

Electromagnetic symmetry dislocations

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Singular optics studies light's topological defects, pioneered by the discovery that phase vortex lines, strands of destructive interference, naturally occur in scalar wave fields. General electromagnetic waves, however, are described by three-dimensional vectors making scalar phase vortices, which depend on the choice of coordinate basis, less meaningful. Polarization singularities instead capture the vector texture of nonparaxial light with separate descriptions for the electric and magnetic fields. But these polarization textures cannot be fundamental because electromagnetic laws can be expressed not only by separate electric and magnetic fields, but by linear combinations of the two. We propose fundamental, basis-independent topological features generic in electromagnetic fields: one- and two-dimensional structures that relate to time-averaged symmetries held locally by the combined electric- and magnetic-field polarization geometry.

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Introduction. Scalar dislocation lines (or phase vortices) have long been identified as general phenomena in scalar wave superpositions [1,2], wherein perfect destructive interference of the complex scalar field ϕ , the total field of the superposition, defines stable one-dimensional line structures in three real-space dimensions. Satisfying two real conditions $\text{Re}\{\phi\} = \text{Im}\{\phi\} = 0$, which can informally be associated with the singularity's codimension (2), a dislocation line disturbs the surrounding field such that its phase angle, $\arg \phi$, smoothly realizes all values between 0 and 2π an integer number of times along any circuit containing the line in isolation. Scalar, monochromatic electromagnetic waves can be described by a single complex scalar field whose dislocation lines leave behind imprints in the streamlines of momentum density. But in nonparaxial monochromatic waves, both electric- and magnetic-field phasors are three-dimensional vectors spanning $\mathbb{C}^3$ and only in their individual scalar components do dislocation lines exist, by themselves unable to effect a mechanical consequence in the vector field as a whole. They depend on the choice of basis in $\mathbb{C}^3$ and in that sense are not fundamental; for instance, phase singularities of the electric field's complex Cartesian components $E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}} + E_z\hat{\mathbf{z}}$ are different to those in spherical components $E_r\hat{\mathbf{r}} + E_\theta\hat{\boldsymbol{\theta}} + E_\phi\hat{\boldsymbol{\phi}}$.

The natural extension of phase singularities to complex vector fields like $\mathbf{E}$ appears when considering lines of polarization [3–8], most importantly the circular and linear polarization singularities known as C lines and L lines. Polarization is independent of whatever co-ordinate basis represents the electric or magnetic field. Since many materials

tend to break electric-magnetic (EM) symmetry, a description of light's topological skeleton using electric [9,10] or magnetic [11,12] polarization singularities and other quantities is usually most relevant (and experimentally measurable) yet, particularly in the presence of matter breaking other symmetries or in free space, not strictly fundamental. What is almost always neglected is that polarization *is* basis dependent, but within an adjacent vector space.

In monochromatic light of frequency ω , the electromagnetic bispinor $\boldsymbol{\psi}$ [13–17] (or "wave function") shows us how to combine the electric and magnetic fields in the $\mathbb{C}^2 \times \mathbb{C}^3 \times \mathbb{L}^2$ space:

$$\boldsymbol{\psi}_{\text{EM}}(\mathbf{r}) = \frac{1}{2} \begin{pmatrix} \sqrt{\epsilon_0} \mathbf{E}(\mathbf{r}) \\ \sqrt{\mu_0} \mathbf{H}(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} \mathbf{F}_e(\mathbf{r}) \\ \mathbf{F}_m(\mathbf{r}) \end{pmatrix}. \quad (1)$$

The EM subscript indicates that the bispinor is expressed in an electric-magnetic basis; the bispinor's entries separate the phasors $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$, which can be thought of as complex vector-valued components of the $\mathbb{C}^2 \times \mathbb{C}^3 \times \mathbb{L}^2$ vector $\boldsymbol{\psi}(\mathbf{r})$. We will now drop explicit $\mathbf{r}$ dependence of quantities for tidiness, except for necessary emphasis later on. We could equivalently express (1) as $\boldsymbol{\psi}_{\text{EM}} = \mathbf{F}_e \otimes \hat{\mathbf{e}} + \mathbf{F}_m \otimes \hat{\mathbf{m}}$, making more explicit that it is represented in this EM basis and with basis vectors $\hat{\mathbf{e}}$ and $\hat{\mathbf{m}}$ which, distinguishing the two rows of (1), describe the orientation of $\boldsymbol{\psi}$ in $\mathbb{C}^2$. The familiar $\mathbb{C}^3$ vector $\mathbf{E}$ we call the electric field, not by coincidence of notation, but specifically because it is associated with the unit vector $\hat{\mathbf{e}}$.

