Erratum: Classical simulation of non-Gaussian bosonic circuits [Phys. Rev. A 110, 042402 (2024)]

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This document lists a number of corrections to the paper titled "Classical simulation of non-Gaussian bosonic circuits" published in Phys. Rev. A **110**, 042402 (2024). These corrections do not invalidate the main conclusions: all results are valid with the present modifications. The most significant update concerns the proof of Lemma IV.2 and consequently the runtime of the algorithm $\mathsf{fastnorm}_E$ which improves from $O(\chi n^3 E p_f^{-1}\varepsilon^{-3})$ to $O(\chi E p_f^{-1}\varepsilon^{-3})$. Importantly, the runtime remains linear in χ (which is related to the Gaussian rank) and polynomial in the remaining relevant parameters.

Text that appeared in the original version and is to be removed is shown in red with the strikethrough, while the corrected or newly added material is shown in blue. Corrections are presented in the order they appear in the article.

We thank Dhruva Sambrani and Matas Garbaravičius for drawing our attention to most of these corrections. The generalization of the proof of Lemma 4.2 to n modes presented in Item (17) is due to Matas Garbaravičius.

Corrections:

(1) On page 2, the squeezing operator $S(z)$ is defined for any real parameter $z \in \mathbb{R}$ instead of $z \geqslant 0$. Corrected version:

> (iii) a single-mode squeezing operator $S_j(z)$, $z \in \mathbb{R}$ applied to the jth mode, for $j \in [n]$, or

(2) On page 3, Theorem II.2 has a new runtime due to considering the possibly negative squeezing parameters. Specifically, both the upper bound on the mean photon number and the runtime depend on $e^{2|z_{\text{tot}}|}$ instead of $e^{2z_{\text{tot}}}$. Corrected version:

> We note that this quantity provides the upper bound $Ne^{2|z_{\text{tot}}|}$ on the mean photon number of the final state $|\Psi_{\text{final}}\rangle := U^{(s)}\cdots U^{(1)}|\Psi\rangle$ (...)
>
> It has a runtime bounded by $O[s\chi n^3 N e^{2|z_{\text{tot}}|}/(\varepsilon^3 p_f)]$.

(3) On page 5 the indices in the expression $[R_j, R_k] = i\Omega_{k,\ell} I$ are corrected to

$$[R_j, R_k] = i\Omega_{j,k} I.$$

(4) On page 5, Eq. (10) is corrected to match the field operator ordering used throughout the paper, changing from $R = (Q_1, \ldots, P_1, \ldots)$ to $R = (Q_1, P_1, \ldots)$. Corrected version:

> According to Williamson's theorem [38] (see also [39, Appendix 6]), any symmetric matrix $\Gamma \in \mathsf{Mat}_{2n\times 2n}(\mathbb{R})$ can be written as (see, e.g., [40, Theorem 8.11])
>
> $$\Gamma = S\left(\bigoplus_{j=1}^{n}\begin{pmatrix} d_j & 0 \\ 0 & d_j \end{pmatrix}\right)S^T,$$
>
> $d_j \in \mathbb{R}$ for $j \in [n]$ and $S \in Sp(2n)$ where $Sp(2n)$ denotes the group of $2n \times 2n$ real symplectic matrices, i.e., S satisfies $S\Omega S^T = \Omega$.

(5) In Table I on page 6, the matrices are corrected to

$$\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix},$$

$$\begin{pmatrix} \cos\omega & 0 & 0 & \sin\omega \\ 0 & \cos\omega & -\sin\omega & 0 \\ 0 & \sin\omega & \cos\omega & 0 \\ -\sin\omega & 0 & 0 & \cos\omega \end{pmatrix},$$

$$\begin{pmatrix} e^{z} & 0 \\ 0 & e^{-z} \end{pmatrix}.$$

(6) On page 6, Eq. (14) includes a displacement by $s \in \mathbb{R}^{2n}$ previously missing. Corrected version:

