

## Quantum interferometry in external gravitational fields

Thomas B. Mieling <sup>1,\*</sup>, Thomas Morling <sup>2</sup>, Christopher Hilweg <sup>1</sup> and Philip Walther <sup>1,3</sup>

<sup>1</sup>*University of Vienna, Faculty of Physics and Research Network TURIS, Boltzmanngasse 5, 1090 Vienna, Austria*

<sup>2</sup>*University of Vienna, Faculty of Physics, Vienna Doctoral School in Physics (VDSP) and Research Network TURIS, Boltzmanngasse 5, 1090 Vienna, Austria*

<sup>3</sup>*Institute for Quantum Optics and Quantum Information (IQOQI) Vienna, Austrian Academy of Sciences, Boltzmanngasse 3, 1090 Vienna, Austria*



(Received 30 July 2025; accepted 12 November 2025; published 8 December 2025)

Current models of quantum interference experiments in external gravitational fields lack a common framework: while matter-wave interferometers are commonly described using the Schrödinger equation with a Newtonian potential, gravitational effects in quantum optics are modeled using either post-Newtonian metrics or highly symmetric exact solutions to Einstein's field equations such as those of Schwarzschild and Kerr. To coherently describe both kinds of experiments, this paper develops a unified framework for modeling quantum interferometers in general stationary space-times. This model provides a rigorous description and coherent interpretation of the effects of classical gravity on quantum probes.

DOI: [10.1103/8dsx-jt6b](https://doi.org/10.1103/8dsx-jt6b)

### I. INTRODUCTION

A central question in current theoretical physics is how quantum theory and general relativity can be unified into a consistent overarching framework. While quantum field theory (QFT) provides a generally covariant extension of quantum mechanics to flat and curved space-times, this framework treats gravity as a classical background field. There are multiple approaches to formulating a quantum theory of gravity, such as loop quantum gravity [1], string theory [2], and others [3–6], but also proposals for theories in which the gravitational field remains classical [7,8]. The lack of consensus on the accuracy of these models is strongly tied to the absence of direct experimental tests: The energies required to study quantum gravity effects are beyond current technological capabilities and could remain so for the coming decades [9,10].

Nevertheless, innovative experiments and proposals have been put forward to investigate gravitational effects in quantum physics, utilizing both massive and massless quantum systems under low-energy conditions in weak gravitational fields. In particular, the so-called BMV experiments, as originally proposed by Bose *et al.* [11] as well as Marletto and Vedral [12] aim at detecting the emergence of entanglement between two quantum particles that interact with each other solely through gravity. There is an ongoing debate about what could be inferred from the observation of such gravitationally mediated entanglement [10,13–17]. A complementary approach is to study the behavior of quantum systems in classical gravitational fields and to explore the gravitationally

influenced dynamics of entangled quantum states. Unlike BMV experiments, this approach does not seek evidence for the quantum nature of gravity but tests the validity of various formulations of the equivalence principle and related phenomena such as the gravitational redshift with quantum resources.

The first experimental investigations in this direction focused on the behavior of massive quantum particles in Earth's gravitational field [18–20]. These groundbreaking experiments showed that neutrons in spatial superpositions exhibit measurable phase shifts due to gravitational acceleration. Although the results can be interpreted in general relativistic terms [21], they are also adequately explained using the Schrödinger equation with a Newtonian gravitational potential [10,22–24]. Subsequent experiments with matter-wave interferometers further explored the response of matter waves to gravitational fields [25–30], even revealing phase shifts induced by gravity gradients [31]. Photonic experiments, on the other hand, could investigate gravitational effects on quantum systems in hitherto unexplored aspects by contrasting matter-wave interferometers with similar setups in which the probes propagate at relativistic speeds [32–36], building large-scale interferometers using satellites [37–39], and using entangled multiphoton states [40,41].

Current theoretical descriptions of massive and massless quantum probes do not share a common framework. Specifically, the literature on gravitational photon interferometers predominantly uses Einsteinian models of gravity (see, e.g., Refs. [34–36,42]). In the literature on gravitational matter-wave interferometers, on the other hand, it is not uncommon to describe the gravitational field using Newtonian methods even when discussing Einsteinian concepts such as the equivalence principle, proper time, or space-time curvature (see, e.g., Refs. [31,43,44]). This discrepancy illustrates the ongoing challenge of finding adequate descriptions of gravitational effects in quantum systems even if quantum gravity effects are neglected.

\*Contact author: [thomas.mieling@univie.ac.at](mailto:thomas.mieling@univie.ac.at)

The aim of this paper is to provide a consistent and general method for describing quantum interference in external gravitational fields. This is based on the Einsteinian description of gravity using Lorentzian geometry, but does not require Einstein's field equations to hold. In our model, the external gravitational field is assumed to be stationary, but it need not be static nor weak. Despite this level of generality, the resulting equations are not significantly more complex than in Newtonian models, but offer greater clarity in their physical interpretation.

This paper is organized as follows. After introducing the general mathematical framework for describing experiments in stationary space-times in Sec. II, we provide general expressions for the phase evolution in interferometric setups in Sec. III. The conversion from phase differences to detection probabilities for both bosonic and fermionic particles is provided in Sec. IV. Our framework is then applied to a variety of optical and matter-wave experiments in Secs. V and VI, respectively. Throughout this paper, Latin indices  $i, j, \dots$  range from 1 to 3, whereas Greek indices  $\mu, \nu, \dots$  range from 0 to 3, and indices occurring once as a subscript and once as a superscript are summed over (Einstein summation convention). The metric signature is  $(-, +, +, +)$  and natural constants such as  $c$ , the speed of light in vacuum, and  $\hbar$ , the reduced Planck constant, are kept explicit.

## II. GEOMETRY OF STATIONARY SPACE-TIMES

This section provides a general framework for describing stationary space-times that forms the basis for the models of concrete experiments developed in the following sections. The position taken here is that it is both physically more transparent and mathematically simpler to use a geometric and covariant formalism than to use perturbative weak-field expansions from the outset. Although explicit expressions are presented in adapted coordinates, the geometric approach taken here helps to clarify interpretational issues arising from general covariance.

*a. Time translations.* The condition for a space-time  $(\mathcal{M}, g)$  to be stationary can be formulated in terms of time-translation invariance. Specifically, such an invariance can be described by a smooth action of the additive group of real numbers  $\mathbf{R}$  on the manifold  $\mathcal{M}$ , here denoted by  $\Phi : \mathbf{R} \times \mathcal{M} \rightarrow \mathcal{M}$  with  $(\chi, p) \mapsto \Phi_\chi(p)$ , such that (i) the orbits of  $\Phi$  are timelike, and (ii) all transformations  $\Phi_\chi : \mathcal{M} \rightarrow \mathcal{M}$  are isometries of the metric  $g$ . The infinitesimal generator of  $\Phi$  is known as the associated Killing vector field  $\mathcal{K}$  (see Sec. 6.2 of Ref. [45]) whose components, in local coordinates, are given by  $\mathcal{K}^\mu = d\Phi_\chi^\mu/d\chi|_{\chi=0}$ .

Examples in flat space-time include inertial motion, rigid rotation, and hyperbolic motion (see Fig. 1 for an illustration). Similarly, the exterior Schwarzschild geometry admits multiple notions of time translations: they include translations in the temporal coordinate of the Schwarzschild chart, combinations of such translations with rotations in the angular coordinates of the same chart, as well as translations in the temporal coordinate of the Gullstrand-Painlevé chart [46]. Thus, depending on the degree of symmetry, a space-time may admit multiple notions of isometric time translations (and, in general, no single such notion is “preferred”).

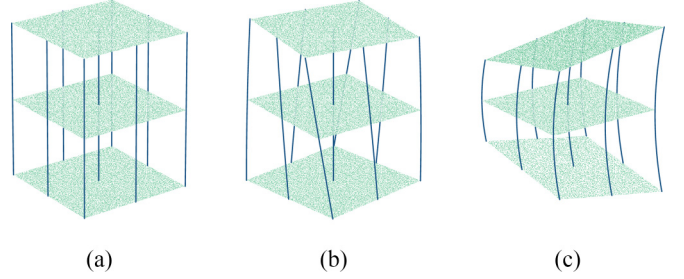


FIG. 1. Graphical illustration of different notions of isometric time translations in flat space-time. The blue lines indicate the orbits of the group action  $\Phi$ . The central green surface is a plane of simultaneity for an inertial system; the surfaces above and below indicate how such a surface is deformed by different notions of time translation: it is (a) shifted in parallel under inertial motion, (b) twisted under rotary motion, and (c) tilted under hyperbolic motion.

*b. Lapse and acceleration.* The lapse function  $\zeta = \sqrt{-g_{\mu\nu}\mathcal{K}^\mu\mathcal{K}^\nu}/c$  measures the rate of flow of proper time  $\tau$  along the orbits of  $\Phi$  via  $d\tau = \zeta d\chi$ . Hence, the four-velocity of these orbits is given by  $v^\mu = \mathcal{K}^\mu/\zeta$ . In the following, the flow parameter  $\chi$  will be assumed to have units of time, which implies that  $\zeta$  is dimensionless. This flow parameter can be rescaled according to  $\chi \mapsto \chi' = \chi/\alpha$  (with constant  $\alpha > 0$ ), which induces  $\mathcal{K}^\mu \mapsto \mathcal{K}'^\mu = \alpha\mathcal{K}^\mu$  and  $\zeta \mapsto \zeta' = \alpha\zeta$ . As a consequence, the lapse can be set to unity at any given point. However, it is not possible to set  $\zeta = 1$  on open regions unless the orbits of  $\Phi$  are inertial. This is because derivatives of  $\zeta$  determine the four-acceleration via  $a_\mu = c^2(\partial_\mu\zeta)/\zeta$ . Defining the logarithmic lapse as  $\varphi = c^2 \ln \zeta$ , this can be written as  $a_\mu = \partial_\mu\varphi$ .

*c. Synchronization and Lorentz factors.* Two events  $p_1, p_2 \in \mathcal{M}$  are said to be simultaneous relative to  $\Phi$  if there exists a connecting curve  $\alpha \mapsto x^\mu(\alpha)$  satisfying  $g_{\mu\nu}\dot{x}^\mu\mathcal{K}^\nu = 0$ . More generally, the one-form  $g_{\mu\nu}\mathcal{K}^\nu$  does not only define a local notion of simultaneity, but also provides a measure of proper time ratios (that depends on the choice of  $\Phi$ ). Specifically, setting  $\Gamma_\mu = -g_{\mu\nu}\mathcal{K}^\nu/(c^2\zeta)$ , the Lorentzian gamma factor that relates the rate of proper time along integral curves of  $v^\mu = \mathcal{K}^\mu/\zeta$  to that along any other timelike curve with four-velocity  $u^\mu$  is given by  $\gamma(v, u) = \Gamma_\mu u^\mu$  [clearly, one has  $\gamma(v, v) = \Gamma_\mu v^\mu = 1$ ]. Based on the relative Lorentz factor  $\gamma(v, u)$ , the relative speed  $V_{\text{rel}}(v, u)$  can be defined using the standard formula  $\gamma(v, u) = 1/\sqrt{1 - V_{\text{rel}}(v, u)^2/c^2}$ .

*d. Shift and twist.* Locally, one can construct coordinates  $(t, x^i)$  such that  $\Phi$  acts as  $\Phi_\chi(t, x^i) = (t + \chi, x^i)$ , which is equivalent to  $\mathcal{K}^0 = 1$  and  $\mathcal{K}^i = 0$  (see Sec. 1.4c of Ref. [47]). In any such coordinate system one thus has  $\mathcal{K}_0 = -c^2\zeta^2$ . The components  $\mathcal{K}_i$ , however, are not fully determined by the geometry of the problem and thus exhibit a gauge redundancy. Specifically, redefining the temporal coordinate as  $t \mapsto t' = t + \theta$ , where  $\theta$  is a function of the “spatial coordinates”  $x^i$  alone, the induced transformation reads  $\mathcal{K}_i \mapsto \mathcal{K}'_i = \mathcal{K}_i - \mathcal{K}_0\partial_i\theta$ . Defining the dimensionless shift (relative to a given coordinate system) as  $\xi_i = -c\mathcal{K}_i/\mathcal{K}_0$ , this can be written as  $\xi_i \mapsto \xi'_i = \xi_i + \partial_i\lambda$ , where  $\lambda = c\theta$ . This means that  $\xi_i$  (and hence also  $\mathcal{K}_i$  and  $g_{0i}$ ) can be chosen to vanish at any given point. However, the twist  $\Xi_{ij} = c\zeta(d\xi)_{ij} = c\zeta(\partial_i\xi_j - \partial_j\xi_i)$ ,

which is invariant under such transformations, provides an obstruction to setting these quantities to zero on open regions. This tensor has a geometric interpretation: the condition  $\Xi_{ij} = 0$  is equivalent to  $\mathcal{K}^\mu$  being hypersurface-orthogonal (see Appendix B of Ref. [48]). Hence,  $\Xi_{ij}$  quantifies the obstruction to clock synchronization along closed spatial paths, which gives rise to the Sagnac effect. Space-times admitting Killing fields with vanishing twist are said to be *static*, but in generic *stationary* space-times one has  $\Xi_{ij} \neq 0$ .

*e. Spatial metric and curvature.* Since the lapse  $\zeta$  and the shift  $\xi_i$  determine the metric coefficients  $g_{0\mu}$  in adapted coordinates, it remains to study the spatial components  $g_{ij}$ . These coefficients, however, transform nontrivially under the gauge transformations described above,  $g_{ij} \mapsto g'_{ij} = g_{ij} - \zeta^2[\partial_i \lambda \partial_j \lambda + \xi_i \partial_j \lambda + \xi_j \partial_i \lambda]$ , and thus do not admit an invariant geometric interpretation. Instead, a gauge-invariant version is given by  $l_{ij} = g_{ij} + \zeta^2 \xi_i \xi_j$ . This “spatial metric” admits the fully covariant expression  $l_{\mu\nu} = g_{\mu\nu} + \mathcal{K}_\mu \mathcal{K}_\nu / (c\zeta)^2$ , which shows that  $l_{ij}$  describes the metric  $g_{\mu\nu}$  in the orthogonal complement of  $\mathcal{K}^\mu$ . By a suitable choice of spatial coordinates  $x^i$ , one may set  $l_{ij}$  equal to the Kronecker delta  $\delta_{ij}$  at any given point or along any given spatial curve (using, for example, spatial Riemann coordinates or Fermi coordinates), but the extension to open neighborhoods is possible if and only if the spatial curvature tensor, here denoted by  $P_{ijkl}$ , vanishes (see Chap. 7 of Ref. [49]).

*f. Metric decomposition.* The constructions described so far show that any stationary metric admits local adapted coordinates  $(t, x^i)$  such that the time-translation map  $\Phi$  acts as  $\Phi_\chi(t, x^i) = (t + \chi, x^i)$  and the space-time metric takes the form

$$g = -\zeta^2(cdt - \xi_i dx^i)^2 + l_{ij} dx^i dx^j, \quad (1)$$

where the lapse  $\zeta$ , shift  $\xi_i$ , and spatial metric  $l_{ij}$  are independent of the temporal coordinate  $t$ . The remaining coordinate freedom covers space-dependent translations in  $t$  (inducing  $\xi_i \mapsto \xi_i + \partial_i \lambda$ ), and  $t$ -independent spatial diffeomorphisms (under which  $\zeta$ ,  $\xi_i$ , and  $l_{ij}$  transform as covariant tensors of rank 0, 1, and 2, respectively).

*g. Frames of reference.* While it is common practice to define stationary metrics by the condition of admitting coordinates with respect to which  $g_{\mu\nu}$  can be written in the form (1), it is worth noting that such a description specifies not only the metric  $g_{\mu\nu}$ , but also a notion of time translations (namely, translations in the temporal coordinate  $t$ ). Indeed, if the isometry group of  $(\mathcal{M}, g)$  has multiple dimensions, the same metric tensor may take different forms depending on the choice of Killing field with respect to which the metric decomposition is carried out. For example, the expressions

$$g_I = -(cdt)^2 + dx^2 + dy^2 + dz^2, \quad (2a)$$

$$g_R = -\left[1 - \frac{r^2 \omega^2}{c^2}\right] \left[cdt - \frac{(r\omega/c) r d\phi}{1 - r^2 \omega^2 / c^2}\right]^2 + dr^2 + \frac{r^2 d\phi^2}{1 - r^2 \omega^2 / c^2} + dz^2, \quad (2b)$$

$$g_H = -\left[1 + \frac{az}{c^2}\right]^2 (cdt)^2 + dx^2 + dy^2 + dz^2 \quad (2c)$$

all describe the flat Minkowski geometry (or parts thereof), but define different notions of time translations whose orbits are depicted in Fig. 1. In this sense, Eqs. (2a)–(2c) can be interpreted as defining different frames of reference. The situation is similar in the (exterior) Schwarzschild and Kerr geometries where “standard” time translations can be combined with rotations to produce further notions of time translations. The possibility of there being multiple stationary reference systems is important when describing experiments “at rest” in a gravitational field as this notion becomes ambiguous in such scenarios. In such a case, the system considered as being at rest defines one notion of time translations and hence singles out one particular Killing field. Relative to any other stationary reference system, such an experiment is in motion and potentially accelerated. The transformation between such reference systems is similar to that known from the special theory of relativity since, at each space-time event, the four-velocities of two Killing orbits are related by a Lorentz transformation, but in general these transformations are not constant and may hence vary with space and time.

