

Experimental evidence-based sub-Rayleigh source discrimination

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We propose a Bayesian evidence-based inference framework built on relative belief ratios and apply it to discriminating between one and two incoherent optical point sources using spatial-mode demultiplexing (SPADE). Unlike the Helstrom measurement, SPADE requires no collective detection and is optimal for asymptotically large samples. Our method avoids *ad hoc* statistical constructs and draws inference directly from the information contained in the data, with all assumptions entering only through the likelihood model and prior beliefs. Using experimental evidence, we demonstrate the superior resolving performance of SPADE over direct imaging from a perspective that generalizes to multiple sources and offers a practical, robust approach to analyzing quantum-enhanced superresolution.

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I. INTRODUCTION

Quantum hypothesis testing stands as a cornerstone of quantum information science [1–3], addressing a central challenge: how to distinguish quantum states (or channels) with maximal precision. At its heart lies the quest to identify the ultimate quantum limits of such distinguishability.

The foundational works of Helstrom [4] and Holevo [5] laid the theoretical bedrock for this field, deriving optimal strategies and fundamental bounds for discriminating between two nonorthogonal quantum states. While the two-state problem is now well understood, generalizing these insights to more complex scenarios remains an open challenge, with optimal measurements known only in special cases [6]. Over the years, various approaches have been developed to identify measurements tailored to different performance metrics [7], ranging from minimum-error and maximum-confidence approaches [8] to unambiguous state discrimination [9–15].

In optical imaging, a common hypothesis-testing problem is source discrimination, i.e., determining whether an im-

age originates from one or two pointlike sources [16–20]. This task is especially relevant in advanced imaging contexts such as exoplanet detection [21,22] and fluorescence microscopy [23–25]. Yet when the separation between sources is smaller than the system point-spread function [26], the performance of direct imaging plummets [27]. In this sub-Rayleigh regime, quantum-inspired measurement strategies that exploit the optimality of modal projection for parameter estimation [28] have emerged as powerful alternatives to conventional imaging. Seminal work by Tsang and collaborators [29–31] on estimating the separation of incoherent sources triggered a broad line of research [32–39], driven in large part by the development of spatial-mode demultiplexing (SPADE) [40]. Subsequent work also established the optimality of SPADE for source discrimination [41]. However, these promising results often rely on idealized measurement conditions. In practice, SPADE performance is vulnerable to various imperfections, such as misalignments and fabrication defects, that introduce modal crosstalk and degrade measurement fidelity [42–51].

In this article, we address these limitations by adopting a Bayesian, evidence-based approach to state discrimination of light sources. Recasting the problem as one of hypothesis testing, we employ relative belief (RB) analysis [52–54], previously applied in quantum information processing to construct optimal credible regions for quantum state estimation and incomplete tomography with limited data [55–60]. RB analysis requires only three essential components: a well-specified model, the observed measurement data, and prior distributions, with optional checks for prior bias.

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This approach avoids *ad hoc* statistical assumptions, such as significance levels and p -values commonly invoked in conventional hypothesis testing, and readily accommodates model-specific hypotheses, making it suitable for many applications. While frequentist methods based on Fisher information [61] are valuable for assessing measurement and estimator optimality, the construction of associated confidence regions is generally valid only in the asymptotic limit of many repeated measurements. The RB framework, by contrast, remains meaningful with limited data, providing a quantification of statistical evidence without large-sample approximations. This enables direct evaluation of hypothesis plausibility across data regimes. All assumptions enter transparently through the likelihood model and chosen priors, with no auxiliary assumptions.

The rest of this article is organized as follows. Section II summarizes the preliminaries of the relative-belief analysis. Then, our experimental setup is described in Sec. III. The single-source versus two-source discrimination problem and its solution through RB analysis are presented in Sec. IV, along with a toy example with some natural constraints. The relative-belief framework is further applied to a generalized problem of multiparameter estimation in Sec. V. Finally, we conclude in Sec. VI.

II. RELATIVE-BELIEF ANALYSIS

Let us recall the basics of RB. It provides a mathematical framework to quantify statistical evidence across a set of competing hypotheses, rather than relying solely on posterior probabilities. Consider a set of K hypotheses about a quantum system. Before conducting any experiment, we may assign prior probabilities $\pi(k)$ reflecting our initial beliefs or knowledge, possibly informed by earlier results. After collecting data $\mathbb{D}$ from an experiment, these beliefs are updated to posterior probabilities $\pi(k|\mathbb{D})$, representing our revised beliefs in each hypothesis via Bayes's rule,

$$\pi(k|\mathbb{D}) = \frac{L(\mathbb{D}|k)\pi(k)}{\sum_{k'=1}^K L(\mathbb{D}|k')\pi(k')}. \quad (1)$$

Here, $L(\mathbb{D}|k)$ is the likelihood of observing the dataset $\mathbb{D}$ assuming that hypothesis k is true. Then RB directly compares the prior and posterior beliefs inasmuch as

$$\text{RB}(k|\mathbb{D}) = \frac{\pi(k|\mathbb{D})}{\pi(k)}, \quad (2)$$

which measures whether our belief in hypothesis k has increased due to the data. A value $\text{RB}(k|\mathbb{D}) > 1$ indicates that the data support hypothesis k —the value k is plausible.

However, an RB ratio above 1 does not convey how strongly we should believe in what the evidence indicates. That strength is measured by a posterior probability. For two possible states, it is the posterior of the hypothesized state. When there is evidence in favor of the hypothesis, a high posterior probability indicates strong evidence in favor, while, when there is evidence against, a small posterior indicates strong evidence against. When there are more than two possible states, a comparison among the RB ratios is made, and the

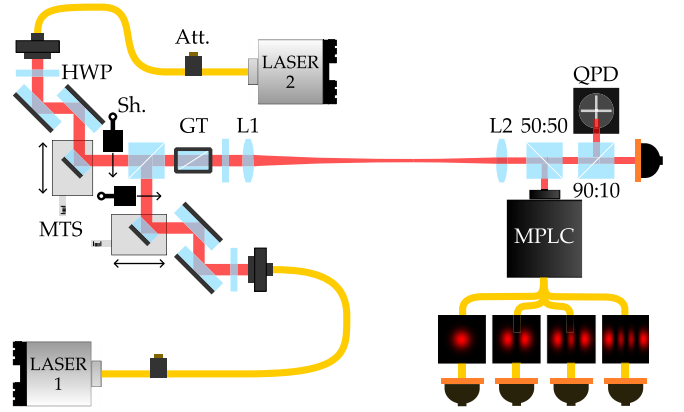


FIG. 1. Schematic diagram of the measurement setup (for details see text).

strength is measured by the evidence strength,

$$E_k = \sum_{k' \neq k} \pi(k'|\mathbb{D}) \Theta(\text{RB}_k - \text{RB}_{k'}), \quad (3)$$

where $\Theta(\cdot)$ is the Heaviside step function with $\Theta(0) \equiv 0$.

The RB framework introduces a natural threshold at $\text{RB}_k = 1$, distinguishing hypotheses that are supported by the evidence from those that are not. This framework contrasts with traditional hypothesis-testing methods, such as reliance on p -values, that are designed to reject a null hypothesis rather than quantify evidence for an alternative. Commonly used thresholds (e.g., $p = 0.05$) are arbitrary and not dictated by the data [62,63]. RB instead emphasizes the support the data provide for each hypothesis rather than rejection of a null.