> Gaussian unitary operators $U : \mathcal{B}(\mathcal{H}_n) \to \mathcal{B}(\mathcal{H}_n)$ map field operators R_j, $j \in [2n]$ to affine transformations thereof according to
>
> $$UR_jU^\dagger = \left(\sum_{k=1}^{2n} S_{j,k}R_k\right) + s_j \quad \text{with} \quad S \in Sp(2n) \text{ and } s \in \mathbb{R}^{2n}.$$

(7) In Item (iii) on page 6, the squeezing operator $S(z)$ is defined for any real parameter $z \in \mathbb{R}$, instead of $z \geqslant 1$. Corrected version:

> (iii) single-mode squeezing operators $S_j(z)$, $z \in \mathbb{R}$ applied to a mode $j \in [n]$;

(8) On page 7, in Eq. (19) the phase $\exp[i\mathrm{Im}(\alpha^T\overline{\beta})]$ is a multiplicative phase factor that should not be part of the definition of the coherent state $|\alpha'\rangle$. Corrected version:

> More precisely, for $\beta \in \mathbb{C}^n$, we have
>
> $$D(\beta)|\alpha\rangle = \exp[i\mathrm{Im}(\alpha^T\overline{\beta})]|\alpha - \beta\rangle$$
>
> for all $\alpha \in \mathbb{C}^n$.

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(9) On page 7, there is a factor of $1/\pi^k$ missing. Corrected version:

$$\rho'_{AB} = \frac{1}{\pi^k p(\alpha)} \Pi_\alpha \rho_{AB} \Pi_\alpha$$

(10) Equation (24) on page 7, the covariance matrix Γ_M should be the identity matrix I. Corrected version:

$$d' = [\hat{d}(\alpha),\, s_B + \Gamma_{AB}^T (\Gamma_A + I)^{-1} (\hat{d}(\alpha) - s_A)]$$

(11) On page 9, d'_1 and d'_2 appear in the statement of Lemma 3.4, instead of d_1 and d_2. Corrected version:

$$\langle \psi_3, D(\alpha)\psi_1 \rangle \langle \psi_1, \psi_2 \rangle \langle \psi_2, \psi_3 \rangle = \frac{\exp\left(-{d'_1}^T W_1 d'_1 - {d'_2}^T W_2 d'_2 + {d'_1}^T W_3 d'_2 - \hat{d}(\alpha)^T \Omega^T \Gamma_5 \Omega \hat{d}(\alpha) - i\hat{d}(\alpha)^T \Omega^T (W_4 d'_1 + W_5 d'_2 + d_3)\right)}{\sqrt{\det[\Omega^T(\Gamma_2 + \Gamma_3)\Omega/2] \det[\Omega^T(\Gamma_1 + \Gamma_4)\Omega/2]}}$$

(12) On page 10, the squeezing parameter z may take negative values. Corrected version:

> (3) $(z, j) \in \mathbb{R} \times [n]$ is a description of the single-mode squeezing operator $S_j(z)$.
> Because of this, we build an algorithmic subroutine called squeezing which, given the description $\Delta \in \mathsf{Desc}_n$ of a Gaussian state $|\psi\rangle$, a parameter $z \in \mathbb{R}$ (…)

(13) On page 11, in the input to the algorithm squeezing the parameter z may take negative values. Corrected version:

ALGORITHM. squeezing.

Input: Δ, z, j
where $\begin{cases} \Delta = (\Gamma, \alpha, r) \in \mathsf{Desc}_n, \\ z \in \mathbb{R}, \\ j \in [n]. \end{cases}$
Output: $(\Gamma', \alpha', r') \in \mathsf{Desc}_n$

