Complementary probes of gravitational radiation states

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(Received 9 July 2025; accepted 22 September 2025; published 20 October 2025)

We demonstrate that statistical fluctuations in resonant radiation detectors operating in homodyne and heterodyne modes offer additional, complementary information to that obtained from their direct operation as click detectors. We use this to refine tests of the coherent-state hypothesis, which is of special interest for gravitational wave fields.

DOI: 10.1103/83tt-tt57

I. INTRODUCTION

Experiments over the last century have revealed that nature obeys the laws of quantum mechanics to great accuracy in many circumstances. But few of these experiments significantly involve gravity, and thus far no observed phenomenon in gravity has required elucidation using its (presumed) quantum dynamics. The relatively recent observation of gravitational radiation [1,2] plausibly opens new possibilities, since it gives direct access to the gravitational quanta in great abundance. It has been suggested that statistical fluctuations in gravitational wave signals might be promising in this regard [3–7].

Classical or semiclassical calculations of the interactions between matter and gravitational radiation implicitly assume a coherent-state description for the gravitational radiation field. For some—but not all—analogous problems in electrodynamics, this can be justified formally using the so-called optical equivalence theorem [8]. Thus, phenomena that indicate deviations from the *coherent-state hypothesis* for the gravitational radiation field would force us away from that classicality assumption and require that we bring in a richer quantum description [3–5,7,9–12].

Recently [7] we suggested that the statistics of resonant bar detectors, when they function as quantized "click" detectors [6], are promising in this regard. Here we extend that work to consider interferometric quantum phase measurement strategies [13–26] using resonant detectors. These methods access additional information that is complementary, in the sense of quantum theory, to click detection.

Discrimination of sub-Poissonian quantum states of gravitational radiation is especially challenging [7,27,28]. We find that use of homodyne techniques, which bring in phase sensitivity, and heterodyne techniques, which enhance selected quadratures, can be helpful in this regard.

Our proposed tests of acoherence rely on the foundational work of Ref. [6]. There it was suggested that a stimulated absorption probability of order unity can be achieved for gravitational radiation using challenging, but feasible, parameters for resonant bar detector designs. The stimulated absorption rate is $\gamma_s = \gamma_0 \langle a^\dagger a \rangle$ in the single-mode approximation for the gravitational radiation field with annihilation operator a. Here γ_0 is the spontaneous emission rate. The spontaneous emission rate (γ_0) and stimulated absorption rate for single gravitons (γ_s for the $0 \to 1$ transition) can be estimated for the fundamental acoustic mode of resonant mass detectors to be [6]

$$\begin{aligned} \gamma_0 &= \frac{8GML^2\omega^4}{\pi^4 c^5}, \quad \langle a^\dagger a \rangle = \frac{h_0^2 c^5}{32\pi\hbar G\omega^2}, \quad \text{and} \\ \gamma_s &= \gamma_0 \langle a^\dagger a \rangle \\ &= \frac{h_0^2 ML^2\omega^2}{4\pi^5\hbar}, \end{aligned} \tag{1}$$

where G is Newton's constant, M and L are the mass and length of the resonant mass detector, c is the speed of sound, and h_0 is the amplitude of the incoming gravitational radiation field. Here ω is the resonant frequency of the bar detector, and $\omega = \pi v_s/L$ for the fundamental acoustic mode of a Weber bar, where v_s is the speed of sound for the material used. Hence the stimulated absorption rate can be reexpressed in terms of material parameters as $\gamma_s = \frac{h_0^2 M v_s^2}{4\pi^3\hbar}$.

When one estimates for typical Weber bars as was done in Ref. [6], $\gamma_0 \approx 10^{-33}$ Hz. This can be compared to the spontaneous emission rate for single gravitons from a single hydrogen atom, that is approximately 10^{-40} Hz [29–31]. In comparison, the resonant frequencies for such bars can

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also be in the kilohertz range, where we know that gravitational waves exist with observable strain amplitudes $h_0 \approx 10^{-22}$ to 10^{-21}. For $\omega/(2\pi)$ in the range 100 to 200 Hz, this represents $\langle a^\dagger a\rangle \approx 10^{36}$ gravitons [31]. Therefore, a stimulated absorption probability $\gamma_s \Delta t \sim \gamma_0 \langle a^\dagger a\rangle \Delta t$ of order unity can be achieved in practice within the millisecond timescales ($\Delta t \approx 1$ ms) that are relevant to the experiment [6,32].

The product $\gamma_0 \Delta t$ represents the graviton-to-phonon conversion efficiency. It is a very small number, but gets compensated by the number of gravitons in the incoming radiation field, making it feasible to observe quantum jumps induced in resonant mass detectors. For click detectors that monitor quantum jumps stimulated by the gravitational radiation, the signal is already at the level of $\gamma_0 \Delta t \langle a^\dagger a\rangle$ as discussed. Our previous work has shown that the observable excess noise in counts is equal to $(\gamma_0 \Delta t)^2 Q \langle a^\dagger a\rangle$, where Q is the Mandel's Q parameter [7]. Thus, if the gravitational radiation field is in a quantum-mechanical state for which $Q \sim O(\langle a^\dagger a\rangle)$, then the excess noise becomes observable too. We showed that a variety of plausibly relevant states, including squeezed and thermal states, have that feature.

One objective of the present work is to show that even when Mandel's Q is bounded, as it is for sub-Poissonian states where $-1 \leqslant Q < 0$, the quantum-mechanical noise characteristics can be amplified by choosing appropriate quantum phase measurement strategies which complement click detectors. We will show in the upcoming sections that the excess quantum noise observable in phase-quadrature measurements can be of the same order as $\gamma_0 \Delta t \langle a^\dagger a\rangle$. This linear dependence on the excess noise in $\gamma_0 \Delta t$ for quadrature measurements is also not surprising, as the variances of $\hat{X}$ and $\hat{P}$ have the same number (two) of creation and annihilation operators as the number operator, $a^\dagger a$. Therefore, the noise in quadrature measurements can be of the same order of magnitude as the signal in number (click) measurements. To further substantiate this, note that for quadrature measurements, the signal is actually proportional to $\sqrt{\gamma_0 \Delta t}$, the square root of the graviton-to-phonon conversion efficiency.