*h. Time of flight and traveled distances.* Given any reference system as described above, key observables include elapsed times  $T$  and traveled distances  $L$  of various world lines  $\alpha \mapsto x^\mu(\alpha)$ , where  $\alpha$  is any quantity parametrizing the world line. Setting  $\Gamma_\mu = -\mathcal{K}_\mu / (c^2 \zeta)$  and  $l_{\mu\nu} = g_{\mu\nu} + c^2 \Gamma_\mu \Gamma_\nu$  as above, these quantities are given by  $dT/d\alpha = \Gamma_\mu dx^\mu / d\alpha$  and  $dL/d\alpha = \sqrt{l_{\mu\nu} (dx^\mu / d\alpha)(dx^\nu / d\alpha)}$ . In adapted coordinates, these expressions take the form

$$\frac{dT}{d\alpha} = \zeta \left( \frac{dt}{d\alpha} - \frac{\xi_i}{c} \frac{dx^i}{d\alpha} \right), \quad (3a)$$

$$\frac{dL}{d\alpha} = \sqrt{l_{ij} \frac{dx^i}{d\alpha} \frac{dx^j}{d\alpha}}, \quad (3b)$$

and the proper time  $\tau$  along a timelike curve can be computed using  $(d\tau/d\alpha)^2 = (dT/d\alpha)^2 - (dL/d\alpha)^2 / c^2$ .

*i. Space-time curvature.* Equations (3a) and (3b) are frame dependent as the quantities involved are derived from a specific Killing vector field  $\mathcal{K}^\mu$ . In situations where there are multiple Killing vector fields, it is useful to identify quantities that are frame independent.<sup>1</sup> Such invariant observables include proper times of specific world lines, but also the four-dimensional Riemann tensor  $R_{\mu\nu\rho\sigma}$  describing the curvature of the space-time metric  $g_{\mu\nu}$ . Its components can be derived from the lapse, shift, and spatial geometry. In particular, in local spatial Riemann coordinates with  $\zeta = 1$  and  $\xi_i = 0$  (these conditions can be satisfied simultaneously at any given point using the coordinate transformations described above),

<sup>1</sup>The question of frame independence as formulated here is vacuous for space-times admitting only a single notion of time translations such as those discussed in Refs. [50–52]. Such geometries thus admit “preferred” frames and the properties of the unique (up to rescaling) Killing field are characteristic geometric invariants despite not being derived from the curvature tensor.

TABLE I. Summary of notation and interpretation of tensorial objects characterizing space-time and its decomposition based on a notion of time-translation invariance.

| Symbol                 | Name                 | Gauge freedom                                                  | Significance                                                                                                  |
|------------------------|----------------------|----------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------|
| $g_{\mu\nu}$           | Space-time metric    |                                                                | Determines the geometry of the space-time manifold $\mathcal{M}$                                              |
| $\mathcal{K}^\mu$      | Killing vector       | $\mathcal{K}^\mu \rightarrow (\text{const}) \mathcal{K}^\mu$   | Infinitesimal generator of time translations                                                                  |
| $\zeta$                | Lapse function       | $\zeta \rightarrow (\text{const}) \zeta$                       | Measure of proper time along the orbits of $\mathcal{K}^\mu$                                                  |
| $\varphi$              | Logarithmic lapse    | $\varphi \rightarrow \varphi + \text{const}$                   | Analog of the Newtonian potential                                                                             |
| $\Gamma_\mu$           | Lorentz form         |                                                                | Determines the Lorentzian gamma factors of four-velocities                                                    |
| $a_i$                  | Acceleration         |                                                                | Measure of $\mathcal{K}^\mu$ orbits deviating from space-time geodesics<br>Primary observable for gravimeters |
| $\xi_i$                | Shift vector         | $\xi_i \rightarrow (\text{const})(\xi_i + \partial_i \lambda)$ | Measure for the deviation from clock synchronization                                                          |
| $\Xi_{ij}$             | Twist tensor         |                                                                | Obstruction to clock synchronization along loops<br>Primary observable for Sagnac interferometers             |
| $l_{ij}$               | Spatial metric       |                                                                | Projection of $g_{\mu\nu}$ to the orthogonal complement of $\mathcal{K}^\mu$                                  |
| $P_{ijkl}$             | Spatial curvature    |                                                                | Measure of the deviation from Euclidean space                                                                 |
| $R_{\mu\nu\rho\sigma}$ | Space-time curvature |                                                                | Measure of the deviation from Minkowskian space-time<br>Primary observable for gradiometers                   |

its components are given by

$$R_{0i0j} = c^2 \partial_i \partial_j \zeta + \frac{1}{4} \Xi_{ik} \Xi_{jl} \delta^{kl}, \quad (4a)$$

$$R_{0ijk} = \frac{1}{2} [(\partial_i \zeta) \Xi_{jk} + \partial_i \Xi_{jk}] - \frac{1}{2} [\Xi_{ij} \partial_k \zeta - \Xi_{ik} \partial_j \zeta], \quad (4b)$$

$$R_{ijkl} = P_{ijkl} + \frac{3}{4} \Xi_{ij} \Xi_{kl} / c^2. \quad (4c)$$

Of course, for the metrics given in Eqs. (2a)–(2c) these equations yield  $R_{\mu\nu\rho\sigma} = 0$ . In particular, Eq. (4c) shows that curved spatial geometries can arise in flat space-time only relative to stationary frames of reference that have a nonzero twist tensor  $\Xi_{ij}$ . Moreover, Eq. (4a) shows that in static reference frames (where  $\Xi_{ij} = 0$ ), gravity gradients *do not* directly coincide with curvature components since

$$R_{0i0j} = g_i g_j / c^2 - \gamma_{ij}, \quad (5)$$

where  $g_i = -a_i$  is the local gravitational acceleration relative to the reference system [see Eq. (13) below], and  $\gamma_{ij} = \partial_j g_i = \partial_i g_j$  is the tidal tensor. On Earth, the gravity gradient  $\gamma_{zz} \approx 3 \times 10^{-6} \text{ s}^{-2}$  far exceeds the other term  $g^2/c^2 \approx 10^{-15} \text{ s}^{-2}$ . This justifies the identification of gravity gradients with curvature in Ref. [43]. Note, however, that this identification is not universally valid: the reference system associated to Eq. (2c) has a nonzero  $g_i$  and  $\gamma_{ij}$  despite having  $R_{0i0j} = 0$ .

*j. Summary.* Table I summarizes the notation and interpretation of the various geometric quantities introduced so far. Note that some of the nomenclature is not uniform throughout the literature. In particular, the notions of lapse, shift, and spatial metric have different meanings in the Arnowitt-Deser-Misner formalism and coincide with the quantities used here only for *static* reference frames (see Sec. 2.7 of Ref. [53]). Moreover, the shift (or a rescaled version) is also known as the gravitomagnetic potential [54], while the Lorentz form (or variants thereof in Cartan theory) is often referred to as the clock form [55]. Some other sources, however, do not use any names for these quantities (see Sec. XIV.3.2 of Ref. [56]).

### III. PHASE EVOLUTION AND RAY TRAJECTORIES

This section shows how phase evolutions in interferometers that are “at rest” in external gravitational fields can be described concisely using the general framework of Sec. II without the need for weak-field approximations.

*a. Dispersion relations and transport laws.* The phases at two points are generally related by

$$\Psi(p_2) = \Psi(p_1) + \int_\Gamma K_\mu dx^\mu, \quad (6)$$

where  $\Gamma$  is any curve connecting  $p_1$  with  $p_2$ , and  $K_\mu = \partial_\mu \Psi$  is the (angular) wave covector.  $K_\mu$  typically satisfies a dispersion relation and a transport law along a vector field  $\bar{K}^\mu$  [the (angular) wave vector, which may differ from  $K^\mu \equiv g^{\mu\nu} K_\nu$  as is illustrated below], the explicit forms of which depend on the details of the problem. Specifically, the analysis here is concerned with the following three cases:

(1) For freely moving massive particles,  $P_\mu = \hbar K_\mu$  can be interpreted as the four-momentum. The mass  $m > 0$  determines the dispersion relation  $g^{\mu\nu} P_\mu P_\nu = -m^2 c^2$ , and the corresponding transport law is  $V^\mu \nabla_\mu P_\nu = 0$ , where  $V^\mu = g^{\mu\nu} P_\nu / m \equiv (\hbar/m) \bar{K}^\mu$  is the four-velocity and  $\nabla_\mu$  denotes the Levi-Civita derivative associated to  $g_{\mu\nu}$ . This transport equation is equivalent to the geodesic equation  $V^\nu \nabla_\nu V^\mu = 0$ .

(2) For light propagation in vacuum, the dispersion relation reads  $g^{\mu\nu} K_\mu K_\nu = 0$ . In this case  $K_\mu$  is parallel-transported along the wave vector  $\bar{K}^\mu = g^{\mu\nu} K_\nu$ , i.e.,  $\bar{K}^\mu \nabla_\mu K_\nu = 0$ , which is equivalent to the geodesic equation  $\bar{K}^\nu \nabla_\nu \bar{K}^\mu = 0$ .

(3) For light propagation in a linear isotropic dielectric with four-velocity  $v^\mu$  and refractive index  $n$ , on the other hand,  $K_\mu$  satisfies the eikonal equation  $l^{\mu\nu} K_\mu K_\nu = n^2 (v^\mu K_\mu)^2 / c^2$ , where  $l^{\mu\nu} = g^{\mu\nu} + v^\mu v^\nu / c^2$  is the (contravariant) spatial metric relative to  $v^\mu$ . This is equivalent to the standard formula  $k^2 = n^2 \omega^2 / c^2$  in the local rest frame of  $v^\mu$ . The corresponding transport law is more intricate than in the case described above: using Gordon’s optical metric  $\bar{g}^{\mu\nu} = l^{\mu\nu} - (n/c)^2 v^\mu v^\nu$  [57], defined such that the dispersion relation takes the form  $\bar{g}^{\mu\nu} K_\mu K_\nu = 0$ , and

denoting the corresponding Levi-Civita derivative by  $\bar{\nabla}_\mu$ , the transport equation can be written as  $\bar{K}^\mu \bar{\nabla}_\mu K_\nu = 0$ , where  $\bar{K}^\mu = \bar{g}^{\mu\nu} K_\nu$  [58].

Note that the equations for light propagation in vacuum can be obtained from those for massive particles using the massless limit  $m \rightarrow 0$ , and also from those for light propagation in media using the vacuum limit  $n \rightarrow 1$ .

The analysis in this paper excludes the case of guided motion of massive particles. The reason for this omission is the lack of a comprehensive relativistic framework for modeling the phase evolution in scenarios considered in experimental proposals such as Ref. [59] or implemented in current experiments [29,30,60]. While the semiclassical phase evolution of charged particles is well understood [61], such Wentzel-Kramers-Brillouin (WKB) models currently do not apply to uncharged particles with electromagnetic dipole moments as relevant, e.g., for the description of atoms in optical lattices. On the other hand, the motion of such particles can be described using the Mathisson-Papapetrou-Dixon equations [62–64] or related models [65], but these equations do not determine the phase evolution along the trajectories. Hence, further theory development is needed to provide a coherent relativistic description of such experiments.

*b. Phase evolution along rays.* Equation (6) simplifies significantly if  $p_1$  and  $p_2$  lie on the same flow line of  $\bar{K}^\mu$ . Setting  $\Gamma$  to be such a ray along which one has  $dx^\mu = \bar{K}^\mu d\alpha$  for some parameter  $\alpha$ , one obtains  $\Psi(p_2) - \Psi(p_1) = \int_{\alpha_1}^{\alpha_2} K_\mu \bar{K}^\mu d\alpha$ . Along the rays, one thus has

$$d\Psi = 0 \quad \text{for light,} \quad (7a)$$

$$d\Psi = -\omega_c d\tau \quad \text{for matter,} \quad (7b)$$

where  $\tau$  denotes proper time along a timelike curve and  $\omega_c = mc^2/\hbar$  is the (angular) Compton frequency. It is worth noting that Eq. (7a) applies both to light propagation in vacuum and to light propagation in linear isotropic media. In the latter case, the rays are *timelike* and thus have  $d\tau \neq 0$ , but it would be incorrect to use Eq. (7b) with  $\omega_c$  replaced by the optical frequency relative to any observer (cf. Appendix A of Ref. [35]). The implications of Eq. (7b) in matter-wave gravimeters were the subject of a debate [59,66–69]. In the gravimeters described in Sec. VIB below, the analysis shows that the interfering rays have *equal* proper times. In such cases, the Compton frequency plays no significant role in the observed phase shifts.

*c. Redshift.* In general, the angular wave covector  $K_\mu$  can be decomposed as

$$K_\mu = -\omega \Gamma_\mu + k \hat{n}_\mu, \quad (8)$$

where  $\omega$  is the angular frequency relative to the given reference system and  $\hat{n}_\mu$  is a spatial unit covector (satisfying  $K^\mu \hat{n}_\mu = 0$  and  $l^{\mu\nu} \hat{n}_\mu \hat{n}_\nu = 1$ ). The angular wave number  $k$  is then determined by the dispersion relation. As freely propagating light and matter are modeled as following geodesics, the quantity  $\omega_* = -K_\mu \bar{K}^\mu = \zeta \omega$  is a constant of motion along the rays. This “angular Killing frequency” is also conserved along light rays in optical media, provided that the optical properties are time independent. For arbitrary points  $p_i$  and  $p_j$  along a ray, the redshift factor  $z_{ij}$ , defined as  $1 + z_{ij} = \omega(p_j)/\omega(p_i)$ , can thus be expressed in terms of the

logarithmic lapse  $\varphi$  as

$$z_{ij} = \exp(\Delta\varphi_{ij}/c^2) - 1 \\ = \Delta\varphi_{ij}/c^2 + \frac{1}{2} \Delta\varphi_{ij}^2/c^4 + O(\Delta\varphi_{ij}^3/c^6), \quad (9)$$

where  $\Delta\varphi_{ij} = \varphi(p_i) - \varphi(p_j)$ . Equation (9) is an exact result in stationary space-times and the power-series expansion converges absolutely for all  $\Delta\varphi_{ij}$ . In particular, this formula does not require the weak-field approximation  $|\varphi| \ll c^2$ .

*d. Stationary solutions.* While the transport equations described above imply that  $\omega_* = \zeta \omega$  is constant along the rays, this does not imply that  $\omega_*$  is globally constant. However, this additional condition is satisfied by stationary solutions, i.e., those satisfying  $\Psi(\Phi_\chi(p)) = \Psi(p) - \omega_* \chi$  for all  $p \in M$  and  $\chi \in \mathbf{R}$  (here,  $\Phi$  denotes the time-translation map introduced in Sec. II). This means that the frequency  $\omega = \omega_*/\zeta$  relative to the orbits of  $\Phi$  is constant along these timelike world lines. For such stationary solutions, Eq. (6) reduces to a particularly simple form: if  $\Gamma$  is any curve connecting  $p_1 = (t_1, x_1)$  with  $p_2 = (t_2, x_2)$ , one obtains

$$\psi(x_2, x_1) := \Psi(t_2, x_2) - \Psi(t_1, x_1) + \omega_*(t_2 - t_1) \\ = \int_\gamma k \hat{n}_i dx^i + \frac{\omega_*}{c} \int_\gamma \xi_i dx^i, \quad (10)$$

where  $\gamma$  is the spatial projection of  $\Gamma$  (a curve connecting  $x_1$  with  $x_2$ ). The explicit form of these integrals depends on the dispersion relation: for light one has

$$\psi(x_2, x_1) = \frac{1}{c} \int_\gamma n \omega \hat{n}_i dx^i + \frac{\omega_*}{c} \int_\gamma \xi_i dx^i, \quad (11)$$

while for matter one has

$$\psi(x_2, x_1) = \frac{1}{\hbar c} \int_\gamma \sqrt{E^2 - m^2 c^4} \hat{n}_i dx^i + \frac{E_*}{\hbar c} \int_\gamma \xi_i dx^i, \quad (12)$$

where  $E = \hbar \omega$  is the energy (relative to the given stationary reference frame), and  $E_* = \hbar \omega_* = \zeta E$  is the corresponding constant of motion. Note that the vacuum limit  $n \rightarrow 1$  in Eq. (11) produces the same result as the massless limit  $m \rightarrow 0$  in Eq. (12).