$S \leftarrow S$ given in the "single-mode squeezing" row of Table I.
$\Gamma' \leftarrow S\Gamma S^T$ ▷ covariance matrix of $S_j(z)|\psi\rangle$
$\Gamma'' \leftarrow SS^T$ ▷ covariance matrix of $S_j(z)|\alpha\rangle$
$\alpha' \leftarrow \alpha$
$\alpha'_j \leftarrow \alpha_j \cosh(z) - \overline{\alpha_j} \sinh(z)$
$d_1, d_2, d_3 \leftarrow \hat{d}(\alpha')$
▷ displacement vectors of $S_j(z)|\alpha\rangle$, $|\alpha'\rangle$, $S_j(z)|\psi\rangle$
$(\Gamma_1, \Gamma_2, \Gamma_3) \leftarrow (\Gamma'', I, \Gamma')$
▷ covariance matrices of $(S_j(z)|\alpha\rangle, |\alpha\rangle, S_j(z)|\psi\rangle)$
$u \leftarrow \overline{r}$ ▷ compute $\langle S_j(z)\psi, S_j(z)\alpha \rangle$
$v \leftarrow 1/\sqrt{\cosh(z)}$ ▷ compute $\langle S_j(z)\alpha, \alpha' \rangle$
$r' \leftarrow$ overlaptriple$(\Gamma_1, \Gamma_2, \Gamma_3, d_1, d_2, d_3, u, v, 0)$
▷ compute $\langle \alpha', S_j(z)\psi \rangle$
return (Γ', α', r')

(14) On page 11, in Lemma III.6 and its proof the squeezing parameter z may take negative values. Corrected version:

Lemma 0.1. The algorithm squeezing : $\mathsf{Desc}_n \times \mathbb{R} \times [n] \to \mathsf{Desc}_n$ runs in time $O(n^3)$. The algorithm is such that

$$|\psi[\mathsf{squeezing}(\Delta, z, j)]\rangle = S_j(z)|\psi(\Delta)\rangle$$

for all $\Delta \in \mathsf{Desc}_n$, $z \in \mathbb{R}$ and $j \in [n]$. That is, the output of the algorithm is a description of the state $S_j(z)|\psi(\Delta)\rangle$.

Let us show that the tuple Δ' is the description of the evolved state $S_j(z)|\psi(\Delta)\rangle$ for any $\Delta \in \mathsf{Desc}_n$, $z \in \mathbb{R}$, and $j \in [n]$.

(15) In the proof of Lemma III.6 on page 11, we add "and covariance matrix SS^T" in the following sentence:

> The state $S_j(z)|\alpha\rangle$ has displacement vector $\hat{d}(\alpha)$ and covariance matrix SS^T.

(16) The algorithm applyunitary on page 12 is updated as a consequence of Item (8). Lines 8 and 21 in the published version are modified to match the Lines 8 and 24 below, respectively. Furthermore, we add the Lines 9, 15, and 21 below.

ALGORITHM. applyunitary.

Input: Δ, Δ_U, ℓ
where $\begin{cases} \Delta = (\Gamma, \alpha, r) \in \mathsf{Desc}_n, \\ \Delta_U \text{is a description of a Gaussian unitary}, \\ \ell \in \{\mathsf{displacement}, \mathsf{phaseshift}, \mathsf{beamsplitter}, \mathsf{squeezing}\} \end{cases}$
Output: A description of a pure Gaussian state

1: **If** $\ell =$ squeezing
2: $(z, j) \leftarrow \Delta_U$ ▷ $\Delta_U = (z, j)$ is a description of $U = S_j(z)$
3: **return** squeezing(Δ, z, j)
4: **end if**
5: **If** $\ell =$ displacement **then**
6: $\beta \leftarrow \Delta_U$ ▷ $\Delta_U = \beta$ is a description of $U = D(\beta)$
7: $S \leftarrow I$
8: $\alpha' \leftarrow \alpha - \beta$
9: $r' \leftarrow \exp[i\mathrm{Im}(\alpha^T \overline{\beta})]r$
10: **end if**
11: **If** $\ell =$ phaseshift **then**
12: $(\phi, j) \leftarrow \Delta_U$ ▷ $\Delta_U = (\phi, j)$ is a description of $U = F_j(\phi)$
13: $S \leftarrow S$ given in the "phase shift" row of Table I
14: $\alpha' \leftarrow$ evaluate Eq. (20)
15: $r' \leftarrow r$
16: **end if**
17: **If** $\ell =$ beamsplitter **then**
18: $(\omega, j, k) \leftarrow \Delta_U$ ▷ $\Delta_U = (\omega, j, k)$ is a description of $U = B_{j,k}(\omega)$
19: $S \leftarrow S$ given in the "beamsplitter" row of Table I
20: $\alpha' \leftarrow$ evaluate Eq. (21)
21: $r' \leftarrow r$
22: **end if**
23: $\Gamma' \leftarrow S\Gamma S^T$ ▷ covariance matrix of $U|\psi\rangle$
24: **return** (Γ', α', r')