More generally, we demonstrate that measurement of complementary quantum noise characteristics gives additional insight into the character of radiation fields, and can bring to light additional phenomena—specifically, violations of the coherent-state hypothesis—that transcend classical physics.

II. FRAMEWORK

For later use let us briefly recall some essential results from Ref. [7]. Given the rotating wave and single mode approximations, the problem of resonant detection of radiation fields involves the interaction Hamiltonian obeying

$$H_I \Delta t = \hbar \sqrt{\gamma_0 \Delta t}\,(a^\dagger b + b^\dagger a). \tag{2}$$

The mode b is the detector (acoustic mode of the resonant mass detector), the mode a represents the gravitational radiation field in the single mode approximation, and γ_0 is the spontaneous emission rate of the detector for the field quantum (graviton). With this model, we revisit the graviton counting statistics framework developed in Ref. [7] to set the stage for the complementary phase detection strategies that follow. A generic quantum state of the field ρ and vacuum of the detector $|0\rangle\langle 0|$ evolve as [7]

$$\begin{aligned} &e^{-\frac{i}{\hbar} H_I \Delta t} \rho \otimes |0\rangle\langle 0| e^{\frac{i}{\hbar} H_I \Delta t} \\ &\quad = \int d^2\alpha P(\alpha) |\alpha \cos(\sqrt{\gamma_0 \Delta t})\rangle\langle \alpha \cos(\sqrt{\gamma_0 \Delta t})| \otimes | \\ &\qquad - i\alpha \sin(\sqrt{\gamma_0 \Delta t})\rangle\langle -i\alpha \sin(\sqrt{\gamma_0 \Delta t})|, \end{aligned} \tag{3}$$

where we have used the Sudarshan-Glauber diagonal P representation for the arbitrary state of the radiation field, ρ [8,33]:

$$\rho = \int d^2\alpha P(\alpha) |\alpha\rangle\langle\alpha|. \tag{4}$$

For click detections, we obtain the probability P_n for registering n clicks, given by [7]

$$\begin{aligned} P_n &= \mathrm{tr}_F \left\{ \langle n| e^{-\frac{i}{\hbar} H_I \Delta t} (\rho \otimes |0\rangle\langle 0|) e^{\frac{i}{\hbar} H_I \Delta t} |n\rangle \right\} \\ &= \frac{[\sin^2(\sqrt{\gamma_0 \Delta t})]^n}{n!} \int d^2\alpha P(\alpha) |\alpha|^{2n} e^{-|\alpha|^2 \sin^2(\sqrt{\gamma_0 \Delta t})}. \end{aligned} \tag{5}$$

From this, one can estimate that the average number of clicks is given by

$$\bar{n} = \sum_{n=0}^{\infty} n P_n = \sin^2(\sqrt{\gamma_0 \Delta t}) \langle a^\dagger a\rangle \approx \gamma_0 \Delta t \langle a^\dagger a\rangle, \tag{6}$$

and the variance

$$(\Delta n)^2 \approx \bar{n} + (\gamma_0 \Delta t)^2 Q \langle a^\dagger a\rangle, \tag{7}$$

where Q is Mandel's Q parameter defined as

$$Q = \frac{\langle (\Delta \hat{N})^2\rangle - \langle \hat{N}\rangle}{\langle \hat{N}\rangle}, \tag{8}$$

where $\hat{N} = a^\dagger a$. Another helpful test statistic relevant at low count rates is the ratio [7]

$$R = \frac{2 P_2 P_0}{P_1^2} \approx 1 + Q/\langle \hat{N}\rangle. \tag{9}$$

See also Ref. [34]. For a coherent state, $Q = 0$, and it can be noted that the observable detector variance $(\Delta n)^2 = \bar{n}$ for the resonant detector. We also see that $R = 1$ for coherent states. However, certain quantum-mechanical states would show observable departures from a coherent state based on the global statistics or the ratio test. We see that if $\gamma_0 \Delta t \langle a^\dagger a\rangle$ is of order unity (which is satisfied by appropriate resonant bar detectors even for gravitational radiation having energy densities comparable to the LIGO-band gravitational waves [6]), as well as if Mandel's Q parameter is of order $\langle a^\dagger a\rangle$, then such departures will become visible as excess quantum noise in the resonant detector [7]. Our earlier work pointed out that thermal states, and highly squeezed vacuum states which are super-Poissonian, show observable deviations from coherent states based on the above acoherence criteria proposed for click detectors [7].

Related ratios and inequalities involving photocurrent probabilities to infer the nonclassicality of light are discussed in Ref. [34], probing regimes where $0 \leqslant R < 1$. However, unlike for light, sub-Poissonian states of gravitational radiation, for which the Mandel Q is restricted to $-1 \leqslant Q < 0$, are difficult to discriminate from coherent states using resonant mass

detectors operating as click detectors; graviton click detectors only respond at very high intensities ($\langle\hat{N}\rangle \gg 1$), and hence one would, in practice, observe $R \approx 1$ for highly populated sub-Poissonian states of the gravitational radiation.

The smaller variance of sub-Poissonian nature for measurements in the number basis implies that appropriate quantum measurements of phase will reveal the excess noise in these quantum states. Here we explore phase-sensitive (homodyne) and phase-preserving (heterodyne) measurements that capture the excess phase noise. Thus we provide additional tests of acoherence that are complementary to our earlier work [7] and access to a regime (sub-Poissonian noise) that is widely regarded as the gold standard for verifying the quantum nature of radiation fields.

