

Decoherence and Brownian motion of a polarizable particle near a medium

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Optically levitated nanoparticles are ideal experimental testbeds for investigating macroscopic superpositions and microscopic thermodynamics. Integrating such levitated nanoparticles with photonic structures can enable strong coupling between their center-of-mass motion and guided photonic modes, facilitating enhanced control and probing of their motion. When coupling a particle to a photonic structure, such as a waveguide, the effects of fluctuations become prominent at nanoscales. In this work, we analyze the classical and quantized center-of-mass motion of a polarizable particle interacting with the fluctuations of the electromagnetic field in the presence of a medium. We derive a position localization master equation for the particle's quantized center of mass, and examine its classical center-of-mass momentum diffusion, elucidating correspondences between classical and quantum Brownian motion of polarizable particles near media. We study the decoherence rate of the particle in the presence of a planar surface as a function of temperature and distance from the surface, comparing it to common sources of decoherence. Our results are pertinent to experiments aimed at preparing levitated nanospheres in macroscopic quantum states and investigating their Brownian dynamics.

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I. INTRODUCTION

Testing the laws of quantum mechanics with macroscopic systems is a goal of foundational importance. Optically levitated dielectric nanospheres offer a promising experimental platform for realizing large superpositions of massive objects [1]. Optical levitation allows for precise control, such as the trapping and cooling of particles, while maintaining isolation from the environment, due to the absence of mechanical clamping. As a milestone toward the creation of macroscopic quantum states [2,3], experiments have achieved ground-state cooling of levitated nanospheres as massive as $\sim 10^8$ amu [4–7]. Furthermore, recent experiments have demonstrated tunable interactions between two optically trapped nanospheres, paving the way to realizing correlated macroscopic quantum systems [8–11].

When levitated nanoparticles are coupled to photonic structures, the near-field interactions with guided photonic

modes enable enhanced control of the nanoparticles' mechanical motion. For instance, experiments with trapped nanospheres coupled to photonic crystals and membranes [12] at subwavelength distances have demonstrated strong optomechanical (OM) couplings of up to $g_{\text{OM}}/2\pi \approx 9$ kHz [13] and feedback cooling down to ≈ 1 K [12]. Such systems are pertinent to microscopic thermodynamics [14,15], short-range tests of Newtonian gravity [16], and force sensing [17–19].

Integrating nanoparticles with photonic structures generates additional sources of decoherence, which poses a critical challenge in preparing macroscopic superpositions of nanoparticles. The ever-present fluctuations of the electromagnetic (EM) field represent an unavoidable source of decoherence. Bringing a nanoparticle close to the surface of a photonic structure increases both the beneficial coupling to relevant photonic modes, and the detrimental interactions with the fluctuating EM field. In present experiments the coherence of levitated nanospheres is limited by the scattering of gas molecules off the nanoparticle [2,3,20]. As future experiments achieve lower background gas pressures, fluctuation-induced decoherence is poised to become the primary barrier to realizing macroscopic quantum superpositions [21].

To understand the decoherence induced by EM field fluctuations in nanospheres close to surfaces, we derive a master equation for the quantized center of mass of a levitated nanoparticle interacting with a thermal field in the

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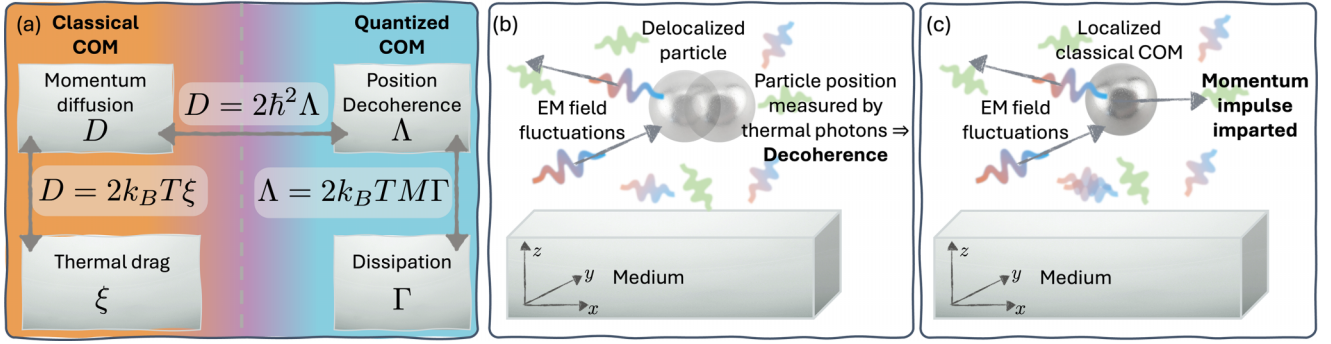


FIG. 1. (a) Connections between Brownian motion of the classical and quantized center-of-mass motion of a polarizable point particle immersed in a thermal EM environment. For the classical center of mass, momentum diffusion D and thermal drag coefficient ξ [34–36] are related by the fluctuation-dissipation relation (FDR) ($D = 2k_B T \xi$). Similarly, the decoherence rate (Λ) and dissipation (Γ) for the quantized center of mass are connected via the FDR ($\Lambda = 2k_B T M \Gamma$) [37]. Additionally, the center-of-mass decoherence rate Λ is related to the momentum diffusion D via the relation $D = 2\hbar^2 \Lambda$ [38,39]. Schematic of a polarizable particle interacting with a thermal field near a planar medium depicting its (b) quantized center-of-mass decoherence and (c) increase in momentum uncertainty.

presence of a medium. Fluctuation-induced decoherence of point charges and charge distributions near surfaces has been studied theoretically [22–28] and experimentally [29–31]. In this work, by contrast, we focus on the decoherence of a neutral, polarizable particle with an *induced dipole*, a model commonly used to describe trapped nanospheres [4,8]. We derive a quantum Brownian motion (QBM) master equation describing the fluctuation-induced dynamics of the quantized center of mass of a polarizable particle near a medium, and we analyze how the resulting decoherence depends on experimentally relevant parameters. While previous works have analyzed such QBM dynamics [21] in the presence of external fields, here we assume that there is no classical field present, which allows us to analyze the dissipative dynamics of the particle’s center-of-mass motion exclusively due to fluctuations of the field. Such decoherence dynamics can be relevant in experimental situations wherein a trap field might be switched off to allow for coherent expansion of the quantized center-of-mass wave packet [32,33].

When a particle interacts with a thermal EM field, the fluctuations that cause its quantized center of mass to decohere also induce Brownian dynamics for its classical center of mass, as shown in Fig. 1. The momentum diffusion associated with the classical center-of-mass Brownian motion is known to be related to its quantized center-of-mass decoherence rate [38,40]. We revisit this connection between Brownian motion and decoherence, in the context of a polarizable particle near a medium, by deriving a Fokker-Planck equation from our master equation. We demonstrate that the decoherence rate Λ and the diffusion rate D are inextricably linked via the relation $D = 2\hbar^2 \Lambda$, similar to the free-space case [39].

The rest of the paper is organized as follows. In Sec. II, we introduce a model of a neutral, polarizable particle interacting with the quantized EM field near a medium. In Sec. III A, we derive a master equation for the particle’s quantized center-of-mass motion. This equation describes both the fluctuation-induced dissipation and decoherence, and the Casimir-Polder potential. In Sec. III B we derive a classical Fokker-Planck-type phase-space equation from the master equation of Sec. III A, and we establish a link between the

decoherence and dissipation experienced by the quantized center of mass, and the momentum diffusion and drag experienced by the classical center of mass. In Sec. III C, we analyze decoherence and dissipation for the specific case of planar motion of the particle near a planar medium. Section IV discusses the momentum diffusion and thermal drag of the classical center-of-mass motion. We summarize and discuss our findings in Sec. V.