(12) The proof of Theorem III.7 on page 13 is updated as a consequence of Item (8). Corrected version:

> The vector $\alpha' \in \mathbb{C}^n$ computed in Lines 8, 14, and 20 labels the coherent state $|\alpha'\rangle$ in
>
> $$U|\alpha\rangle = e^{i\delta}|\alpha'\rangle,$$
>
> with $\delta = 0$ for U a phase shift or a beam splitter and $\delta = \mathrm{Im}(\alpha^T\overline{\beta})$ for U the displacement operator $D(\beta)$.
> (...)
>
> Finally, we set
>
> $$\begin{aligned} r' &= r \\ &= \langle \alpha, \psi(\Delta)\rangle \\ &= \langle U\alpha, U\psi(\Delta)\rangle \\ &= \langle \alpha', U\psi(\Delta)\rangle \end{aligned}$$
>
> for U a phase shift or a beam splitter. For U the displacement operator $D(\beta)$ we set instead
>
> $$\begin{aligned} r' &= \exp[i\mathrm{Im}(\alpha^T\overline{\beta})]r \\ &= \exp[i\mathrm{Im}(\alpha^T\overline{\beta})]\langle \alpha, \psi(\Delta)\rangle \\ &= \exp[i\mathrm{Im}(\alpha^T\overline{\beta})]\langle U\alpha, U\psi(\Delta)\rangle \\ &= \langle \alpha', U\psi(\Delta)\rangle, \end{aligned}$$
>
> where, in the last equality, we use
>
> $$\begin{aligned} U|\alpha\rangle = D(\beta)|\alpha\rangle &= \exp[i\mathrm{Im}(\alpha^T\overline{\beta})]|\alpha - \beta\rangle \\ &= \exp[i\mathrm{Im}(\alpha^T\overline{\beta})]|\alpha'\rangle \end{aligned}$$
>
> by Eq. (19) and Line 9. Observe this leads Δ' to satisfy Item (1) in Definition III.1.

The updates in Items (13) to (16) are a consequence of Item (9).

(13) In Eq. (41) on page 13 there is a factor of $1/\sqrt{\pi^k}$ missing. Corrected version:

$$|\psi'\rangle = \frac{1}{\sqrt{\pi^k p(\beta)}}\Pi_\beta|\psi\rangle$$

(14) On Line 10 in Algorithm postmeasure on page 13, there is a factor of $1/\sqrt{\pi^k}$ missing. Corrected version:

ALGORITHM. postmeasure.

Input: Δ, β

where $\begin{cases} \Delta = \left(\Gamma = \begin{pmatrix} \Gamma_A & \Gamma_{AB} \\ \Gamma_{AB}^T & \Gamma_B \end{pmatrix}, \alpha = (\alpha_A, \alpha_B), r\right) \in \mathsf{Desc}_n \\ \beta \in \mathbb{C}^k \end{cases}$

Output: $(\Gamma', \alpha', r') \in \mathsf{Desc}_n$

1: $(s_A, s_B) \leftarrow [\hat{d}(\alpha_A), \hat{d}(\alpha_B)]$
2: $\Gamma' \leftarrow$ evaluate Eq. (23)
3: $s' \leftarrow$ evaluate Eq. (24)
4: $\alpha' \leftarrow \hat{d}^{-1}(s')$
5: $(\Gamma_1, \Gamma_2, \Gamma_3) \leftarrow (\Gamma, \Gamma', I)$
▷ covariance matrices of $[|\psi(\Delta)\rangle, |\psi'\rangle, |\alpha'\rangle]$
6: $(d_1, d_2, d_3) \leftarrow (s, s', s')$
▷ displacements of $[|\psi(\Delta)\rangle, |\psi'\rangle, |\alpha'\rangle]$
7: $\lambda \leftarrow \alpha - \alpha'$
8: $u \leftarrow \exp[i\mathrm{Im}(\alpha'^T\overline{\alpha})]r$
9: $p \leftarrow \mathsf{prob}(\Delta, \beta)$
10: $v \leftarrow \sqrt{\pi^k p}$
11: $\overline{r'} \leftarrow \mathsf{overlaptriple}(\Gamma_1, \Gamma_2, \Gamma_3, d_1, d_2, d_3, u, v, \lambda)$
▷ compute $\langle \alpha', \psi'\rangle$
12: **return** (Γ', α', r')