*e. Ray trajectories.* Full integration of Eqs. (7a) and (7b) or Eqs. (11) and (12) requires knowledge of the rays. Here, we distinguish two cases: (i) guided motion and (ii) free motion. Guided motion applies, for example, to light propagating through optical fibers. In this case, the spatial trajectories are determined by the experimental setup and the temporal part of the corresponding rays is determined by the local speed of light, i.e., by the fiber’s refractive index. Free motion describes freely falling matter and also free light propagation in vacuum, in which case the rays are modeled as space-time geodesics. Their spatial projections satisfy the equation

$$\frac{d^2 x^i}{dT^2} + \left\{ \begin{matrix} i \\ jk \end{matrix} \right\} \frac{dx^j}{dT} \frac{dx^k}{dT} = \left( l^{ij} - \frac{1}{c^2} \frac{dx^i}{dT} \frac{dx^j}{dT} \right) g_j - \Xi_j^i \frac{dx^j}{dT}, \quad (13)$$

where  $T$  is a time parameter along the curve that is defined by Eqs. (3), i.e.,  $dT = \Gamma_\mu dx^\mu$ . Here,  $\{ \begin{smallmatrix} i \\ jk \end{smallmatrix} \}$  denote the Christoffel symbols of the spatial metric  $l_{ij}$  and  $g_i = -a_i = -\partial_i \varphi$  is the negative acceleration of the stationary Killing orbits. This equation shows that massive particles that are initially at rest

in the considered reference frame experience an acceleration that is equal to  $g_i$ . Note that, despite this close analogy to Newtonian gravity, Eq. (13) is an exact result that holds in arbitrary stationary space-times. As such, this equation generally does not admit closed-form solutions, implying that exact or approximate solutions for specific experiments must be computed on a case-by-case basis. Explicit expressions are discussed in Secs. V and VI.

#### IV. DETECTION PROBABILITIES FROM PHASE SHIFTS

The methods outlined in Sec. III describe the deterministic evolution of the phase function  $\Psi$ . While the phase is generally considered as being not directly observable, *phase differences* arising from interfering rays determine detection probabilities. This section summarizes the main equations relevant to the description of interferometric gravimeters and gradiometers using single-particle and multiparticle states following either bosonic or fermionic statistics.

For a general interferometer with  $N_{\text{in}}$  inputs and  $N_{\text{out}}$  outputs, let  $|\text{in}_i\rangle$  denote a (normalized) single-particle excitation of the input mode  $i$ . In the Heisenberg picture, this state *equals* a superposition of single-particle excitations  $|\text{out}_{k|i}\rangle$  of the various output modes  $k$ . Modeling potential losses using an additional “fictitious output port” that remains undetected, one thus has

$$|\text{in}_i\rangle = \sum_{k=1}^{N_{\text{out}}} |\text{out}_{k|i}\rangle + |\text{loss}_i\rangle. \quad (14)$$

The corresponding equation for ladder operators takes the form<sup>2</sup>

$$a_i^\dagger = \sum_k b_{k|i}^\dagger + c_i^\dagger. \quad (15)$$

By construction, the input vectors  $|\text{in}_i\rangle$  are orthonormal, and the output vectors satisfy  $\langle \text{out}_{k|i} | \text{out}_{l|j} \rangle = 0$  for  $k \neq l$  since excitations at different outputs are perfectly distinguishable. For  $k = l$ , however, the inner products

$$u_{k|ij} = \langle \text{out}_{k|i} | \text{out}_{k|j} \rangle \equiv \langle 0 | b_{k|i} b_{k|j}^\dagger | 0 \rangle \quad (16)$$

have a nontrivial structure that depends on the details of the interferometer. A simple model of a Mach-Zehnder interferometer with symmetric 50:50 beam splitters and equal losses in both arms is obtained by setting

$$(u_{1|ij}) = \frac{1}{2} \mathcal{A} \begin{pmatrix} 1 - \mathcal{V} \cos \Delta \Psi & + \mathcal{V} \sin \Delta \Psi \\ + \mathcal{V} \sin \Delta \Psi & 1 + \mathcal{V} \cos \Delta \Psi \end{pmatrix}, \quad (17a)$$

$$(u_{2|ij}) = \frac{1}{2} \mathcal{A} \begin{pmatrix} 1 + \mathcal{V} \cos \Delta \Psi & - \mathcal{V} \sin \Delta \Psi \\ - \mathcal{V} \sin \Delta \Psi & 1 + \mathcal{V} \cos \Delta \Psi \end{pmatrix}, \quad (17b)$$

<sup>2</sup>Equation (15) is reminiscent of Eq. (8) in Ref. [70] and Eq. (39) in Ref. [71], both of which relate ladder operators corresponding to different frequency spectra rather than spatial modes. The criticism of these equations expressed in Ref. [72] does not apply to Eq. (15) as this equation describes a transformation of the structure  $N \rightarrow N^2 + N$  (expressing the  $N$  input operators in terms of  $N^2$  output operators and  $N$  loss operators), rather than  $N \rightarrow N$  as considered in Refs. [70,71].

where  $\mathcal{A}$  and  $\mathcal{V}$  are parameters describing the overall transmission probability and the fringe visibility, respectively, and  $\Delta \Psi$  takes the form

$$\Delta \Psi = \Delta \Psi_\zeta + \Delta \Psi_\xi + \Delta \Psi_*, \quad (18)$$

where  $\Delta \Psi_\zeta$  and  $\Delta \Psi_\xi$  depend, respectively, only on the lapse and the shift along the rays  $\gamma_I$  and  $\gamma_{II}$  as

$$\Delta \Psi_\zeta = \int_{\gamma_I} k_I \hat{n}_I dx^i - \int_{\gamma_{II}} k_{II} \hat{n}_{II} dx^i, \quad (19a)$$

$$\Delta \Psi_\xi = \frac{\omega_*}{c} \int_{\gamma_*} \xi_i dx^i. \quad (19b)$$

Here, the wave vectors  $k_I$  and  $k_{II}$  depend on the dispersion relation [see Eqs. (11) and (12)], and  $\gamma_*$  denotes the closed path obtained by concatenating  $\gamma_I$  with the reverse of  $\gamma_{II}$ . (If  $\gamma_*$  encloses an area  $A$ , Stokes’ theorem implies  $\Delta \Psi_\xi = \int_A \omega \Xi_{ij} dx^i dx^j / c^2$ .) Finally,  $\Delta \Psi_*$  accounts for all additional phase shifts arising, for example, from optical mirrors and phase shifters (which are commonly used to introduce adjustable bias phases) or from the interaction of massive particles with matter or electromagnetic radiation [73]. Explicit expressions for  $\Delta \Psi$  depend on the experiment under consideration (see Secs. V and VI below).

For both bosonic and fermionic particles, the probability of detecting one particle at output 1 after it being injected into input 1 is given by

$$P_1 = \frac{1}{2} \mathcal{A} [1 - \mathcal{V} \cos(\Delta \Psi)]. \quad (20)$$

The coincide probability of detecting one particle at each of the outputs after two identical particles have been injected at the two inputs depends on the statistics:

$$P_2^{\text{bos}} = \frac{1}{2} \mathcal{A}^2 [1 + \mathcal{V}^2 \cos(2\Delta \Psi)], \quad (21a)$$

$$P_2^{\text{ferm}} = \frac{1}{2} \mathcal{A}^2 [1 - \mathcal{V}^2]. \quad (21b)$$

In general, gravity influences both the phase shift  $\Delta \Psi$  and the visibility  $\mathcal{V}$ . However, since the former is typically more sensitive to gravitational effects than the latter [34,40,74], the following analysis focuses on  $\Delta \Psi$  and treats  $\mathcal{V}$  as a constant.

Equations (19) show that, in the current model, all detection probabilities are expressible in terms of integrals of the lapse and shift along rays. Since other geometric quantities such as the curvature tensor  $R_{\mu\nu\rho\sigma}$  do not enter explicitly, information about them can only be obtained by indirect means, e.g., using Eqs. (4). This absence of a direct curvature coupling is the result of the WKB approximation underlying the calculations of Sec. III: more detailed analyses in Refs. [75–77] show that Maxwell’s equations imply curvature-induced corrections to the dispersion relation, but such higher-order corrections are inaccessible to current experiments and are thus beyond the scope of this paper.

Having laid out the general methods for computing detection probabilities for quantum interferometers in a gravitational field, Secs. V and VI describe concrete experiments involving massless and massive particles, respectively.

## V. OPTICAL EXPERIMENTS

This section does not follow the chronological development of the subject but rather presents various experimental proposals sorted according to the mathematical complexity of their models. For simplicity, the calculations presented here mainly focus on *static* frames of reference and treat shift-induced effects perturbatively.

### A. Pound-Rebka experiment

In their 1960 experiment, Pound and Rebka determined the gravitational redshift in  $\gamma$  rays (that were emitted in the decay of  $^{57}\text{Co}$  to  $^{57}\text{Fe}$ ) by measuring the difference in frequency over a height difference of  $h = 74 \text{ ft} \approx 22.6 \text{ m}$  [78]. The result was found to be compatible with the theoretical prediction

$$z = \frac{\nu_h - \nu_0}{\nu_0} \approx \Delta\phi_{0h}/c^2 \approx -gh/c^2 \approx -2.46 \times 10^{-15}, \quad (22)$$

which is a direct application of Eqs. (9). Since this experiment measured the shift in  $\gamma$ -ray frequencies relative to nuclear transition frequencies [79], a debate arose whether the redshift is to be interpreted as a gravitational effect on light—as suggested by the paper title “Apparent weight of photons” [78]—or a gravitational effect on the emitting and absorbing atoms—as suggested by the title of the follow-up paper, “Effect of gravity on nuclear resonance,” by Pound and Snider [80] (see, e.g., Ref. [79] and references therein). Note, however, that the current definition of the second [81] fixes the frequency of light emitted in the ground-state hyperfine transition of  $^{133}\text{Cs}$ . Consequently, in a (hypothetical) variant of the Pound-Rebka experiment involving such radiation, gravitational effects on the emission frequencies of the atoms are excluded *by definition*, thus leading to the alternative description of the redshift as a gravitational effect on light propagation.

The prevalent view that the Pound-Rebka experiment can be described—on the whole or at least in its central aspects—classically, i.e., without quantum theory [34,39,82–85], has motivated the development of alternative experiments that could distinguish more clearly between the predictions of quantum optics in curved space-times and its classical or flat limits.

### B. Tanaka’s proposal

In his 1983 paper, Tanaka suggested using a fiber-optic Mach-Zehnder interferometer for measuring a phase shift that is induced on light by the gravitational redshift [33]. A simplified schematic representation of such an interferometer is provided in Fig. 2. In this setup, a coherent beam of light is injected on the left and split into two beams that traverse fiber-optic spools at different heights (the two spatial trajectories are denoted by  $\gamma_I$  and  $\gamma_{II}$ , respectively), after which the rays interfere at a fiber coupler before being detected. Rotating the interferometer about the horizontal allows modulating the gravitational phase difference between the two rays following the prescribed trajectories. Based on Eqs. (10) and (11), one finds the lapse-induced phase shift for stationary solutions to

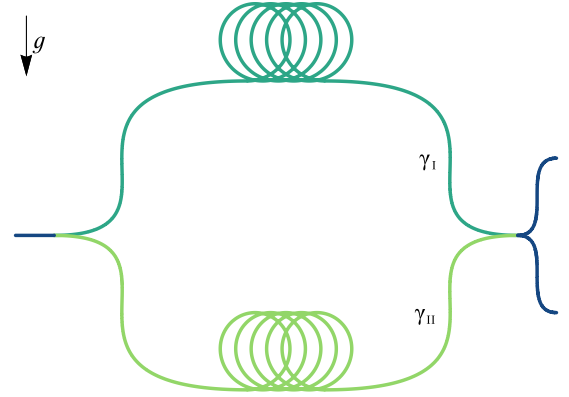


FIG. 2. Simplified schematic of the fiber-optic interferometer proposed in Ref. [33] where a light ray is split into two paths that traverse fiber spools of equal length. The resulting phase shift at the output depends on the vertical separation between the two fiber spools via the redshift.

be given by

$$\Delta\Psi_\zeta = \frac{n\omega_*}{c} \left[ \int_{\gamma_I} \frac{dl}{\zeta} - \int_{\gamma_{II}} \frac{dl}{\zeta} \right], \quad (23)$$

where  $\omega_*$  is the Killing frequency. This ray-optic result agrees with more rigorous fiber-optic calculations up to higher-order curvature corrections that are negligible in most applications [36,77,86]. If both fiber spools have equal length  $l$  and the lapse is approximately constant along each of the two spools, the phase shift  $\Delta\Psi_T$  arising from small vertical separations  $h$  can be approximated by

$$\begin{aligned} \Delta\Psi_T &\approx -ghn\omega/c^3 \\ &\approx -10^{-6} \text{ rad } (h/\text{m}) (l/\text{km}) (\lambda_0/\mu\text{m})^{-1}, \end{aligned} \quad (24)$$

where  $n \approx 1.5$  is the refractive index of glass and  $\lambda_0 = 2\pi c/\omega$  is the wavelength of light in vacuum (see also Ref. [35]).

After Ref. [33] originally suggested performing the experiment with continuous-wave laser output, more recent papers suggested using single photons, pairs of entangled photons, and other nonclassical states of light instead [35,36,40,87]. With appropriately chosen bias phases [specifically  $\pi/(2N)$  for  $N$ -photon NOON states], and if the Sagnac effect is negligible, the single-photon and two-photon detection probabilities take the form

$$P_1 \approx \frac{1}{2} \mathcal{A} [1 - \mathcal{V} ghn\omega/c^3], \quad (25a)$$

$$P_2 \approx \frac{1}{2} \mathcal{A} [1 + 2\mathcal{V}^2 ghn\omega/c^3]. \quad (25b)$$

Whereas the single-photon detection probabilities are identical to classical field intensities [88], the nonclassical factor of 2 in  $P_2$  makes the two-photon coincidence experiment immune to such criticism. As a consequence, two-photon interferometry experiments of this kind could demonstrate gravitationally induced quantum interference without a classical analog.

In Ref. [33], such an experiment was interpreted as a “twin paradox for photons.” However, contrary to the standard “twin paradox,” the rays interfering at the second fiber coupler

(where the world lines intersect) *do not* pass the first fiber coupler at the same time. Indeed, the phase shift given above is fully attributable to such a temporal offset [34,74]. This is consistent with Eq. (7a), which shows that the rays acquire no gravitational phases while propagating through the fibers.

While the formulas given here directly apply to static frames of references, their use in stationary frames (where  $\xi_i \neq 0$ ) requires some caution because the shift-induced phase difference  $\Delta\Psi_\xi$  is generally non-negligible. Using Eqs. (19) the ratio  $|\Delta\Psi_\xi/\Delta\Psi_\zeta|$  can be estimated by  $2(\cos\vartheta)cA\Omega/(gh)$ , where  $A$  is the effective area of the surface enclosed by the interferometer and  $\vartheta$  is the angle between the corresponding surface normal and the local rotation vector  $\Omega^i$ . This shows that, on Earth, the Sagnac phase shift exceeds the gravitationally induced one by a factor of about  $4 \times 10^3 (\cos\vartheta)A/(lh)$ . As a consequence, the isolation of  $\Delta\Psi_\zeta$  poses a severe experimental challenge. (For recent experiments on the Sagnac effect in quantum fiber interferometry, see, e.g., Refs. [89–92]. Their detailed description, however, is beyond the scope of this paper.)

### C. Stodolsky's proposal

In his 1979 paper, Stodolsky proposed a free-space Mach-Zehnder interferometer for measuring the gravitationally induced phase shift on light [32] (this was presented as an optical variant of the experiment done by Colella, Overhauser, and Werner, which is described in Sec. VIA below, and predates Tanaka's proposal described above). A simplified schematic of the setup is provided in Fig. 3. In contrast to Tanaka's setup, light is not guided through optical fibers, but follows free-fall trajectories between the beam splitters and the mirrors. Explicit integration of the geodesic equation (13) in spatial Riemann coordinates with constant  $g_i$  shows that the rays interfering at the second beam splitter ( $D$ ) pass the first beam splitter ( $A$ ) at different times and different positions. To leading order in the acceleration  $g$ , these offsets are given by

$$\Delta t_s = -(ghl/c^3)(\cot^2\theta - 1), \quad (26a)$$

$$\Delta x_s = -(ghl/c^2)\cot^2\theta, \quad (26b)$$

$$\Delta z_s = -(ghl/c^2)\cot\theta, \quad (26c)$$

where  $\theta$  is the inclination of the mirrors and beam splitters that are separated horizontally by the length  $l$  and vertically by the height  $h$ . Here and in the following,  $\Delta x^\mu = x_1^\mu - x_{II}^\mu$  refers to the coordinate separation between the upper ray  $\gamma_I$  ( $ABD$ ) and the lower ray  $\gamma_{II}$  ( $ACD$ ) at the respective events of entering the interferometer.<sup>3</sup> Since the phases are constant along the rays, the phase shift at  $D$  is given by the “separation phase”  $\Psi(x_I) - \Psi(x_{II}) \approx K_\mu \Delta x^\mu$  [see Eq. (6)]. Using  $(K_\mu) \approx (-\omega, \omega/c, 0, 0)$ , the lapse-induced phase shift thus takes the form

$$\Delta\Psi_s \approx -ghl\omega/c^3, \quad (27)$$

<sup>3</sup>The coordinate displacement arising here can be formalized geometrically using the notion of geodesic separation vectors, but this is not necessary for the purpose of computing the difference in phase between the two events with coordinates  $x_1^\mu$  and  $x_{II}^\mu$  using Eq. (6).