Naturally, the basis of the bispinor may be changed; its entries admit linear combinations of $\mathbf{E}$ and $\mathbf{H}$ [17]. For example, a basis of left- or right-handed pure-helicity fields $\boldsymbol{\psi}_{\text{RL}} = \mathbf{F}_R \otimes \hat{\mathbf{R}} + \mathbf{F}_L \otimes \hat{\mathbf{L}}$ [17–20] can be chosen,

$$\boldsymbol{\psi}_{\text{RL}} = \begin{pmatrix} \mathbf{F}_R \\ \mathbf{F}_L \end{pmatrix} = \frac{1}{2\sqrt{2}} \begin{pmatrix} \sqrt{\epsilon_0} \mathbf{E} + i\sqrt{\mu_0} \mathbf{H} \\ \sqrt{\epsilon_0} \mathbf{E} - i\sqrt{\mu_0} \mathbf{H} \end{pmatrix}, \quad (2)$$

in which the basis vectors $\hat{\mathbf{R}} = (\hat{\mathbf{e}} - i\hat{\mathbf{m}})/\sqrt{2}$ and $\hat{\mathbf{L}} = (\hat{\mathbf{e}} + i\hat{\mathbf{m}})/\sqrt{2}$ correspond to the rows of $\boldsymbol{\psi}_{\text{RL}}$, or even a parallel and

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antiparallel field pair $\psi_{\text{PA}} = \mathbf{F}_p \otimes \hat{\mathbf{p}} + \mathbf{F}_a \otimes \hat{\mathbf{a}}$,

$$\psi_{\text{PA}} = \begin{pmatrix} \mathbf{F}_p \\ \mathbf{F}_a \end{pmatrix} = \frac{1}{2\sqrt{2}} \begin{pmatrix} \sqrt{\varepsilon_0} \mathbf{E} + \sqrt{\mu_0} \mathbf{H} \\ \sqrt{\varepsilon_0} \mathbf{E} - \sqrt{\mu_0} \mathbf{H} \end{pmatrix}, \quad (3)$$

where $\hat{\mathbf{p}} = (\hat{\mathbf{e}} + \hat{\mathbf{m}})/\sqrt{2}$ and $\hat{\mathbf{a}} = (\hat{\mathbf{e}} - \hat{\mathbf{m}})/\sqrt{2}$. In each representation prefactors have been chosen such that the inner product $\psi^\dagger \cdot \psi$ produces the time-averaged electromagnetic energy density. The vectors $\mathbf{F}_{\text{R/L}}$ are similar to Riemann-Silberstein vectors [21–23], but here are defined using complex phasors $\mathbf{E}$ and $\mathbf{H}$.

The primary significance of Eqs. (1)–(3) is that the very laws of electromagnetism—Maxwell’s equations and the Lorentz force law, in free space and in media with arbitrary time dependence [see [17] and Supplemental Material (SM) [24]]—may undergo a change of basis in $\mathbb{C}^2$ to be expressed not by $\mathbf{E}$ and $\mathbf{H}$, but by any pair of vector fields $\mathbf{F}_i$ and $\mathbf{F}_j$. We see this as a strong motivation to search for topological features beyond the $\mathbb{C}^3$ space occupied by phasors and their polarization singularities. It is tempting to argue that the standard EM basis for these laws is the most meaningful because $\mathbf{E}$ and $\mathbf{H}$ are responsible for separately measurable “electric” and “magnetic” light-matter interactions and because only one of the two fields can support monopoles. Perhaps this is because we are so accustomed to (both theoretically and experimentally) dealing with matter that breaks EM symmetry, having approximately vacuum permeability ($\mu \approx 1$) but a distinctly nonvacuum permittivity ($\varepsilon \neq 1$), yet other asymmetries of matter can be better accommodated by different bispinor bases. In a system of dual particles ($\varepsilon = \mu$) breaking parity symmetry, for example, $\mathbf{F}_R$ and $\mathbf{F}_L$ would be separately measurable and the RL basis, not the EM basis, would offer a more natural representation of electromagnetic laws. Were we to restrict ourselves to studying the polarization topology of $\mathbb{C}^3$ fields like $\mathbf{E}$ and $\mathbf{H}$, like in most of the singular optics literature to date, then there is surely no reason not to devote attention to the polarization structure of (for example) $\mathbf{F}_R$ or $\mathbf{F}_a$ which, too, contain polarization singularities of their own that do not generally coincide with those of other $\mathbb{C}^3$ fields (see SM [24]). The spatial polarization structure of $\mathbf{F}_R$ or $\mathbf{F}_a$ is only thus far neglected by the literature because, for these fields to engage with matter and reveal themselves in a measurement, they require broken symmetries of matter that are less common in experiments—at least, *for now*.