(15) In Theorem III.8 on page 13, there is a factor of $1/\sqrt{\pi^k}$ missing. Corrected version:

$$|\psi[\mathsf{postmeasure}(\Delta, \beta)]\rangle = \frac{1}{\sqrt{\pi^k p(\beta)}}\Pi_\beta|\psi(\Delta)\rangle.$$

(16) In the proof of Theorem III.8 on page 14, there is a factor of $1/\sqrt{\pi^k}$ missing. Corrected version:

$$\begin{aligned} v &= \sqrt{\pi^k p} \\ &= \frac{1}{\sqrt{\pi^k p}}\|\Pi_\beta\psi(\Delta)\|^2 \\ &= \langle \psi(\Delta), \psi'\rangle \\ &= \langle \psi_1, \psi_2\rangle. \end{aligned}$$

(17) Lemma IV.2 on page 15. The published proof of Lemma IV.2 is limited to single-mode states. Here, we provide a minimal generalization to n-mode states, along with the corrected parameters R and L valid for any $n \in \mathbb{N}$. This correction does not change the main implication: the runtime of the algorithm $\mathsf{fastnorm}_E$ in Lemma IV.3 remains polynomial in the relevant parameters. Corrected version:

Lemma IV.2. Suppose $E > 0$ and

$$\langle \Psi', H\Psi'\rangle \leqslant E \quad \text{where} \quad \Psi' = \Psi/\|\Psi\| .$$

Let $\varepsilon > 0$ and $p_f \in (0, 1)$ be given. Set

$$R := \sqrt[2n]{\frac{E}{\varepsilon}} \quad \text{and} \quad L := \left\lceil \frac{4\pi^n E}{n! p_f \varepsilon^3} \right\rceil.$$

Then

$$\Pr[(1-\varepsilon)\|\Psi\|^2 \leqslant \overline{X} \leqslant (1+\varepsilon)\|\Psi\|^2] \geqslant 1 - p_f.$$

Proof. We write the Hamiltonian as

$$H = 2\sum_{j=1}^{n}(a_j^\dagger a_j + I) = 2\sum_{j=1}^{n} a_j a_j^\dagger$$

such that

$$\langle \Psi, H\Psi\rangle = \frac{1}{\pi^n}\int_{\mathbb{C}^n} 2\|\alpha\|^2 |\langle \alpha, \Psi\rangle|^2 d\alpha,$$

where we use [cf. Eq. (C1) in Appendix C for the single-mode case]

$$\sum_{j=1}^{n} a_j a_j^\dagger = \frac{1}{\pi^n}\int_{\mathbb{C}^n} \|\alpha\|^2 |\alpha\rangle\langle\alpha|\, d\alpha.$$

Define

$$p(\alpha) = \frac{1}{\pi^n}\frac{|\langle \alpha, \Psi\rangle|^2}{\|\Psi\|^2} \quad \text{for} \quad \alpha \in \mathbb{C}^n.$$

Then p is a probability density function (the one associated with the outcome distribution when applying the heterodyne measurement to the normalized state $\Psi/\|\Psi\|$), and we have

$$\frac{\langle \Psi, H\Psi\rangle}{2\|\Psi\|^2} = \mathbb{E}_{\alpha\sim p}[\|\alpha\|^2],$$

where $\|\alpha\|^2 = \sum_{j=1}^{n} |\alpha_j|^2$. In particular, Markov's inequality implies that

$$\Pr_{\alpha\sim p}[\|\alpha\|^2 \geqslant R^{2n}] \leqslant \frac{\langle \Psi, H\Psi\rangle}{2R^{2n}\|\Psi\|^2}.$$