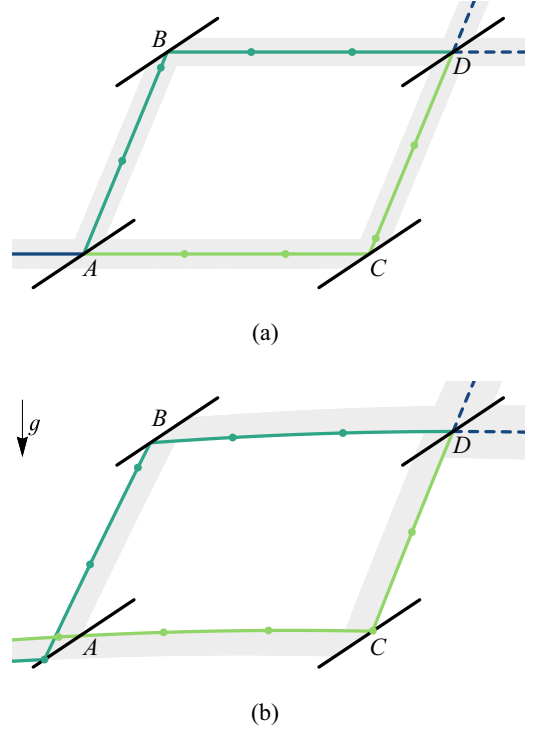


FIG. 3. Schematic representation of optical free-fall trajectories in a Mach-Zehnder interferometer, with dots marking which points along the two rays reach the point  $D$  simultaneously. If the system is inertial, interfering rays at  $D$  originate from the same point on the beam splitter  $A$ , which they pass at the same time (a). This no longer applies in uniformly accelerated interferometers, as is the case in gravimeters (b). Instead, the interfering rays must have a spatial and temporal offset at the first beam splitter  $A$  in order to reach the final beam splitter at the same space-time event. The gray area indicates the minimal beam width required to observe the gravitationally induced phase shift.

which is identical to the phase shift in Tanaka's setup with  $n = 1$ . Hence, even though the two setups differ in the details of light propagation (guided motion within optical fibers is generally not geodesic), the resulting phase shifts and detection probabilities coincide.

The point of view that *geodesic motion* of light in the interferometer plays no essential role is supported by comparison with the analysis of Ref. [34]. The rays considered there are lightlike curves of constant radial coordinate  $r$  in the Schwarzschild chart. Such rays fail to be geodesic unless  $r = 3GM/c^2$  (see Sec. 6.3 of Ref. [48]), a condition that rules out the comparison of two such rays at different  $r$  and is impossible to satisfy on Earth (the photon sphere of the Schwarzschild geometry with  $M$  equal to Earth's mass would lie at  $r \approx 1.3$  cm). Nonetheless, the resulting phase shift agrees with Eqs. (26) to the considered level of approximation. However, these observations do not imply that the geometric details of the interferometer are wholly irrelevant. Indeed, as shown in Appendix A, imbalances in the arm lengths can significantly alter the phase shift. For example, the assumption that the interferometer arms have equal *coordinate lengths* in the frequently used post-Newtonian gauge

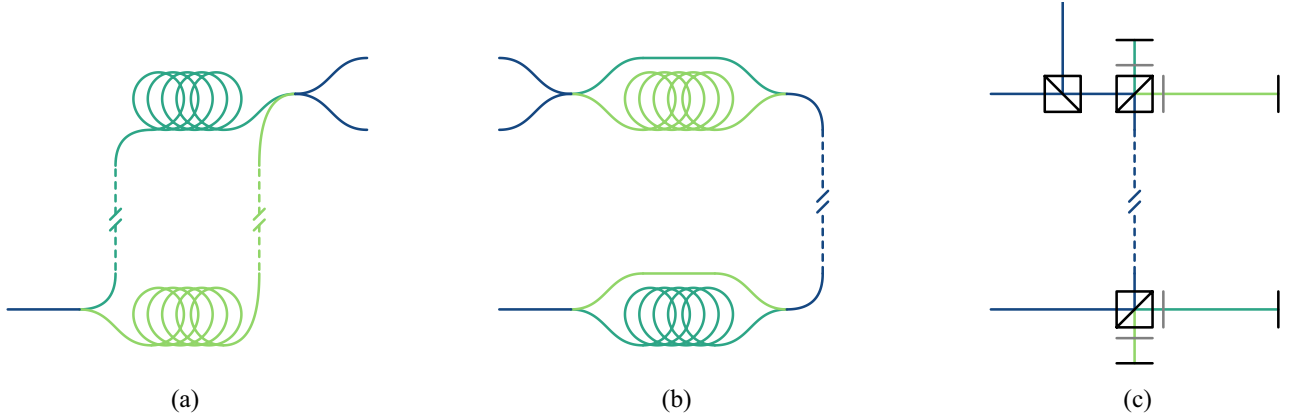


FIG. 4. Schematic representation of satellite experiments to measure the gravitationally induced phase shift on light. (a) The setup is a large-scale version of Tanaka's setup (Sec. V B) and requires two uplinks. The setups (b) and (c), on the other hand, use a scheme similar to time-bin encoding for which a single uplink suffices. All these setups, however, yield the same lapse-induced phase shift.

corresponds to having unequal *physical lengths* and hence leads to deviations from Eq. (27) (see also Ref. [82]).

Similarly to Tanaka's setup, the relative size of the shift-induced phase difference can be estimated by the ratio  $2(\cos \vartheta)c\Omega/g \approx 4 \times 10^3 \cos \vartheta$ . The isolation of the lapse-induced phase shift thus requires special alignments relative to the local rotation axis or alternative compensation or subtraction schemes.

#### D. Proposed satellite experiments

In 2012, Rideout *et al.* proposed measuring the gravitationally induced phase shift of light in a satellite experiment [38]. Subsequent papers described variants of this proposal [10,36,37,39,40,93–95], whose setups are sketched in Fig. 4. In all considered cases, the lapse-induced phase shift takes the form

$$\Delta\Psi_\zeta = \frac{n\omega_* l}{c} \left[ \frac{1}{\zeta_s} - \frac{1}{\zeta_g} \right] = n l \omega_g z_{gs}/c \approx n l \omega_g (\varphi_g - \varphi_s)/c^3, \quad (28)$$

where  $n$  is the refractive index of the fiber-optic delay lines in the setups in Figs. 4(a) and 4(b), and  $n = 1$  in the setup in Fig. 4(c). The parameter  $l$  denotes the length of the delay lines,  $\omega_g$  denotes the optical frequency at the ground station, and  $z_{gs}$  is the redshift factor [10,37–40]. However, the lapse-induced phase shift is not the only relevant phase contribution in satellite experiments [96]. Indeed, in setups of the type in Fig. 4(a) the large area enclosed by the interferometer produces a significant shift-induced phase difference  $\Delta\Psi_\xi$ , and, moreover, in all setups shown in Fig. 4 the satellite's motion gives rise to an additional phase shift  $\Delta\Psi_D$  via the Doppler effect [93]. Future implementations of such experimental setups thus require compensation schemes to isolate  $\Delta\Psi_\zeta$  [39,94].

#### E. Main experimental challenges

All optical experiments described above ultimately depend on the magnitude of the redshift as given by Eq. (9). A direct observation of this effect with single photons requires either a large height difference between the source and receiver, or

the use of short wavelengths, as was done in the Pound-Rebka experiment. For example, at telecommunication wavelengths (1550 nm), the absolute value of the gravitational frequency shift is about 18 000 times smaller than that for the  $\gamma$  rays used in the Pound-Rebka experiment (86 pm). In interferometric experiments using single- or multiphoton states, the gravitational redshift manifests itself as a phase shift between the paths at different altitudes. By extending the path lengths, for example, using optical fibers, this phase shift becomes detectable even at telecom wavelengths and laboratory-scale height differences [34,35].

A key experimental challenge is the isolation of the gravitational signal from phase noise introduced by the light-guiding medium. Any differential change in the optical path length leads to phase shifts that cannot be distinguished from the signal of interest within the same frequency band. To stabilize the interferometer arms, a coherent light source at a frequency distinct from that of the photons in the quantum state can be used. This wavelength-division multiplexing method allows for two different approaches:

(1) The stabilization laser can be used to maintain a constant phase difference between the two arms, regardless of their vertical separation. This approach compensates for the gravitational redshift experienced by the laser and thus only allows to bound any potential difference in the gravitationally induced phase shifts between the laser and the single- or multiphoton quantum states, i.e., the quantity  $\epsilon = \Delta\Psi_\zeta^{\text{laser}} - \Delta\Psi_\zeta^{\text{photon}}$ . (Note, however, that the theoretical prediction  $\epsilon = 0$  is independent of the value of  $g$  and, moreover, does not depend on the validity of Einstein's field equations.)

(2) Alternatively, after initial calibration, the two interferometer arms can be stabilized independently. This enables direct measurements of the lapse-induced phase shift of the quantum states as a function of height, assuming sufficient phase stability in both arms independently.

Independent stabilization places severe bounds on laser stability and accuracy. This can be exemplified by the satellite experiments using time-bin encoding shown in Figs. 4(b) and 4(c). The first constraint arises because different lasers are used to stabilize the two unbalanced interferometers, one

at the ground station and one on the satellite. The difference in their central frequencies,  $\Delta\nu$ , must not exceed the gravitational redshift on the signal photons,  $|\Delta\nu| < \nu_0 |\Delta\phi|/c^2$ . For experiments on Earth, this is a severe constraint as it leads to an admissible difference in laser frequencies of only a few tens of millihertz per meter of height difference, while for satellite experiments the acceptable difference relaxes to several kilohertz. The second constraint arises from the wavelength differences between the stabilization lasers and the photons whose redshift is to be measured. In each unbalanced Mach-Zehnder interferometer, the induced phase difference between the stabilization laser and the quantum-state photons varies with the arm length difference  $\delta l$  between the long arm common to both rays and the short reference arm. Any difference in length imbalance between the ground ( $\delta l_g$ ) and the satellite station ( $\delta l_s$ ) leads to a phase shift between the interfering time-bin amplitudes and must be smaller than the gravitational phase,  $2\pi n |\nu_0 - \nu_{\text{laser}}| |\delta l_s - \delta l_g|/c < |\Delta\Psi_g|$ . This constraint is much less demanding than the one mentioned above, but still requires attention for Earthbound experiments. For 10-km-long fiber spools separated in altitude by 20 m, the frequency difference between telecom-wavelength photons and the laser should be smaller than about 4 MHz for  $|\delta l_g - \delta l_s| = 1$  mm. Since this is too narrow for filtering, the stabilization laser needs to be injected from the other end of the interferometer.

Although optical fibers allow for a comparatively simple way of increasing the interferometric area by extending the optical path length in each arm, they also introduce significant challenges. Losses in the fibers and other optical components can severely limit the number of detected photons and thus the precision in the estimation of the phase shift due to shot noise. High losses thus increase the duration of the measurement and therefore the stability requirements on the experiments. The simultaneous use of quantum states and stabilization lasers within the same optical fiber can also introduce noise via several mechanisms [97]. Spontaneous Raman scattering of photons from the laser into the quantum channel and fundamental thermal noise are among the most difficult to deal with. A potential solution is to use (doubly) nested antiresonant nodeless fibers (DNANFS) [98] instead of standard single mode fibers. In DNANFS, more than 99% of the optical mode can propagate in vacuum with greatly reduced losses [99] and nonlinear effects [100].

Several methods exist to separate the lapse-induced phase from phase shifts induced by the Sagnac effect. The conceptually simplest approach is to orient the interferometer such that its surface normal is aligned at an angle of  $\vartheta = \pi/2$  with respect to Earth's rotation axis. While straightforward in principle, this configuration is only practical for small-scale, laboratory-based experiments. For schemes using laser stabilization to maintain a constant phase relationship between the two arms, the Sagnac effect is common to both the coherent state and the single- or multiphoton quantum states and thus does not contribute to the differential output signal. When optical fibers are used, they can be coiled so that the winding direction is reversed halfway through the fiber length. This configuration ensures that light in both halves of the fiber experiences equal and opposite phase shifts, resulting in a net-zero Sagnac contribution.

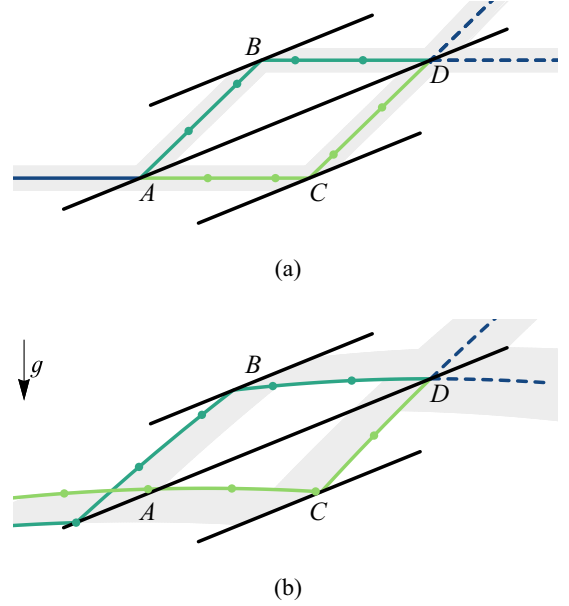


FIG. 5. Schematic representation of the neutron interferometer proposed by Overhauser and Colella in Ref. [101]. (a) If the system is inertial, interfering rays at  $D$  originate from the same point on the beam splitter  $A$ , which they pass at the same time. (b) In uniformly accelerated interferometers, as is the case in gravimeters, this no longer applies. The dots indicate which points along the two rays reach the point  $D$  simultaneously and the gray region shows the minimal beam width required to observe phase shifts in the presence of gravity.

## VI. MATTER-WAVE EXPERIMENTS

Whereas gravitationally induced phase shifts have not yet been observed in optical experiments, analogous effects in matter-wave experiments have already been measured. This section describes the main characteristics of these experiments using the general methods developed in Secs. III and IV. Similarly to the analyses in Sec. V above, the calculations presented here mainly focus on static frames of reference for simplicity of exposition.

### A. Colella-Overhauser-Werner experiment

In their 1974 paper, Overhauser and Colella proposed a neutron-interferometry experiment for demonstrating gravitationally induced phase shifts [101], a simplified schematic of which is shown in Fig. 5. In this setup, a neutron beam is split, reflected, and recombined by diffraction at three parallel slabs cut from a single crystal of silicon. The expected phase shift in the Overhauser-Colella setup is given by

$$\Delta\Psi_{\text{oc}} = -\frac{ghl}{\hbar v^2} \frac{mv}{\sqrt{1-v^2/c^2}} = -\frac{ghlm}{\hbar v} [1 + O(v^2/c^2)], \quad (29)$$

where the height difference  $h$  depends on the base length  $l$  and the inclination angle  $\theta$  of the crystal slabs,  $h = l \sin(2\theta)$ , and  $v$  is the neutron's velocity. The leading-order term in Eq. (29) was originally derived using the semiclassical approximation of the Schrödinger equation and assuming the neutron rays to be straight lines [101]. Similar derivations of

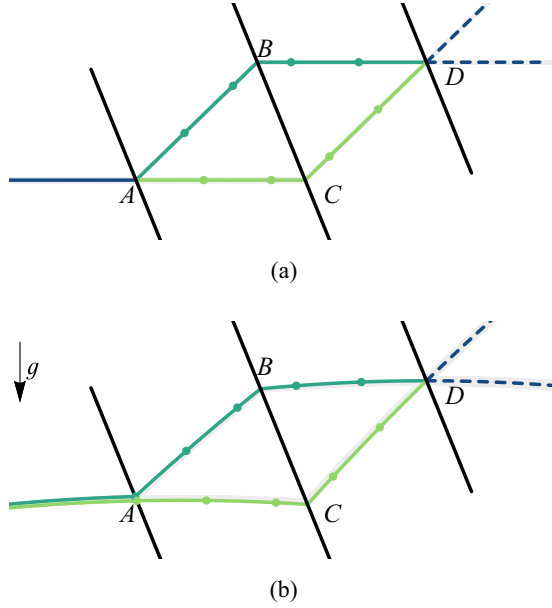


FIG. 6. Schematic representation of the neutron interferometer described by Colella, Overhauser, and Werner in Ref. [18]. In the absence of gravity (a), the rays form a rhombus similar to that in the setup proposed by Overhauser and Colella, see Fig. 5(a), but in the presence of gravity (b) the free-fall trajectories differ from those shown in Fig. 5(b). As a consequence, the minimal beam width and the offsets at the first beam splitter (indicated by the gray band and the dots along the rays) are reduced. The overall phase shift, however, is the same in both setups.

the phase shift using straight rays are given, for example, in Refs. [10,21,102–104]. In his 1998 paper, Mannheim showed that—by taking into account the bending of the rays under the influence of gravity—the phase shift can be explained by the fact that rays interfering at the output must have reached the central crystal slab at different times [22]. Specifically, the temporal and spatial offsets of the rays shown in Fig. 5(b) are given by

$$\Delta t_{oc} = -\frac{ghl}{v^3} \left[ \cot^2 \theta + 1 - 2 \frac{v^2}{c^2} \right], \quad (30a)$$

$$\Delta x_{oc} = -\frac{ghl}{v^2} \cot^2 \theta, \quad (30b)$$

$$\Delta z_{oc} = -\frac{ghl}{v^2} \cot \theta, \quad (30c)$$

as can be calculated by explicitly integrating the geodesic equation (13) if the spatiotemporal extent of the experiment is sufficiently small so that curvature terms are negligible. In the limit  $v \rightarrow c$ , the offsets given in Eqs. (30) reproduce those for light stated in Eqs. (26). Similarly, the phase shift  $\Delta \Psi_{oc}$  reproduces the optical result  $\Delta \Psi_s = -ghl\omega/c$  provided one replaces the relativistic particle momentum  $mv/\sqrt{1-v^2/c^2}$  with the optical analog  $\hbar k = \hbar\omega/c$ .