III. MEASUREMENT SETUP

Our setup, schematized in Fig. 1, employs two independent, fiber-coupled cw lasers at 1550 nm to generate coherent states of light. Each output passes through a polarization controller and fiber attenuator (Att.) for independent control of polarization and optical power. The resulting incoherent modes are collimated into free space as Gaussian beams with waist $w_1 \simeq 1.135$ mm. Each beam goes through a half-wave plate (HWP) for proper polarization orientation (in anticipation of the filter) and is then reflected by a mirror on a motorized translation stage (MTS), allowing fine adjustment of transverse displacement and precise control over spatial separation at the beam splitter. The combined beams are projected through a Glan-Thomson polarizer (GT) to ensure a common linear polarization, rendering the sources indistinguishable. A telescope (L1, L2) images and mode matches the beams for optimal coupling into a spatial-mode demultiplexer with waist $w_0 \simeq 320$ μm .

For demultiplexing, we use a multiplane light converter (MPLC; PROTEUS-C from Cailabs) that decomposes the incoming light into the Hermite-Gauss (HG) mode basis. Each mode is subsequently coupled into a single-mode fiber. We use six of the ten available MPLC outputs and simultaneously measure their intensities using photodiodes. HG_{00} , HG_{10} , HG_{20} , and HG_{30} are used for the test. In the preliminary stage dedicated to alignment and mode matching, we

rely on modes HG_{10} , HG_{01} , HG_{20} , and HG_{02} . To accurately characterize the parameters of a given scene, we employ a quadrant photodiode (QPD) along with an additional photodiode to sequentially measure the position and power of each source. This is achieved by closing one source at a time using shutters (Sh.). In our protocol, we make no assumptions about the specific form of experimental imperfections, relying instead on robust model calibration. Any remaining subjective assumptions enter only through the explicit specification of the likelihood and the priors, rather than through *ad hoc* thresholds or large-sample approximations.

To account for the setup imperfections and variations in the object profiles, the demultiplexed mode intensities are recorded as functions of the positions of the sources. These data are then calibrated using high-degree polynomial fits. The measured intensities are subject to random fluctuations and are recorded as voltage signals from the detectors, with a sampling rate of 10 kHz. Concretely, after the detected signals are normalized by the total detected intensity $P(t)$, the sample mean estimates the mean intensity vector $\mathbf{I}$, while sample covariance matrices acquired over 1 s for each calibration position of the two sources are pooled to estimate the per-sample (per-time-bin) noise covariance Σ_{calib} . A more thorough noise model as a function of position and total power was used in [64].

A typical hypothesis for the image is modeled as an incoherent mixture of the calibrated, mode-sorted intensities $\tilde{\mathbf{I}}_1(x)$ and $\tilde{\mathbf{I}}_2(x)$ of the two sources, where each $\tilde{\mathbf{I}}(x) = (\tilde{I}^{(0)}(x) \tilde{I}^{(1)}(x) \tilde{I}^{(2)}(x) \tilde{I}^{(3)}(x))^T$ is a column of four calibrated HG spatial-mode intensity distributions in x . This intensity is

$$\tilde{\mathbf{I}}_d(x_c) := q\tilde{\mathbf{I}}_1(x_c - d/2) + (1 - q)\tilde{\mathbf{I}}_2(x_c + d/2), \quad (4)$$

where x_c is the centroid relative to the demultiplexer reference, d is the signed separation between the sources, and q quantifies their brightness imbalance. For notational simplicity, the position x in the intensities will be omitted henceforth.

The likelihood density is the joint conditional probability over the $M = 4$ HG modes, $L_d = \pi(\mathbf{I}|\tilde{\mathbf{I}}_d)$, where $\mathbf{I} = (I^{(0)} I^{(1)} I^{(2)} I^{(3)})^T$ collects the observed intensities. We take this density to be the multivariate Gaussian $\exp[-\frac{1}{2}(\mathbf{I} - \tilde{\mathbf{I}}_d)^T \cdot \Sigma^{-1} \cdot (\mathbf{I} - \tilde{\mathbf{I}}_d)] / \sqrt{(2\pi)^M \text{Det}\{\Sigma\}}$, where Σ is the empirical, generally nondiagonal noise covariance among the four HG modes. To match the calibration noise to the measurement conditions, the calibration covariance is rescaled as $\Sigma = \Sigma_{\text{calib}} \langle 1/Q^2 \rangle_{\text{meas}} / (\langle 1/P^2 \rangle_{\text{calib}} N_{\text{meas}})$, where $Q(t)$ is the total power during measurement and N_{meas} is the number of time bins averaged. All the analyses presented here use $N_{\text{meas}} = 1$, i.e., a single sample time bin. We now address two tasks: source discrimination and parameter estimation.

IV. SOURCE DISCRIMINATION

First, we consider the source-discrimination problem with three mutually exclusive hypotheses: a single source located at $x_c \mp d/2$ alone or both together, with a brightness imbalance q between them as specified in the model (4). Since d , x_c , and q are unknown, they are the nuisance parameters of the model. The three hypotheses are constructed by partitioning the nuisance-parameter space, finely discretized over the

bounded windows specified in Appendix A, into the three hypothesis regions through the priors.

The treatment of these nuisance parameters warrants comment. What is conventionally reported as the maximum-likelihood estimate (MLE) of a parameter of interest is, in fact, a profile MLE: The full likelihood is maximized over the nuisance parameters, and the remaining component is quoted. The profile likelihood is not, however, the likelihood of any sampling model, and profile estimates can fail outright—in the Neyman-Scott problem they remain inconsistent no matter how much data are collected. Marginalization avoids this. Integrating the nuisance parameters against their priors, as detailed in Appendix A, yields a genuine sampling model, $L(\mathbb{D}|k) = \int dq dx dy L(\mathbb{D}|k, q, x, y) \pi(q, x, y|k)$, and since $\text{RB}_k \propto L(\mathbb{D}|k)$, maximizing the relative-belief ratio maximizes the true likelihood. The RB estimate is thus an MLE for a well-defined model, and the plausible region $\{k : \text{RB}_k > 1\}$ is a bona fide likelihood region—a guarantee that confidence regions, whose coverage is a property of the procedure across hypothetical repetitions rather than of the region obtained, do not provide.

For the brightness imbalance, the one-source hypotheses receive uniform priors on $q \in [0, q_0] \cup [1 - q_0, 1]$, and the two-source hypothesis receives a uniform prior on the complementary interval. For source separation, the two-source hypothesis receives a uniform prior on $|d| > d_0$, while the strip $|d| \leq d_0$ at intermediate q is divided between the two one-source hypotheses at $q = 1/2$. The tolerances q_0 and d_0 favoring the one-source hypotheses are motivated by experimental as well as computational limitations common to all analyses. The prior on the centroid location is uniform over the same x_c window for all three hypotheses. The corresponding likelihoods are obtained by averaging over these priors (Appendix A). Evidence from RB analysis is then used to accept all plausible hypotheses, rather than commit to the most likely.