It is our purpose to find singular structures that have an intrinsic meaning to electromagnetic waves and which are not subject to perception by a detector with a specific sensitivity to $\mathbf{E}$, $\mathbf{H}$ or some linear combination. We find six such structures relating to fundamental symmetries held locally by the geometry of the electromagnetic field vectors, averaged over time, which therefore have a basis-independent physical significance. They do not rely on the presence of a particular kind of detector to justify their meaning and for this reason we argue them to be more fundamental than polarization singularities. Half of these structures are phase singularities in complex scalar fields (codimension 2), while the other three are two-dimensional surfaces (codimension 1) and five of them are to our knowledge unrecognized. The remaining structure, a time-averaged version of the Riemann-Silberstein vortex, sta-

tionary in monochromatic light [25–28], we do not believe is understood in relation to symmetries held by field vectors. We will derive these results initially in monochromatic light; however, we have generalized our definitions to time-varying fields using analytic signals, which have an intuitive interpretation for waves with well-resolved phase and envelope. In this way all six structures can be found in nonmonochromatic fields and three of them are Lorentz invariant.

Results. Each of our proposed structures are located where the electromagnetic field vectors are locally symmetric to the effect of one or a sequence of parity-inversion $\hat{P}$, time-reversal $\hat{T}$, or duality operations $\hat{D}$, when averaged over one cycle. An operator $\hat{O}$ transforms in a general basis the bispinor $\psi = (\mathbf{F}_i, \mathbf{F}_j)^T$ according to

$$\hat{O}\psi = \psi' = \begin{pmatrix} \mathbf{F}'_i \\ \mathbf{F}'_j \end{pmatrix}. \quad (4)$$

The action of $\hat{P}$, $\hat{T}$, and $\hat{D}$ on the components of the bispinor ψ depends on its chosen basis; in the EM representation (1), for example, the symmetry operators in $\mathbb{C}^2$ reflect or rotate the bispinor and possibly conjugate its components [14]:

$$\{\hat{P}, \hat{T}, \hat{D}\}\psi_{\text{EM}} = \frac{1}{2} \left\{ \begin{pmatrix} -\sqrt{\varepsilon_0} \mathbf{E} \\ \sqrt{\mu_0} \mathbf{H} \end{pmatrix}, \begin{pmatrix} \sqrt{\varepsilon_0} \mathbf{E}^* \\ -\sqrt{\mu_0} \mathbf{H}^* \end{pmatrix}, \begin{pmatrix} \sqrt{\mu_0} \mathbf{H} \\ -\sqrt{\varepsilon_0} \mathbf{E} \end{pmatrix} \right\}. \quad (5)$$

We define the electromagnetic field at a position $\mathbf{r}_0$ to be locally symmetric with respect to a certain transformation $\hat{O}$ if the transformed component coefficients of the bispinor, $\mathbf{F}'_i$ and $\mathbf{F}'_j$, can be superimposed with the original coefficients, pretransformation, via a unitary transformation $\hat{R}$:

$$\begin{pmatrix} \hat{R} & 0 \\ 0 & \hat{R} \end{pmatrix} \psi'(\mathbf{r}_0) = \psi(\mathbf{r}_0). \quad (6)$$

It is important to specify why we have designated $\hat{R}$ as a unitary, rather than orthogonal, matrix. Were $\hat{R}$ a real rotation matrix belonging to $\text{SO}(3)$, then any transformation $\hat{R}\mathbf{F}$ would preserve the shape of the polarization ellipse swept out by $\mathbf{F}$, while changing the ellipse’s orientation. For there to exist an $\text{SO}(3)$ matrix in place of $\hat{R}$ that can satisfy (6) places too many constraints on $\mathbf{F}_i$ and $\mathbf{F}_j$; the four vectors, $\mathbf{p}_{i,j}$ and $\mathbf{q}_{i,j}$, where $\mathbf{F}_{i,j} = \mathbf{p}_{i,j} + i\mathbf{q}_{i,j}$, must all be eigenvectors of $\hat{R}$, with specific eigenvalues dictated by the operator $\hat{O}$. This only occurs under exceptional circumstances and means the field is symmetric with respect to the transformation $\psi' = \hat{O}\psi$ for all values of time.

If instead $\hat{R}$ is designated an $\text{SU}(3)$ matrix, then polarization ellipses are generally distorted after transforming $\hat{R}\mathbf{F}_i$ and $\hat{R}\mathbf{F}_j$. However, unitarity of $\hat{R}$ ensures that all scalar time-averaged quantities (which are complex inner products) associated with the relationship between vectors $\mathbf{F}_i$ and $\mathbf{F}_j$ as they oscillate [e.g., averages of the instantaneous difference in their square lengths and their orientation, characterized by $|\mathcal{F}_i(t)|^2 - |\mathcal{F}_j(t)|^2$ and $\mathcal{F}_i(t) \cdot \mathcal{F}_j(t)$] are preserved after the transformation. We refer to these relationships between the vectors $\mathbf{F}_i$ and $\mathbf{F}_j$ during oscillation as their “geometry.” If (6) is satisfied by a unitary matrix $\hat{R}$, then there is nothing distinguishing the time-averaged geometry of $\mathbf{F}_i$ and $\mathbf{F}_j$ before