II. MODEL

Let us consider a linearly polarizable particle interacting with the quantized EM field in the presence of a medium, as shown in Fig. 1. We assume that all constituents of the system, i.e., the medium, the particle, and the field, are in thermal equilibrium at temperature T . The total Hamiltonian describing the system is given by $\hat{H} = \hat{H}_M + \hat{H}_F + \hat{H}_I$, where $\hat{H}_M = \frac{\hat{\mathbf{p}}^2}{2M}$ is the Hamiltonian governing the free dynamics for the particle’s quantized center-of-mass position $\hat{\mathbf{r}}$ and momentum $\hat{\mathbf{p}}$, M being the mass of the particle.

$$\hat{H}_F = \int d\omega \int d^3r' \sum_{\lambda=e,m} \hbar \omega \hat{\mathbf{f}}_{\lambda}^{\dagger}(\mathbf{r}', \omega) \cdot \hat{\mathbf{f}}_{\lambda}(\mathbf{r}', \omega) \quad (1)$$

represents the Hamiltonian for the quantized EM field in the presence of a medium as described in the macroscopic quantum electrodynamics formalism [41,42]. The operators $\hat{\mathbf{f}}_{\lambda}^{(\dagger)}(\mathbf{r}', \omega)$ represent the creation and annihilation operators for the polaritonic excitations of the field+matter system in the presence of media at frequency ω , position $\mathbf{r}'$, with $\lambda = e, m$ characterizing the quantum noise polarization (e) or magnetization (m). These operators obey the canonical commutation relations $[\hat{f}_{\lambda}(\mathbf{r}, \omega)_i, \hat{f}_{\lambda'}^{\dagger}(\mathbf{r}', \omega')_j] = \delta(\omega - \omega') \delta(\mathbf{r} - \mathbf{r}') \delta_{\lambda\lambda'} \delta_{ij}$, where the indices $i, j = \{x, y, z\}$ refer to the spatial directions.

The interaction Hamiltonian between the particle and the EM field is

$$\hat{H}_I = - \int_{V,r} d^3s \hat{\mathbf{P}}(\mathbf{s}) \cdot \hat{\mathbf{E}}(\mathbf{s}), \quad (2)$$

where $\mathbf{s}$ is a point in the interior of the polarizable particle, centered around position $\mathbf{r}$ and integrated over its volume V . $\hat{\mathbf{E}}(\mathbf{s})$ is the electric field and $\hat{\mathbf{P}}(\mathbf{s})$ is the polarization density induced by the field.

The quantized electric field near a surface is given by [41]

$$\hat{\mathbf{E}}(\mathbf{r}) = \int d\omega \int d^3r' \sum_{\lambda=e,m} [\bar{G}_\lambda(\mathbf{r}, \mathbf{r}', \omega) \cdot \hat{\mathbf{f}}_\lambda(\mathbf{r}', \omega) + \hat{\mathbf{f}}_\lambda^\dagger(\mathbf{r}', \omega) \cdot \bar{G}_\lambda^\dagger(\mathbf{r}, \mathbf{r}', \omega)], \quad (3)$$

where the coefficients $\bar{G}_\lambda(\mathbf{r}, \mathbf{r}', \omega)$ are proportional to the Green tensor $\bar{G}(\mathbf{r}, \mathbf{r}', \omega)$ of the EM field, accounting for the presence of the surface [41,43] (see Appendix A for details).

We consider the size of the particle to be small compared to the characteristic wavelengths of the EM field, such that it can be described as pointlike. The total polarization induced in the particle by the above electric field is thus $\hat{\mathbf{P}}(\mathbf{r}) \equiv \int_{V,r} d^3s \hat{\mathbf{P}}(\mathbf{s}) \approx V \hat{\mathbf{P}}(\mathbf{r})$, such that

$$\hat{\mathbf{P}}(\mathbf{r}) = \int d\omega \int d^3r' \sum_{\lambda=e,m} [\alpha(\omega) \bar{G}_\lambda(\mathbf{r}, \mathbf{r}', \omega) \cdot \hat{\mathbf{f}}_\lambda(\mathbf{r}', \omega) + \alpha^*(\omega) \hat{\mathbf{f}}_\lambda^\dagger(\mathbf{r}', \omega) \cdot \bar{G}_\lambda^\dagger(\mathbf{r}, \mathbf{r}', \omega)], \quad (4)$$

where $\alpha(\omega)$ is the linear polarizability of the particle, which we will further assume to be isotropic.

We expand the position of the particle $\hat{\mathbf{r}}_p = \mathbf{r}_0 + \hat{\mathbf{r}}$ around the classical center-of-mass position $\mathbf{r}_0$ up to first order in quantized center-of-mass displacement $\hat{\mathbf{r}}$. We obtain $\hat{H}_I \approx \hat{H}_I^{(1)} + \hat{H}_I^{(2)}$, with

$$\hat{H}_I^{(1)} = -\frac{1}{2} \hat{\mathbf{P}}(\mathbf{r}_0) \cdot \hat{\mathbf{E}}(\mathbf{r}_0), \quad (5)$$

$$\hat{H}_I^{(2)} = -\frac{1}{2} \hat{\mathbf{r}} \cdot \nabla [\hat{\mathbf{P}}(\mathbf{r}_0) \cdot \hat{\mathbf{E}}(\mathbf{r}_0)]. \quad (6)$$

Here, we assume the long-wavelength limit for interaction between the particle and thermal field, where $\lambda_T = \hbar c/k_B T \gg \Delta r$, with λ_T being the characteristic thermal photon wavelength at temperature T and Δr being the spread of the particle wave function in the x - y plane [40]. Thus, the thermal field does not vary appreciably over the range of the wave function.¹ The first term ($\hat{H}_I^{(1)}$) describes the interaction between the radiation and the induced polarization of the particle, which in turn affects the particle's *classical* center of mass. The second term ($\hat{H}_I^{(2)}$) describes the interaction between the *quantized* center-of-mass motion, EM field, and the particle's polarization altogether.

III. QUANTIZED CENTER-OF-MASS DYNAMICS

A. QBM master equation

We now describe the quantized center-of-mass dynamics resulting from the interaction between the particle and fluctuations of the EM field. In contrast with Ref. [21], the model

studied here does not include any externally applied fields. Reference [21] described the quantum Brownian motion of a polarizable particle due to the interaction between the laser field and the fluctuations in the particle's polarization, and between the thermal EM field fluctuations and the laser-induced particle polarization. In contrast, here we analyze exclusively the dissipative dynamics due to fluctuations, i.e., there is no external drive acting as a mean field about which the interaction is linearized. Experimentally, this allows us to consider decoherence in a setting wherein a trap field might be switched off to allow for coherent expansion of the quantized center-of-mass wave packet [32,33].