In their 1975 paper, Colella, Overhauser, and Werner (COW) reported on the experimental observation of gravitationally induced phase shifts in single neutrons using a setup whose geometry differs from that originally proposed by Overhauser and Colella [18] (see Fig. 6 for a simplified

schematic). Compared to the Overhauser-Colella setup shown in Fig. 5, the incoming rays are sent toward one of the outer crystal slabs, rather than the middle one. As a consequence, the free-fall trajectories in the Overhauser-Colella setup cross, but those in the COW setup do not [cf. Figs. 5(b) and 6(b)]. Similarly to the Overhauser-Colella setup, the rays interfering at the output must have reached the first crystal slab at different times and at different locations:

$$\Delta t_{cow} = -\frac{ghl}{v^3} \left[ \tan^2 \theta + 1 - 2 \frac{v^2}{c^2} \right], \quad (31a)$$

$$\Delta x_{cow} = -\frac{ghl}{v^2} \tan^2 \theta, \quad (31b)$$

$$\Delta z_{cow} = -\frac{ghl}{v^2} \tan \theta \quad (31c)$$

[cf. Eq. (30)]. The overall phase shift, however, is identical to that of the previous setup:

$$\Delta \Psi_{cow} = -\frac{ghl}{\hbar v^2} \frac{mv}{\sqrt{1-v^2/c^2}} = -\frac{ghlm}{\hbar v} [1 + O(v^2/c^2)]. \quad (32)$$

Similarly to the Overhauser-Colella phase shift, there are multiple derivations for Eq. (32) based on various approximations, differing, in particular, in whether the rays are taken to be straight lines or free-fall parabolas (cf., e.g., Refs. [10,105]). Equation (32) can be generalized to account for the nonzero thickness of the crystal slabs [18,106,107] but the detailed description of such nongravitational effects is beyond the scope of this paper.

Contrary to the optical experiments described in Secs. VB and VC, the parameters entering Eqs. (29) and (32) are not independent since the diffraction of the rays must satisfy the Bragg-Laue condition  $\mathbf{n} \cdot (\mathbf{p}_{out} - \mathbf{p}_{in}) = \hbar \mathbf{n} \cdot \mathbf{k}_{lattice}$ , where  $\mathbf{k}_{lattice}$  is a reciprocal lattice vector and  $\mathbf{n}$  is a collinear unit vector.<sup>4</sup> Using this notation, for  $v \ll c$ , Eq. (32) can be written in the form

$$\Delta \Psi_{cow} = -\mathbf{g} \cdot \mathbf{k}_{lattice} T_*^2, \quad (33)$$

where  $T_* = l/v$  is the time of uninterrupted free fall (up to negligible corrections of higher order) (see also Appendix A of Ref. [67]). The observation that the parameters  $h$ ,  $l$ , and  $m$  in Eqs. (29) and (32) are constrained to a two-dimensional parameter space resolves the tension between the statement that “the phase shifts in atomic beam and neutron interferometry in general do depend on the mass” [108], on the one hand, and that “the phase shift is independent of the mass of the atoms (neutrons) or the associated Compton frequency” [67] on the other.

Similarly to the optical experiments described above, the relative size of the shift-induced phase difference can be

<sup>4</sup>Specifically, if  $d_{lattice}$  denotes the lattice constant along  $\mathbf{n}$ ,  $\mathbf{k}_{lattice}$  must be an integer multiple of  $(2\pi/d_{lattice})\mathbf{n}$ , with the absolute value of this integer coefficient being commonly referred to as the order of the diffraction process (the diffraction in the COW experiment was of first order). In practice, reflection also occurs for momenta that do not satisfy the diffraction condition *exactly*, with the finite range of admissible  $\mathbf{p}_{in}$  being known as the Darwin plateau.

estimated by the ratio  $2(\cos \vartheta)v\Omega/g$ . For the COW experiment, for example, this evaluates to  $\sim 4 \times 10^{-2} \cos \vartheta$ . The reason for this suppression of shift-induced effects compared to optical setups lies in the velocity ratio  $v/c \ll 1$ . Hence, decreasing the particle's velocity reduces the relative error produced by neglecting the shift-induced phase.

As shown in Eqs. (11) and (12), the gravitationally induced phase shift in neutrons is larger than that in photons by a factor that is given by the ratio of the optical wavelength to the neutron's de Broglie wavelength. As a result, gravitational effects can be measured using interferometers with arm lengths of only a few centimeters [20], in contrast to the kilometer-scale interferometers required for optical experiments, as discussed above. However, current results from COW experiments on neutron interferometry can also be derived using a purely Newtonian gravitational description and, as such, do not provide further insight into the combined framework of quantum theory and general relativity. In order to measure effects indicative of general relativistic theory such as space-time curvature or Lense-Thirring precession, a much larger interaction area would be required, since the corrections from higher-order gravitational effects are on the order of  $10^{-9}$  of the COW phase [109]. Recent advancements in neutron interferometry include the fabrication of large, defect-free silicon crystals or use of other materials for neutron diffraction [110,111], very cold neutron sources [112], and improvements in environmental shielding and detection [113]. While these are increasing the sensitivity of neutron interferometry, it is still limited by small interaction areas, pushing gravitational research with massive particles toward atomic-fountain experiments that allow for a much larger interaction length (see Sec. VIB). Alternative proposals include focusing on using the neutron's spin precession in a magnetic field as a “clock” to track the proper time of the neutron's evolution, and thus allowing measurement of the proper time difference across the gravitational field [114]. However, these effects would be minuscule ( $\sim 10^{-10}$  of the COW phase for a magnetic field of 10 T), and are thus out of reach for current technologies. An in-depth discussion on possible neutron experiments involving gravity is provided in Chap. 8 of Ref. [115].

### B. Atomic fountains

Whereas the interferometers described above enclose nonzero areas, in atomic-fountain experiments the particle motion is commonly approximated as being restricted to one spatial dimension [73,116]. More generally, if motion takes place in the vicinity of a spatial curve, one can locally erect spatial Fermi coordinates  $(x, y, z) \equiv (x^\alpha, z)$ , relative to which the spatial metric  $l_{ij}$  takes a particularly simple form [77]. If the reference curve is a spatial geodesic (as is relevant, for example, for radial motion in spherically symmetric geometries), one has

$$l = [1 - P_{\alpha\gamma\beta} x^\alpha x^\beta](dz)^2 + [\delta_{\alpha\beta} - \frac{1}{3}P_{\alpha\gamma\beta\delta} x^\gamma x^\delta]dx^\alpha dx^\beta + O[(\delta_{\alpha\beta} x^\alpha x^\beta)^{3/2}], \quad (34)$$

where the components of the spatial curvature tensor  $P_{ijkl}$  are evaluated on the reference curve (along which  $x^\alpha = 0$ ). If motion takes place only along the reference curve, the exact

equations of motion (13) in a static reference frame take the form

$$\ddot{z} = -(1 - \dot{z}^2/c^2)\varphi'(z), \quad (35a)$$

$$\dot{t} = \exp[-\varphi(z)/c^2], \quad (35b)$$

where dots indicate derivatives with respect to the temporal parameter  $T$ . Although these equations generally do not admit closed-form solutions, for practical applications it suffices to use short-time expansions of the form

$$t(T) = t_0 + \frac{e^{-\varphi(z_0)/c^2}}{\tilde{\gamma}_0} \left\{ \tilde{\gamma}_0 T - \frac{(\tilde{\gamma}_0 T)^2}{2!} \frac{p_0}{mc} \frac{\varphi'_0}{c} - \frac{(\tilde{\gamma}_0 T)^3}{3!} \left[ \frac{p_0^2}{m^2 c^2} \varphi''_0 - \left( 1 + \frac{p_0^2}{m^2 c^2} \right) \frac{(\varphi'_0)^2}{c^2} \right] + O(T^4) \right\}, \quad (36a)$$

$$z(T) = z_0 + \frac{p_0}{m} \tilde{\gamma}_0 T - \frac{(\tilde{\gamma}_0 T)^2}{2!} \varphi'_0 - \frac{(\tilde{\gamma}_0 T)^3}{3!} \frac{p_0}{m} \left( \varphi''_0 + 2 \frac{(\varphi'_0)^2}{c^2} \right) + O(T^4), \quad (36b)$$

$$\tau(T) = \tau_0 + \tilde{\gamma}_0 T + \frac{(\tilde{\gamma}_0 T)^2}{2!} \frac{p_0}{mc} \frac{\varphi'_0}{c} + \frac{(\tilde{\gamma}_0 T)^3}{3!} \left[ \frac{p_0^2}{m^2 c^2} \varphi''_0 - \left( 1 - \frac{p_0^2}{m^2 c^2} \right) \frac{(\varphi'_0)^2}{c^2} \right] + O(T^4), \quad (36c)$$

where subscript zeros indicate initial values at  $T = 0$ ,  $p = m\dot{z}/\sqrt{1 - \dot{z}^2/c^2}$  denotes the momentum, and  $\tilde{\gamma} = 1/\sqrt{1 + p^2/m^2 c^2}$  is the reciprocal Lorentz factor. For simplicity of exposition, the following analysis is carried out in the limit  $c \rightarrow \infty$ , in which case Eqs. (36) reproduce the trajectories of Newtonian mechanics. Corrections beyond this limiting case are described in Appendix B.

Whereas Eqs. (11) and (12) suffice for computing the phase shifts in the experiments described above, this is no longer the case here since the atom-light interactions produce nontrivial phase shifts. Specifically, the interaction of the atoms with  $\pi$  and  $\pi/2$  pulses produces additional phase shifts of the form

$$\Delta\Psi_* = \sum_i [n_i^I \Psi_{\text{EM}}(t_i^I, z_i^I) - n_i^{\text{II}} \Psi_{\text{EM}}(t_i^{\text{II}}, z_i^{\text{II}})]. \quad (37)$$

Here,  $i$  extends over all atom-light interactions at the coordinates  $(t_i^I, z_i^I)$  in the upper arm and at  $(t_i^{\text{II}}, z_i^{\text{II}})$  in the lower arm,  $\Psi_{\text{EM}}$  denotes the phase of the electromagnetic field, and the integers  $n_i^I$  and  $n_i^{\text{II}}$  depend on the details of the interaction [28,73]: for Bragg transitions, the absolute values of  $n_i^I$  and  $n_i^{\text{II}}$  indicate the order of the transition, whereas  $|n_i^I| = |n_i^{\text{II}}| = 1$  for Raman transitions, and the signs of  $n_i^I$  and  $n_i^{\text{II}}$  indicate whether the atom's momentum along the gradient of  $\Psi_{\text{EM}}$  increases or decreases in the process of the interaction. A heuristic derivation of Eq. (37) can be found in Sec. 3.2 of Ref. [117]; more detailed derivations for Bragg and Raman transitions are provided in Refs. [118] and [119], respectively. If Raman transitions are used, additional phase shifts arise if the two interfering rays spend unequal times in the various internal energy states. However, since the experiments

considered here have equal time intervals between the light-matter interactions, these additional phase contributions vanish [67].

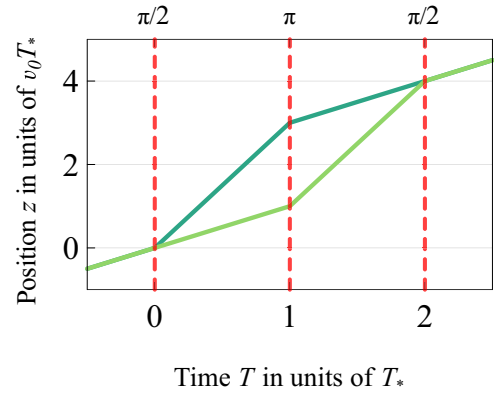
*a. Gravimetry.* In their 1991 paper, Kasevich and Chu reported on the first atomic-fountain experiment capable of functioning as a gravimeter [25]. While this experiment used stimulated Raman transitions (see Ref. [26] for a detailed model of the experiment), in 2013, Altin *et al.* reported on a refined experiment using Bragg transitions instead [28]. Simplified space-time diagrams of these experiments are shown in Figs. 7(a) and 7(b): a  $\pi/2$  pulse at  $T = 0$  acts as a beam splitter to produce atomic wave packets that propagate upward with different velocities  $v_0$  and  $v_0 + \Delta v_r$ , respectively, where  $\Delta v_r = n\hbar k_{\text{EM}}/m$  is the recoil velocity (in which  $n \equiv n_1^1$  denotes the order of the diffraction process). After the “interrogation time”  $T_*$ , the wave packets are subject to a  $\pi$  pulse that changes the velocities by  $\pm n\hbar k_{\text{EM}}/m$ . The wave packets meet at  $T = 2T_*$ , where a final  $\pi/2$  pulse brings them to equal final velocities in each output arm, after which interference is observed. Integrating Eq. (7b) along the trajectories given by Eqs. (36) shows that  $\Delta\Psi_\zeta$  vanishes if gravity gradients and  $v^2/c^2$  corrections are negligible. This is in agreement with calculations based on the Schrödinger equation for a particle in a linear gravitational potential [69]. Assuming low velocities, i.e.,  $v \ll c$ , and an electromagnetic phase of the form  $\Psi_{\text{EM}} = \omega_{\text{EM}}(z/c - t)$ , one obtains

$$\Delta\Psi_\zeta = 0, \quad (38a)$$

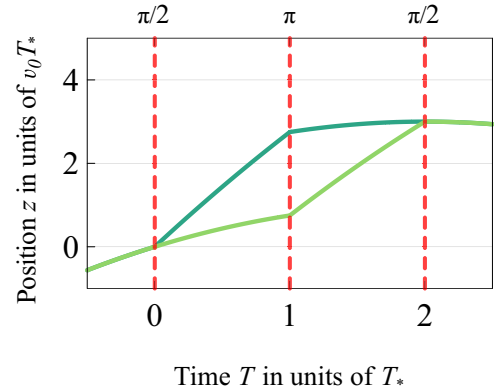
$$\Delta\Psi_* = -gm\Delta v_r T_*^2/\hbar^2 = -n\mathbf{g} \cdot \mathbf{k}_{\text{EM}} T_*^2, \quad (38b)$$

where  $g = d\phi/dz$  is the vertical gravitational acceleration. Hence, the overall phase shift is structurally identical to that of Eq. (33). The interpretation of this result was debated in the literature [66–69, 120–123]. Specifically, in view of Eq. (7b), Müller *et al.* interpreted the phase shift as a measure of proper time tracked by a “Compton clock” [66]. However, Sinha and Samuel [68] as well as Giulini [69] argued that Eq. (7b) does not describe any observable periodic changes in internal degrees of freedom. Moreover, Wolf *et al.* showed that both interfering rays have equal proper times [67, 120]. This last observation is consistent with  $\Delta\Psi_\zeta = 0$  derived here.

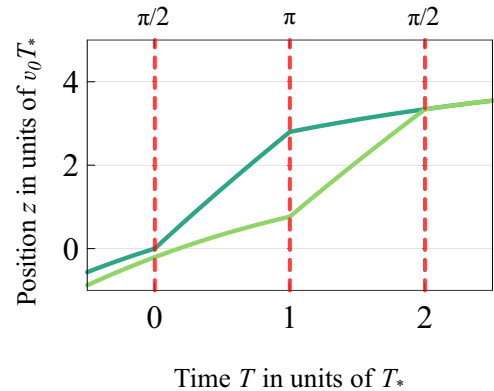
In a subsequent experiment, Asenbaum *et al.* [124] compared high-precision phase measurements from an atom cloud of  $^{87}\text{Rb}$  to that of  $^{85}\text{Rb}$  in order to measure differential gravitational accelerations between the two isotopes. The results showed no deviations from universality of free fall to the level of  $10^{-12}g_{\text{std}}$ , where  $g_{\text{std}}$  denotes the standard acceleration of gravity. The atom clouds ( $\sim 10^5$  atoms per cloud) underwent evaporative cooling before being launched from the atom trap to a height of 8.6 m. With a total free-fall time of  $2T_* = 1910$  ms, the experiment had a single-shot acceleration sensitivity of  $1.4 \times 10^{-11}g_{\text{std}}$ . Experimental challenges to reach this sensitivity involve an AC-Stark shift generated by the atom launch optics, which can be reduced by improved laser systems; differential magnetic gradients across the drop tower, mitigated significantly by generating a uniform magnetic field across the experiment with a solenoid; and imaging errors, which are addressed by calibration of the atom imaging optics.