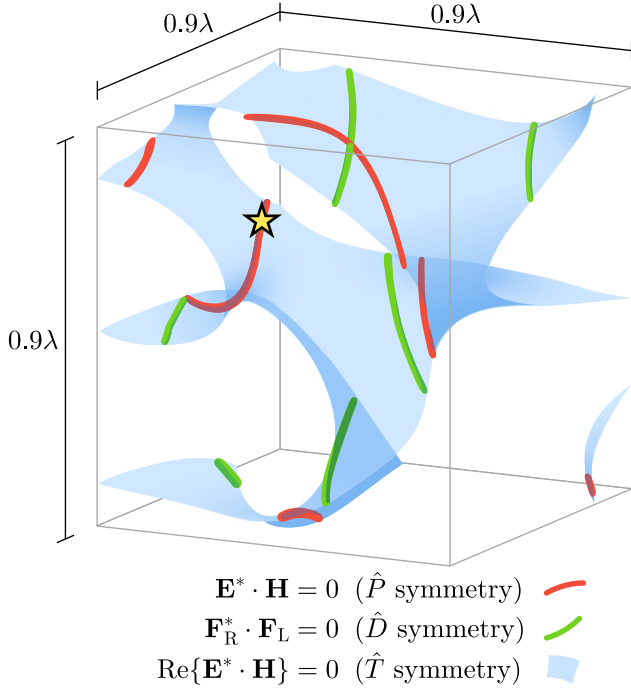


FIG. 1. Cycle-averaged symmetry structures shown in a $(0.9\lambda)^3$ volume produced by interfering four monochromatic, randomly polarized plane waves with random wave-vector directions. The electric and magnetic polarization ellipses (that satisfy $\mathbf{E}^* \cdot \mathbf{H} = 0$) at the location of the star are plotted in Fig. 2.

and after the action of the operator $\hat{O}$. This instead places less-severe constraints on the field that are satisfied naturally in general nonparaxial light.

We have quite deliberately left the bispinor basis unspecified in this section, for it is often possible to choose the bispinor basis so that its entries are eigenvectors of $\hat{O}$ with distinct eigenvalues, meaning

$$\hat{O}\mathbf{F}_i = \lambda_1 \mathbf{F}_i, \quad \hat{O}\mathbf{F}_j = \lambda_2 \mathbf{F}_j. \quad (7)$$

Equation (5) has an example of this behavior for $\hat{O} = \hat{P}$ in the EM bispinor basis. Symmetry with respect to $\hat{O}$ means that $\mathbf{F}_i$ and $\mathbf{F}_j$ are also eigenvectors of $\hat{R}$ with the same eigenvalues $\lambda_{1,2}$, where $\lambda_1 \neq \lambda_2$. Since the three eigenvectors of an $\text{SU}(3)$ matrix form an orthonormal basis, if $\mathbf{F}_i$ and $\mathbf{F}_j$ are to be eigenvectors of $\hat{R}$ with different eigenvalues, then $(\mathbf{F}_i, \mathbf{F}_j) = \mathbf{F}_i^* \cdot \mathbf{F}_j = 0$.

Whatever the choice of bispinor basis, the quantity $\mathbf{F}_i^* \cdot \mathbf{F}_j$ is a complex scalar field. Such is the nature of any complex scalar field in three dimensions that the zeros of $\mathbf{F}_i^* \cdot \mathbf{F}_j$ are one-dimensional structures forming just as naturally in nonparaxial fields as polarization singularities. The topological structures relating to time-averaged symmetry of the local field vectors with respect to certain fundamental symmetry operators behave in precisely this way, as shown in Fig. 1. We define these structures next, beginning with $\hat{P}$.

a. Parity, duality, and time reversal. According to (5), the electric- and magnetic-field phasors are eigenvectors of the parity operator with eigenvalues ∓ 1 ,

$$\hat{P}\mathbf{E} = -\mathbf{E}, \quad \hat{P}\mathbf{H} = \mathbf{H}, \quad (8)$$

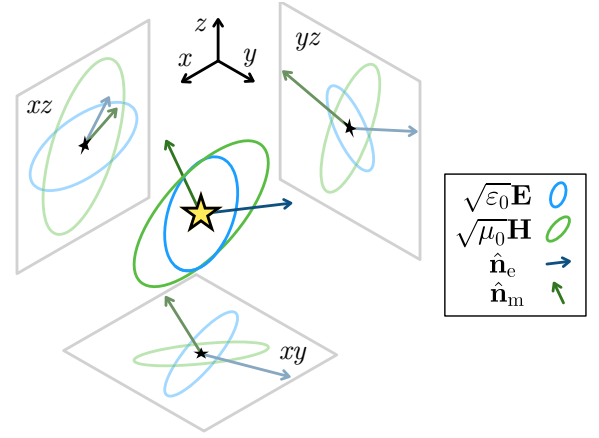


FIG. 2. The electric and magnetic polarization ellipses and normal vectors at the location of the star in Fig. 1, along with projections onto the xy , yz , and xz planes. Despite their nondistinct 3D orientation, these ellipses satisfy $\mathbf{E}^* \cdot \mathbf{H} = 0$ and are cycle-averaged $\hat{P}$ symmetric.