The Hamiltonian $\hat{H}_I^{(2)}$ for the interaction between the particle's center of mass and the fluctuations of the field can be written in the interaction picture as

$$\begin{aligned} \tilde{H}_I^{(2)}(t) &\equiv e^{-i(\hat{H}_M + \hat{H}_F)t/\hbar} \hat{H}_I^{(2)} e^{i(\hat{H}_M + \hat{H}_F)t/\hbar} \\ &= -\tilde{\mathbf{r}}(t) \cdot \tilde{\mathbf{B}}(t), \end{aligned} \quad (7)$$

where we define the operators $\tilde{\mathbf{r}}(t) \equiv e^{-i\hat{H}_M t/\hbar} \hat{\mathbf{r}} e^{i\hat{H}_M t/\hbar}$ and

$$\begin{aligned} \tilde{\mathbf{B}}(t) &\equiv e^{-i\hat{H}_F t/\hbar} \hat{\mathbf{B}} e^{i\hat{H}_F t/\hbar} \\ &= \frac{1}{2} [\partial_t \hat{\mathbf{P}}(\mathbf{r}_0, t) \cdot \hat{\mathbf{E}}(\mathbf{r}_0, t) + \hat{\mathbf{P}}(\mathbf{r}_0, t) \cdot \partial_t \hat{\mathbf{E}}(\mathbf{r}_0, t)], \end{aligned} \quad (8)$$

with $\hat{\mathbf{B}} = [\frac{1}{2} \nabla (\hat{\mathbf{P}}(\mathbf{r}_0) \cdot \hat{\mathbf{E}}(\mathbf{r}_0))]$ as the bath operator. We note that the bath operator is nonlinear in the field fluctuations, capturing particle-field interactions in the absence of an external field, which is distinct from the case studied in Ref. [21]. The dynamics of the center-of-mass density matrix, $\hat{\rho}_M = \int d^2r \int d^2r' \rho_M(\mathbf{r}, \mathbf{r}', t) |\mathbf{r}\rangle \langle \mathbf{r}'|$, with $\{\mathbf{r}, \mathbf{r}'\}$ pertaining to motion of the particle around $\mathbf{r}_0$, is given by the second-order Born-Markov master equation [37]:

$$\begin{aligned} \frac{d\hat{\rho}_M(t)}{dt} &= -\frac{i}{\hbar} \text{Tr}_B [\tilde{H}_I^{(2)}(t), \hat{\rho}_M(0) \otimes \hat{\rho}_B] - \frac{1}{\hbar^2} \text{Tr}_B \\ &\times \int_0^\infty d\tau [\tilde{H}_I^{(2)}(t), [\tilde{H}_I^{(2)}(t-\tau), \hat{\rho}_M(t) \otimes \hat{\rho}_B]]. \end{aligned} \quad (9)$$

The first term in Eq. (9) is typically neglected on account of representing a physically irrelevant modification to the system Hamiltonian; here, it corresponds to the Casimir-Polder force along the direction of motion [44].

We can simplify the above master equation to obtain the QBM master equation in the Born-Markov approximation. After transforming back to the Schrödinger picture we get:

$$\begin{aligned} \frac{d\hat{\rho}_M(t)}{dt} &= -\frac{i}{\hbar} [\hat{H}_M, \hat{\rho}_M(t)] - \frac{i}{\hbar} \sum_i C_i^{(1)} [\hat{r}_i, \hat{\rho}_M(0)] \\ &- \frac{1}{2\hbar^2} \int_0^\infty d\tau \sum_{ij} \left[i\mathcal{D}_{ij}(\tau) [\hat{r}_i, \{\hat{r}_j, \hat{\rho}_M(t)\}] \right. \\ &- i\mathcal{D}_{ij}(\tau) \frac{\tau}{M} [\hat{r}_i, \{\hat{p}_j, \hat{\rho}_M(t)\}] - \mathcal{N}_{ij}(\tau) \\ &\times [\hat{r}_i, [\hat{r}_j, \hat{\rho}_M(t)]] + \mathcal{N}_{ij}(\tau) \frac{\tau}{M} [\hat{r}_i, [\hat{p}_j, \hat{\rho}_M(t)]] \Big], \end{aligned} \quad (10)$$

¹This assumption precludes the short-wavelength limit of the radiation where one would observe a diffusive Brownian motion for the particle's center of mass.

where the indices i, j indicate displacements and momenta along the respective axes. The first-order Casimir-Polder force, as well as the dissipation and noise kernels, are defined via the expectation value and two-time correlation functions of the bath operator $\tilde{\mathbf{B}}(t)$ as

$$C_i^{(1)} \equiv \langle \tilde{B}_i(t) \rangle, \quad (11)$$

$$\mathcal{D}_{ij}(\tau) \equiv i \langle [\tilde{B}_i(t), \tilde{B}_j(t - \tau)] \rangle, \quad (12)$$

$$\mathcal{N}_{ij}(\tau) \equiv \langle \{\tilde{B}_i(t), \tilde{B}_j(t - \tau)\} \rangle. \quad (13)$$

At this point we assume that the particle's center-of-mass momentum is small; specifically, we assume $\frac{p}{M} \Delta t \ll \Delta r$, where p is the magnitude of the momentum, Δr is the spread of the particle's wave function, and Δt is the timescale over which the particle interacts with its environment. If we now note that the magnitude of the second-to-last term in Eq. (10) is $\sim \Delta r^2 \mathcal{N}(\tau)$ and the magnitude of the last term is $\sim \frac{\Delta r p}{M} \Delta t \mathcal{N}(\tau)$, then we see that their ratio is $\sim \frac{p \Delta t}{\Delta r M}$. Since $\frac{p}{M} \Delta t \ll \Delta r$, this ratio is much smaller than 1, which means that we can neglect the last term in Eq. (10) and arrive at

$$\begin{aligned} \frac{d\hat{\rho}_M}{dt} \approx & -\frac{i}{\hbar} [\hat{H}_M, \hat{\rho}_M(t)] - \frac{i}{\hbar} \sum_i C_i^{(1)} [\hat{r}_i, \hat{\rho}_M(0)] \\ & - \frac{i}{\hbar} \sum_{ij} C_{ij}^{(2)} [\hat{r}_i \hat{r}_j, \hat{\rho}_M(t)] - \frac{i}{\hbar} \sum_{ij} \Gamma_{ij} [\hat{r}_i, \{\hat{p}_j, \hat{\rho}_M(t)\}] \\ & - \sum_{ij} \Lambda_{ij} [\hat{r}_i, [\hat{r}_j, \hat{\rho}_M(t)]]. \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{d\rho_M(\mathbf{r}, \mathbf{r}', t)}{dt} = & \frac{i\hbar}{2M} \left(\frac{\partial^2}{\partial r_i^2} - \frac{\partial^2}{\partial r_i'^2} \right) \rho_M(\mathbf{r}, \mathbf{r}', t) - \frac{i}{\hbar} \sum_i C_i^{(1)} (r_i - r_i') \rho_M(\mathbf{r}, \mathbf{r}', 0) - \frac{i}{\hbar} \sum_{ij} C_{ij}^{(2)} (r_i r_j - r_i' r_j') \rho_M(\mathbf{r}, \mathbf{r}', t) \\ & - \sum_{ij} \Gamma_{ij} (r_i - r_i') \left(\frac{\partial}{\partial r_j} - \frac{\partial}{\partial r_j'} \right) \rho_M(\mathbf{r}, \mathbf{r}', t) - \sum_{ij} \Lambda_{ij} (r_i - r_i') (r_j - r_j') \rho_M(\mathbf{r}, \mathbf{r}', t). \end{aligned} \quad (18)$$

Now, we take the Wigner-Weyl transform of the above equation by making the substitution $r_i = \bar{r}_i + \frac{a_i}{2}$ and $r_i' = \bar{r}_i - \frac{a_i}{2}$ and using $W(\mathbf{r}, \mathbf{p}) = \int d\mathbf{a} e^{i\mathbf{p}\cdot\mathbf{a}} / \hbar \rho(\mathbf{r} + \mathbf{a}/2, \mathbf{r} - \mathbf{a}/2)$ [37,40]. After relabeling $\bar{r}_i$ as r_i we obtain the quantum Fokker-Planck equation:

$$\begin{aligned} \frac{\partial W}{\partial t} = & - \sum_i \frac{p_i}{M} \frac{\partial W}{\partial r_i} - \sum_i C_i^{(1)} \frac{\partial W}{\partial p_i} - 2 \sum_{ij} C_{ij}^{(2)} r_i \frac{\partial W}{\partial p_j} \\ & + \sum_{ij} 2\Gamma_{ij} \frac{\partial(p_j W)}{\partial p_i} + \sum_{ij} \Lambda_{ij} \hbar^2 \frac{\partial W}{\partial p_i \partial p_j}. \end{aligned} \quad (19)$$