III. PHASE SENSITIVE QUANTUM MEASUREMENTS (HOMODYNE DETECTION)

We first consider the case of phase-sensitive quantum measurements, also known as homodyne detection. For homodyne detection, the objective is to measure along one direction (given phase) in the phase space. This can be achieved by using a local oscillator as a phase reference to the signal at the same frequency. While an arbitrary direction can be chosen, we pick this direction as the $\hat{x}$ direction for simplicity, where the measurement operators are simply the projectors $|x\rangle\langle x|$ on the detector, for the observable $\hat{x} = x_0(b + b^\dagger)/\sqrt{2}$: $\hat{x}|x\rangle = x|x\rangle$ [14,15,17–20,25]. We obtain the following probabilities for observing a given quadrature x:

$$\begin{aligned} P_D(x) &= \mathrm{Tr}_F \langle x_D| e^{-\frac{i}{\hbar}H_I \Delta t} \rho \otimes |0\rangle\langle 0| e^{\frac{i}{\hbar}H_I \Delta t} |x_D\rangle \\ &= \frac{1}{\sqrt{\pi}x_0} \int d^2\alpha P(\alpha) e^{-\frac{[x-\sqrt{2}x_0 \mathrm{Im}(\alpha)\sin(\sqrt{\gamma_0\Delta t})]^2}{x_0^2}}, \end{aligned} \tag{10}$$

where $x_0 \propto \sqrt{\hbar}$ is the zero-point length of the detector mode. Here, we have also made use of the relation $\mathrm{Re}[-i\alpha \sin(\sqrt{\gamma_0\Delta t})] = \mathrm{Im}(\alpha)\sin(\sqrt{\gamma_0\Delta t})$. We find that

$$\begin{aligned} \langle\hat{x}\rangle &= \sqrt{2}x_0 \sin(\sqrt{\gamma_0\Delta t}) \int d^2\alpha P(\alpha)\mathrm{Im}(\alpha) \\ &= \sqrt{2}x_0 \sin(\sqrt{\gamma_0\Delta t})\langle \mathrm{Im}(\alpha)\rangle. \end{aligned} \tag{11}$$

Similarly, we have

$$\begin{aligned} \langle\hat{x}^2\rangle &= \frac{1}{2}x_0^2 + 2x_0^2 \sin^2(\sqrt{\gamma_0\Delta t}) \int d^2\alpha P(\alpha)\mathrm{Im}(\alpha)^2 \\ &= \frac{1}{2}x_0^2 + 2x_0^2 \sin^2(\sqrt{\gamma_0\Delta t})\langle \mathrm{Im}(\alpha)^2\rangle. \end{aligned} \tag{12}$$

We obtain that the variance is given by

$$\begin{aligned} \langle(\Delta\hat{x})^2\rangle &= \langle\hat{x}^2\rangle - \langle\hat{x}\rangle^2 = \tfrac{1}{2}x_0^2 + 2x_0^2 \sin^2(\sqrt{\gamma_0\Delta t})[\langle \mathrm{Im}(\alpha)^2\rangle \\ &\quad - \langle \mathrm{Im}(\alpha)\rangle^2] \\ &= x_0^2\big[\tfrac{1}{2} + \sin^2(\sqrt{\gamma_0\Delta t})\big(\langle(\Delta\hat{P})^2\rangle - \tfrac{1}{2}\big)\big]. \end{aligned} \tag{13}$$

We have made use of the result

$$2[\langle \mathrm{Im}(\alpha)^2\rangle - \langle \mathrm{Im}(\alpha)\rangle^2] = \langle(\Delta\hat{P})^2\rangle - \tfrac{1}{2}, \tag{14}$$

where $\hat{P} = (a - a^\dagger)/(i\sqrt{2})$ and $\hat{X} = (a + a^\dagger)/\sqrt{2}$ are the quadratures of the field represented in dimensionless units (see the Appendix). We can now estimate the observable quantum noise for different quantum states of the radiation field. First we estimate the noise observable for a coherent state $\rho = |\alpha\rangle\langle\alpha|$ for which $\langle(\Delta\hat{P})^2\rangle = 1/2$. We find that

$$\langle(\Delta\hat{x})^2\rangle = \frac{x_0^2}{2}. \tag{15}$$

For a thermal state of the gravitational radiation with average thermal quanta n_{th}, the P function and the momentum variance are given by

$$P_{\mathrm{th}}(\alpha) = \frac{1}{\pi n_{\mathrm{th}}} e^{-|\alpha|^2/n_{\mathrm{th}}}, \quad \langle(\Delta\hat{P})^2\rangle = \frac{2n_{\mathrm{th}} + 1}{2}. \tag{16}$$

We obtain

$$\begin{aligned} \langle(\Delta\hat{x})^2\rangle &= \tfrac{1}{2}x_0^2 + 2x_0^2 \sin^2(\sqrt{\gamma_0\Delta t})[\langle \mathrm{Im}(\alpha)^2\rangle - \langle \mathrm{Im}(\alpha)\rangle^2] \\ &= \tfrac{1}{2}x_0^2 + 2x_0^2 \sin^2(\sqrt{\gamma_0\Delta t})\frac{n_{\mathrm{th}}}{2} \\ &= x_0^2\big[n_{\mathrm{th}} \sin^2(\sqrt{\gamma_0\Delta t}) + \tfrac{1}{2}\big]. \end{aligned} \tag{17}$$

If the number of thermal gravitons is of the same order such that the resonant detector would register a click, $\langle a^\dagger a\rangle \sin^2(\sqrt{\gamma_0\Delta t}) \approx \gamma_0\Delta t\langle a^\dagger a\rangle \sim O(1)$, we see that the quantum noise of gravitons in the thermal state also shows a similar order of magnitude difference in the excess noise in homodyne measurements. For Weber bars used as click detectors for gravitons as proposed in Ref. [6], stimulated absorption probability $\gamma_0\langle a^\dagger a\rangle\Delta t$ of order unity can be achieved for gravitational radiation with energy densities comparable to the LIGO band. Hence it appears that such tests are feasible within reasonable time windows ($\Delta t \approx 1$ ms). While click detection strategy was proposed only recently, homodyne detection has the added advantage that it is similar in spirit to Weber's classic, more straightforward detection strategy [35,36].