which suggests that the scalar dislocation line defined by $\mathbf{E}^* \cdot \mathbf{H} = 0$ locates positions in an electromagnetic field where the field vectors are, cycle-averaged, locally parity symmetric. It is important to stress the word *locally* here, because we do not consider the inversion of the whole spatial distribution of the field by transformation of the position vector $\mathbf{r} \rightarrow -\mathbf{r}$. To instead specify the local parity of the field vectors at a given position $\mathbf{r}_0$, we designate $\mathbf{r}_0$ as the inversion center (which is after all an arbitrary choice).

It is often tempting to connect the chirality of the field vectors with simplistic features of electric and magnetic polarization ellipses, which are restricted in the paraxial regime where $\text{Re}\{\mathbf{E}^* \cdot \mathbf{H}\}$ is zero. Towards the nonparaxial regime, however, it becomes extremely difficult to picture the specific relationship between electric- and magnetic-field vectors and their time evolution that means they are time-averaged $\hat{P}$ symmetric. It is possible for both electric and magnetic fields to be elliptically polarized, and their polarization ellipses oriented indistinctly relative to each other like in Fig. 2, and still satisfy $\mathbf{E}^* \cdot \mathbf{H} = 0$. The dislocation $\mathbf{E}^* \cdot \mathbf{H} = 0$ is associated with the simultaneous vanishing of helicity density, $h = -\text{Im}\{\mathbf{E}^* \cdot \mathbf{H}\}/(2\omega c)$ [14,29,30], and the reactive helicity density, $h_{\text{reac}} = \text{Re}\{\mathbf{E}^* \cdot \mathbf{H}\}/(2\omega c)$ [14,31]. As we shall see, the real part of $\mathbf{E}^* \cdot \mathbf{H}$, being time-odd, determines time-averaged $\hat{T}$ symmetry of the field vectors and is therefore a falsely chiral quantity [32].

Next, the duality operator $\hat{D}$, for which the vectors $\mathbf{F}_R$ and $\mathbf{F}_L$ from (2) are eigenvectors with eigenvalues $\mp i$,

$$\hat{D}\mathbf{F}_R = -i\mathbf{F}_R, \quad \hat{D}\mathbf{F}_L = i\mathbf{F}_L, \quad (9)$$

indicating that time-averaged duality symmetry of the field vectors occurs when $\mathbf{F}_R^* \cdot \mathbf{F}_L$ is zero. Notably, the two real scalar conditions satisfied here, $\text{Re}\{\mathbf{F}_R^* \cdot \mathbf{F}_L\} = \text{Im}\{\mathbf{F}_R^* \cdot \mathbf{F}_L\} = 0$, are time-averaged versions of the fundamental invariants that vanish on Riemann-Silberstein vortices [26], strands of instantaneously plane-wave-like, null electromagnetic field [25,27] that move through space rapidly [33]. Cycle-averaging introduced by the definition $\mathbf{F}_R^* \cdot \mathbf{F}_L = 0$ renders the vortex

stationary, as has been recognized [26]. All paraxial light approaches $\mathbf{F}_R^* \cdot \mathbf{F}_L = 0$.

Finally, we discuss the time-reversal operator $\hat{T}$. Unlike $\hat{P}$ and $\hat{D}$, none of the $\mathbb{C}^3$ vectors from (1) to (3) are eigenvectors of $\hat{T}$ due to complex conjugation. Nonetheless, the PA representation of ψ (3) presents an interesting scenario where

$$\hat{T}\mathbf{F}_p = \mathbf{F}_a^*, \quad \hat{T}\mathbf{F}_a = \mathbf{F}_p^*. \quad (10)$$

If the vector geometry is cycle-averaged, time-reversal symmetric, then a unitary matrix $\hat{R}$ should be able to undo the above transformation from right to left, but because a unitary transformation cannot change the length of a vector, it must be that $|\mathbf{F}_p| = |\mathbf{F}_a|$ or, alternatively, that $|\mathbf{F}_p|^2 - |\mathbf{F}_a|^2 \propto \text{Re}\{\mathbf{E}^* \cdot \mathbf{H}\} = 0$.