Consider the random variable $X = X_R$ defined by choosing α uniformly from $B_R(0)$ and outputting

$$X = \frac{R^{2n}}{n!}|\langle \alpha, \Psi\rangle|^2.$$

The random variable X has the expectation value

$$\begin{aligned}\mathbb{E}[X] &= \frac{n!}{\pi^n R^{2n}}\int_{B_R(0)} \frac{R^{2n}}{n!}|\langle \alpha, \Psi\rangle|^2 d\alpha \\ &= \frac{1}{\pi^n}\int_{B_R(0)} |\langle \alpha, \Psi\rangle|^2 d\alpha \\ &= \|\Psi\|^2 \int_{B_R(0)} p(\alpha) d\alpha,\end{aligned}$$

and it follows from Eq. (45) and the fact that p is a probability density function on $\mathbb{C}^n$ that

$$\|\Psi\|^2\left(1 - \frac{\langle \Psi, H\Psi\rangle}{2R^{2n}\|\Psi\|^2}\right) \leqslant \mathbb{E}[X] \leqslant \|\Psi\|^2.$$

We have

$$\begin{aligned}\mathbb{E}[X^2] &= \frac{n!}{\pi^n R^{2n}}\int_{B_R(0)} \frac{R^{4n}}{(n!)^2}|\langle \alpha, \Psi\rangle|^4 d\alpha \\ &= \frac{\pi^n R^{2n}}{n!}\|\Psi\|^4 \int_{B_R(0)} p(\alpha)^2 d\alpha \\ &\leqslant \frac{\pi^n R^{2n}}{n!}\|\Psi\|^4,\end{aligned}$$

where we used that $\int_{B_R(0)} p(\alpha)^2 d\alpha \leqslant \int_{B_R(0)} p(\alpha) d\alpha \leqslant 1$. Thus

$$\begin{aligned}\mathrm{Var}(X) &\leqslant \mathbb{E}[X^2] - \mathbb{E}[X]^2 \\ &\leqslant \frac{\pi^n R^{2n}}{n!}\|\Psi\|^4.\end{aligned}$$

Let $X_1, \ldots, X_L$ be i.i.d. realizations of X and

$$\overline{X} := \frac{1}{L}\sum_{j=1}^{L} X_j.$$

Then

$$\mathbb{E}[\overline{X}] = \mathbb{E}[X]$$

$$\mathrm{Var}(\overline{X}) = \mathrm{Var}(X)/L \leqslant \frac{\pi^n R^{2n}}{n!}\|\Psi\|^4/L,$$

and it follows by Chebyshev's inequality that

$$\Pr[|\overline{X} - \mathbb{E}[\overline{X}]| \geqslant \kappa\|\Psi\|^2] \leqslant \frac{\mathrm{Var}(\overline{X})}{\kappa^2\|\Psi\|^4} = \frac{\pi^n R^{2n}}{n!\kappa^2 L}.$$

Since

$$\begin{aligned}|\overline{X} - \|\Psi\|^2| &\leqslant |\overline{X} - \mathbb{E}[\overline{X}]| + |\|\Psi\|^2 - \mathbb{E}[\overline{X}]| \\ &= |\overline{X} - \mathbb{E}[\overline{X}]| + |\|\Psi\|^2 - \mathbb{E}[X]| \\ &\leqslant |\overline{X} - \mathbb{E}[\overline{X}]| + \|\Psi\|^2\frac{\langle \Psi, H\Psi\rangle}{2R^{2n}\|\Psi\|^2}\end{aligned}$$

by Eq. (46), it follows that

$$\Pr\left[|\overline{X} - \|\Psi\|^2| \geqslant \left(\varepsilon/2 + \frac{\langle \Psi, H\Psi\rangle}{2R^{2n}\|\Psi\|^2}\right)\|\Psi\|^2\right] \leqslant \frac{4\pi^n R^{2n}}{n!\varepsilon^2 L},$$

where we substitute $\varepsilon/2$ for κ in Eq. (47).