Analogous to the master equation [Eq. (14)], the second and the third terms correspond to the first- and second-order Casimir-Polder potentials. The fourth term corresponds to friction and the fifth term represents the momentum diffusion coefficient of the quantized center of mass, which is therefore

The first term on the right describes unitary evolution under the free-particle Hamiltonian. The second term corresponds to the first-order Casimir-Polder potential; the third term represents a harmonic potential resulting from the second-order Casimir-Polder interaction between the particle and medium [45], with

$$C_{ij}^{(2)} \equiv \frac{1}{2\hbar} \int_0^\infty d\tau \mathcal{D}_{ij}(\tau). \quad (15)$$

The fourth term represents dissipation or friction on the quantized center of mass, with [37]:

$$\Gamma_{ij} \equiv \frac{1}{2M\hbar} \int_0^\infty d\tau \tau \mathcal{D}_{ij}(\tau). \quad (16)$$

The last term stands for decoherence that acts to localize the center of mass in the position basis, with a localization rate [37]

$$\Lambda_{ij} \equiv \frac{1}{2\hbar^2} \int_0^\infty d\tau \mathcal{N}_{ij}(\tau). \quad (17)$$

We will now investigate these effects for the specific example of a polarizable particle near a planar medium.

B. Fokker-Planck equation

To illustrate how decoherence and dissipation (Λ_{ij} and Γ_{ij}) are linked to Brownian motion for the classical center of mass, we now derive a phase-space equation from the master equation [46]. To this end, we first write Eq. (14) in the position basis as

related to the decoherence coefficient in Eq. (14) as

$$2\hbar^2 \Lambda_{ij} = \frac{\langle \Delta \hat{p}_i \Delta \hat{p}_j \rangle_q}{\Delta t}, \quad (20)$$

where the subscript “ q ” refers to a quantum expectation value. The above relation can be seen as a manifestation of the complementarity of the position and momentum variables. Such a relation was established in Ref. [47] for general QBM dynamics.

Classically, the diffusion coefficient is defined as

$$D_{ij} = \frac{\langle \Delta p_i \Delta p_j \rangle_c}{\Delta t}, \quad (21)$$

where the subscript “ c ” denotes a classical average. Invoking the Ehrenfest theorem [48] to form a correspondence between classical and quantum expectation values, we establish a relation between momentum diffusion for the classical center of

mass and decoherence of the quantized center of mass:

$$\frac{\langle \Delta p_i \Delta p_j \rangle_c}{\Delta t} = \frac{\langle \Delta \hat{p}_i \Delta \hat{p}_j \rangle_q}{\Delta t} \Rightarrow D_{ij} = 2\hbar^2 \Lambda_{ij}. \quad (22)$$

This relation reflects the fact that both decoherence and momentum diffusion arise due to the scattering of thermal photons. All the conclusions drawn in this section hold for an arbitrary macroscopic body.

We can motivate the result $D_{ij} = 2\hbar^2 \Lambda_{ij}$ [Eq. (22)] through a back-of-the-envelope calculation, treating momentum diffusion as the cumulative effect of many random “kicks” delivered to a Brownian particle. These kicks produce momentum changes $\cdots \delta \mathbf{p}_{n-1}, \delta \mathbf{p}_n, \delta \mathbf{p}_{n+1} \cdots$, which sum to $\Delta \mathbf{p} \equiv \sum_n \delta \mathbf{p}_n$ over a time Δt . For sufficiently many kicks, $\Delta \mathbf{p}$ becomes a random sample from the Gaussian distribution

$$\eta(\Delta \mathbf{p}) = \sqrt{\frac{|K|}{2\pi \Delta t}} \exp \left[-\frac{\sum_{ij} K_{ij} \Delta p_i \Delta p_j}{2\Delta t} \right], \quad (23)$$

where $K \equiv D^{-1}$ is the inverse of the momentum diffusion matrix given by $D_{ij} = \langle \Delta p_i \Delta p_j \rangle / \Delta t$.

To evaluate the decoherence produced by these kicks, note that when a quantum system undergoes a sudden momentum change $\delta \mathbf{p}$, its density matrix $\hat{\rho}$ transforms as $\hat{\rho} \rightarrow e^{-i\delta \mathbf{p} \cdot \hat{\mathbf{r}}/\hbar} \hat{\rho} e^{i\delta \mathbf{p} \cdot \hat{\mathbf{r}}/\hbar}$. Many such changes, summing to $\Delta \mathbf{p}$ over time Δt , produce the change (in the position representation)

$$\rho(\mathbf{r}, \mathbf{r}', t + \Delta t) = e^{-i\Delta \mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')/\hbar} \rho(\mathbf{r}, \mathbf{r}', t), \quad (24)$$

with $\Delta \mathbf{p} \equiv \sum_n \delta \mathbf{p}_n$. Averaging over $\eta(\Delta \mathbf{p})$ [Eq. (23)] leads to the expression

$$\begin{aligned} \rho(\mathbf{r}, \mathbf{r}', t + \Delta t) &= \langle e^{-i\Delta \mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')/\hbar} \rangle \rho(\mathbf{r}, \mathbf{r}', t) \\ &= e^{-\sum_{ij} D_{ij} (r_i - r'_i)(r_j - r'_j) \Delta t / 2\hbar^2} \rho(\mathbf{r}, \mathbf{r}', t), \end{aligned} \quad (25)$$

describing decoherence in the position representation.

Now, consider the effects of the decoherence term in Eq. (18):

$$\frac{\partial}{\partial t} \rho(\mathbf{r}, \mathbf{r}', t) = - \sum_{ij} \Lambda_{ij} (r_i - r'_i)(r_j - r'_j) \rho(\mathbf{r}, \mathbf{r}', t). \quad (26)$$

Upon integrating, we obtain the expression

$$\rho(\mathbf{r}, \mathbf{r}', t + \Delta t) = e^{-\sum_{ij} \Lambda_{ij} (r_i - r'_i)(r_j - r'_j) \Delta t} \rho(\mathbf{r}, \mathbf{r}', t), \quad (27)$$

which has the same form as Eq. (25). Comparing the two gives $D_{ij} = 2\hbar^2 \Lambda_{ij}$.

C. Planar medium

While the QBM master equation [Eq. (9)] describes motion of a polarizable particle interacting with a thermal field near an arbitrary medium, as a concrete example we consider the motion of the particle in the x - y plane ($\hat{\mathbf{r}} = \hat{x}\mathbf{e}_x + \hat{y}\mathbf{e}_y$) near a planar medium as depicted in Fig. 1. In this case, we utilize the statistical independence of the electric field $\hat{\mathbf{E}}(\mathbf{r})$ and its spatial derivative $\partial_i \hat{\mathbf{E}}(\mathbf{r})$ for a thermal state of the EM field [39]:

$$\langle \hat{\mathbf{E}}_p(\mathbf{r}_0, t) \partial_i \hat{\mathbf{E}}_k(\mathbf{r}_0, t - \tau) \rangle = 0, \quad (28)$$

which allows one to simplify each term of the master equation [Eq. (14)] as follows.

The first term in Eq. (14) corresponds to the Casimir-Polder force along direction $i \in \{x, y\}$. Since near a planar surface, the force depends only on the vertical distance z from the surface, the corresponding forces parallel to the surface are zero: $\langle \hat{\mathbf{E}}_p(\mathbf{r}) \partial_i \hat{\mathbf{E}}_k(\mathbf{r}) \rangle \rightarrow 0$ [41].