At the same time, unitarity of $\hat{R}$ means the inner product of the vectors pre- and post- $\hat{R}$ must be preserved, $\langle \mathbf{F}_a^*, \mathbf{F}_p^* \rangle = \langle \mathbf{F}_p, \mathbf{F}_a \rangle$, which in this case is trivially true. That means the single real condition $|\mathbf{F}_p|^2 - |\mathbf{F}_a|^2 \propto \text{Re}\{\mathbf{E}^* \cdot \mathbf{H}\} = 0$ is sufficient to ensure that $\hat{R}$ exists and therefore indicates cycle-averaged $\hat{T}$ symmetry of the field vectors. Since $\text{Re}\{\mathbf{E}^* \cdot \mathbf{H}\} = 0$ is also included in the conditions $\mathbf{E}^* \cdot \mathbf{H} = 0$ and $\mathbf{F}_R^* \cdot \mathbf{F}_L = 0$, the lines of time-averaged $\hat{P}$ and $\hat{D}$ symmetry are contained in the surfaces of time-averaged $\hat{T}$ symmetry, as shown in Fig. 1.

b. *Additional symmetries.* It is natural now to ask what the zeros of the complex scalar $\mathbf{F}_p^* \cdot \mathbf{F}_a$ correspond to. In fact the $\mathbb{C}^3$ vectors in the PA basis are eigenvectors of the combined $\hat{P}\hat{D}$ operator,

$$\hat{P}\hat{D}\mathbf{F}_p = -\mathbf{F}_p, \quad \hat{P}\hat{D}\mathbf{F}_a = \mathbf{F}_a, \quad (11)$$

implying that $\mathbf{F}_p^* \cdot \mathbf{F}_a = 0$ corresponds generically to lines of cycle-averaged $\hat{P}\hat{D}$ symmetry.

Meanwhile, symmetry with respect to other compound operators, namely $\hat{P}\hat{T}$ and $\hat{D}\hat{T}$, can be identified by using the EM and RL representations of ψ :

$$\hat{D}\hat{T}\sqrt{\varepsilon_0}\mathbf{E} = -\sqrt{\mu_0}\mathbf{H}^*, \quad \hat{D}\hat{T}\sqrt{\mu_0}\mathbf{H} = -\sqrt{\varepsilon_0}\mathbf{E}^*, \quad (12)$$

$$\hat{P}\hat{T}\mathbf{F}_R = -\mathbf{F}_L^*, \quad \hat{P}\hat{T}\mathbf{F}_L = -\mathbf{F}_R^*. \quad (13)$$

Neither $\mathbf{E}$ nor $\mathbf{H}$ are eigenvectors of $\hat{D}\hat{T}$ and, likewise, $\mathbf{F}_R$ nor $\mathbf{F}_L$ are eigenvectors of $\hat{P}\hat{T}$. But the behavior of (12) and (13) is similar to that of (10) and with the same arguments as in the text which follows (that $\hat{R}$ cannot change the length of the vector it transforms and must preserve inner products), we can associate time-averaged $\hat{D}\hat{T}$ symmetry with $\varepsilon_0|\mathbf{E}|^2 - \mu_0|\mathbf{H}|^2 = 0$ and $\hat{P}\hat{T}$ symmetry with $|\mathbf{F}_R|^2 - |\mathbf{F}_L|^2 \propto \text{Im}\{\mathbf{E}^* \cdot \mathbf{H}\} = 0$. These real scalar conditions define different two-dimensional surfaces corresponding to cycle-averaged $\hat{P}\hat{T}$ symmetry and $\hat{D}\hat{T}$ symmetry.

Even number combinations of $\hat{P}$, $\hat{D}$, and $\hat{T}$ may commute or anticommute, for example, $\hat{P}\hat{D}\psi = -\hat{D}\hat{P}\psi$, though the conditions for time-averaged symmetry with respect to either permutation of the same two operators in a combination is the same. Remarkably, expanding the expression $\mathbf{F}_p^* \cdot \mathbf{F}_a = 0$ using the definitions of $\mathbf{F}_p$ and $\mathbf{F}_a$ in (3), we see that $\varepsilon_0|\mathbf{E}|^2 - \mu_0|\mathbf{H}|^2 = 0$ and $|\mathbf{F}_R|^2 - |\mathbf{F}_L|^2 \propto \text{Im}\{\mathbf{E}^* \cdot \mathbf{H}\} = 0$ are also satisfied, meaning the dislocation line $\mathbf{F}_p^* \cdot \mathbf{F}_a = 0$ is also symmetric with respect to any even number combination of $\hat{P}$, $\hat{D}$, and $\hat{T}$ operators.

c. *Time-dependent dislocations.* It is relatively straightforward to identify symmetry dislocations in time-dependent, nonmonochromatic light by using complex analytic signals to describe the electromagnetic field. Though only occasionally used in optics [34], analytic signals offer a powerful generalization of phasors to fields containing multiple frequency components and lead to a time-dependent bispinor occupying the same $\mathbb{C}^2 \times \mathbb{C}^3 \times \mathbb{L}^2$ vector space as before.