Averaged over bins, the $\text{RB} > 1$ criterion identifies the true illumination case in every panel of Fig. 2(a) for all q_0 scanned. Identifying the two-source hypothesis with case 3 data becomes challenging as d_0 increases, with a larger parameter space assigned to one-source hypotheses. Here, the maximum-likelihood (ML) decision is the hypothesis with the largest marginal likelihood. Resolved bin by bin in Fig. 2(b), the maximum likelihood fails in 35.74% bins, yet RB accepts the true hypothesis as plausible in all of them, preventing a wrong decision at a tolerance of $d_0 = 0.12 \mu\text{m}$. This value is not chosen to favor RB, but rather, it is where the one-source or two-source decision is most ambiguous and read off the data from the d_0 scan in Fig. 7 in Appendix A. As a representative example, consider the results for the first bin where ML is wrong but RB accepts the true hypothesis. The posterior probabilities for the “src 2” and “combined” hypotheses were 0.27 and 0.54, respectively, whereas ML incorrectly chose src 2 for this bin. However, maximizing posteriors alone can similarly lead to wrong decisions in principle, but by accepting all plausible hypotheses RB lacks such a deficiency. Moreover, Fig. 2(b) shows that RB accepts at most two of three hypotheses, so none of the bins were deemed completely ambiguous (or uninformative), even at $d_0 = 0.12 \mu\text{m}$, where the decision is most ambiguous (Appendix A).

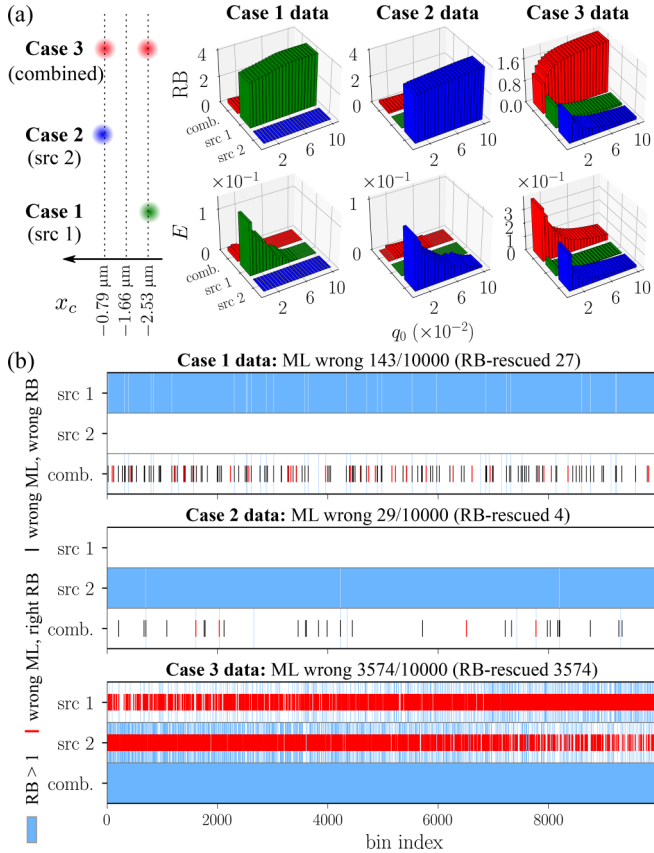


FIG. 2. (a) Two individual sources (src 1 and src 2), located at positions not aligned with the demultiplexer, illuminate the scene either individually or simultaneously (combined, with brightness ratio $q \simeq 0.5$). The respective source centroids x_c are marked, with source separation $d \simeq 1.74 \mu\text{m}$ in the combined case. RB-based source discrimination is used for all three illumination cases. The space (q, x_c, d) is partitioned by tolerances q_0 around $q \in \{0, 1\}$ and d_0 around $d = 0$: Brightness within q_0 of 0 or 1 goes to the corresponding one-source hypothesis, as does the strip $|d| \leq d_0$ (split between the two at $q = 1/2$), and the rest go to the two-source hypothesis. Integration runs over $|x_c| \leq w_0/2$ (151 points) and $|d| \leq w_0$ (0.005- μm spacing), with q marginalized in closed form (Appendix A). Rows show RB and the evidence strength E versus $q_0 \in [0.005, 0.1]$ at fixed $d_0 = 0.12 \mu\text{m}$. With priors $1/2$ (two sources) and $1/4$ (each one source), bars average RB and E over the first 100 of the 10 000 time bins taken in 1 s at 10 kHz ($\sim 8 \times 10^9$ photons per 0.1-ms bin). The Rayleigh limit is $\sim w_0$ (320 μm). (b) The same decision resolved binwise at $q_0 = 0.005$ and $d_0 = 0.12 \mu\text{m}$, for which the decision is most ambiguous (Appendix A): Blue marks every hypothesis with $\text{RB} > 1$, red (black) indicates the bins where ML chose wrongly while RB did (did not) deem the true hypothesis plausible.

As a toy example, consider a simpler problem with fixed centroid and brightness, and see how the success of RB analysis depends on separation without marginalization to quantify superresolution in this framework. For data generated by two sources separated by a distance d , the RB for the two-source hypothesis is $\text{RB}_d = L_d / [\pi(1S)L_0 + \pi(2S)L_d]$, where we assume uniform priors $\pi(1S) = \pi(2S) = 1/2$ for complete ignorance. The condition $\text{RB}_d > 1$ decides for two

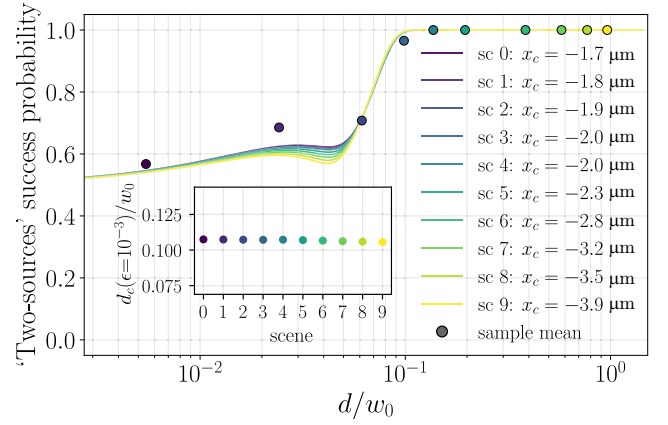


FIG. 3. Sensitivity of RB in discriminating one-source and two-source hypotheses without marginalization over the whole space. Empirical success frequency from all 10 000 time-bin samples of the experimental data is plotted (markers) for two sources separated by d across various scenes with $q \simeq 0.5$. The solid curve is the theoretical p_{succ} of Eq. (5) evaluated at the calibrated $\mathbf{b}$ and Σ . The empirical points lie near the theoretical d_c . The inset shows the per-scene d_c/w_0 at $\epsilon = 10^{-3}$ obtained by numerically inverting Eq. (5), all well below the Rayleigh limit, of the order of $w_0 \simeq 320 \mu\text{m}$. This batch of experiments used $\sim 8 \times 10^9$ photons in 0.1 ms.

sources with success rate

$$p_{\text{succ}} = \frac{1}{2} [1 + \text{erf}(\sqrt{\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b}} / (2\sqrt{2}))], \quad (5)$$

where $\mathbf{b} := \tilde{\mathbf{I}}_d - \tilde{\mathbf{I}}_0$ is the difference of the model mode-intensity vectors in Eq. (4) under the two-source and one-source hypotheses, evaluated at the same centroid x_c and brightness imbalance q , and $\text{erf}(x)$ is the error function (Appendix B). For the equal priors used here, $\text{RB}_d > 1$ reduces to the simple likelihood comparison $L_d > L_0$. The super-resolution advantage over direct imaging therefore reflects the information content of SPADE itself rather than a tuned acceptance threshold, which is illustrated by Fig. 3. What RB adds is a single decision rule, fixed once the priors and likelihood are stated, that delineates the limits of the superresolution claims.