The coefficients $C_{ij}^{(2)}$, Γ_{ij} , and Λ_{ij} as defined in Eqs. (15)–(17), simplify to (see Appendixes B and C for details)

$$\begin{aligned} C_{ij}^{(2)} &= \frac{\hbar \mu_0^2}{2\pi} \sum_{pk=\{x,y,z\}} \int_0^\infty d\omega \omega^4 |\alpha(\omega)|^2 (2n_{\text{th}}(\omega) + 1) \\ &\quad \times [\text{Re}\{\partial_i G_{pk}(\mathbf{r}_0, \mathbf{r}_0, \omega) \partial_j\} \text{Im}\{G_{pk}(\mathbf{r}_0, \mathbf{r}_0, \omega)\} \\ &\quad + \text{Re}\{G_{pk}(\mathbf{r}_0, \mathbf{r}_0, \omega)\} \text{Im}\{\partial_j G_{pk}(\mathbf{r}_0, \mathbf{r}_0, \omega) \partial_i\}] \end{aligned} \quad (29)$$

$$\begin{aligned} \Gamma_{ij} &= \frac{\hbar \mu_0^2}{\pi M} \int_0^\infty d\omega \omega^4 |\alpha(\omega)|^2 \frac{\partial n_{\text{th}}(\omega)}{\partial \omega} \\ &\quad \times \sum_{p,k=\{x,y,z\}} \text{Im}[G_{pk}(\mathbf{r}_0, \mathbf{r}_0, \omega)] \text{Im}[\partial_i G_{pk}(\mathbf{r}_0, \mathbf{r}_0, \omega) \partial_j] \end{aligned} \quad (30)$$

$$\begin{aligned} \Lambda_{ij} &= \frac{2\mu_0^2}{\pi} \int d\omega \omega^4 |\alpha(\omega)|^2 n_{\text{th}}(\omega) (n_{\text{th}}(\omega) + 1) \\ &\quad \times \sum_{p,k=\{x,y,z\}} \text{Im}[G_{pk}(\mathbf{r}_0, \mathbf{r}_0, \omega)] \text{Im}[\partial_i G_{pk}(\mathbf{r}_0, \mathbf{r}_0, \omega) \partial_j], \end{aligned} \quad (31)$$

where $n_{\text{th}}(\omega) = \frac{1}{e^{\hbar\omega/k_B T} - 1}$ is the Bose-Einstein distribution at temperature T , and $\partial_i \bar{G}(\mathbf{r}_1, \mathbf{r}_2, \omega) \partial_j$ represents the double derivative of the Green tensor ($\bar{G}(\mathbf{r}_1, \mathbf{r}_2, \omega)$) with respect to r_{1i} and r_{2j} .

We note a few salient features of the above expressions for the center-of-mass decoherence and dissipation:

(1) For n_{th} given by the Bose-Einstein distribution $n_{\text{th}}(\omega) = \frac{1}{e^{\hbar\omega/(k_B T)} - 1} \Rightarrow \frac{\partial n_{\text{th}}(\omega)}{\partial \omega} = \frac{\hbar}{k_B T} n_{\text{th}}(\omega) (n_{\text{th}}(\omega) + 1)$, which yields [from Eqs. (31) and (30)]

$$2M\Gamma_{ij} k_B T = \hbar^2 \Lambda_{ij}, \quad (32)$$

in accordance with the fluctuation-dissipation relation between the quantized center-of-mass decoherence and dissipation, as shown in Fig. 1(a). Conversely, instead of assuming n_{th} to be the Bose-Einstein distribution, one could use the fluctuation-dissipation relation to relate the dissipation and decoherence coefficients, thus solving for n_{th} , and deriving the Bose-Einstein distribution for the field near a medium. While the Planck spectrum can generally assume a different form near boundaries [49], we show that for the system under consideration, the overall energy spectrum of the field changes due to its field mode distribution, not the thermal state statistics [$n_{\text{th}}(\omega)$] [39].

(2) Due to planar symmetry, the dissipation and decoherence coefficients can be simplified to

$$\Gamma_{ij} = \Gamma \delta_{ij} = (\Gamma^{\text{free}} + \Gamma^{\text{s}}) \delta_{ij}, \quad (33)$$

$$\Lambda_{ij} = \Lambda \delta_{ij} = (\Lambda^{\text{free}} + \Lambda^{\text{s}}) \delta_{ij}, \quad (34)$$

where $\Lambda^{\text{free}}, \Gamma^{\text{free}}$ are the contributions due to free-space scattering processes and $\Lambda^{\text{s}}, \Gamma^{\text{s}}$ are the surface-induced

contributions, arising from free space and scattering parts of the total Green tensor (see Appendix A). Λ^{free} , which corresponds to the decoherence due to thermal radiation in the absence of any media, becomes

$$\Lambda^{\text{free}} = \frac{1}{18\pi^3\epsilon_0^2c^8} \int_0^\infty d\omega \omega^8 |\alpha(\omega)|^2 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1), \quad (35)$$

in agreement with the previous expressions of free-space decoherence in thermal fields obtained via the scattering theory formalism [20,39,40].

(3) Decoherence Λ_{ij} [Eq. (31)] can be connected to the decoherence for an oscillating dipole as derived in Ref. [27], by replacing the permanent dipole with an induced one.

(4) The center-of-mass decoherence and dissipation vanish at zero temperature, implying that there is no dissipative dynamics purely from quantum fluctuations in the absence of thermal photons, to second order in the particle-field interaction. For finite temperature, the decoherence and dissipation increase with medium losses determined by $\text{Im} \bar{G}(\mathbf{r}_0, \mathbf{r}_0, \omega)$.

(5) The expressions of dissipation and decoherence scale with $\sim n_{\text{th}}^2(\omega) + n_{\text{th}}(\omega)$. The $n_{\text{th}}(\omega)$ and $n_{\text{th}}^2(\omega)$ terms correspond, respectively, to the “particle-like” and “wavelike” contributions of the field [50].

(6) Last, compared to the QBM master equation of a polarizable particle in the presence of an external drive [21], the effective spectral density of a purely thermal environment scales nonlinearly with the thermal occupation $n_{\text{th}}(\omega)$ and the surface loss $\sim \text{Im} \bar{G}$.

D. Decoherence near a planar metallic surface

We now analyze the decoherence rate for a silica nanoparticle of radius $r \approx 100$ nm near a gold half-space medium. Such a scenario is considered in Ref. [51]. We describe the metal as a Drude-model conductor with permittivity:

$$\epsilon_S(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}, \quad (36)$$

with plasma frequency $\omega_p \approx 1.37 \times 10^{16}$ Hz and a loss parameter $\gamma \approx 5.31 \times 10^{13}$ Hz [52].