The analytic extension (denoted by tilde) of the bispinor is in the EM basis,

$$\tilde{\psi}(\mathbf{r}, t) = \frac{1}{2} \begin{pmatrix} \sqrt{\varepsilon_0} \tilde{\mathcal{E}}(\mathbf{r}, t) \\ \sqrt{\mu_0} \tilde{\mathcal{H}}(\mathbf{r}, t) \end{pmatrix}, \quad (14)$$

where to define an analytic signal, for example, $\tilde{\mathcal{E}}(\mathbf{r}, t) = \mathcal{E}(\mathbf{r}, t) + i\mathfrak{h}[\mathcal{E}(\mathbf{r}, t)]$, where $\mathfrak{h}[\cdot]$ denotes the Hilbert transform, we take the spectrum of the physically measurable field $\mathcal{E}(\mathbf{r}, t)$ and discard negative frequency components. Because $\tilde{\psi}(\mathbf{r}, t)$ remains in the same vector space, $\mathbb{C}^2 \times \mathbb{C}^3 \times \mathbb{L}^2$, as the monochromatic bispinor, we can subject it to the same transformations as we did the monochromatic bispinor and its vector valued components between (4) and (13) and arrive at time-dependent definitions of the electromagnetic field's symmetry dislocations (see SM [24] for more detail). What in monochromatic light were time-averaged quantities now correspond to envelopes—effectively ‘local’ time-averages—of instantaneous products with non-trivial time-dependence. For example, the analytic dot product $\tilde{\mathcal{E}}^* \cdot \tilde{\mathcal{H}}$ averages the instantaneous signal $\mathcal{E} \cdot \mathcal{H}$ over a timescale long enough to smooth out fast oscillations of the signal, but short enough to capture its variation in amplitude over time. The local time-average interpretation holds if there are well-resolved sinusoidal oscillations in the signal. We emphasize, however, that in any kind of time-dependent waves analytic signals remain well defined, as do our time-dependent definitions of symmetry dislocations—only the interpretation becomes less intuitive. The operators $\hat{P}$, $\hat{D}$, and $\hat{T}$ act in the same way on the analytic bispinor:

$$\{\hat{P}, \hat{T}, \hat{D}\} \tilde{\psi}_{\text{EM}} = \frac{1}{2} \left\{ \begin{pmatrix} -\sqrt{\varepsilon_0} \tilde{\mathcal{E}} \\ \sqrt{\mu_0} \tilde{\mathcal{H}} \end{pmatrix}, \begin{pmatrix} \sqrt{\varepsilon_0} \tilde{\mathcal{E}}^* \\ -\sqrt{\mu_0} \tilde{\mathcal{H}}^* \end{pmatrix}, \begin{pmatrix} \sqrt{\mu_0} \tilde{\mathcal{H}} \\ -\sqrt{\varepsilon_0} \tilde{\mathcal{E}} \end{pmatrix} \right\}, \quad (15)$$

though what should also be made clear is our interpretation of the time-reversal transformation.

Time reversal inverts signals over the entire time axis by transforming $t \rightarrow -t$. In monochromatic light this can be simulated as in (5) by conjugating the $\mathbf{E}$ and $\mathbf{H}$ phasors (and inverting the pseudovector $\mathbf{H}$), changing the sense of rotation of polarization ellipses. Conjugating an analytic signal with general time dependence like in (15) is certainly not equivalent to substituting $t \rightarrow -t$, but can be interpreted as a *time-local* time reversal, exactly analogous to the parity operator that acts on the field vectors without a global space inversion. One appreciates this by looking at the spectra of an analytic signal and its time-reversed version, for instance, $\tilde{\mathcal{E}}$:

$$\tilde{\mathcal{E}}(t) = 2 \int_0^\infty \mathbf{E}(\omega) e^{-i\omega t} d\omega, \quad (16)$$

$$\tilde{\mathcal{E}}(-t) = \tilde{\mathcal{E}}_{\text{rev}}(t) = 2 \int_0^\infty \mathbf{E}(-\omega) e^{-i\omega t} d\omega. \quad (17)$$

TABLE I. General definitions of symmetry dislocations in time-dependent electromagnetic fields, which are zeros of complex or real scalar fields (second column) defined using complex analytic signals and where $\tilde{\mathcal{F}}_{R/L} = (\sqrt{\epsilon_0}\tilde{\mathcal{E}} \pm i\sqrt{\mu_0}\tilde{\mathcal{H}})/(2\sqrt{2})$ and $\tilde{\mathcal{F}}_{p/a} = (\sqrt{\epsilon_0}\tilde{\mathcal{E}} \pm \sqrt{\mu_0}\tilde{\mathcal{H}})/(2\sqrt{2})$. The generic real-space dimension of these structures in nonparaxial fields is in column three and their possible Lorentz invariance in column four. Monochromatic definitions can be recovered by changing each analytic signal for the corresponding phasor, e.g., $\tilde{\mathcal{E}} \rightarrow \mathbf{E}$.