For an upper bound $\epsilon > 0$ on the failure probability, the critical separation d_c at which $1 - p_{\text{succ}}(d_c) = \epsilon$ is obtained numerically by inverting Eq. (5) on the calibrated $\mathbf{b}(d)$ and Σ . Upon expanding $\tilde{\mathbf{I}}_1(x_c - d/2)$ and $\tilde{\mathbf{I}}_2(x_c + d/2)$ about x_c , the $O(d)$ contribution to $\mathbf{b}$ is $\mathbf{c}_1 d$, with $\mathbf{c}_1 := [-q\tilde{\mathbf{I}}'_1(x_c) + (1-q)\tilde{\mathbf{I}}'_2(x_c)]/2$, which vanishes exactly for nearly identical sources at $q \simeq 0.5$ by symmetry. If we neglect this $O(d)$ term together with $O(d^3)$ corrections for small d , d_c is given by

$$d_c \approx \left\{ \frac{8 [\text{erf}^{-1}(1 - 2\epsilon)]^2}{\mathbf{c}_2^\top \cdot \Sigma^{-1} \cdot \mathbf{c}_2} \right\}^{1/4}, \quad (6)$$

with $\mathbf{c}_2 := [q\tilde{\mathbf{I}}''_1(x_c) + (1-q)\tilde{\mathbf{I}}''_2(x_c)]/8$, the second derivatives of the calibrated mode profiles (Appendix C).

The RB analysis serves as a Bayesian postprocessing that extracts the additional information revealed by modal decomposition, providing a data-driven criterion for distinguishing two sources from one by returning the full data-supported

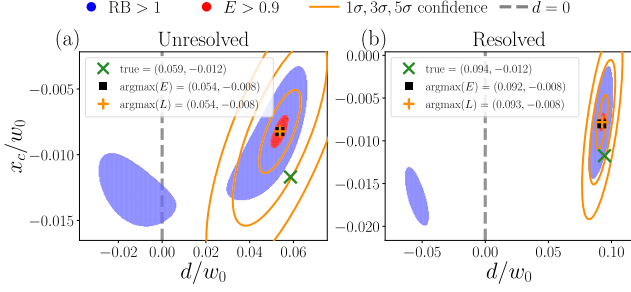


FIG. 4. Parameter estimation for equally bright sources ($q \simeq 0.5$), with (d, x_c) scanned on a 300×300 grid with uniform priors. The search space is iteratively refined from sufficiently large global bounds $|d| \leq 1.25w_0$ and $|x_c| \leq 1.25w_0/2$. The blue region is the plausible region with $\text{RB} > 1$, and the red region marks $E > 0.9$. The orange contours mark off n - σ confidence ellipses from the inverse of observed Fisher information at the maximum-likelihood point. Results are shown for a single time-bin sample for two scenes: (a) unresolved and (b) resolved, with evidence from flux as large as 5×10^{10} photons in 0.1 ms. In (a), the confidence regions can exclude $d = 0$ even though the RB plausible region includes small separations down to $d = 0$, reflecting the data-driven small-sample uncertainty, which the local quadratic curvature of the log likelihood misses.

plausible region for the source parameters rather than a maximum-likelihood point estimate, which fails in Fig. 2(b) where RB still retains the true hypothesis. This criterion surpasses the classical resolution limits imposed by the Rayleigh criterion [65,66], which states that two Gaussian objects can be resolved with direct imaging only when their separation exceeds the sum of their half-widths, which roughly scales with the beam waist w_0 here. The per-scene numerical inversion of Eq. (5) gives $d_c/w_0 \simeq 0.11$ ($d_c \simeq 35 \mu\text{m}$ at $w_0 \simeq 320 \mu\text{m}$) at $\epsilon = 10^{-3}$ (inset of Fig. 3), well below the Rayleigh limit.

V. MULTIPARAMETER ESTIMATION

The RB paradigm allows flexible assessment of hypothesis plausibility. For instance, if the scene contains two sources, we may estimate either the plausible interval of the source separation d or that of the centroid position x_c individually (single-parameter estimation), provided the other parameter is reasonably known. If neither parameter can be reliably ascertained, RB can be used to find the plausible region of both unknown parameters simultaneously (multiparameter estimation). This is achieved by scanning hypotheses over a finely divided interval (for a single parameter) or a region (for multiple parameters), with the search window iteratively refined from a broad initialization.

Figure 4 shows the distinctive feature of RB: It produces the full plausible region directly, without invoking the asymptotic behavior of hypothetical unmeasured data dictated by the likelihood model. For Gaussian likelihoods with abundant data, the maximum-evidence estimate coincides with the maximum-likelihood point, but the RB region may be wider than the high-confidence region predicted by the observed Fisher-information matrix, reflecting genuine data-driven uncertainty. Figure 5 shows that this single-bin analysis is reproducible across three batches whose photon flux drops by

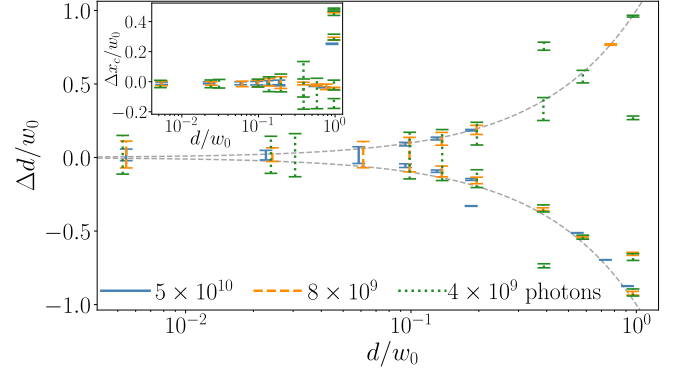


FIG. 5. Single-time-bin RB plausible regions across three batches with progressively decreasing total photon flux (5×10^{10} , 8×10^9 , 4×10^9 photons in 0.1 ms). For each of 30 measurement scenes (10 per batch), the first 100 of the 10 000 time bins acquired in 1 s at 10 kHz are analyzed independently, and the union of $\text{RB} > 1$ intervals over these bins is plotted against the true source separation for plausible separations d/w_0 and plausible centroid locations x_c/w_0 (inset). SPADE with a single MPLC produces a likelihood that has reflection symmetry about d , which gives ideal RB plausible intervals consisting of the two symmetric branches.

an order of magnitude over three batches, with the calibrated model fitting the data less tightly at lower flux.

VI. CONCLUSION

Optimal state discrimination underpins quantum information processing, with direct implications for quantum imaging. Superresolution techniques like SPADE have redefined the limits for resolving closely spaced sources, with potential impact on applications from astronomy to biological microscopy. Traditionally, when the problem is viewed as parameter estimation, the analysis has relied on quantum Fisher information and the quantum Cramér-Rao bound, particularly for evaluating the fluctuations in the estimates of the source separation. Alternatively, state discrimination has also been framed as a hypothesis-testing problem using statistical significance measures such as the p value for inference. However, both the QCRB-based and conventional hypothesis-testing approaches to statistical analysis have come under increasing scrutiny for relying on implicit and often *ad hoc* assumptions that are not dictated by the data themselves.

We introduced an evidence-based framework for statistical inference grounded in the RB paradigm to address these limitations. This approach quantifies support for a hypothesis or model parameter by comparing the posterior to the prior, providing evidence for a particular value precisely when the data raise the belief that it is the true value. In addition to identifying supported hypotheses, RB naturally ranks competing alternatives through the strength of this evidence.

In this way, RB treats both types of state-discrimination problems within a single, unified framework. Crucially, this framework remains valid even in limited-data regimes and extends seamlessly to both single- and multiparameter estimation problems, offering a promising path toward analyzing more complex scenarios, including multisource discrimination. The same machinery extends naturally to any

other source statistics, at lower photon fluxes, with photon-number-resolving detection and to other quantum-imaging scenarios.

