions and dissipation (5) and (6). In fact, we find identical results for a model with only *local* dissipation (6), even though the nonlocal contributions are *a priori* non-negligible (see SM [56]).

Finally, we recall that the far-field emission can only probe the Brillouin zone for $|k_z| \leq k_0$, where $|k_z| = k_0$ corresponds to a direction of emission parallel to the chain [$\phi = \pm\pi/2$ in Fig. 1(a)]. Nevertheless, there is a finite window of momenta in which such a broadening can be observed, as shown in Fig. 3(b). Changing the drive direction provides a natural way around this constraint.

Drive direction, atomic polarization. The triplet line shape also does not depend significantly on specific choices of drive direction, as shown here, and atomic polarization discussed in SM [56]. Figure 3(c) shows the momentum-resolved propagator at $k_z = k_0$ for different drive directions. These are accounted for within DMFT by a gauge transformation $\sigma_j^+ e^{ik_{d,z}aj} \rightarrow \sigma_j^+$ that trades the drive phases with a shift in momentum of dipolar interactions and dissipation $V_{k_z} \rightarrow V_{k_z - k_{d,z}}$ and $\Gamma_{k_z} \rightarrow \Gamma_{k_z - k_{d,z}}$. Both in DMFT and ED the Green's functions depend little on drive direction apart that they are shifted in momentum by $k_{d,z}$. This allows to scan the Brillouin zone by varying the drive direction, keeping the detection direction fixed.

Experimentally relevant parameter regimes. The model studied finds natural experimental realizations [13, 18, 45, 46], where optical lattices or tweezers are used to trap the atoms in ordered arrays (see also the discussion in Ref. [7]). The broadened Mollow triplet is more pronounced for small lattice

spacings (large peak widths) and large drive strengths (large peak separation), such as considered in previous figures, but these parameters are limited in experiments. The main experimental challenge is to reach deep subwavelength regimes, and the smallest lattice spacing of $k_0 a = 4.4$ was achieved experimentally in Refs. [13,14], using ^{87}Rb atoms in a 2D square optical lattice.

In Fig. 4 we report DMFT calculations with parameters that are relevant for this experimental setup. We note that the Mollow triplet broadening is already noticeable for the lattice spacing achieved in the experiment. The characteristic line shape will be instead better resolved in the next generation of experiments that will realize shorter lattice spacings, such as those proposed in Refs. [45,46].

Conclusions. We showed that the spectrum of light emitted by strongly driven atomic arrays is characterized by a modified Mollow triplet whose line shape is determined by dipolar interactions, to some extent independent of lattice geometry,

emission, and drive directions. This sets atomic arrays apart from disordered ensembles and noninteracting emitters and would provide a clear experimental signature of strong coupling in nonlinear regimes.

We have also introduced a dynamical mean-field theory approach for atomic arrays, paving the way for investigating a variety of strongly correlated phenomena in these systems. Our steady-state approach could be extended to the transient dynamics [48], where superradiance or the generation of sub-radiant states could be investigated [73].

The data (code) to reproduce the results of the manuscript will be provided by the author upon request.

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