Time-averaged symmetry	Scalar field zero	Dim.	Lorentz invariant?
$\hat{P}$	$\tilde{\mathcal{E}}^* \cdot \tilde{\mathcal{H}} = 0$	1	no
$\hat{D}$	$\tilde{\mathcal{F}}_R^* \cdot \tilde{\mathcal{F}}_L = 0$	1	yes
$\hat{T}$	$ \tilde{\mathcal{F}}_p ^2 - \tilde{\mathcal{F}}_a ^2 = 0$	2	yes
$\hat{P}\hat{D}$	$\tilde{\mathcal{F}}_p^* \cdot \tilde{\mathcal{F}}_a = 0$	1	no
$\hat{P}\hat{T}$	$ \tilde{\mathcal{F}}_R ^2 - \tilde{\mathcal{F}}_L ^2 = 0$	2	no
$\hat{D}\hat{T}$	$\epsilon_0 \tilde{\mathcal{E}} ^2 - \mu_0 \tilde{\mathcal{H}} ^2 = 0$	2	yes

At the inversion center $t = 0$, the integrands of (16) and (17) simplify to $\mathbf{E}(\omega)d\omega$ and $\mathbf{E}(-\omega)d\omega$. Due to Hermitian symmetry, $\mathbf{E}(-\omega) = \mathbf{E}^*(\omega)$, the time-reversed signal's spectrum at $t = 0$ is equivalent to that of the conjugate of the original analytic signal. Since we can shift the inversion center to any instant t_0 along the time axis via $t \rightarrow t - t_0$, we can interpret $\hat{T}\tilde{\mathcal{E}}(t) \rightarrow \tilde{\mathcal{E}}^*(t)$ as a time-local time-reversal operation on the electric-field vector.

General, time-dependent definitions of symmetry dislocations—the main results of this work—are given in Table I. Three of these structures are Lorentz invariant because they are constructed of one or both of the fundamental invariants which for analytic signals are $\epsilon_0\tilde{\mathcal{E}}^* \cdot \tilde{\mathcal{E}} - \mu_0\tilde{\mathcal{H}}^* \cdot \tilde{\mathcal{H}}$ and $\text{Re}\{\tilde{\mathcal{E}}^* \cdot \tilde{\mathcal{H}}\}$ (see SM [24]).

Discussion. We identified six fundamental topological structures permeating electromagnetic fields on which by the local field vector geometry preserve certain fundamental symmetries when averaged over time. Of the six structures summarized in Table I, three are phase vortices amounting to lines in real space, while the other three are defined by real scalar zeros, corresponding to surfaces. We derived the conditions in Table I for general, time-dependent, and nonparaxial fields, where each structure forms naturally and stably. All structures

map to axial lines or planes within the recently proposed "electromagnetic symmetry sphere," a Bloch sphere that characterizes the distribution of total energy density among (in appropriate units) helicity density, reactive helicity density, and reactive energy density [17].

There is a subtle correspondence between the $\hat{D}$ operator and the charge conjugation operator $\hat{C}$ used in quantum theory, which is that $\hat{D}^2 \equiv \hat{C}$, causing an inversion of charge density and accordingly the electric- and magnetic-field vectors; $\hat{C}\psi = -\psi$. The effect of $\hat{C}$ on the bispinor is rather trivial compared to that of $\hat{P}$, $\hat{D}$, and $\hat{T}$, which might explain why it does not play such an important role in classical electromagnetism as in quantum theory.

Phase and polarization singularities can be sculpted in such a way as to create exceptional, possibly skyrmionic topologies [35–39], in which information can be encoded. We expect that the local symmetry structures in this work could be manipulated in much the same way and perhaps provide additional degrees of freedom for encoding topological information in structured light. The ability to locate strands and surfaces in electromagnetic fields corresponding to certain local symmetries of the field vectors could also be useful in the context of exotic light-matter interactions that relate to broken symmetries of matter. Quantities $\text{Im}\{\mathbf{E}^* \cdot \mathbf{H}\}$, $\epsilon_0|\mathbf{E}|^2 - \mu_0|\mathbf{H}|^2$, and $\text{Re}\{\mathbf{E}^* \cdot \mathbf{H}\}$ are associated with chiral, electric-magnetic, and magnetoelectric effects in small particles [14,40]. Our symmetry-dislocation perspective, which relates to how these quantities are distributed over space, could be useful for structuring waves so as to elicit these special light-matter interactions.

Topological defects are ubiquitous in physical systems and similarities to optical singularities can be drawn from defects in acoustics [41,42], water waves [43], linearized gravity [2,44], or even nematic liquid crystals [45–47]. When a wave field has nontrivial behavior under certain symmetry operators, a similar methodology to that which we have used could be applied to other wave fields to uncover analogous local symmetry structures.

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