In the binary, equal-prior case the criterion $RB > 1$ collapsed to a likelihood-ratio test for the simple analytical demonstration, but we saw that for the multihypothesis and parameter-estimation scenarios with practical models, it provides a single decision rule that does not invoke large-sample approximations and one that reports every hypothesis the data support rather than only the most likely one, which recovers the true hypothesis in all the trials where maximum likelihood errs at the most ambiguous tolerance. These features position RB as a flexible statistical-inference tool in quantum imaging and beyond.

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DATA AVAILABILITY

The data that support the findings of this article are available upon reasonable request from the authors.

APPENDIX A: LIKELIHOODS FOR THE SOURCE-DISCRIMINATION PROBLEM

Here, we explain how to evaluate the relative beliefs of two one-source hypotheses and a single two-source hypothesis of the source-discrimination problem where the brightness imbalance q , the centroid location x , and the source separation y are the nuisance parameters of the likelihood model. The integration variables x and y play the role of the centroid x_c

and separation d in the main text. We treat y as signed to account for the swap between the two source locations and integrate the likelihood density weighted by priors as

$$L_{2S} := \int_0^1 dq \int dx \int dy \pi(q, x, y) L(\mathbb{D}|q, x, y) \quad (A1)$$

for the two-source hypothesis over suitable prior ranges. In this manner, we do not assume any particular (q, x, y) triplet when using RB analysis for the two-source hypothesis. On the other hand, for the one-source hypotheses, the priors would be profiles of $\pi(q, x, y)$ at $q = 0$ (or 1) and $y = 0$, and hence,

$$L_{1S} := \int dx \frac{\pi(0(\text{or } 1), x, 0)}{\int dx' \pi(0(\text{or } 1), x', 0)} L(\mathbb{D}|q = 0(\text{or } 1), x, y = 0), \quad (A2)$$

where $q = 1$ (0) refers to the single source being src 1 (src 2), consistent with the model $\tilde{\mathbf{I}}_d(x_c) = q\tilde{\mathbf{I}}_1(x_c - d/2) + (1 - q)\tilde{\mathbf{I}}_2(x_c + d/2)$, where $\tilde{\mathbf{I}}_1$ corresponds to src 1.

If, however, we are ignorant about the specific kind of single source, we may consider the likelihood

$$L_{1S} := \int dx \left[\frac{\pi(0, x, 0)}{\mathcal{N}} L(\mathbb{D}|0, x, 0) + \frac{\pi(1, x, 0)}{\mathcal{N}} L(\mathbb{D}|1, x, 0) \right], \quad (A3)$$

where $\pi(0, x, 0)/\mathcal{N}$ and $\pi(1, x, 0)/\mathcal{N}$ are the priors assigned to the source types (src 1 or 2 here) and where the normalization is

$$\mathcal{N} := \int dx' [\pi(0, x', 0) + \pi(1, x', 0)]. \quad (A4)$$

Next, consider distinct one-source hypotheses along with tolerances q_0 on the brightness imbalance q around the one-source values 0 and 1 and d_0 on the source separation y around 0. With y signed, this partition fully covers the joint (q, y) nuisance space. Each of the brightness intervals $[0, q_0]$ and $[1 - q_0, 1]$ extends across all y , and the small- $|y|$ strip at intermediate q is split between the two one-source hypotheses at $q = 1/2$.

The three likelihoods then read

$$\begin{aligned} L_{\text{src } 1} &= \frac{1}{\mathcal{N}_{\text{src } 1}} \left[\int_{1-q_0}^1 dq \int dx \int dy \pi(q, x, y) L(\mathbb{D}|q, x, y) \right. \\ &\quad \left. + \int_{1/2}^{1-q_0} dq \int dx \int_{|y| < d_0} dy \pi(q, x, y) L(\mathbb{D}|q, x, y) \right], \\ L_{\text{src } 2} &= \frac{1}{\mathcal{N}_{\text{src } 2}} \left[\int_0^{q_0} dq \int dx \int dy \pi(q, x, y) L(\mathbb{D}|q, x, y) \right. \\ &\quad \left. + \int_{q_0}^{1/2} dq \int dx \int_{|y| < d_0} dy \pi(q, x, y) L(\mathbb{D}|q, x, y) \right], \\ L_{\text{combined}} &= \frac{1}{\mathcal{N}_{\text{combined}}} \int_{q_0}^{1-q_0} dq \int dx \int_{|y| > d_0} dy \pi(q, x, y) \\ &\quad \times L(\mathbb{D}|q, x, y), \end{aligned} \quad (A5)$$

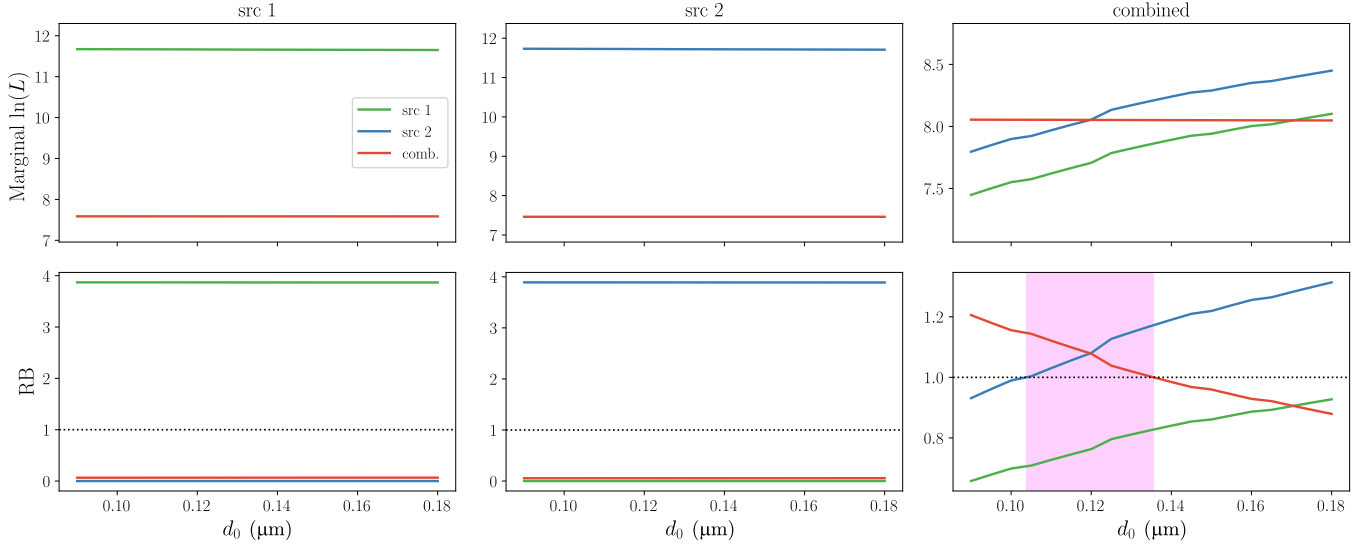


FIG. 6. Dependence of the three-hypothesis marginal likelihoods and relative beliefs on the separation tolerance d_0 for the first time bin in Fig. 2(b) of the main text. Top row: marginal log likelihoods $\ln(L)$ for the three hypotheses {src 1, src 2, combined}, evaluated from Eq. (A5) with q marginalized in closed form from Eq. (A9), $q_0 = 0.005$; centroid window $|x_c| \leq w_0/2$; and separation window $|d| \leq w_0$. Bottom row: the corresponding relative beliefs RB; the dotted line is $RB = 1$. Columns are the three illumination cases (src 1, src 2, combined). The shaded region marks the region in which RB analysis accepts a one-source hypothesis as plausible along with the combined one. The minor fluctuations in the curves may be attributed to finite resolution of the Riemann grid used for the integration, limited by the computational resources.

with the normalizations

$$\mathcal{N}_H = \int_{\text{region } H} dq dx dy \pi(q, x, y), \quad (\text{A6})$$

where $H \in \{\text{src 1, src 2, combined}\}$ so that each prior $\pi(q, x, y)/\mathcal{N}_H$ integrates to unity over its region. We set $d_0 = 0.12 \mu\text{m}$, so the small- $|y|$ strip retains finite measure. In the joint limit $q_0, d_0 \rightarrow 0$ the one-source hypotheses recover the Dirac- δ priors at $q \in \{0, 1\}$, $y = 0$.