Thus Eq. (48) can be rewritten using the definition $\Psi' = \Psi/\|\Psi\|$ as

$$\Pr\left[|\overline{X} - \|\Psi\|^2| \geqslant \left(\varepsilon/2 + \frac{\langle \Psi', H\Psi'\rangle}{2R^{2n}}\right)\|\Psi\|^2\right] \leqslant \frac{4\pi^n R^{2n}}{n!\varepsilon^2 L}.$$

Equation (49) implies that

$$\Pr[(1-\varepsilon)\|\Psi\|^2 \leqslant \overline{X} \leqslant (1+\varepsilon)\|\Psi\|^2] \geqslant 1 - p_f$$

for any pair (R, L) such that

$$R \geqslant \sqrt[2n]{\frac{\langle \Psi', H\Psi'\rangle}{\varepsilon}} \quad \text{and} \quad L \geqslant \frac{4\pi^n R^{2n}}{n! p_f \varepsilon^2}.$$

It is clear that if $E \geqslant \langle \Psi', H\Psi'\rangle$ is an upper bound on the energy of the normalized state Ψ', then the choice

$$R := \sqrt[2n]{\frac{E}{\varepsilon}} \quad \text{and} \quad L := \left\lceil \frac{4\pi^n E}{n! p_f \varepsilon^3} \right\rceil$$

satisfies condition (50). This implies the claim. ■

The remaining changes, apart from Item (22), are a consequence of the new proof of Lemma IV.2 for general n given in Item (17).

(18) Equation (43) on page 15 is corrected to

$$X_j = \frac{R^{2n}}{n!}|\langle \alpha, \Psi\rangle|^2.$$

(19) In the Algorithm $\mathsf{fastnorm}_E$ on page 16, the Lines 2 and 9 are corrected as follows, as a consequence of the correction in Item (17). Corrected version:

ALGORITHM. $\mathsf{fastnorm}_E$.

Input: $\{c_j, \Delta_j\}_{j=1}^{\chi}, \varepsilon, p_f$
where $\begin{cases}\{c_j, \Delta_j = (\Gamma_j, d_j, r_j)\}_{j=1}^{\chi} \in (\mathbb{C} \times \mathsf{Desc}_n)^{\chi}, \\ \varepsilon > 0, p_f \in (0,1), E > 0\end{cases}$
Output: $\overline{X} \in \mathbb{C}$
1: $R \leftarrow \sqrt[2n]{\frac{E}{\varepsilon}}$
2: $L \leftarrow \lceil \frac{4\pi^n E}{n! p_f \varepsilon^3} \rceil$
3: **for** $\ell \leftarrow 1$ to L **do**
4: $\alpha \leftarrow$ randomly sample from the Lebesgue measure on $B_R(0) \subset \mathbb{C}^n$
5: $\Delta \leftarrow [I, d(\alpha), 1]$
6: **for** $j \leftarrow 1$ to χ **do**
7: $o_j \leftarrow \mathsf{overlap}(\Delta, \Delta_j)$
8: **end for**
9: $X_l \leftarrow R^{2n}/n! |\sum_{j=1}^{\chi} c_j o_j|^2$
10: **end for**
11: $\overline{X} \leftarrow \frac{1}{L}\sum_{\ell=1}^{L} X_\ell$
12: **return** $\overline{X}$

(20) Theorem IV.3 on page 16. The corrected runtime is smaller; we also added a new reference that is relevant here. This is a consequence of the corrected generalization to arbitrary integers n in Lemma IV.2. Corrected version:

Theorem IV.3. The algorithm $\mathsf{fastnorm}_E$ given in Item (19) achieves the following. Given as input a tuple $\{c_j, \Delta(\psi_j)\}_{j\in[\chi]}$ specifying a linear combination $|\Psi\rangle = \sum_{j=1}^{\chi} c_j |\psi_j\rangle \in \mathcal{H}_n$ of Gaussian states ψ_j, $j \in [\chi]$ where

$$\langle \Psi', H\Psi' \rangle \leqslant E \quad \text{with} \quad \Psi' = \Psi/\|\Psi\|,$$

and parameters $\varepsilon > 0$ and $p_f \in (0,1)$, the output $\overline{X}$ of $\mathsf{fastnorm}_E$ satisfies

$$(1-\varepsilon)\|\Psi\|^2 \leqslant \overline{X} \leqslant (1+\varepsilon)\|\Psi\|^2,$$

with probability at least $1 - p_f$. The algorithm has runtime $O(\frac{\chi E}{p_f \varepsilon^3})$.