The polarizability of the nanospheres we assume to be described by the Clausius-Mosotti relation:

$$\alpha(\omega) = 3\epsilon_0 V \left(\frac{\epsilon_N(\omega) - 1}{\epsilon_N(\omega) + 2} \right), \quad (37)$$

where V is the volume of the nanosphere and ϵ_N is the permittivity of nanosphere, which we describe as a Lorentz-model dielectric:

$$\epsilon_N(\omega) = 1 + \sum_{i=1,2} \frac{\omega_{p_i}^2}{\omega_i^2 - \omega^2 - i\gamma_i\omega}, \quad (38)$$

where the transition frequencies, plasma frequencies, and loss parameters are $\omega_{t1} = 1.32 \times 10^{14}$ Hz, $\omega_{t2} = 2.72 \times 10^{16}$ Hz, $\omega_{p1} = 1.75 \times 10^{14}$ Hz, $\omega_{p2} = 2.96 \times 10^{16}$ Hz, $\gamma_1 = 4.28 \times 10^{13}$ Hz, and $\gamma_2 = 8.09 \times 10^{15}$ Hz [53]. The surface-induced decoherence Λ^s simplifies to (see Appendix C 1 for

details)

$$\Lambda^s = \frac{2|\alpha(0)|^2\pi^3\gamma}{105\epsilon_0^2c^5\omega_p^2} \frac{1}{z^3} \left(\frac{k_B T}{\hbar} \right)^7 + \frac{|\alpha(0)|^2\pi\gamma}{15\epsilon_0^2c^3\omega_p^2} \frac{1}{z^5} \left(\frac{k_B T}{\hbar} \right)^5 + \frac{3|\alpha(0)|^2\gamma^2}{64\omega_p^4\epsilon_0^2\pi} \frac{1}{z^8} \left(\frac{k_B T}{\hbar} \right)^3. \quad (39)$$

Thus, near a gold surface the surface-mediated decoherence exhibits three different scalings with particle-surface separation z and field temperature T :

$$z \gg \sqrt{\frac{7}{2}} \frac{\lambda_T}{\pi} : \{z^{-3}, T^7\} \text{ (far field)}, \quad (40)$$

$$\left(\frac{45\gamma c \lambda_T^2}{64\omega_p^2\pi^2} \right)^{1/3} \lesssim z \lesssim \sqrt{\frac{7}{2}} \frac{\lambda_T}{\pi} : \{z^{-5}, T^5\} \text{ (intermediate)}, \quad (41)$$

$$z \ll \left(\frac{45\gamma c \lambda_T^2}{64\omega_p^2\pi^2} \right)^{1/3} : \{z^{-8}, T^3\} \text{ (near field)}. \quad (42)$$

Similar expressions can be derived for the remaining coefficients, as detailed in Appendixes C 2 and D.

Figure 2(a) shows the decoherence rate close to a gold surface for three different temperatures compared with three different decoherence mechanisms: gas scattering [40], thermal radiation, and laser scattering [21]. For low temperatures the decoherence is dominated by gas scattering and laser scattering, whereas at room temperature thermal radiation is the dominant decoherence mechanism. Furthermore, at room temperature the typical thermal wavelength is smaller than the radius of the sphere. Thus, for realistic distances beyond 100 nm from the surface where the sphere does not touch the surface, the sphere is always in the far-field regime, and Λ^s is dominated by the first term in Eq. (39), scaling as z^{-3} . Going to lower temperatures increases the typical thermal wavelength and pushes the far-field regimes further away from the surface. At $T \approx 1$ K the intermediate regime, scaling as z^{-5} starts to appear, and for very low temperatures the near-field regime, which scales as z^{-8} , appears. Figure 2(b) shows the scaling of the various decoherence mechanisms with temperature at a distance of $z = 10 \mu\text{m}$ from the surface. At this distance the decoherence is dominated by gas and laser scattering at temperatures below ~ 1 K and by thermal radiation above $T \sim 1$ K. Figure 3 shows the three surface contributions to the decoherence coefficient [Eq. (39)] as a function of distance and temperature.

In Figs. 4(a) and 4(b) we summarize the comparison to other sources of decoherence by plotting the dominant decoherence source for a fixed laser intensity for the parameter's space spanned by pressure and temperature as well as by the distance from the surface and the temperature. It can be seen that for our model at realistic pressures, surface-induced decoherence effects are only dominant at very small distances from the surface. Surface effects are thus not expected to play a role at experimentally relevant distances from the surface. We also note that the assumption that the whole system is in thermal equilibrium is somewhat unrealistic, since optically trapped particles are typically heated up by the laser drive.

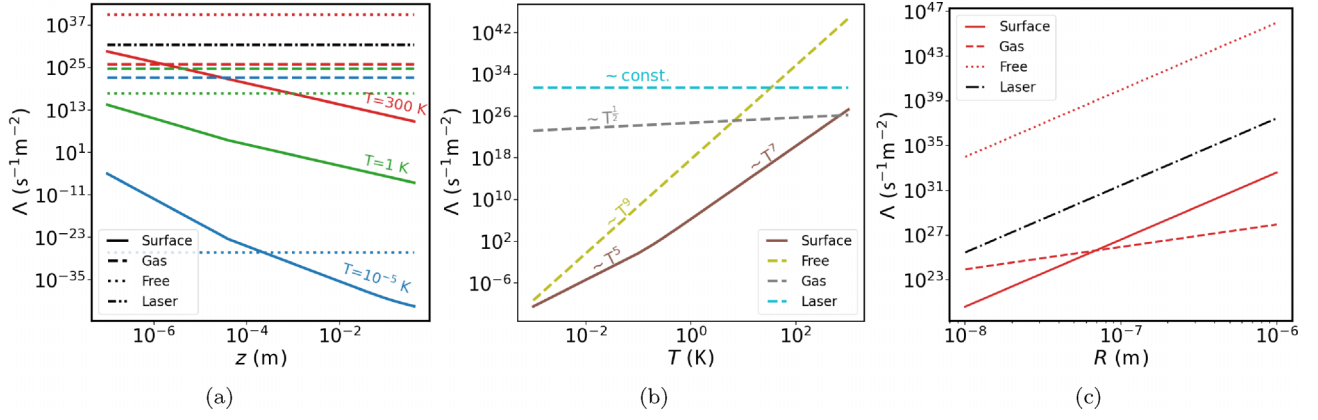


FIG. 2. (a) The decoherence rate of a dielectric polarizable silica particle with radius $r \approx 100$ nm near a gold surface as a function of distance z from the surface. The solid lines represent the surface-induced decoherence at three different temperatures, red: $T = 300$ K, green: $T = 1$ K, and blue: $T = 10^{-5}$ K. We assume all constituents of the model, i.e., the surface, particle, EM field, and the ambient gas, to be at thermal equilibrium. For comparison we also plot the decoherence rate due to three standard sources for each temperature: (1) free-space thermal photon scattering (dotted curves), (2) background gas scattering (dashed curves) at pressure $P = 10^{-8}$ mbar, and (3) laser scattering (dashed-dotted line) with a wavelength of 1064 nm, a power of 0.5 W, and a beam waist of $0.73 \mu\text{m}$, which amounts to an intensity of $2.9 \times 10^7 \text{ W/cm}^2$. (b) The decoherence rate of a dielectric polarizable particle at distance $z = 10 \mu\text{m}$ from a gold surface as a function of temperature T . For comparison we plot the decoherence rate due to blackbody radiation, gas scattering at $P = 10^{-8}$ mbar, and laser scattering with a wavelength of 1064 nm and an intensity of $2.9 \times 10^7 \text{ W/cm}^2$. The labels above the lines indicate the scaling with temperature of the various decoherence mechanisms. (c) The decoherence rate as a function of nanosphere radius at $T = 300$ K, $P = 10^{-8}$ mbar. It should be noted that optical trapping of spheres with a radius smaller than 50 nm becomes difficult because of the decrease in trap depth.

A more careful description of the system would include this nonequilibrium scenario; see for example Ref. [15].