(a)



(b)



(c)

FIG. 7. Schematic space-time diagrams of atomic-fountain interferometers (a) in an inertial system, (b) in a uniform gravitational field, and (c) in an inhomogeneous gravitational field. Atoms are launched into a vacuum tube and manipulated using a sequence of laser pulses. In the absence of gravity gradients, both rays take equal amounts of proper time regardless of the vertical acceleration  $g$ . The strength and form of the gravitational field determines the position of the light-matter interactions and thus also the phase shifts acquired in the course of these interaction events. If the relative recoil velocities are in the ratio 1:–2:1, the rays interfering at the output are separated at the input by a height difference that depends on the gravity gradient (b) and (c).

*b. Gradiometry.* Using multiple gravimeters at different heights, it is possible to measure gravity gradients  $\gamma = dg/dz$  [27,43,73,125–127] and even second spatial derivatives of the gravitational acceleration [128]. Additionally, it is possible to extend the free-fall heights of atomic-fountain experiments so that higher-order terms in Eqs. (36) become relevant. Specifically, if  $T^3$  terms are retained in the perturbative expansion and if the same electromagnetic pulse sequence is used as for the gravimeters described above, the rays interfering at the output must have a vertical offset  $\Delta z_0 = \gamma \Delta v_r T_*^3 + O(T_*^4)$  at the input [129] [see Fig. 7(c)]. In this case (assuming, as above,  $v \ll c$ ), the total phase shift takes the form  $\Delta\Psi = \Delta\Psi_\zeta + \Delta\Psi_*$  with

$$\Delta\Psi_\zeta = \frac{1}{2}\gamma m \Delta v_r^2 T_*^3 / \hbar + O(T_*^4), \quad (39a)$$

$$\Delta\Psi_* = -gm \Delta v_r T_*^2 / \hbar - \gamma m \Delta v_r (v_0 + \Delta v_r) T_*^3 / \hbar + O(T_*^4), \quad (39b)$$

in agreement with calculations based on the Schrödinger equation with a Newtonian potential [130]. Alternatively, the frequencies of the electromagnetic pulses can be adjusted (depending on the gravity gradient  $\gamma$ ) to make the wave packets overlap at both the input and the output. In this case, the total phase shift takes the same value as given here [67]. Such a gravity-gradient-induced phase shift was measured by Overstreet *et al.* in 2022 [31].

Equations (39) also show that gravity gradients can introduce systematic errors in measurements of  $g$ . For high-precision gravimetry, additional gravity-gradient terms of order  $T_*^4$  must also be accounted for (see Appendix C).

Atomic-fountain experiments of this kind are candidates for many future gravitational experiments. One such proposal suggests using atomic fountains to detect gravitational waves (GWs), allowing measurement of signals out of reach for current optical detection systems [131–133]. These include both terrestrial and space-based detection systems, consisting of multiple atom interferometers measuring variations in the timing of the laser pulses caused by a GW passing through the interferometer. Other space-based atom interferometry experiments have been proposed for further tests of equivalence principles and for dark matter searches [134]. Recent advancements in atom lattice traps are allowing the possibility of gravitational measurements with trapped atoms [30]. These experiments have the advantage of extended interaction times of tens of seconds. However, a theoretical description of these guided atom evolutions is beyond the scope of this paper and is one avenue for future expansion of the framework described here.

## VII. DISCUSSION

In this paper, we developed a unified theoretical framework for describing quantum interference in both massless and massive systems in the presence of external stationary gravitational fields. Compared to perturbative weak-field expansions, our geometric approach offers a simpler and more transparent description, reducing interpretational ambiguities:

(1) Our geometric description does not rely on any flat reference geometry and thus eliminates complications in mod-

eling experimental setups using charts for which coordinate differences and lengths computed using a flat reference metric  $\eta_{\mu\nu}$  differ from physical lengths computed using the full metric  $g_{\mu\nu}$  (cf., e.g., Ref. [86]).

(2) This approach makes it evident that details of the spatial metric tensor do not matter for the description of the experiments described here. In particular, none of the experiments discussed here are sensitive to the parameter  $\gamma_{\text{PPN}}$  arising in the parametrized post-Newtonian (PPN) framework. Previous derivations of this statement were restricted to optical interferometry and required significantly more extensive computations [36,82].

(3) Similarly, the general equations describing the phase evolution of massive and massless particles do not involve the space-time curvature tensor. Hence, curvature components can only be inferred by indirect means, e.g., by varying external parameters such as the spatial extent of the interferometer. In this context, we derived an exact formula that relates specific components of the space-time curvature tensor to the gravitational acceleration and its gradient [see Eq. (5)]. This provides a rigorous basis for the commonly used heuristic of interpreting gravity gradients as an indicator for curvature.

(4) Moreover, our computation of lapse-induced phases in photons does not rely on heuristic methods such as effective refractive indices arising from a gravitational potential, nor does it require assigning to photons an effective gravitational mass.

We derived general formulas for the phase evolution of different quantum systems and used them to analyze various experimental configurations. For experiments with photons, we showed that free-space and fiber-based interferometers yield essentially the same detectable lapse-induced phase shift. In both scenarios, the use of entangled quantum states provides insights into the gravitational coupling as the resulting detection probabilities cannot be mimicked by classical field intensities. In experiments where gradients of the lapse-induced phase shifts are measurable, such setups enable the direct observation of specific components of the Riemann curvature tensor with nonclassical probes. Our geometric approach also yields predictions for phase shifts in experiments that probe gravity using unguided matter waves instead of photons. The phase shifts we derived for Mach-Zehnder, Bonse-Hart, and atomic-fountain configurations not only agree with previous results obtained assuming either Newtonian gravity or linearized Einsteinian gravity, but our method also extends these results to arbitrary stationary gravitational fields. The interpretation of the phase shifts arising for both the optical and matter-wave experiments follows naturally from our description. While gravitational phase shifts in optical interferometers arise directly from the lapse function, the situation is more subtle for matter-wave interferometers. In matter-wave gravimeters, the lapse-induced phase shift vanishes and the observed phase originates from the interaction with light pulses at different locations that depend on the gravitational acceleration. However, when gravity gradients are taken into account, lapse-induced phase shifts reemerge in addition to the phase from the light-matter interaction and are directly proportional to the gravity gradient. Our methods can be extended to include internal degrees of freedom and also guided motion of massive particles.

## ACKNOWLEDGMENTS

We thank Maximilian Ofner and Francesco Giovineti for useful discussions. This research is funded by the European Union (ERC, GRAVITES, Project No. 101071779). T.M. is supported by the Vienna Doctoral School in Physics (VDSP).

Views and opinions expressed are those of the authors only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.

## DATA AVAILABILITY

No data were created or analyzed in this study.

## APPENDIX A: PHASE SHIFTS IN LINEARIZED GRAVITY

In linearized gravity, the metric tensor is decomposed as  $g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon_{\mu\nu}$ , where  $\eta_{\mu\nu}$  is the flat Minkowskian metric, and squares and higher powers of the metric perturbation  $\varepsilon_{\mu\nu}$  are assumed to be negligible (see Chap. 4.4 of Ref. [48]). In this setting, versions of Eqs. (27) and (32) can be derived by integrating Eq. (6) along curves other than the actual rays: denoting by  $K_\mu^0$  the “unperturbed” wave covector satisfying  $\eta^{\mu\nu} K_\mu^0 K_\nu^0 = -m^2 c^2 / \hbar^2$  (with  $m = 0$  for light), the eikonal equation implies that the “perturbed” wave covector in linearized gravity can be written as

$$K_\mu = K_\mu^0 + \frac{1}{2} \varepsilon_{\mu\nu} \bar{K}_0^\nu + \kappa_\mu, \quad (\text{A1})$$

where  $\bar{K}_0^\mu = \eta^{\mu\nu} K_\nu^0$  is the “unperturbed” wave vector and  $\kappa_\mu$  is any one-form satisfying  $\bar{K}_0^\mu \kappa_\mu = 0$ . For any two points  $p_1$  and  $p_2$  lying along an integral curve  $\Gamma$  of  $\bar{K}_0^\mu$ , Eq. (6) yields

$$\Psi(p_2) = \Psi(p_1) + \int_\Gamma \left[ K_\mu^0 + \frac{1}{2} \varepsilon_{\mu\nu} \bar{K}_0^\nu \right] dx^\mu, \quad (\text{A2})$$

in which  $\kappa_\mu$  does not contribute because of its orthogonality to  $\bar{K}_0^\mu$  (see Sec. 8 of Ref. [32]). When using this equation, care must be taken to use appropriate curves along which the integrals are to be evaluated. In particular, these curves *cannot* be prescribed independently of  $\varepsilon_{\mu\nu}$ . This can be seen, for example, by studying the transformation of the integral arising in Eq. (A2) under linearized diffeomorphisms, under which  $\varepsilon_{\mu\nu}$  changes as  $\delta \varepsilon_{\mu\nu} = \partial_\mu \lambda_\nu + \partial_\nu \lambda_\mu$ . If the curve  $\Gamma$  were invariant under such gauge transformations, then the integral  $I_\Gamma = \int_\Gamma [K_\mu^0 + \frac{1}{2} \varepsilon_{\mu\nu} \bar{K}_0^\nu] dx^\mu$  would transform as

$$\delta I_\Gamma = (\bar{K}^\mu \lambda_\mu)(p_2) - (\bar{K}^\mu \lambda_\mu)(p_1), \quad (\text{A3})$$

where  $p_1$  and  $p_2$  are the initial and final end points of  $\Gamma$ , respectively. This shows that gauge-invariant predictions for phase shifts can be computed using Eq. (A2) only if the coordinate expressions for the curves involved are gauge *dependent*.

To illustrate this point, consider Eq. (A2) applied to null rays with (angular) coordinate frequency  $\omega$  propagating along straight coordinate lines joining the points

$$(x_A^i) = (0, 0, 0), \quad (x_B^i) = (0, 0, h), \quad (\text{A4a})$$

$$(x_C^i) = (l, 0, 0), \quad (x_D^i) = (l, 0, h). \quad (\text{A4b})$$

If  $l_{ij} = \delta_{ij}$ , the parameters  $h$  and  $l$  accurately represent the physical arm lengths of the interferometer. Integrating Eq. (A2) in the linear potential  $\varphi = gz$ , one obtains the same result as in Eq. (27). If, instead, the metric is given by  $l_{ij} = (1 + 2gz/c^2)\delta_{ij}$ , which corresponds to a post-Newtonian metric with a linear potential (see Sec. 18.4 of Ref. [135]), then the geodesic distances of the “horizontal” segments are unequal:

$$l_{AC} \approx l, \quad l_{BD} \approx (1 + gh/c^2)l. \quad (\text{A5})$$

This mismatch causes an additional phase shift  $\Delta\Psi_{\text{mismatch}} = -\omega(l_{AC} - l_{BD})/c \approx \omega gh/c^3$ , so that the overall phase shift is twice as large as that in Eq. (27):

$$\Delta\Psi_{\text{imbalanced}} \approx -2gh\omega/c^3. \quad (\text{A6})$$

Identical coordinate expressions for the spatial curves in Eq. (A2) thus yield different phase shifts depending on the explicit form of  $\varepsilon_{\mu\nu}$ , despite the fact that the two fields are related by a gauge transformation, namely, that generated by

$$\lambda_x = gxz, \quad \lambda_y = gyz, \quad \lambda_z = \frac{1}{2}g(z^2 - x^2 - y^2). \quad (\text{A7})$$

This analysis also shows that experimental tests of Eq. (27) require precise control over the interferometer’s arm lengths, as relative imbalances of  $gh/c^2 \approx 10^{-16}(h/\text{m})$  may double or cancel the phase shift induced by the gravitational redshift.

## APPENDIX B: FSL CORRECTIONS IN ATOMIC INTERFEROMETERS

Because of the finiteness of the speed of light (FSL), the optical pulses in an atomic fountain must be “chirped” to account for frequency shifts between the stationary reference frame and the atoms [73], and the  $\pi$  transitions in the two interferometer arms are not simultaneous [136–139]. The influence of these FSL effects on the interferometric phase can be computed efficiently using the “retarded” time  $\mathcal{T}(t, z)$  that measures at what proper time a light signal at zero altitude must be emitted in order for it to reach the point  $(t, z)$ . This function satisfies the eikonal equation

$$g^{\mu\nu}(\partial_\mu \mathcal{T})(\partial_\nu \mathcal{T}) = 0 \quad (\text{B1})$$

with boundary conditions  $\mathcal{T}(t, z = 0) = t e^{\varphi(0)/c^2}$  and can be computed perturbatively in  $z$ :

$$\begin{aligned} \mathcal{T}(t, z) = & t e^{\varphi(0)/c^2} - \frac{z}{c} + \frac{z^2}{2!} \frac{\varphi'(0)}{c^3} \\ & + \frac{z^3}{3!} \left( \frac{\varphi''(0)}{c^3} - \frac{\varphi'(0)^2}{c^5} \right) + O(z^4). \end{aligned} \quad (\text{B2})$$

Since the electromagnetic phase is constant along null rays, the eikonal is of the form  $\Psi_{\text{EM}}(t, z) = f(\mathcal{T}(t, z))$ , where  $f$  describes the waveform as a function of proper time at  $z = 0$ , and hence  $df/d\mathcal{T}$  is the (negative) frequency emitted at  $z = 0$  that is continuously adjusted to be resonant for the internal transition of the moving atoms. The condition that the various light-matter interactions arise from light rays emitted at pre-determined values of  $\mathcal{T}$  can be implemented by parametrizing the world lines by  $\mathcal{T}$  instead of  $T$ . Explicitly, the relation between the free-fall time  $T$  and differences in retarded time

$\mathcal{T}$  is given by

$$\begin{aligned} T = \Delta\mathcal{T} & \left( 1 + \frac{p_0}{mc} + \frac{p_0^2}{m^2c^2} + \frac{\varphi(z_0) - \varphi(0)}{c^2} \right) \\ & - \frac{(\Delta\mathcal{T})^2}{2!} \left( \frac{\varphi'(z_0)}{c} + \frac{p_0}{mc} \frac{\varphi'(z_0)}{c} \right) \\ & - \frac{(\Delta\mathcal{T})^3}{3!} \left( \frac{p_0}{mc} \varphi''(z_0) + \frac{p_0^2}{m^2c^2} \varphi''(z_0) - 2 \frac{\varphi'(z_0)^2}{c^2} \right) \\ & + O((\Delta\mathcal{T})^4) + O(c^{-3}). \end{aligned} \quad (\text{B3})$$

Extending the analysis in Sec. VIB to account for these corrections leads to a phase shift of the form

$$\begin{aligned} \Delta\Psi_\zeta = & -\frac{\hbar k^2}{2mc} \varphi_0' T_*^2 - \left( 1 + \frac{4p_0}{mc} + \frac{3\hbar k}{mc} \right) \frac{\hbar k^2}{2m} \varphi_0'' T_*^3 \\ & + O(T_*^4) + O(c^{-3}), \end{aligned} \quad (\text{B4})$$

$$\begin{aligned} \Delta\Psi_* = & -\left( 1 + \frac{p_0}{mc} \right) k \varphi_0' T_*^2 - \left( 1 + \frac{2p_0}{mc} \right) \frac{k p_0}{m} \varphi_0'' T_*^3 \\ & + \frac{k}{c} (\varphi_0')^2 T_*^3 + O(T_*^4) + O(c^{-3}). \end{aligned} \quad (\text{B5})$$

In isolation, the results for  $\Delta\Psi_\zeta$  and  $\Delta\Psi_*$  do not converge to those given in Eqs. (39). This is because even though the world lines with  $c < \infty$  converge to the Newtonian trajectories as  $c \rightarrow \infty$ , the eikonal along them evolves as  $d\Psi = -mc^2 d\tau/\hbar$ , whose  $c^2$  dependence makes the result depend on higher-order corrections to the trajectories. However, the observable phase shift  $\Delta\Psi = \Delta\Psi_\zeta + \Delta\Psi_*$  converges to that of Eqs. (39) as  $c \rightarrow \infty$ , thus leading to unambiguous predictions.

More comprehensive models can be obtained by taking into account the finite interaction times of the atoms with the electromagnetic pulses. This leads to further  $1/c$  contributions to the total phase (see, e.g., Ref. [139]).