We evaluate these integrals with the brightness fraction q marginalized in closed form (see below) and the remaining nuisance parameters summed on uniform grids: $|x| \leq w_0/2$ with 151 points and the signed separation y with a spacing of $0.005 \mu\text{m}$ over a fixed window $|y| \leq w_0$, equivalent to confining each source to $|x \pm y/2| \leq w_0$. The two spacings differ by design: The boundary between the hypotheses lies at $|y| = d_0$, so the y grid must resolve d_0 , whereas x is integrated over the same window with the same prior in all three hypotheses, so its discretization enters each likelihood alike. We have verified that the reported relative beliefs are converged in the number of x points.

The q integration is carried out in closed form, which lets us reach dense (x, y) resolution at low cost. As the model is linear in q ,

$$\tilde{\mathbf{I}}_y(x) = q \tilde{\Delta}(x, y) + \tilde{\mathbf{I}}_2(x + y/2), \quad (\text{A7})$$

where $\tilde{\Delta}(x, y) := \tilde{\mathbf{I}}_1(x - y/2) - \tilde{\mathbf{I}}_2(x + y/2)$, the exponent of the Gaussian likelihood is quadratic in q . With the observed intensity column $\mathbf{I}(\mathbb{D})$, we set

$$\Delta := \mathbf{I} - \tilde{\mathbf{I}}_2(x + y/2) \quad (\text{A8})$$

and define $A := \Delta^\top \Sigma^{-1} \Delta$, $B := \Delta^\top \Sigma^{-1} \tilde{\Delta}$, and $C = \tilde{\Delta}^\top \Sigma^{-1} \tilde{\Delta}$, so that the integral over any q interval $[a, b]$

evaluates to

$$\int_a^b dq L(\mathbb{D}|q, x, y) = \frac{e^{B^2/(2C)} - A/2}{2\sqrt{(2\pi)^{M-1} C \text{Det}\{\Sigma\}}} \times \left[\text{erf}\left(\frac{b-B/C}{\sqrt{2/C}}\right) - \text{erf}\left(\frac{a-B/C}{\sqrt{2/C}}\right) \right]. \quad (\text{A9})$$

Then the relative beliefs of the three hypotheses using the marginal likelihoods are

$$\begin{aligned} \text{RB}_{\text{src 1}} &= \frac{L_{\text{src 1}}}{\pi(\text{src 1})L_{\text{src 1}} + \pi(\text{src 2})L_{\text{src 2}} + \pi(\text{combined})L_{\text{combined}}}, \\ \text{RB}_{\text{src 2}} &= \frac{L_{\text{src 2}}}{\pi(\text{src 1})L_{\text{src 1}} + \pi(\text{src 2})L_{\text{src 2}} + \pi(\text{combined})L_{\text{combined}}}, \\ \text{RB}_{\text{combined}} &= \frac{L_{\text{combined}}}{\pi(\text{src 1})L_{\text{src 1}} + \pi(\text{src 2})L_{\text{src 2}} + \pi(\text{combined})L_{\text{combined}}}. \end{aligned} \quad (\text{A10})$$

We adopt the discrete hypothesis priors $\pi(\text{src 1}) = \pi(\text{src 2}) = 1/4$ and $\pi(\text{combined}) = 1/2$, weighting one source and two sources equally as the binary alternative and splitting the one-source weight evenly between the two one-source labels.

Figure 6 illustrates this three-hypothesis analysis for the first time bin in Fig. 2(b) of the main text for each of the three illumination cases (only src 1 on, only src 2 on, and

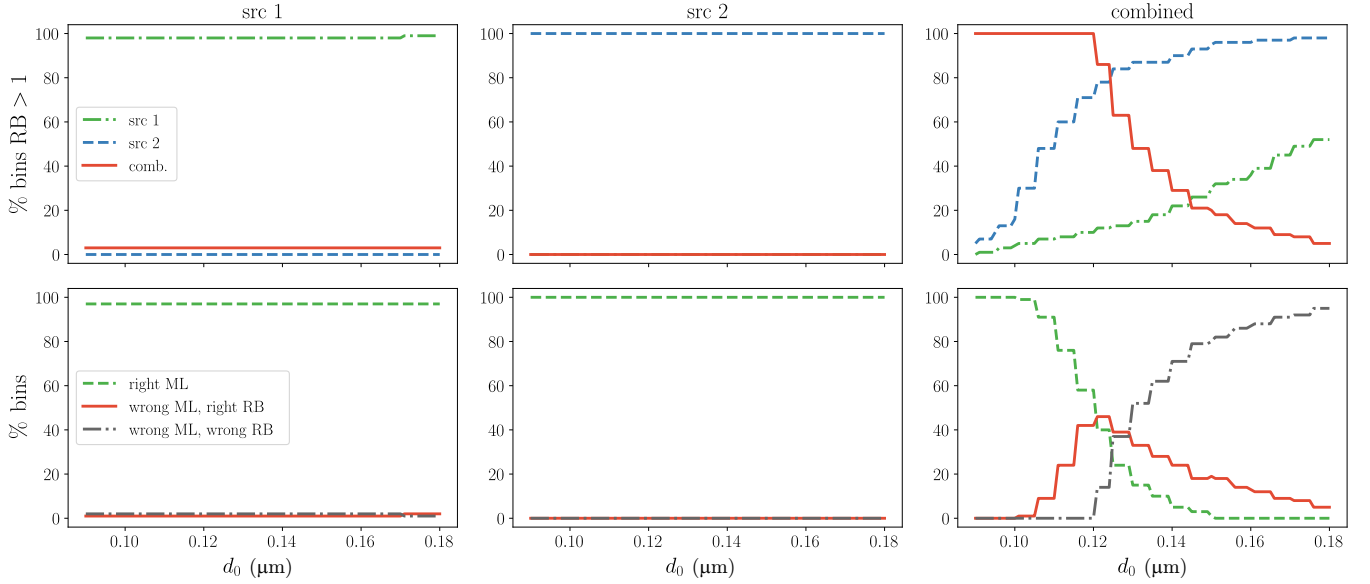


FIG. 7. Aggregate maximum-likelihood and RB decision outcomes as a function of the separation tolerance d_0 over the first 100 time bins in Fig. 2(b) of the main text, complementing the single-bin view in Fig. 6. As before, q is marginalized in closed form from Eq. (A9), with $q_0 = 0.005$, centroid window $|x_c| \leq w_0/2$, and separation window $|d| \leq w_0$. Columns are the three illumination cases (src 1, src 2, combined). Top row: the percentage of bins in which each hypothesis is deemed plausible ($RB > 1$) for src 1 (green dash-dotted line), src 2 (blue dashed line), and the combined case (red solid line). Bottom row: the percentage of bins in each decision category, namely, a correct ML choice (green dashed line), a wrong ML choice for which RB still retains the true hypothesis as plausible (red solid line), and a wrong ML choice for which RB also fails to retain the truth (gray dash-dotted line). For the combined case the accepted hypotheses and the decision outcomes shift markedly near the operating point $d_0 = 0.12 \mu\text{m}$, where the combined hypothesis ceases to be accepted in most bins and the fraction of bins with a wrong ML choice but a plausible RB verdict is largest. Thus, for tolerances motivated by our priors around this value, the benefit of RB over ML is prominent.

both combined). Figure 6 shows how the marginal likelihoods and relative beliefs vary with the prior choice of tolerance d_0 assigned in favor of the one-source hypotheses. Figure 7 aggregates this analysis over the first 100 time bins, showing the fraction of bins in which each hypothesis is accepted and the resulting ML and RB decision outcomes as a function of d_0 .