IV. CLASSICAL CENTER-OF-MASS BROWNIAN MOTION

We now turn to the Brownian motion properties of the classical center of mass of the particle interacting with the thermal field near a surface. The force acting on the classical

center of mass of the particle along the i axis is derived from the first-order Hamiltonian $\hat{H}_I^{(1)}$ as

$$\hat{F}_i = -\partial_i \hat{H}_I^{(1)} = \tilde{\mathcal{B}}_i(t), \quad (43)$$

which is the same as the i th component of the bath operator in Eq. (8). The operator nature of the force results from the quantized EM field exerting a force on the particle's classical center of mass. The resulting force is averaged over the

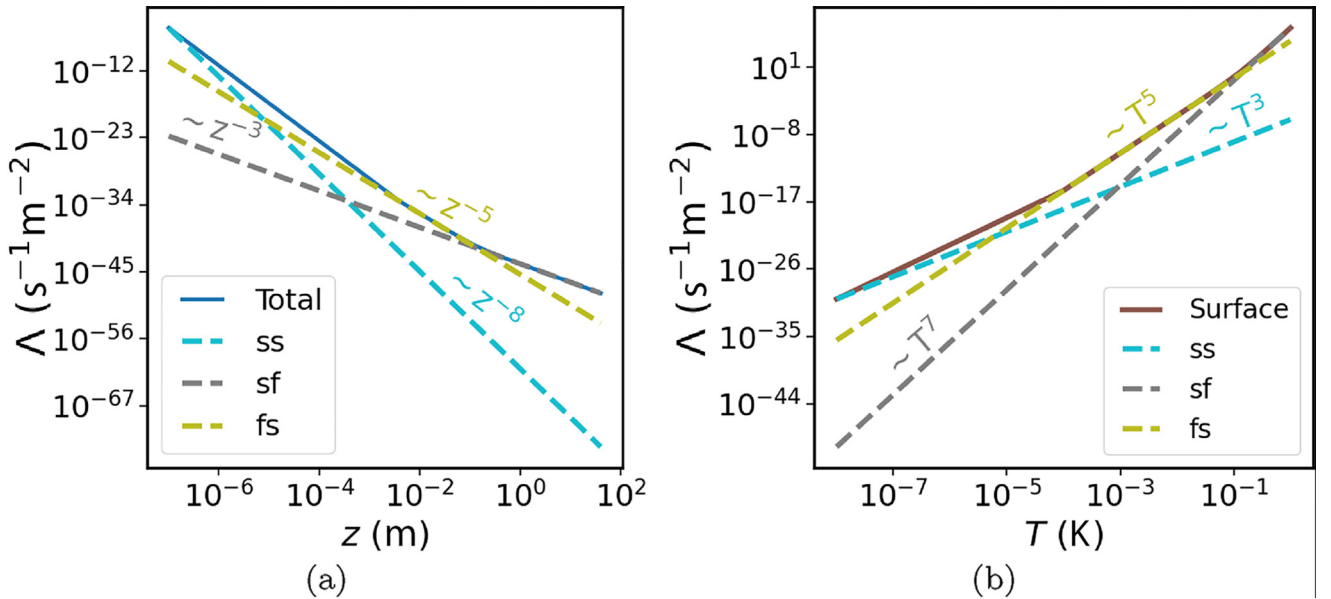


FIG. 3. (a) The different contributions to the distance scaling of Λ^s at $T = 10^{-5}$ K. The three terms in Eq. (39) are labeled as ss, sf, and fs according to their origin as terms depending on $\bar{G}^s - \bar{G}^s$, $\bar{G}^s - \bar{G}^f$, and $\bar{G}^f - \bar{G}^s$ in Eq. (39), where $\bar{G}^f$ and $\bar{G}^s$ are defined in Appendix A. At $T = 10^{-5}$ K, Λ^s scales as z^{-3} in the far field, as z^{-5} in the intermediate regime, and as z^{-8} in the near field. (b) The different contributions to the temperature scaling of Λ^s .

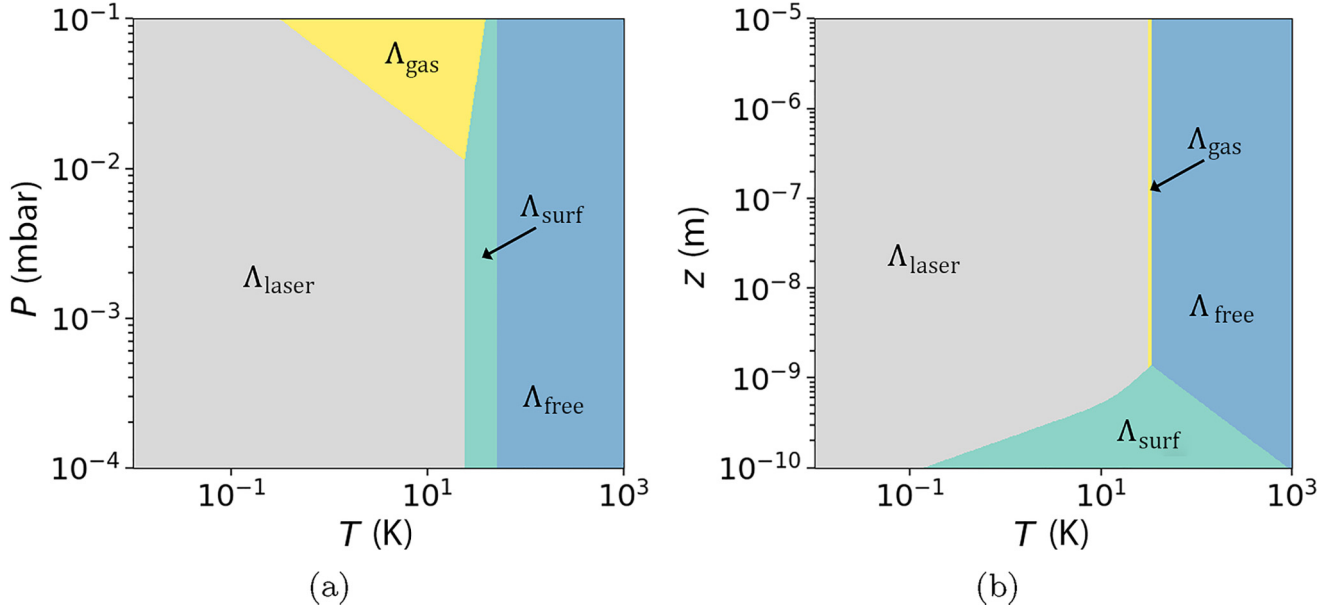


FIG. 4. (a) Dominant contributions to the total decoherence rate in the parameter space spanned by the pressure P and temperature T . The laser is assumed to have a wavelength of 1064 nm and an intensity of $2.9 \times 10^7 \text{ W/cm}^2$ as before. The distance is taken to be 1 nm from the surface, which illustrates that one needs to go to very small distances from the surface to see surface-induced decoherence effects, where finite-size effects would come into play. Therefore, surface effects are not expected to play a role at experimentally relevant distances from the surface. (b) Dominant contributions to the total decoherence rate in the parameter space spanned by the distance from the surface z and the temperature T . The pressure is set to 10^{-2} mbar. It can be seen that surface-induced decoherence effects only play a role at very small distances from the surface.

thermal density matrix of the field, leading to a net momentum impulse of zero, such that $\langle \Delta p_i \rangle = -\int_0^{\Delta t} d\tau \langle \tilde{B}_i(\tau) \rangle = 0$. The corresponding momentum correlator is thus found from the two-time correlation of the forces as

$$\langle \Delta p_i \Delta p_j \rangle = \int_0^{\Delta t} d\tau_1 \int_0^{\Delta t} d\tau_2 \langle \tilde{B}_i(\tau_1) \tilde{B}_j(\tau_2) \rangle. \quad (44)$$

In the presence of a planar medium, we see that the momentum diffusion for the particle's classical center-of-mass diffusion can be found as the rate of increase in the momentum uncertainty with time, $D_{ij} \equiv \frac{\langle \Delta p_i \Delta p_j \rangle}{\Delta t}$, as (see Appendix D for details)

$$D_{ij} = \frac{4\hbar^2 \mu_0^2}{\pi} \int_0^\infty d\omega \omega^4 n_{\text{th}}(\omega) (n_{\text{th}}(\omega) + 1) |\alpha(\omega)|^2 \times \sum_{p,k=\{x,y,z\}} \text{Im}[G_{pk}(\mathbf{r}_0, \mathbf{r}_0, \omega)] \text{Im}[\partial_i G_{pk}(\mathbf{r}_0, \mathbf{r}_0, \omega) \partial_j]. \quad (45)$$

As shown in the previous section, the above expression for momentum diffusion is related to the decoherence rate in Eq. (31) as $D_{ij} = 2\hbar^2 \Lambda_{ij} = 2\hbar^2 \Lambda \delta_{ij}$.