We can also generalize our results for a generic Gaussian state (a displaced, squeezed, thermal state; see the Appendix):

$$\begin{aligned} \langle(\Delta\hat{x})^2\rangle = x_0^2\Bigg[\Bigg\{\frac{2n_{\mathrm{th}} + 1}{2}[\cosh(2r) + \sinh(2r)\cos(\phi)] - \frac{1}{2}\Bigg\} \\ \times \sin^2(\sqrt{\gamma_0\Delta t}) + \frac{1}{2}\Bigg]. \end{aligned} \tag{18}$$

Consider gravitational radiation in a squeezed vacuum state: $n_{\mathrm{th}} = 0$ and $\phi = 0$. In this case, we get

$$\begin{aligned} \langle(\Delta\hat{x})^2\rangle = x_0^2\Bigg[\Bigg\{\frac{1}{2}\exp(2r) - \frac{1}{2}\Bigg\}\sin^2(\sqrt{\gamma_0\Delta t}) + \frac{1}{2}\Bigg] \to \frac{x_0^2}{2} \\ \times \Bigg[\cos^2(\sqrt{\gamma_0\Delta t}) + \exp(2r)\sin^2(\sqrt{\gamma_0\Delta t})\Bigg], \end{aligned} \tag{19}$$

which again suggests that excess quantum noise along the other quadrature is observable, since $\exp(2r)\sin^2(\sqrt{\gamma_0\Delta t}) \sim \exp(2r)\gamma_0\Delta t \sim O(1)$ for large squeezing such that $\langle a^\dagger a\rangle = \sinh(r)^2 \sim \exp(2r) \approx 10^{36}$, which corresponds to number of gravitons with energy densities comparable to LIGO band gravitational waves.

In the special case $\phi = \pi$ one obtains

$$\langle(\Delta\hat{x})^2\rangle = x_0^2\left[\left\{\frac{1}{2}\exp(-2r) - \frac{1}{2}\right\}\sin^2(\sqrt{\gamma_0\Delta t}) + \frac{1}{2}\right]. \tag{20}$$

Then in the limit $r \to \infty$ we find

$$\langle(\Delta\hat{x})^2\rangle = x_0^2\left[\left\{\frac{1}{2}\exp(-2r) - \frac{1}{2}\right\}\sin^2(\sqrt{\gamma_0\Delta t}) + \frac{1}{2}\right] \to \frac{x_0^2}{2} \times \left[1 - \sin^2(\sqrt{\gamma_0\Delta t})\right]. \tag{21}$$

Here noise below vacuum noise is difficult to measure, since $\gamma_0\Delta t$ alone is very small. This difficulty was also pointed out recently in Refs. [27,28].

It is interesting to see what results we find for a Fock (number) state that is maximally sub-Poissonian. For a number state $|n\rangle$ we know that

$$\langle(\Delta\hat{P})^2\rangle = \langle(\Delta\hat{X})^2\rangle = \frac{2n+1}{2}. \tag{22}$$

Hence, we obtain that

$$\langle(\Delta\hat{x})^2\rangle = x_0^2\left[\frac{1}{2} + \sin^2(\sqrt{\gamma_0\Delta t})n\right]. \tag{23}$$

If the number of gravitons making up the gravitational radiation in this context has energy densities comparable to that of LIGO band gravitational waves ($n \approx 10^{36}$), we can have $n\sin^2(\sqrt{\gamma_0\Delta t}) \approx n\gamma_0\Delta t$ of order unity for typical Weber bar detectors. This suggests that for our phase-sensitive quantum measurement strategy, the deviation in the excess noise produced when compared to a coherent state of the radiation field can be of order unity. This should be compared to statistical noise in direct click detection, which does not discriminate between Fock and coherent states [7].

While our tests are based on the excess quantum noise produced (i.e., in the second moment), it can be noted that the average signal also exhibits complementary features depending on the quantum state of the radiation field, and the detection strategy used [37].

IV. HETERODYNE DETECTION

Heterodyne detection corresponds to projecting the field onto the coherent-state basis, described by measurement operators [16]

$$K_\beta = \frac{1}{\sqrt{\pi}}|\beta\rangle\langle\beta|. \tag{24}$$

The factor of π comes from the overcompleteness of coherent states. For the interaction Hamiltonian we have considered, the probability of measuring a given coherent state $|\beta\rangle$ is given by

$$\begin{aligned} P_D(\beta) &= \frac{1}{\pi}\mathrm{Tr}_F\langle\beta_D|e^{-\frac{i}{\hbar}H_I\Delta t}\rho\otimes|0\rangle\langle 0|e^{\frac{i}{\hbar}H_I\Delta t}|\beta_D\rangle \\ &= \frac{1}{\pi}\int d^2\alpha P(\alpha)e^{-|\beta+i\alpha\sin(\sqrt{\gamma_0\Delta t})|^2}. \end{aligned} \tag{25}$$

A. Statistics of quadratures

From $P_D(\beta)$ given in Eq. (25), we can estimate the moments of measured quadrature outcomes:

$$\begin{aligned} \langle\mathrm{Re}(\beta)\rangle &= \int d^2\beta\,\mathrm{Re}(\beta)\frac{1}{\pi}\int d^2\alpha P(\alpha)e^{-|\beta+i\alpha\sin(\sqrt{\gamma_0\Delta t})|^2} = \sin(\sqrt{\gamma_0\Delta t})\int d^2\alpha P(\alpha)\mathrm{Im}(\alpha) = \sin(\sqrt{\gamma_0\Delta t})\langle\mathrm{Im}(\alpha)\rangle, \\ \langle\mathrm{Re}(\beta)^2\rangle &= \frac{1}{2} + \sin^2(\sqrt{\gamma_0\Delta t})\int d^2\alpha P(\alpha)\mathrm{Im}(\alpha)^2 = \frac{1}{2} + \sin^2(\sqrt{\gamma_0\Delta t})\langle\mathrm{Im}(\alpha)^2\rangle, \\ \langle\mathrm{Im}(\beta)\rangle &= \int d^2\beta\,\mathrm{Im}(\beta)\frac{1}{\pi}\int d^2\alpha P(\alpha)e^{-|\beta+i\alpha\sin(\sqrt{\gamma_0\Delta t})|^2} = -\sin(\sqrt{\gamma_0\Delta t})\int d^2\alpha P(\alpha)\mathrm{Re}(\alpha) = -\sin(\sqrt{\gamma_0\Delta t})\langle\mathrm{Re}(\alpha)\rangle, \\ \langle\mathrm{Im}(\beta)^2\rangle &= \frac{1}{2} + \sin^2(\sqrt{\gamma_0\Delta t})\int d^2\alpha P(\alpha)\mathrm{Re}(\alpha)^2 = \frac{1}{2} + \sin^2(\sqrt{\gamma_0\Delta t})\langle\mathrm{Re}(\alpha)^2\rangle. \end{aligned} \tag{26}$$