APPENDIX B: SUCCESS PROBABILITY OF SOURCE DISCRIMINATION

This Appendix analyzes a toy problem of source discrimination in which two hypotheses are tested: a single one-source alternative and a single two-source alternative at a particular source separation d . We are interested here in understanding the sensitivity of the RB analysis in correctly identifying this two-source hypothesis.

The relative belief of two sources separated by a distance d is

$$RB_d = \frac{L_d}{\pi(1S)L_0 + \pi(2S)L_d}, \quad (\text{B1})$$

where $\pi(1S)$ and $\pi(2S)$ are the prior probabilities assigned to the one-source and two-source hypotheses and

$$L_d = \pi(\mathbf{I}|\tilde{\mathbf{I}}_d) \quad (\text{B2})$$

is the joint likelihood density of observing the intensities $\mathbf{I} = (I^{(0)} I^{(1)} \dots I^{(M-1)})^\top$ in M measured modes $j = 0, 1, 2, \dots, M-1$, with $\tilde{\mathbf{I}}_d$ being the model intensities. In the experiment, the number of modes is $M = 4$.

The model intensities are defined as the mixture

$$\tilde{\mathbf{I}}_d := q\tilde{\mathbf{I}}_1(x_c - d/2) + (1 - q)\tilde{\mathbf{I}}_2(x_c + d/2), \quad (\text{B3})$$

where $\tilde{\mathbf{I}}_1(x)$ and $\tilde{\mathbf{I}}_2(x)$ are the calibrated intensities as a function of position x for the two different sources of interest.

The conditional probability densities $\pi(\mathbf{I}|\tilde{\mathbf{I}}_d)$ appearing in the likelihood density are taken to be Gaussian,

$$\pi(\mathbf{I}|\tilde{\mathbf{I}}_d) = \exp\left[-\frac{1}{2}(\mathbf{I} - \tilde{\mathbf{I}}_d)^\top \cdot \boldsymbol{\Sigma}^{-1} \cdot (\mathbf{I} - \tilde{\mathbf{I}}_d)\right] / \sqrt{(2\pi)^M \text{Det}\{\boldsymbol{\Sigma}\}}, \quad (\text{B4})$$

with the covariance matrix $\boldsymbol{\Sigma}$ and $\text{Det}\{\boldsymbol{\Sigma}\}$ denoting its determinant.

The success probability of correctly identifying a two-source alternative using relative belief is given by

$$p_{\text{succ}} = \int (d\mathbf{I}) L_d \Theta(RB_d - 1), \quad (\text{B5})$$

wherein we substitute the expression from above.

The prior probabilities $\pi(1S) = 1/2 - \delta$ and $\pi(2S) = 1/2 + \delta$ for $-1/2 < \delta < 1/2$ give relative beliefs

$$RB_d = \frac{2L_d}{L_0 + L_d + 2\delta(L_d - L_0)}. \quad (\text{B6})$$

The condition $RB_d > 1$ simplifies as follows to a likelihood-ratio test independent of the priors:

$$\begin{aligned} 2L_d &> L_0 + L_d + 2\delta(L_d - L_0) \\ &\Rightarrow (1 - 2\delta)L_d > (1 - 2\delta)L_0 \\ &\Rightarrow L_d > L_0. \end{aligned} \quad (\text{B7})$$

Substituting the likelihood density from Eq. (B2) and the conditional probability densities from Eq. (B4), we rewrite the condition as

$$\begin{aligned} e^{-\frac{1}{2}(\mathbf{I}-\tilde{\mathbf{I}}_d)^\top \cdot \Sigma^{-1} \cdot (\mathbf{I}-\tilde{\mathbf{I}}_d)} &> e^{-\frac{1}{2}(\mathbf{I}-\tilde{\mathbf{I}}_0)^\top \cdot \Sigma^{-1} \cdot (\mathbf{I}-\tilde{\mathbf{I}}_0)} \\ (\mathbf{I}-\tilde{\mathbf{I}}_d)^\top \cdot \Sigma^{-1} \cdot (\mathbf{I}-\tilde{\mathbf{I}}_d) &< (\mathbf{I}-\tilde{\mathbf{I}}_0)^\top \cdot \Sigma^{-1} \cdot (\mathbf{I}-\tilde{\mathbf{I}}_0) \\ \Rightarrow \mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{X} + \frac{1}{2}\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b} &> 0, \end{aligned} \quad (\text{B8})$$

where we defined $\mathbf{b} := \tilde{\mathbf{I}}_d - \tilde{\mathbf{I}}_0$ and a new random variable, $\mathbf{X} := \mathbf{I} - \tilde{\mathbf{I}}_d$.

The success probability from Eq. (B5) can therefore be rewritten as

$$\begin{aligned} p_{\text{succ}} &= \frac{1}{\sqrt{(2\pi)^M \text{Det}\{\Sigma\}}} \int (d\mathbf{X}) e^{-\frac{1}{2}\mathbf{X}^\top \cdot \Sigma^{-1} \cdot \mathbf{X}} \\ &\quad \times \Theta(\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{X} + \frac{1}{2}\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b}/2). \end{aligned} \quad (\text{B9})$$

Now, using the integral representation of the Heaviside step function,

$$\Theta(x) = \lim_{\epsilon \rightarrow 0^+} \int \frac{d\tau}{2\pi i} \frac{e^{ix\tau}}{\tau - i\epsilon}, \quad (\text{B10})$$

we obtain

$$p_{\text{succ}} = \lim_{\epsilon \rightarrow 0^+} \int \frac{d\tau}{2\pi i(\tau - i\epsilon)} \frac{1}{\sqrt{(2\pi)^M \text{Det}\{\Sigma\}}} \int (d\mathbf{X}) e^{-\frac{1}{2}\mathbf{X}^\top \cdot \Sigma^{-1} \cdot \mathbf{X} + i\tau(\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{X} + \frac{1}{2}\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b})}. \quad (\text{B11})$$

The usual multivariable Gaussian integral identity

$$\frac{1}{\sqrt{(2\pi)^M \text{Det}\{\Sigma\}}} \int (d\mathbf{x}) e^{-\frac{1}{2}\mathbf{x}^\top \cdot \Sigma^{-1} \cdot \mathbf{x} + \mathbf{a}^\top \cdot \mathbf{x}} = e^{\frac{1}{2}\mathbf{a}^\top \cdot \Sigma \cdot \mathbf{a}} \quad (\text{B12})$$

gives

$$p_{\text{succ}} = \lim_{\epsilon \rightarrow 0^+} \int \frac{d\tau}{2\pi i(\tau - i\epsilon)} e^{-\frac{\tau^2}{2}\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b} + \frac{i\tau}{2}\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b}}. \quad (\text{B13})$$

The above equation can be further simplified by using the identity

$$\frac{1}{i(\tau - i\epsilon)} = \int_0^\infty dy e^{-iy(\tau - i\epsilon)} \quad (\text{B14})$$

for $\epsilon > 0$, which leads to

$$\begin{aligned} p_{\text{succ}} &= \lim_{\epsilon \rightarrow 0^+} \int_0^\infty dy e^{-\epsilon y} \int \frac{d\tau}{2\pi} e^{-\frac{\tau^2}{2}\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b} - i\tau(y - \mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b}/2)} \\ &= \frac{1}{\sqrt{2\pi\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b}}} \int_0^\infty dy e^{-\frac{(y - \mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b}/2)^2}{2\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b}}}, \end{aligned} \quad (\text{B15})$$

where the Gaussian integral identity from Eq. (B12) was used again.