### APPENDIX C: $T_*^4$ CORRECTIONS IN ATOMIC INTERFEROMETERS

The perturbative expansion of Sec. VIB can also be extended to arbitrary order in the time of flight. For example, the fourth-order corrections to Eqs. (36) are given by

$$\begin{aligned} t(T)_{[4]} = & -\frac{e^{-\varphi(z_0)/c^2}}{\tilde{\gamma}_0} \frac{(\tilde{\gamma}_0 T)^4}{4!} \frac{p_0}{mc} \\ & \times \left[ \frac{p_0^2}{m^2c^2} c \varphi_0''' - \left( 4 + \frac{3p_0^2}{m^2c^2} \right) \frac{\varphi_0'' \varphi'}{c} \right. \\ & \left. + \left( 1 + \frac{p_0^2}{m^2c^2} \right) \frac{(\varphi')^3}{c^3} \right], \end{aligned} \quad (\text{C1})$$

$$\begin{aligned} z(T)_{[4]} = & -\frac{(\tilde{\gamma}_0 T)^4}{4!} \left[ \frac{p_0^2}{m^2} \varphi_0''' - \left( 1 - \frac{6p_0^2}{m^2c^2} \right) \varphi_0'' \varphi' \right. \\ & \left. - \left( 2 - \frac{4p_0^2}{m^2c^2} \right) \frac{(\varphi')^3}{c^2} \right], \end{aligned} \quad (\text{C2})$$

$$\begin{aligned} \tau(T)_{[4]} = & \frac{(\tilde{\gamma}_0 T)^4}{4!} \frac{p_0}{mc} \left[ \frac{p_0^2}{m^2c^2} c \varphi_0''' - \left( 5 - \frac{p_0^2}{m^2c^2} \right) \frac{(\varphi_0')^3}{c^3} \right. \\ & \left. - \left( 4 - \frac{3p_0^2}{m^2c^2} \right) \frac{\varphi_0'' \varphi_0}{c} \right]. \end{aligned} \quad (\text{C3})$$

In the limit  $c \rightarrow \infty$ , this results in the following correction to the phase shifts given in Eqs. (39):

$$\begin{aligned} \Delta\Psi_\zeta = & \frac{\hbar k^2}{2m} \varphi_0'' T_*^3 + \frac{7\hbar k^2 v_0}{12m} \varphi_0''' T_*^4 \\ & + \frac{\hbar^2 k^3}{3m^2} \varphi_0''' T_*^4 + O(T_*^5), \end{aligned} \quad (\text{C4})$$

$$\begin{aligned} \Delta\Psi_* = & -k \varphi_0' T_*^2 - k v_0 \varphi_0'' T_*^3 - \frac{\hbar k^2}{m} \varphi_0'' T_*^3 \\ & + \frac{7k}{12} \varphi_0' \varphi_0'' T_*^4 - \frac{7k v_0^2}{12} \varphi_0''' T_*^4 - \frac{7\hbar k^2 v_0}{6m} \varphi_0''' T_*^4 \\ & - \frac{\hbar^2 k^3}{2m^2} \varphi_0''' T_*^4 + O(T_*^5), \end{aligned} \quad (\text{C5})$$

where  $v_0 = p_0/m$ . For a constant gravity gradient, i.e.,  $\varphi''' = 0$ , this reproduces the results of Refs. [130, 140]. Despite being of higher order in  $T_*$ , the contribution  $\frac{7}{12} k \varphi_0' \varphi_0'' T_*^4$  is numerically comparable to the third-order term  $-k v_0 \varphi_0'' T_*^3$  and thus needs to be taken into account in high-accuracy measurements [73]. Writing the overall phase shift as  $\Delta\Psi = -gkT_*^2$ , the effective acceleration  $g$  takes the form

$$\begin{aligned} g = & \varphi_0' + v_0 \varphi_0'' T_* + \frac{\hbar k}{2m} \varphi_0'' T_* - \frac{7}{12} \varphi_0' \varphi_0'' T_*^2 \\ & + \frac{7}{12} v_0^2 \varphi_0''' T_*^2 + \frac{7\hbar k v_0}{12m} \varphi_0''' T_*^2 \\ & + \frac{\hbar^2 k^2}{6m^2} \varphi_0''' T_*^2 + O(c^{-1}) + O(T_*^3), \end{aligned} \quad (\text{C6})$$

which reproduces all significant contributions up to and including fourth order in  $T_*$  listed in Table I of Ref. [117]. As was pointed out in Ref. [141], the quantity  $g$  differs from the gravitational acceleration of the atoms at the time  $T_*$  (see also Refs. [142–144]).

- [1] C. Rovelli, Loop quantum gravity, *Liv. Rev. Relativ.* **11**, 5 (2008).
- [2] A. Sen and B. Zwiebach, String field theory: A review, in *Handbook of Quantum Gravity*, edited by C. Bambi, L. Modesto, and I. Shapiro (Springer Nature, Berlin, 2024), pp. 2385–2600.
- [3] R. Loll, Quantum gravity from causal dynamical triangulations: A review, *Class. Quantum Grav.* **37**, 013002 (2020).

- [4] M. Niedermaier and M. Reuter, The asymptotic safety scenario in quantum gravity, *Living Rev. Relativ.* **9**, 5 (2006).
- [5] S. Surya, The causal set approach to quantum gravity, *Liv. Rev. Relativ.* **22**, 5 (2019).
- [6] R. Penrose and M. A. H. MacCallum, Twistor theory: An approach to the quantisation of fields and space-time, *Phys. Rep.* **6**, 241 (1973).

- [7] R. Penrose, On the gravitization of quantum mechanics 1: Quantum state reduction, *Found. Phys.* **44**, 557 (2014).
- [8] J. Oppenheim, A postquantum theory of classical gravity? *Phys. Rev. X* **13**, 041040 (2023).
- [9] D. Carney, P. C. E. Stamp, and J. M. Taylor, Tabletop experiments for quantum gravity: A user's manual, *Class. Quantum Grav.* **36**, 034001 (2019).
- [10] S. Bose, I. Fuentes, A. A. Geraci, S. M. Khan, S. Qvarfort, M. Rademacher, M. Rashid, M. Toroš, H. Ulbricht, and C. C. Wanjura, Massive quantum systems as interfaces of quantum mechanics and gravity, *Rev. Mod. Phys.* **97**, 015003 (2025).
- [11] S. Bose, A. Mazumdar, G. W. Morley, H. Ulbricht, M. Toroš, M. Paternostro, A. A. Geraci, P. F. Barker, M. S. Kim, and G. Milburn, Spin entanglement witness for quantum gravity, *Phys. Rev. Lett.* **119**, 240401 (2017).
- [12] C. Marletto and V. Vedral, Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity, *Phys. Rev. Lett.* **119**, 240402 (2017).
- [13] M. J. W. Hall and M. Reginatto, On two recent proposals for witnessing nonclassical gravity, *J. Phys. A* **51**, 085303 (2018).
- [14] M. Christodoulou and C. Rovelli, On the possibility of laboratory evidence for quantum superposition of geometries, *Phys. Lett. B* **792**, 64 (2019).
- [15] A. Kent and D. Pitalúa-García, Testing the nonclassicality of spacetime: What can we learn from Bell-Bose *et al.*-Marletto-Vedral experiments?, *Phys. Rev. D* **104**, 126030 (2021).
- [16] T. D. Galley, F. Giacomini, and J. H. Selby, A no-go theorem on the nature of the gravitational field beyond quantum theory, *Quantum* **6**, 779 (2022).
- [17] E. Martín-Martínez and T. R. Perche, What gravity mediated entanglement can really tell us about quantum gravity, *Phys. Rev. D* **108**, L101702 (2023).
- [18] R. Colella, A. W. Overhauser, and S. A. Werner, Observation of gravitationally induced quantum interference, *Phys. Rev. Lett.* **34**, 1472 (1975).
- [19] S. A. Werner, H. Kaiser, M. Arif, and R. Clothier, Neutron interference induced by gravity: New results and interpretations, *Physica B+C (Amsterdam)* **151**, 22 (1988).
- [20] K. C. Littrell, B. E. Allman, and S. A. Werner, Two-wavelength-difference measurement of gravitationally induced quantum interference phases, *Phys. Rev. A* **56**, 1767 (1997).
- [21] K. Varjú and L. H. Ryder, General relativistic treatment of the Colella-Overhauser-Werner experiment on neutron interference in a gravitational field, *Am. J. Phys.* **68**, 404 (2000).
- [22] P. D. Mannheim, Classical underpinnings of gravitationally induced quantum interference, *Phys. Rev. A* **57**, 1260 (1998).
- [23] H. Abele, The neutron: Its properties and basic interactions, *Prog. Part. Nucl. Phys.* **60**, 1 (2008).
- [24] N. Huggett, N. Linnemann, and M. D. Schneider, in *Quantum Gravity in a Laboratory?* edited by R. Dawid and J. Wells, Elements in the Foundations of Contemporary Physics (Cambridge University Press, Cambridge, 2023).
- [25] M. Kasevich and S. Chu, Atomic interferometry using stimulated Raman transitions, *Phys. Rev. Lett.* **67**, 181 (1991).
- [26] M. Kasevich and S. Chu, Measurement of the gravitational acceleration of an atom with a light-pulse atom interferometer, *Appl. Phys. B* **54**, 321 (1992).
- [27] M. J. Snadden, J. M. McGuirk, P. Bouyer, K. G. Haritos, and M. A. Kasevich, Measurement of the Earth's gravity gradient with an atom interferometer-based gravity gradiometer, *Phys. Rev. Lett.* **81**, 971 (1998).
- [28] P. A. Altin, M. T. Johnsson, V. Negnevitsky, G. R. Dennis, R. P. Anderson, J. E. Debs, S. S. Szigeti, K. S. Hardman, S. Bennetts, G. D. McDonald, L. D. Turner, J. D. Close, and N. P. Robins, Precision atomic gravimeter based on Bragg diffraction, *New J. Phys.* **15**, 023009 (2013).
- [29] V. Xu, M. Jaffe, C. D. Panda, S. L. Kristensen, L. W. Clark, and H. Müller, Probing gravity by holding atoms for 20 seconds, *Science* **366**, 745 (2019).
- [30] C. D. Panda, M. J. Tao, M. Ceja, J. Khoury, G. M. Tino, and H. Müller, Measuring gravitational attraction with a lattice atom interferometer, *Nature (London)* **631**, 515 (2024).
- [31] C. Overstreet, P. Asenbaum, J. Curti, M. Kim, and M. A. Kasevich, Observation of a gravitational Aharonov-Bohm effect, *Science* **375**, 226 (2022).
- [32] L. Stodolsky, Matter and light wave interferometry in gravitational fields, *Gen. Relativ. Gravit.* **11**, 391 (1979).
- [33] K. Tanaka, How to detect the gravitationally induced phase shift of electromagnetic waves by optical-fiber interferometry, *Phys. Rev. Lett.* **51**, 378 (1983).
- [34] M. Zych, F. Costa, I. Pikovski, T. C. Ralph, and Č. Brukner, General relativistic effects in quantum interference of photons, *Class. Quantum Grav.* **29**, 224010 (2012).
- [35] C. Hilweg, F. Massa, D. Martynov, N. Mavalvala, P. T. Chruściel, and P. Walther, Gravitationally induced phase shift on a single photon, *New J. Phys.* **19**, 033028 (2017).
- [36] T. B. Mieling, Gupta-Bleuler quantization of optical fibers in weak gravitational fields, *Phys. Rev. A* **106**, 063511 (2022).
- [37] M. Mohageg, L. Mazzarella, C. Anastopoulos, J. Gallicchio, B.-L. Hu, T. Jennewein, S. Johnson, S.-Y. Lin, A. Ling, C. Marquardt, M. Meister, R. Newell, A. Roura, W. P. Schleich, C. Schubert, D. V. Strekalov, G. Vallone, P. Villoresi, L. Wörner, N. Yu *et al.*, The deep space quantum link: Prospective fundamental physics experiments using long-baseline quantum optics, *EPJ Quantum Technol.* **9**, 25 (2022).
- [38] D. Rideout, T. Jennewein, G. Amelino-Camelia, T. F. Demarie, B. L. Higgins, A. Kempf, A. Kent, R. Laflamme, X. Ma, R. B. Mann, E. Martín-Martínez, N. C. Menicucci, J. Moffat, C. Simon, R. Sorkin, L. Smolin, and D. R. Terno, Fundamental quantum optics experiments conceivable with satellites—reaching relativistic distances and velocities, *Class. Quantum Grav.* **29**, 224011 (2012).
- [39] H.-N. Wu, Y.-H. Li, B. Li, X. You, R.-Z. Liu, J.-G. Ren, J. Yin, C.-Y. Lu, Y. Cao, C.-Z. Peng, and J.-W. Pan, Single-Photon interference over 8.4 km urban atmosphere: Toward testing quantum effects in curved spacetime with photons, *Phys. Rev. Lett.* **133**, 020201 (2024).
- [40] T. B. Mieling, C. Hilweg, and P. Walther, Measuring space-time curvature using maximally path-entangled quantum states, *Phys. Rev. A* **106**, L031701 (2022).
- [41] R. Barzel, M. Gündoğan, M. Krutzik, D. Rätzel, and C. Lämmerzahl, Entanglement dynamics of photon pairs and quantum memories in the gravitational field of the earth, *Quantum* **8**, 1273 (2024).
- [42] R. Barzel, D. E. Bruschi, A. W. Schell, and C. Lämmerzahl, Observer dependence of photon bunching: The influence

- of the relativistic redshift on Hong-Ou-Mandel interference, *Phys. Rev. D* **105**, 105016 (2022).
- [43] P. Asenbaum, C. Overstreet, T. Kovachy, D. D. Brown, J. M. Hogan, and M. A. Kasevich, Phase shift in an atom interferometer due to spacetime curvature across its wave function, *Phys. Rev. Lett.* **118**, 183602 (2017).
- [44] R. Y. Chiao, N. A. Inan, M. Scheibner, J. Sharping, D. A. Singleton, and M. E. Tobar, Gravitational Aharonov-Bohm effect, *Phys. Rev. D* **109**, 064073 (2024).
- [45] P. T. Chruściel, *Elements of general relativity*, Compact Textbooks in Mathematics (Birkhäuser, Cham, Switzerland, 2019).
- [46] K. Martel and E. Poisson, Regular coordinate systems for Schwarzschild and other spherical spacetimes, *Am. J. Phys.* **69**, 476 (2001).
- [47] T. Frankel, *The Geometry of Physics*, 3rd ed. (Cambridge University Press, Cambridge, 2012).
- [48] R. M. Wald, *General Relativity* (Chicago University Press, Chicago, 1984).
- [49] J. M. Lee, *Introduction to Riemannian Manifolds*, 2nd ed., Graduate Texts in Mathematics 176 (Springer, Berlin, 2018).
- [50] M. T. Anderson, P. T. Chruściel, and E. Delay, Non-trivial, static, geodesically complete, vacuum space-times with a negative cosmological constant, *J. High Energy Phys.* **10** (2002) 063.
- [51] L. Andersson, R. Beig, and B. Schmidt, Static self-gravitating elastic bodies in Einstein gravity, *Commun. Pure Appl. Math.* **61**, 988 (2008).
- [52] P. T. Chruściel, E. Delay, and P. Klinger, Nonsingular spacetimes with a negative cosmological constant: Stationary solutions with matter fields, *Phys. Rev. D* **95**, 104039 (2017).
- [53] T. W. Baumgarte and S. L. Shapiro, *Numerical Relativity* (Cambridge University Press, Cambridge, 2010).
- [54] M. L. Ruggiero and A. Tartaglia, Gravitomagnetic effects, *Nuovo Cimento B* **117**, 743 (2002).
- [55] P. K. Schwartz, Teleparallel Newton-Cartan gravity, *Class. Quantum Grav.* **40**, 105008 (2023).
- [56] Y. Choquet-Bruhat, *General Relativity and the Einstein Equations* (Oxford University Press, Oxford, 2009).
- [57] W. Gordon, Zur Lichtfortpflanzung nach der Relativitätstheorie, *Ann. Phys.* **377**, 421 (1923).
- [58] J. Ehlers, Zum Übergang von der Wellenoptik zur geometrischen Optik in der allgemeinen Relativitätstheorie, *Z. Naturforsch. A* **22**, 1328 (1967).
- [59] M. A. Hohensee, B. Estey, P. Hamilton, A. Zeilinger, and H. Müller, Force-Free gravitational redshift: Proposed gravitational Aharonov-Bohm experiment, *Phys. Rev. Lett.* **108**, 230404 (2012).
- [60] O. Dobkowski, B. Trok, P. Skakunenko, Y. Japha, D. Groswasser, M. Efremov, C. Marletto, I. Fuentes, R. Penrose, V. Vedral, W. P. Schleich, and R. Folman, Observation of the quantum equivalence principle for matter-waves, [arXiv:2502.14535](https://arxiv.org/abs/2502.14535).
- [61] M. A. Oancea and A. Kumar, Semiclassical analysis of Dirac fields on curved spacetime, *Phys. Rev. D* **107**, 044029 (2023).
- [62] W. G. Dixon, Dynamics of extended bodies in general relativity. I. Momentum and angular momentum, *Proc. R. Soc. London, Ser. A* **314**, 499 (1970).
- [63] W. G. Dixon, Dynamics of extended bodies in general relativity. II. Moments of the charge-current vector, *Proc. R. Soc. London, Ser. A* **319**, 509 (1970).
- [64] W. G. Dixon, Dynamics of extended bodies in general relativity. III. Equations of motion, *Philos. Trans. R. Soc. London A* **277**, 59 (1974).
- [65] S. E. Gralla, A. I. Harte, and R. M. Wald, Rigorous derivation of electromagnetic self-force, *Phys. Rev. D* **80**, 024031 (2009).
- [66] H. Müller, A. Peters, and S. Chu, A precision measurement of the gravitational redshift by the interference of matter waves, *Nature (London)* **463**, 926 (2010).
- [67] P. Wolf, L. Blanchet, C. J. Bordé, S. Reynaud, C. Salomon, and C. Cohen-Tannoudji, Does an atom interferometer test the gravitational redshift at the Compton frequency? *Class. Quantum Grav.* **28**, 145017 (2011).
- [68] S. Sinha and J. Samuel, Atom interferometry and the gravitational redshift, *Class. Quantum Grav.* **28**, 145018 (2011).
- [69] D. Giulini, Equivalence principle, quantum mechanics, and atom-interferometric tests, in *Quantum Field Theory and Gravity*, edited by F. Finster, O. Müller, M. Nardmann, J. Tolksdorf, and E. Zeidler (Birkhäuser, Cham, Switzerland, 2012), pp. 345–370.
- [70] D. E. Bruschi and A. W. Schell, Gravitational redshift induces quantum interference, *Ann. Phys.* **535**, 2200468 (2023).
- [71] L. A. Alanís Rodríguez, A. W. Schell, and D. E. Bruschi, Introduction to gravitational redshift of quantum photons propagating in curved spacetime, *J. Phys.: Conf. Ser.* **2531**, 012016 (2023).
- [72] N. Leber, L. Adrián Alanís Rodríguez, A. Ferreri, A. Wolfgang Schell, and D. E. Bruschi, Limits to the validity of quantum-optical models of the effects of gravitational redshift on photonic quantum states, [arXiv:2502.20521](https://arxiv.org/abs/2502.20521).
- [73] A. Peters, K. Y. Chung, and S. Chu, High-precision gravity measurements using atom interferometry, *Metrologia* **38**, 25 (2001).
- [74] T. B. Mieling, Gupta-Bleuler quantization of the electromagnetic field in curved space-times with applications to gravitational photon interferometry, Doctoral thesis, University of Vienna, 2023, doi: [10.25365/thesis.75758](https://doi.org/10.25365/thesis.75758).
- [75] Q. Exirifard, E. Culf, and E. Karimi, Towards communication in a curved spacetime geometry, *Commun. Phys.* **4**, 171 (2021).
- [76] M. A. Oancea, J. Joudioux, I. Y. Dodin, D. E. Ruiz, C. F. Paganini, and L. Andersson, Gravitational spin Hall effect of light, *Phys. Rev. D* **102**, 024075 (2020).
- [77] T. B. Mieling and M. Hudelist, Fiber optics in curved spacetimes, *Phys. Rev. Res.* **7**, 013162 (2025).
- [78] R. V. Pound and G. A. Rebka, Apparent weight of photons, *Phys. Rev. Lett.* **4**, 337 (1960).
- [79] L. B. Okun, K. G. Selivanov, and V. L. Telegdi, On the interpretation of the redshift in a static gravitational field, *Am. J. Phys.* **68**, 115 (2000).
- [80] R. V. Pound and J. L. Snider, Effect of gravity on nuclear resonance, *Phys. Rev. Lett.* **13**, 539 (1964).
- [81] Bureau international des poids et mesures, Resolution 1 of the 26th CGPM, in *General Conference on Weights and Measures: Proceedings of the 26th meeting of the CGPM* (Bureau international des poids et mesures (BIPM), 2018), pp. 472–474.
- [82] A. Brodutch, A. Gilchrist, T. Guff, A. R. H. Smith, and D. R. Terno, Post-Newtonian gravitational effects in optical interferometry, *Phys. Rev. D* **91**, 064041 (2015).