These results yield the following estimates for the variances:

$$\begin{aligned} \langle\Delta\mathrm{Re}(\beta)^2\rangle &= \tfrac{1}{2} + \sin^2(\sqrt{\gamma_0\Delta t})[\langle\mathrm{Im}(\alpha)^2\rangle - \langle\mathrm{Im}(\alpha)\rangle^2] \\ &= \tfrac{1}{2}\left\{1 + \sin^2(\sqrt{\gamma_0\Delta t})\left[\langle(\Delta\hat{P})^2\rangle - \tfrac{1}{2}\right]\right\}, \end{aligned} \tag{27}$$

$$\begin{aligned} \langle\Delta\mathrm{Im}(\beta)^2\rangle &= \tfrac{1}{2} + \sin^2(\sqrt{\gamma_0\Delta t})[\langle\mathrm{Re}(\alpha)^2\rangle - \langle\mathrm{Re}(\alpha)\rangle^2] \\ &= \tfrac{1}{2}\left\{1 + \sin^2(\sqrt{\gamma_0\Delta t})\left[\langle(\Delta\hat{X})^2\rangle - \tfrac{1}{2}\right]\right\}. \end{aligned} \tag{28}$$

As an example, we consider a squeezed state with $\phi = 0$ and $n_{\mathrm{th}} = 0$. In this case, $\langle(\Delta\hat{P})^2\rangle \to e^{2r}/2$ and $\langle(\Delta\hat{X})^2\rangle \to e^{-2r}/2$. In the limit of large squeezing ($r \to \infty$) we find that

$$\begin{aligned} \langle\Delta\mathrm{Im}(\beta)^2\rangle &= \frac{1}{2}\left\{1 + \sin^2(\sqrt{\gamma_0\Delta t})\left[\langle(\Delta\hat{X})^2\rangle - \frac{1}{2}\right]\right\} \\ &= \frac{1}{2}\left\{1 + \sin^2(\sqrt{\gamma_0\Delta t})\left[\frac{e^{-2r}}{2} - \frac{1}{2}\right]\right\} \\ &\to \frac{1}{2}\cos^2(\sqrt{\gamma_0\Delta t}) \approx \frac{1}{2}, \end{aligned} \tag{29}$$

and the last equality holds since $\gamma_0\Delta t \ll 1$. However, we can consider the noise observable in the conjugate signal. For

$r \gg 1$, we find

$$\begin{aligned}\langle \Delta \mathrm{Re}(\beta)^2 \rangle &= \frac{1}{2}\left\{1 + \sin^2(\sqrt{\gamma_0 \Delta t})\left[\langle (\Delta \hat{P})^2 \rangle - \frac{1}{2}\right]\right\} \\ &= \frac{1}{2}\left\{1 + \sin^2(\sqrt{\gamma_0 \Delta t})\left[\frac{e^{2r}}{2} - \frac{1}{2}\right]\right\} \\ &\to \left[\frac{1}{2}\cos^2(\sqrt{\gamma_0 \Delta t}) + \frac{e^{2r}}{4}\sin^2(\sqrt{\gamma_0 \Delta t})\right]. \quad (30)\end{aligned}$$

This again suggests that a large squeezing can make the excess noise of gravitons visible as noise in one of the quadratures. However, note that the excess noise observable in each quadrature of heterodyne is a factor of 2 smaller than that observable in the homodyne signal (in units where we set $x_0 = 1$).

Next we also consider the detector response to heterodyne quadrature measurements for a maximally sub-Poissonian number state of the radiation field. We obtain

$$\begin{aligned}\langle \Delta \mathrm{Re}(\beta)^2 \rangle &= \tfrac{1}{2} + \sin^2(\sqrt{\gamma_0 \Delta t})[\langle \mathrm{Im}(\alpha)^2 \rangle - \langle \mathrm{Im}(\alpha) \rangle^2] \\ &= \tfrac{1}{2}\left\{1 + \sin^2(\sqrt{\gamma_0 \Delta t})\left[\langle (\Delta \hat{P})^2 \rangle - \tfrac{1}{2}\right]\right\} \\ &= \tfrac{1}{2}\{1 + \sin^2(\sqrt{\gamma_0 \Delta t}) n\}, \quad (31)\end{aligned}$$

$$\begin{aligned}\langle \Delta \mathrm{Im}(\beta)^2 \rangle &= \tfrac{1}{2} + \sin^2(\sqrt{\gamma_0 \Delta t})[\langle \mathrm{Re}(\alpha)^2 \rangle - \langle \mathrm{Re}(\alpha) \rangle^2] \\ &= \tfrac{1}{2}\left\{1 + \sin^2(\sqrt{\gamma_0 \Delta t})\left[\langle (\Delta \hat{X})^2 \rangle - \tfrac{1}{2}\right]\right\} \\ &= \tfrac{1}{2}\{1 + \sin^2(\sqrt{\gamma_0 \Delta t}) n\}. \quad (32)\end{aligned}$$

We arrive at the same conclusion as before. The noise characteristics in the heterodyne signal for a Fock state are similar to those of a thermal state, a factor of 2 smaller when compared to homodyne. Nevertheless, as is evident, both homodyne and heterodyne detection strategies can discriminate a Fock state from a coherent state, while click detectors cannot. Given that click detectors can discriminate a thermal state from a coherent state, these different (phase and number) quantum measurement strategies can be combined to probe the acoherence of gravitational radiation incisively.

Finally, we note that the measurements of power fluctuations in the heterodyne signal tell a different story, that is comparable to click detectors. In Sec. IV B, we show that measuring heterodyne power fluctuations does not allow us to discriminate the number state from a coherent state, while thermal states are discriminable. Here again, combining different measurement strategies helps to map out the quantum state of the radiation field.