Note added. The procedure in Theorem IV.3 can be modified by using a median-of-means argument as done in [30, Sec. 4.1] (see also [31, Lemma 10]). This gives an improved runtime of $O[\chi E \varepsilon^{-3} \log(1/p_f)]$.

(21) In the proof of Theorem IV.3 on page 17, we correct the runtime. Corrected version:

The algorithm has runtime

$$L \times \chi\,\mathsf{time}(\mathsf{overlap}) = O\left(\frac{4n^3\pi^n \chi E}{n! p_f \varepsilon^3}\right) = O\left(\frac{\chi E}{p_f \varepsilon^3}\right),$$

where we use $\mathsf{time}(\mathsf{overlap}) = O(n^3)$ and $n^3\pi^n/n! < 320$ for $n \in \mathbb{N}$.

(22) On page 17. The statement "$\mathrm{tr}[HS_j(z)^\dagger \rho S_j(z)] \leqslant e^{2z}\mathrm{tr}(H\rho)$for all $z > 1$" is generalized such that it applies to any real squeezing parameter $z \in \mathbb{R}$. Corrected version:

$$\mathrm{tr}[HS_j(z)^\dagger \rho S_j(z)] \leqslant e^{2z}\mathrm{tr}(H\rho) \quad \text{for all} \quad z \geqslant 0,$$
$$\mathrm{tr}[HS_j(z)^\dagger \rho S_j(z)] \leqslant e^{-2z}\mathrm{tr}(H\rho) \quad \text{for all} \quad z \leqslant 0.$$

On the other hand, for the squeezing channel with squeezing parameter $z \in \mathbb{R}$ we have

$$\mathrm{Var}_{\rho'} \leqslant e^{2|z|}\mathrm{Var}_\rho.$$

(23) In Lemma IV.4 on page 17, we correct the runtime. Corrected version: The runtime of the algorithm is $O[\chi E/(p_f \varepsilon^3)]$.

(24) Proof of Lemma IV.4 on page 18:

$$\chi(\mathsf{time}(\mathsf{postmeasure}) + \mathsf{time}(\mathsf{prob})) + \mathsf{time}(\mathsf{fastnorm}_E) = O\left(\frac{\chi E}{p_f \varepsilon^3}\right).$$

(25) Equation (56) on page 18 is corrected to

$$\Pr[\|\alpha\|^2 \geqslant R^{2n}] \leqslant \frac{\langle \Psi, H\Psi \rangle}{2R^{2n}}.$$

(26) In Lemma IV.5 on page 18, we correct the runtime. Corrected version:

(1) $\mathcal{T}_\delta \subset B_{\sqrt[2n]{E/\delta}}(0)$, i.e., each $\alpha \in \mathcal{T}_\delta$ has a squared norm bounded by $\|\alpha\|^2 \leqslant E/\delta$.

(2) There is a classical algorithm that, for any $\alpha \in \mathcal{T}_\delta$, outputs an estimate p of $p_\Psi(\alpha)$ which satisfies $p/p_\Psi(\alpha) \in [1-\varepsilon, 1+\varepsilon]$, and its runtime is bounded by $O[\chi E/(\delta p_f \varepsilon^3)]$.

(27) In the proof of Lemma IV.5 on page 18, we make the following corrections:

$$R = \sqrt[2n]{E/\delta},$$
$$\Pr[\|\alpha\|^2 \geqslant R^{2n}] \leqslant \delta/2,$$
$$O\left(\frac{\chi \tilde{E}}{p_f \varepsilon^3}\right) = O\left(\frac{\chi E}{\delta p_f \varepsilon^3}\right).$$