- [83] M. Zych, I. Pikovski, F. Costa, and Č. Brukner, General relativistic effects in quantum interference of “clocks”, *J. Phys.: Conf. Ser.* **723**, 012044 (2016).
- [84] M. Aspelmeyer, Č. Brukner, D. Giulini, and G. Milburn, Focus on gravitational quantum physics, *New J. Phys.* **19**, 050401 (2017).
- [85] S. Pallister, S. Coop, V. Formichella, N. Gampierakis, V. Notaro, P. Knott, R. Azevedo, N. Buchheim, S. de Carvalho, E. Järvelä, M. Laporte, J.-P. Kaikkonen, N. Meshksar, T. Nikkanen, and M. Yttergren, A blueprint for a simultaneous test of quantum mechanics and general relativity in a space-based quantum optics experiment, *EPJ Quantum Technol.* **4**, 2 (2017).
- [86] R. Beig, P. T. Chruściel, C. Hilweg, P. Kornreich, and P. Walther, Weakly gravitating isotropic waveguides, *Class. Quantum Grav.* **35**, 244001 (2018).
- [87] S. Y. Chen and T. C. Ralph, Estimation of gravitational acceleration with quantum optical interferometers, *Phys. Rev. A* **99**, 023803 (2019).
- [88] S. M. Barnett, On single-photon and classical interference, *Phys. Scr.* **97**, 114004 (2022).
- [89] G. Bertocchi, O. Alibart, D. B. Ostrowsky, S. Tanzilli, and P. Baldi, Single-photon Sagnac interferometer, *J. Phys. B* **39**, 1011 (2006).
- [90] M. Toroš, M. Cromb, M. Paternostro, and D. Faccio, Generation of entanglement from mechanical rotation, *Phys. Rev. Lett.* **129**, 260401 (2022).
- [91] M. Cromb, S. Restuccia, G. M. Gibson, M. Toroš, M. J. Padgett, and D. Faccio, Mechanical rotation modifies the manifestation of photon entanglement, *Phys. Rev. Res.* **5**, L022005 (2023).
- [92] R. Silvestri, H. Yu, T. Strömberg, C. Hilweg, R. W. Peterson, and P. Walther, Experimental observation of Earth’s rotation with quantum entanglement, *Sci. Adv.* **10**, eado0215 (2024).
- [93] D. R. Terno, G. Vallone, F. Vedovato, and P. Villoresi, Large-scale optical interferometry in general spacetimes, *Phys. Rev. D* **101**, 104052 (2020).
- [94] D. R. Terno, F. Vedovato, M. Schiavon, A. R. H. Smith, P. Magnani, G. Vallone, and P. Villoresi, Proposal for an optical interferometric measurement of the gravitational redshift with satellite systems, *Phys. Rev. D* **108**, 084063 (2023).
- [95] C.-Y. Lu, Y. Cao, C.-Z. Peng, and J.-W. Pan, Micius quantum experiments in space, *Rev. Mod. Phys.* **94**, 035001 (2022).
- [96] G. Vallone, D. Dequal, M. Tomasin, F. Vedovato, M. Schiavon, V. Luceri, G. Bianco, and P. Villoresi, Interference at the single photon level along satellite-ground channels, *Phys. Rev. Lett.* **116**, 253601 (2016).
- [97] C. Hilweg, D. Shadmany, P. Walther, N. Mavalvala, and V. Sudhir, Limits and prospects for long-baseline optical fiber interferometry, *Optica* **9**, 1238 (2022).
- [98] F. Poletti, Nested antiresonant nodeless hollow core fiber, *Opt. Express* **22**, 23807 (2014).
- [99] Y. Chen, M. Petrovich, E. N. Fokoua, A. Adamu, M. Hassan, H. Sakr, R. Slavík, S. B. Gorajoobi, M. Alonso, R. F. Ando, A. Papadimopoulos, T. Varghese, D. Wu, M. F. Ando, K. Wisniowski, S. Sandoghchi, G. Jasion, D. Richardson, and F. Poletti, Hollow core DNANF optical fiber with <0.11 dB/km loss, in *Optical Fiber Communication Conference (OFC) 2024* (Optica Publishing Group, 2024), p. Th4A.8.
- [100] Z. Liu, B. Karanov, L. Galdino, J. R. Hayes, D. Lavery, K. Clark, K. Shi, D. J. Elson, B. C. Thomsen, M. N. Petrovich, D. J. Richardson, F. Poletti, R. Slavik, and P. Bayvel, Nonlinearity-Free coherent transmission in hollow-core antiresonant fiber, *J. Lightwave Technol.* **37**, 909 (2019).
- [101] A. W. Overhauser and R. Colella, Experimental test of gravitationally induced quantum interference, *Phys. Rev. Lett.* **33**, 1237 (1974).
- [102] J. Anandan, Gravitational and rotational effects in quantum interference, *Phys. Rev. D* **15**, 1448 (1977).
- [103] J. Kuroiwa, M. Kasai, and F. Toshifumi, A treatment of general relativistic effects in quantum interference, *Phys. Lett. A* **182**, 330 (1993).
- [104] A. Galiatdinov and L. H. Ryder, Neutron interference in the Earth’s gravitational field, *Gen. Relativ. Gravit.* **49**, 82 (2017).
- [105] H. Beyer and J. Nitsch, The non-relativistic COW experiment in the uniformly accelerated reference frame, *Phys. Lett. B* **182**, 211 (1986).
- [106] U. Bonse and T. Wroblewski, Dynamical diffraction effects in noninertial neutron interferometry, *Phys. Rev. D* **30**, 1214 (1984).
- [107] M. A. Horne, Neutron interferometry in a gravity field, *Physica B+C (Amsterdam)* **137**, 260 (1986).
- [108] C. Lämmerzahl, On the equivalence principle in quantum theory, *Gen. Relativ. Gravitation* **28**, 1043 (1996).
- [109] J. Anandan, Curvature effects in interferometry, *Phys. Rev. D* **30**, 1615 (1984).
- [110] E. Hadden, M. Fally, Y. Iso, T. Jenke, J. Klepp, A. Kume, K. Umemoto, and Y. Tomita, Holographic nanodiamond-polymer composite grating with unprecedented slow-neutron refractive index modulation amplitude, *Appl. Phys. Lett.* **124**, 071901 (2024).
- [111] M. G. Huber, B. Heacock, I. Taminiau, D. G. Cory, D. Sarenac, R. Valdillez, and D. A. Pushin, Achieving a near-ideal silicon crystal neutron interferometer using submicrometer fabrication techniques, *Phys. Rev. Res.* **6**, 043079 (2024).
- [112] F. Mezei, Very cold neutrons in condensed matter research, *J. Neutron Res.* **24**, 205 (2022).
- [113] D. A. Pushin, M. G. Huber, M. Arif, C. B. Shahi, J. Nsofini, C. J. Wood, D. Sarenac, and D. G. Cory, Neutron interferometry at the National Institute of Standards and Technology, *Adv. High Energy Phys.* **2015**, 687480 (2015).
- [114] M. Zych, F. Costa, I. Pikovski, and Č. Brukner, Quantum interferometric visibility as a witness of general relativistic proper time, *Nat. Commun.* **2**, 505 (2011).
- [115] H. Rauch and S. A. Werner, *Neutron Interferometry*, 2nd ed. (Oxford University Press, Oxford, 2015).
- [116] C. Overstreet, P. Asenbaum, and M. A. Kasevich, Physically significant phase shifts in matter-wave interferometry, *Am. J. Phys.* **89**, 324 (2021).
- [117] J. M. Hogan, D. M. S. Johnson, and M. A. Kasevich, Light-pulse atom interferometry, in *CLXVIII Corso, Rendiconti della Scuola Internazionale di Fisica “Enrico Fermi”*, edited by E. Arimondo, W. Ertmer, W. P. Schleich, and E. M. Rasel (Società Italiana di Fisica, 2009), pp. 411–447.
- [118] A. F. Bernhardt and B. W. Shore, Coherent atomic deflection by resonant standing waves, *Phys. Rev. A* **23**, 1290 (1981).
- [119] K. Moler, D. S. Weiss, M. Kasevich, and S. Chu, Theoretical analysis of velocity-selective Raman transitions, *Phys. Rev. A* **45**, 342 (1992).

- [120] P. Wolf, L. Blanchet, C. J. Bordé, S. Reynaud, C. Salomon, and C. Cohen-Tannoudji, Atom gravimeters and gravitational redshift, *Nature (London)* **467**, E1 (2010).
- [121] H. Müller, A. Peters, and S. Chu, Müller, Peters & Chu reply, *Nature (London)* **467**, E2 (2010).
- [122] P. Wolf, L. Blanchet, C. J. Bordé, S. Reynaud, C. Salomon, and C. Cohen-Tannoudji, Reply to comment on: “Does an atom interferometer test the gravitational redshift at the Compton frequency?”, *Class. Quantum Grav.* **29**, 048002 (2012).
- [123] W. P. Schleich, D. M. Greenberger, and E. M. Rasel, A representation-free description of the Kasevich-Chu interferometer: A resolution of the redshift controversy, *New J. Phys.* **15**, 013007 (2013).
- [124] P. Asenbaum, C. Overstreet, M. Kim, J. Curti, and M. A. Kasevich, Atom-Interferometric test of the equivalence principle at the  $10^{-12}$  level, *Phys. Rev. Lett.* **125**, 191101 (2020).
- [125] J. M. McGuirk, G. T. Foster, J. B. Fixler, M. J. Snadden, and M. A. Kasevich, Sensitive absolute-gravity gradiometry using atom interferometry, *Phys. Rev. A* **65**, 033608 (2002).
- [126] J. B. Fixler, G. T. Foster, J. M. McGuirk, and M. A. Kasevich, Atom interferometer measurement of the Newtonian constant of gravity, *Science* **315**, 74 (2007).
- [127] F. Sorrentino, A. Bertoldi, Q. Bodart, L. Cacciapuoti, M. de Angelis, Y. H. Lien, M. Prevedelli, G. Rosi, and G. M. Tino, Simultaneous measurement of gravity acceleration and gravity gradient with an atom interferometer, *Appl. Phys. Lett.* **101**, 114106 (2012).
- [128] G. Rosi, L. Cacciapuoti, F. Sorrentino, M. Menchetti, M. Prevedelli, and G. M. Tino, Measurement of the gravity-field curvature by atom interferometry, *Phys. Rev. Lett.* **114**, 013001 (2015).
- [129] C. J. Bordé, Theoretical tools for atom optics and interferometry, *C. R. Acad. Sci., Ser. IV* **2**, 509 (2001).
- [130] P. Wolf and P. Tournenc, Gravimetry using atom interferometers: Some systematic effects, *Phys. Lett. A* **251**, 241 (1999).
- [131] S. Dimopoulos, P. W. Graham, J. M. Hogan, M. A. Kasevich, and S. Rajendran, Gravitational wave detection with atom interferometry, *Phys. Lett. B* **678**, 37 (2009).
- [132] G. Arduini, L. Badurina, K. Balazs, C. Baynham, O. Buchmueller, M. Buzio, S. Calatroni, J. P. Corso, J. Ellis, C. Gignat, M. Guinchard, T. Hakulinen, R. Hobson, A. Infantino, D. Lafarge, R. Langlois, C. Marcel, J. Mitchell, M. Parodi, M. Pentella *et al.*, A long-baseline atom interferometer at CERN: Conceptual feasibility study, [arXiv:2304.00614](https://arxiv.org/abs/2304.00614).
- [133] C. Baynham, A. Bertoldi, D. Blas, O. Buchmueller, S. Calatroni, V. Charmandaris, M. L. Chiofalo, P. Cladé, J. Coleman, F. Di Pumpo, J. Ellis, N. Gaaloul, S. Guellati-Khelifa, T. Harte, R. Hobson, M. Holynski, S. Lellouch, L. Lombriser, E. Lopez Asamar, M. Maggiore *et al.*, Letter of intent: AICE-100m atom interferometer experiment at CERN, [arXiv:2509.11867](https://arxiv.org/abs/2509.11867).
- [134] G. M. Tino, A. Bassi, G. Bianco, K. Bongs, P. Bouyer, L. Cacciapuoti, S. Capozziello, X. Chen, M. L. Chiofalo, A. Derevianko, W. Ertmer, N. Gaaloul, P. Gill, P. W. Graham, J. M. Hogan, L. Iess, M. A. Kasevich, H. Katori, C. Klempt, X. Lu *et al.*, SAGE: A proposal for a space atomic gravity explorer, *Eur. Phys. J. D* **73**, 228 (2019).
- [135] C. W. Misner, K. S. Throne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, 1973).
- [136] Y.-J. Tan, C.-G. Shao, and Z.-K. Hu, Finite-speed-of-light perturbation in atom gravimeters, *Phys. Rev. A* **94**, 013612 (2016).
- [137] Y.-J. Tan, C.-G. Shao, and Z.-K. Hu, Relativistic effects in atom gravimeters, *Phys. Rev. D* **95**, 024002 (2017).
- [138] Y.-J. Tan, C.-G. Shao, and Z.-K. Hu, Time delay and the effect of the finite speed of light in atom gravimeters, *Phys. Rev. A* **96**, 023604 (2017).
- [139] C. Niehof, D. Derr, and E. Giese, Finite-speed-of-light effects in atom interferometry: Diffraction mechanisms and resonance conditions, *AVS Quantum Sci.* **7**, 034404 (2025).
- [140] A. Peters, High precision gravity measurements using atom interferometry, Dissertation, Stanford University, 1998.
- [141] A. M. Nobili, Fundamental limitations to high-precision tests of the universality of free fall by dropping atoms, *Phys. Rev. A* **93**, 023617 (2016).
- [142] B. Dubetsky, Comment on “Circumventing Heisenberg’s uncertainty principle in atom interferometry tests of the equivalence principle”, *Phys. Rev. Lett.* **121**, 128903 (2018).
- [143] A. Roura, Roura replies, *Phys. Rev. Lett.* **121**, 128904 (2018).
- [144] A. M. Nobili, A. Anselmi, and R. Pegna, Systematic errors in high-precision gravity measurements by light-pulse atom interferometry on the ground and in space, *Phys. Rev. Res.* **2**, 012036 (2020).