B. Statistics of power in the heterodyne signal

We first estimate the moments of the heterodyne power, which we define as $\hat{J} = b^\dagger b$. For the average, we obtain

$$\begin{aligned}\langle \hat{J} \rangle &= \langle b^\dagger b \rangle_D = \int d^2\beta P_D(\beta)|\beta|^2 \\ &= \frac{1}{\pi}\int d^2\alpha P(\alpha) \int d^2\beta |\beta|^2 e^{-|\beta + i\alpha \sin(\sqrt{\gamma_0 \Delta t})|^2} \\ &= \int d^2\alpha P(\alpha)[1 + \sin^2(\sqrt{\gamma_0 \Delta t})|\alpha|^2] \\ &= 1 + \sin^2(\sqrt{\gamma_0 \Delta t})\langle a^\dagger a \rangle. \quad (33)\end{aligned}$$

Similarly, we can write the variance as

$$\begin{aligned}\langle (\Delta \hat{J})^2 \rangle &= \langle (b^\dagger b)^2 \rangle - \langle b^\dagger b \rangle^2 = \langle b^\dagger b \rangle + \langle (b^\dagger)^2 b^2 \rangle - \langle b^\dagger b \rangle^2 \\ &= \langle |\beta|^2 \rangle_D + \langle |\beta|^4 \rangle_D - \langle |\beta|^2 \rangle_D^2 \\ &= \int d^2\beta P_D(\beta)[|\beta|^2 + |\beta|^4 - \langle |\beta|^2 \rangle^2] \\ &= \frac{1}{\pi}\int d^2\alpha P(\alpha) \int d^2\beta e^{-|\beta + i\alpha \sin(\sqrt{\gamma_0 \Delta t})|^2} \\ &\quad \times [|\beta|^2 + |\beta|^4 - \langle |\beta|^2 \rangle^2]. \quad (34)\end{aligned}$$

We evaluated the first term above already, and the third term is simply the square of the first integral since it is integrating a constant (squared average). The second term gives

$$\begin{aligned}&\frac{1}{\pi}\int d^2\alpha P(\alpha) \int d^2\beta e^{-|\beta + i\alpha \sin(\sqrt{\gamma_0 \Delta t})|^2} |\beta|^4 \\ &= \int d^2\alpha P(\alpha)[2 + 4|\alpha|^2 \sin^2(\sqrt{\gamma_0 \Delta t}) + |\alpha|^4 \sin^4(\sqrt{\gamma_0 \Delta t})] \\ &= 2 + 4\sin^2(\sqrt{\gamma_0 \Delta t})\langle a^\dagger a \rangle + \sin^4(\sqrt{\gamma_0 \Delta t})\langle (a^\dagger)^2 a^2 \rangle. \quad (35)\end{aligned}$$

We can substitute this back, and find

$$\begin{aligned}\langle (\Delta \hat{J})^2 \rangle &= \frac{1}{\pi}\int d^2\alpha P(\alpha) \int d^2\beta e^{-|\beta + i\alpha \sin(\sqrt{\gamma_0 \Delta t})|^2}[|\beta|^2 + |\beta|^4 - \langle |\beta|^2 \rangle^2] \\ &= 2 + 3\sin^2(\sqrt{\gamma_0 \Delta t})\langle a^\dagger a \rangle + \sin^4(\sqrt{\gamma_0 \Delta t})[\langle (a^\dagger)^2 a^2 \rangle - \langle a^\dagger a \rangle^2] \\ &= 2 + 3\sin^2(\sqrt{\gamma_0 \Delta t})\langle a^\dagger a \rangle + \sin^4(\sqrt{\gamma_0 \Delta t})[\langle (a^\dagger a)^2 - a^\dagger a \rangle - \langle a^\dagger a \rangle^2] \\ &= 2 + 3\sin^2(\sqrt{\gamma_0 \Delta t})\langle \hat{N} \rangle + \sin^4(\sqrt{\gamma_0 \Delta t})[\langle (\Delta \hat{N})^2 \rangle - \langle \hat{N} \rangle] \\ &= 2 + 3\sin^2(\sqrt{\gamma_0 \Delta t})\langle \hat{N} \rangle + \sin^4(\sqrt{\gamma_0 \Delta t}) Q\langle \hat{N} \rangle, \quad (36)\end{aligned}$$

where $\hat{N} = a^\dagger a$. For $\gamma_0 \Delta t \ll 1$, we can write $\sin^2(\sqrt{\gamma_0 \Delta t}) \approx \gamma_0 \Delta t$. In this case, we have

$$\langle \hat{J} \rangle \approx 1 + \gamma_0 \Delta t \langle \hat{N} \rangle, \quad \text{and,} \quad \langle (\Delta \hat{J})^2 \rangle \approx 2 + 3\gamma_0 \Delta t \langle \hat{N} \rangle + (\gamma_0 \Delta t)^2 Q\langle \hat{N} \rangle = (3\langle \hat{J} \rangle - 1) + (\gamma_0 \Delta t)^2 Q\langle \hat{N} \rangle. \quad (37)$$

We see that super-Poissonian states which have a large Q can make the noise of the radiation field measurable in heterodyne power fluctuations too. In the gravitational context, we have $\langle\hat{N}\rangle \sin^2(\sqrt{\gamma_0\Delta t}) \approx \gamma_0\Delta t\langle\hat{N}\rangle \sim O(1)$ for Weber bars ($\gamma_0 \approx 10^{-33}$ Hz), and for intense ($\langle\hat{N}\rangle \approx 10^{36}$) gravitational radiation with energy densities comparable to LIGO band graviational waves, within reasonable time windows ($\Delta t \approx$ 1 ms). For $Q \sim O(\langle\hat{N}\rangle)$, we will also be operating in a regime where $(\gamma_0\Delta t)^2 Q\langle\hat{N}\rangle$ is substantial. Hence the excess quantum noise of gravitons in these states will manifest as excess noise in the power of the heterodyne signal.

Finally, note that above we defined the heterodyne power fluctuations based on the operator $\hat{J} = b^\dagger b$. Alternatively one could think of a classical signal $j = |\beta|^2$ and estimate its fluctuations, $(\Delta j)^2 = \langle j^2\rangle - \langle j\rangle^2$. Note the measured fluctuations will therefore be different. We obtain that the mean is the same as before, $\langle j\rangle \approx 1 + \gamma_0\Delta t\langle a^\dagger a\rangle$, and that the variance $\langle(\Delta j)^2\rangle = (2\langle j\rangle - 1) + (\gamma_0\Delta t)^2 Q\langle a^\dagger a\rangle$.