Now using the definite integral

$$\frac{1}{\sqrt{2\pi}\sigma} \int_a^b dx e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{2} \text{erf}\left(\frac{b-\mu}{\sqrt{2}\sigma}\right) - \frac{1}{2} \text{erf}\left(\frac{a-\mu}{\sqrt{2}\sigma}\right), \quad (\text{B16})$$

we finally get

$$p_{\text{succ}} = \frac{1}{2} + \frac{1}{2} \text{erf}\left(\frac{1}{2} \sqrt{\frac{\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b}}{2}}\right). \quad (\text{B17})$$

Equation (B17) is evaluated at known centroid x_c , brightness imbalance q , and signed separation d , all three treated as scene-fixed reference values. In the empirical test of the success probability of identifying the two-source hypothesis in the main text, we estimate the probability from the same window of bins that the test consumes, namely, $x_c = (\langle x_2 + x_1 \rangle / 2)$, $q = \langle P_1 \rangle / (\langle P_1 \rangle + \langle P_2 \rangle)$, and $d = \langle x_2 - x_1 \rangle$, where the brackets denote the time average over the bins of one scene, x are the positions, and P are the measured power, with

the subscripts identifying the sources. The signed convention $d = x_2 - x_1$ removes any ambiguity in $\tilde{\mathbf{I}}_d$ given nonidentical source profiles, so the binary test in Eq. (B17) compares L_d at the known sign of d to L_0 without marginalization.

APPENDIX C: CRITICAL DISTANCE OF SOURCE DISCRIMINATION OF NEARLY INDISTINGUISHABLE GAUSSIAN OBJECTS

Suppose we upper bound the tolerance $\epsilon > 0$ on the failure probability. Then

$$\begin{aligned} 1 - p_{\text{succ}} &\leq \epsilon \\ \Rightarrow \frac{1}{2} - \frac{1}{2} \text{erf}\left(\frac{1}{2} \sqrt{\frac{\mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b}}{2}}\right) &\leq \epsilon \\ \Rightarrow \mathbf{b}^\top \cdot \Sigma^{-1} \cdot \mathbf{b} &\geq 8[\text{erf}^{-1}(1 - 2\epsilon)]^2. \end{aligned} \quad (\text{C1})$$

Recall that

$$\begin{aligned} \mathbf{b} = \tilde{\mathbf{I}}_d - \tilde{\mathbf{I}}_0 &= q[\tilde{\mathbf{I}}_1(x_c - d/2) - \tilde{\mathbf{I}}_1(x_c)] \\ &\quad + (1 - q)[\tilde{\mathbf{I}}_2(x_c + d/2) - \tilde{\mathbf{I}}_2(x_c)]. \end{aligned} \quad (\text{C2})$$

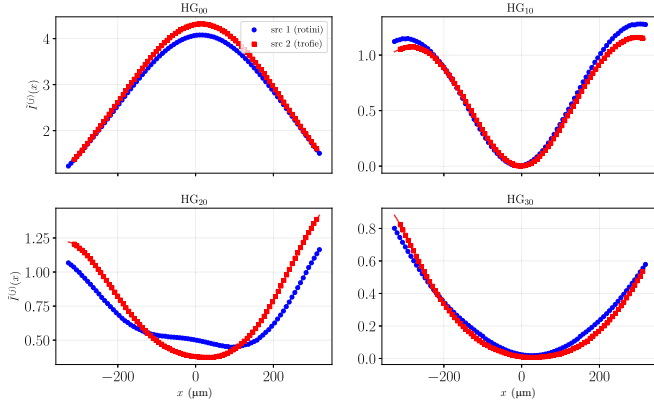


FIG. 8. Calibrated HG mode-intensity profiles $\tilde{I}^{(j)}(x)$ for modes $j = 0, 1, 2, 3$ as a function of source position x (in μm) for src 1 (rotini, blue circles) and src 2 (trofie, red squares). Curves are degree-10 polynomial fits used in all the likelihood evaluations.

Taylor's series of the calibration curves $\tilde{I}_1(x_c - d/2)$ and $\tilde{I}_2(x_c + d/2)$ about the point x_c gives

$$\begin{aligned} \mathbf{b} &= \sum_{n=1}^{\infty} \frac{1}{n!} \left[q \left(-\frac{d}{2} \right)^n \frac{d^n}{dx^n} \tilde{I}_1(x_c) + (1-q) \left(\frac{d}{2} \right)^n \frac{d^n}{dx^n} \tilde{I}_2(x_c) \right], \\ &\cong d\mathbf{c}_1 + d^2\mathbf{c}_2 + O(d^3), \end{aligned} \quad (\text{C3})$$

where $\mathbf{c}_1 := [-q\tilde{I}'_1(x_c) + (1-q)\tilde{I}'_2(x_c)]/2$ and $\mathbf{c}_2 := [q\tilde{I}''_1(x_c) + (1-q)\tilde{I}''_2(x_c)]/8$.

We neglect $O(d^3)$ terms for further analysis, assuming d is small, and thus Eq. (C1) becomes

$$\begin{aligned} (d^2\mathbf{c}_1^\top \cdot \Sigma^{-1} \cdot \mathbf{c}_1 + 2d^3\mathbf{c}_1^\top \cdot \Sigma^{-1} \cdot \mathbf{c}_2 + d^4\mathbf{c}_2^\top \cdot \Sigma^{-1} \cdot \mathbf{c}_2) \\ \gtrsim 8[\text{erf}^{-1}(1-2\epsilon)]^2. \end{aligned} \quad (\text{C4})$$

Note that if the calibrations for the two sources have identical profiles and if the brightness imbalance $q = 1/2$, then $\mathbf{c}_1 \rightarrow 0$, in which case we will have

$$d_c \approx \left\{ \frac{8[\text{erf}^{-1}(1-2\epsilon)]^2}{\mathbf{c}_2^\top \cdot \Sigma^{-1} \cdot \mathbf{c}_2} \right\}^{\frac{1}{4}}. \quad (\text{C5})$$

APPENDIX D: CALIBRATED HG MODE INTENSITY PROFILES

Figure 8 shows the four calibrated mode-intensity profiles $\tilde{I}^{(j)}(x)$ for $j = 0, 1, 2, 3$ as a function of the source position x for src 1 (rotini, blue circles) and src 2 (trofie, red squares). The points are mean voltage values from 1-s averaging windows at 10-kHz sampling, normalized by the simultaneously measured source power. The curves are degree-10 polynomial fits used in all likelihood evaluations through $\tilde{I}_1(x)$ and $\tilde{I}_2(x)$ in Eq. (A5) and the model in the main text.

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