V. CONCLUSION

We have compared different quantum measurement strategies, the measurements of quantum jumps (clicks) and diffusive quantum phase measurements, and discussed how they can be complementary for probing the quantum-mechanical character of gravitational radiation using resonant mass detectors. The feasibility of such tests is justified by recent observations that resonant mass detectors allow us to have stimulated absorption probabilities that are of order unity for gravitational radiation with chirp frequencies and energy densities comparable to the LIGO band [6,32]. Our analysis suggests that the excess quantum-mechanical noise in different quantum-mechanical states of the radiation field must manifest in a complementary manner when subjected to different (number or phase) quantum measurement detection strategies. Highly super-Poissonian states of gravitational radiation, such as thermal states and highly squeezed vacuum states [for which Mandel's $Q \sim O(\langle a^\dagger a\rangle)$], can be discriminated from coherent states using resonant mass detectors operating as click detectors, as they produce excess quantum-mechanical noise in click detection. Thermal states will also show excess noise in resonant mass detectors operating in homodyne or heterodyne protocols, allowing one to check the consistency of the thermality hypothesis. The sub-Poissonian quantum states of gravitational radiation ($-1 \leqslant Q < 0$) effectively collapse to the Poissonian ($Q = 0$) prediction for click detectors, but generally produce excess noise in phase measurements. This behavior occurs when the radiation field is described by a Fock state or certain special quantum-mechanical squeezed states. In those cases, homodyne or heterodyne protocols can still reveal violation of the coherent-state hypothesis. These different detection strategies provide useful information that is complementary in the quantum-mechanical sense to test the coherent-state hypothesis for gravitational radiation. By providing more detailed tomography of the radiation, they determine its state more definitively.

The operation of resonant mass detectors or acoustic bar resonators as click detectors for gravitational radiation at kilohertz frequencies is challenging, given cooling and quality requirements for the bar resonator. But it requires only fairly modest improvements in existing cooling capabilities and quality factors [6]. Much has already been achieved. Quantum measurements that can resolve single phonons have been demonstrated for acoustic modes at much smaller mass scales [38], which could be relevant to detecting gravitational radiation at higher frequencies. This also makes the coherent-state hypothesis testing using resonant mass detectors operating as click detectors a challenging yet achievable goal, with the difficulties involved being at the same level as for inferring a single quantum exchange between resonant mass detectors and gravitational radiation [6,32].

The phase measurements discussed here are more closely aligned with the originally conceived mode of operation for resonant mass detectors [35,36]. Measuring the deviations in relative positions corresponds to phase-sensitive measurements [15,17–23], and phase-preserving measurements correspond to measuring both the relative position and momentum deviations simultaneously [16,24]. To infer quantum characteristics of gravitational radiation, beyond Weber's original approach, we need quantum-noise-limited phase measurements on bar resonators cooled to their quantum-mechanical ground state. While challenging, this is not hopelessly beyond current experimental capabilities. Quantum-noise-limited sensing of resonant mass detectors has long been discussed in the context of resonant mass detectors [21,22], although the original objective was to observe a classical signal. Early works have also looked into possible improvements in the sensitivities when a bar detector is prepared in different quantum-mechanical states [23]. There has also been recent progress in experimental implementations of such detection strategies, especially in the context of resonant mass detectors operating at high frequencies [39]. Analysis of statistical fluctuations in observed measurement records would be facilitated through the use of several resonant mass detectors operating as an array, but this does not appear much more difficult than single-detector operation. Indeed, it might well be easier to manufacture and operate an array of high-Q, ultracold bars as opposed to a single one of equivalent sensitivity. Note that low-frequency modes are associated with the long direction of a bar, so a long cylindrical shape could be favorable.

Transduction of the signal to smaller masses may also be possible [40], which may further enhance the feasibility of the tests proposed here.

Sources for gravitational radiation with quantum-mechanical characteristics are likely to exist in nature. Indeed, quantum effects such as squeezing require nonlinearities at the source, and general relativity is an inherently nonlinear theory. Single mode and multimode squeezing of gravitational radiation has been proposed in some contexts. For instance, cosmological expansion is one paradigm that is expected to squeeze primordial gravitational radiation [41,42]. The simplest of such multimode scenarios we could consider is pair creation in the gravitational context where gravitons are produced in pairs via interactions of the type

$$V_I \sim -ig(a_k a_{-k} - a_k^\dagger a_{-k}^\dagger). \tag{38}$$

Above, g can be thought of as the pump or source mode in the semiclassical approximation, analogous to the pump mode in parametric down conversion. The resultant quantum state of

gravitons will be the two-mode squeezed vacuum state:

$$|\psi\rangle = \exp(-iV_I t)|0\rangle_k|0\rangle_{-k} = \sum_n \frac{[\tanh(gt)]^n}{\cosh(gt)}|n\rangle_k|n\rangle_{-k}. \tag{39}$$

We can use this quantum state to predict the results for a ratio test that is possible with a click detector. Assume that our detector only has access to one of the modes from the pair, say, with wave vector k. To the leading order, we find the following click probabilities: $P_0 \approx 1$, $P_1 = \gamma_0 \Delta t \langle a_k^\dagger a_k\rangle$, and $P_2 \approx (\gamma_0 \Delta t)^2 \langle (a_k^\dagger)^2 a_k^2\rangle/2$. Since $\langle a_k^\dagger a_k\rangle = \sinh^2(gt)$ and $\langle (a_k^\dagger)^2 a_k^2\rangle = 2\sinh^4(gt)$ for the quantum state $|\psi\rangle$, we obtain the ratio $R = 2P_0P_2/P_1^2 = 2$. The result is different from the coherent-state result ($R = 1$), and is equivalent to the result expected for a thermal state with average quanta $n_{\rm th} = \sinh^2(gt)$. It is not surprising, as tracing out one of the modes from a two-mode squeezed vacuum state results in a thermal state for the other mode.

While a systematic analysis of nonlinearities involved can be substantially more challenging close to the merger, it has also been suggested that head-on collisions of black holes may also generate acoherent quantum-mechanical states of this nature [43]. Hence, it is highly suggestive that the nonclassical features of the gravitational radiation can be probed in near-future resonant bar detector-based measurements using the methods we have proposed here. The quantum noise characteristics observable in ground-based resonant mass detectors could reveal important information about various quantum-mechanical effects that are relevant at the sources of gravitational radiation and in propagation, thereby offering a new quantum-mechanical window into the observable universe.

ACKNOWLEDGMENTS

F.W. is supported by the U.S. Department of Energy under Contract No. DE-SC0012567 and by the Swedish Research Council under Contract No. 335-2014-7424. S.K.M. is supported in part by the Swedish Research Council under Contract No. 335-2014-7424 and in part by the Wallenberg Initiative on Networks and Quantum Information (WINQ). We thank Maulik Parikh and Igor Pikovski for stimulating conversations around these subjects.

DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: SUMMARY OF USEFUL RESULTS

Using dimensionless representation for $\hat{X}$ and $\hat{P}$,

$$\hat{X} = \frac{1}{\sqrt{2}}(a + a^\dagger), \quad \hat{P} = \frac{1}{\sqrt{2}i}(a - a^\dagger), \tag{A1}$$

we obtain $\langle\alpha|\hat{X}|\alpha\rangle = \sqrt{2}\mathrm{Re}(\alpha)$ and $\langle\alpha|\hat{X}^2|\alpha\rangle = \mathrm{Re}(\alpha^2) + |\alpha|^2 + 1/2 = 2\mathrm{Re}(\alpha)^2 + 1/2$. Similarly, $\langle\alpha|\hat{P}|\alpha\rangle = \sqrt{2}\mathrm{Im}(\alpha)$ and $\langle\alpha|\hat{P}^2|\alpha\rangle = -\mathrm{Re}(\alpha^2) + |\alpha|^2 + 1/2 = 2\mathrm{Im}(\alpha)^2 + 1/2$. We can use these results to estimate the position and momentum variances in the P representation for an arbitrary density matrix:

$$\rho = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|. \tag{A2}$$

From this we find

$$\begin{aligned}\langle(\Delta\hat{X})^2\rangle &= \frac{1}{2} + 2\int d^2\alpha P(\alpha)\mathrm{Re}(\alpha)^2 \\ &\quad -2\left[\int d^2\alpha P(\alpha)\mathrm{Re}(\alpha)\right]^2 = \frac{1}{2} + 2\langle\Delta\mathrm{Re}(\alpha)^2\rangle, \\ \langle(\Delta\hat{P})^2\rangle &= \frac{1}{2} + 2\int d^2\alpha P(\alpha)\mathrm{Im}(\alpha)^2 \\ &\quad -2\left[\int d^2\alpha P(\alpha)\mathrm{Im}(\alpha)\right]^2 = \frac{1}{2} + 2\langle\Delta\mathrm{Im}(\alpha)^2\rangle.\end{aligned} \tag{A3}$$

This yields

$$\begin{aligned}\langle\Delta\mathrm{Im}(\alpha)^2\rangle &= \tfrac{1}{2}\left[\langle(\Delta\hat{P})^2\rangle - \tfrac{1}{2}\right], \\ \langle\Delta\mathrm{Re}(\alpha)^2\rangle &= \tfrac{1}{2}\left[\langle(\Delta\hat{X})^2\rangle - \tfrac{1}{2}\right].\end{aligned} \tag{A4}$$

For a generic Gaussian state, we can use

$$\langle(\Delta\hat{P})^2\rangle = \frac{2n_{\rm th}+1}{2}[\cosh(2r) + \sinh(2r)\cos(\phi)], \tag{A5}$$

$$\langle(\Delta\hat{X})^2\rangle = \frac{2n_{\rm th}+1}{2}[\cosh(2r) - \sinh(2r)\cos(\phi)]. \tag{A6}$$

It is easy to see that the above results are true for a coherent state and a thermal state. As for a nontrivial check, we may use the following P function used for a squeezed vacuum state [44,45]:

$$P(\alpha) = [e^{O_R}\delta(\alpha_R)][e^{-O_I}\delta(\alpha_I)], \tag{A7}$$

where

$$O_r = \frac{1 - e^{2r}}{8e^{2r}}\frac{\partial^2}{\partial\alpha_R^2}, \tag{A8}$$

$$O_I = \frac{1 - e^{2r}}{8}\frac{\partial^2}{\partial\alpha_I^2}. \tag{A9}$$

We have also denoted $\mathrm{Re/Im}(\alpha) = \alpha_{R/I}$. From this, we can estimate the mean, and the variances. Note that, for the above P function, we can easily verify that the mean is equal to zero. For the variance,

$$\begin{aligned}\int d^2\alpha\alpha_R^2 P(\alpha) &= \int d\alpha_R(\alpha_R)^2[e^{O_R}\delta(\alpha_R)] \\ &= \int d\alpha_R\delta(\alpha_R)[e^{O_R}(\alpha_R)^2] = \frac{1 - e^{2r}}{4e^{2r}}.\end{aligned} \tag{A10}$$

Similarly,

$$\begin{aligned}\int d^2\alpha\alpha_I^2 P(\alpha) &= \int d\alpha_I(\alpha_I)^2[e^{-O_I}\delta(\alpha_I)] \\ &= \int d\alpha_I\delta(\alpha_I)[e^{-O_I}(\alpha_I)^2] = \frac{e^{2r} - 1}{4}.\end{aligned} \tag{A11}$$

From these we find

$$\langle(\Delta\hat{X})^2\rangle = \tfrac{1}{2} + 2\langle\Delta\mathrm{Re}(\alpha)^2\rangle = e^{-2r}/2, \tag{A12}$$

$$\langle(\Delta\hat{P})^2\rangle = \tfrac{1}{2} + 2\langle\Delta\mathrm{Im}(\alpha)^2\rangle = e^{2r}/2. \tag{A13}$$

The result matches explicitly.

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