The above momentum diffusion of a polarizable particle moving through blackbody radiation is accompanied by a drag force of the form $(F_{\text{drag}})_i = -\xi_{ij} v_j$ [34], with v_j as the particle's classical center-of-mass velocity, and ξ_{ij} as the friction coefficient. Using the fluctuation-dissipation theorem $D_{ij} = 2\xi_{ij} k_B T$, we obtain the friction coefficient

as

$$\xi_{ij} = \frac{2\hbar^2 \mu_0^2}{k_B T \pi} \int_0^\infty d\omega \omega^4 n_{\text{th}}(\omega) (n_{\text{th}}(\omega) + 1) |\alpha(\omega)|^2 \times \sum_{p,k=\{x,y,z\}} \text{Im}[G_{pk}(\mathbf{r}_0, \mathbf{r}_0, \omega)] \text{Im}[\partial_i G_{pk}(\mathbf{r}_0, \mathbf{r}_0, \omega) \partial_j], \quad (46)$$

in agreement with previous results on thermal drag near surfaces [54].² As before, using planar symmetry, the diffusion and drag coefficients, $D_{ij} = (D^{\text{free}} + D^s) \delta_{ij}$ and $\xi_{ij} = \gamma \delta_{ij} = (\xi^{\text{free}} + \xi^s) \delta_{ij}$, respectively, can be separated into free-space ($D^{\text{free}}, \xi^{\text{free}}$) and surface-induced (D^s, ξ^s) components. In the free-space limit, we recover

$$D^{\text{free}} = \frac{\hbar^2}{9\pi^3 \epsilon_0^2 c^8} \int_0^\infty d\omega \omega^8 |\alpha(\omega)|^2 n_{\text{th}}(\omega) (n_{\text{th}}(\omega) + 1), \quad (47)$$

$$\xi^{\text{free}} = \frac{\hbar^2}{18k_B T \pi^3 \epsilon_0^2 c^8} \int_0^\infty d\omega \omega^8 |\alpha(\omega)|^2 n_{\text{th}}(\omega) (n_{\text{th}}(\omega) + 1), \quad (48)$$

in agreement with Ref. [39].

V. DISCUSSION

To summarize, we establish a connection between the Brownian motion of the quantum and classical center of mass:

²The friction from thermal radiation is distinct from that due to quantum fluctuations; the two exhibit different velocity scalings [54].

a neutral, polarizable particle interacting with the fluctuations of a thermal EM field near a medium [Fig. 1(a)]. We derive a second-order Born-Markov QBM master equation for the particle's quantized center of mass near an arbitrary macroscopic body, which encompasses the thermal Casimir-Polder force, position decoherence, and dissipation [Eq. (14)]. We analyze this master equation for the specific case wherein the motion of the particle is restricted to a plane parallel to a planar medium, demonstrating that the decoherence and dissipation of the particle increase with the medium losses [Eqs. (31) and (30)]. Further deriving the quantum Fokker-Planck equation corresponding to the QBM master equation [Eq. (19)], we link the quantized center-of-mass decoherence Λ to the classical center-of-mass momentum diffusion D via the simple relation $D = 2\hbar^2\Lambda$. This relation relies on the position decoherence form of the QBM master equation in the "long-wavelength limit," and is a manifestation of the position-momentum complementarity and Ehrenfest theorem. Subsequently, we calculate an explicit expression for the momentum diffusion and classical drag of the polarizable particle in the presence of a planar medium [Eqs. (45) and (46)].

In the free-space limit our decoherence expression agrees with Refs. [39,40], and the classical momentum diffusion and thermal drag near surfaces are in agreement with Refs. [54,55]. The connection between decoherence and diffusion highlighted in Sec. IIIB has been previously pointed out by Ref. [47] for general QBM dynamics.

Loss of spatial coherence of particles near surfaces is also pertinent to matter-wave interferometry [56–61]. Previous works have investigated the effect of fluctuation-induced van der Waals interactions between particles and gratings which causes dephasing [56,62]. Dephasing or phase averaging also occurs when the system interacts with an ensemble of EM sources [63]. This statistical effect occurs as the phases induced on the system state fluctuate due to a changing environment, leading to an apparent loss of coherence after averaging over many runs of the experiment.

In a recent work, it was illustrated that a polarizable particle in uniformly accelerated motion experiences momentum fluctuations in accord with its observed Unruh-Davies temperature [64]. The relation between momentum fluctuations and center-of-mass decoherence suggests that a uniformly accelerating particle prepared in spatial superposition should decohere at a rate determined by the effective Unruh-Davies temperature. Such decoherence and Brownian motion in accelerated frames of reference can further correspond to decoherence of particles in the vicinity of black hole event horizons via the equivalence principle, a subject of growing recent interest [65–68].

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APPENDIX A: THE ELECTROMAGNETIC GREEN TENSOR

The electromagnetic Green tensor $\bar{\bar{G}}(\mathbf{r}_1, \mathbf{r}_2, \omega)$ represents the propagator for the EM field in the presence of media, accounting for how a photon at frequency ω propagates between positions $\mathbf{r}_1$ and $\mathbf{r}_2$. The Green tensor can be found as the solution of the Helmholtz equation [43],

$$\left(\nabla_1 \times \frac{1}{\mu(\mathbf{r}_1, \omega)} \nabla_1 \times - \frac{\omega^2}{c^2} \epsilon(\mathbf{r}_1, \omega) \right) \bar{\bar{G}}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (\text{A1})$$

where $\epsilon(\mathbf{r}, \omega)$ and $\mu(\mathbf{r}, \omega)$ represent, respectively, the spatially dependent dielectric permittivity and magnetic permeability in the presence of a medium. The Green tensor encompasses the effects of surface geometry, optical and material properties of the medium, thereby modifying the density of states of the EM environment. For a general medium the Green tensor can always be split up into a free-space Green tensor $\bar{\bar{G}}^f(\mathbf{r}_1, \mathbf{r}_2, \omega)$ and a scattering Green tensor $\bar{\bar{G}}^s(\mathbf{r}_1, \mathbf{r}_2, \omega)$.

The free-space Green tensor describes the EM field when $\epsilon(\mathbf{r}, \omega) = \epsilon_0$ and $\mu(\mathbf{r}, \omega) = \mu_0$, i.e., when there are no media present, and between the points $\mathbf{r}_1$ and $\mathbf{r}_2$ is given as

$$\bar{\bar{G}}^f(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{e^{ikr}}{4\pi k^2 r^3} \{ f(kr) \mathbb{1} - h(kr) \mathbf{e}_r \otimes \mathbf{e}_r \}, \quad (\text{A2})$$

where $f(x) \equiv 1 - ix - x^2$, $h(x) \equiv 3 - 3ix - x^2$, $r = |\mathbf{r}_1 - \mathbf{r}_2|$, and $\mathbf{e}_r = \mathbf{r}/r$. $\bar{\bar{G}}^f(\mathbf{r}_1, \mathbf{r}_2, \omega)$ includes all the processes that do not involve scattering of any surfaces and is thus independent of any surface properties and only depends on the distance between $\mathbf{r}_1$ and $\mathbf{r}_2$.

The surface Green tensor, which describes the response of the electric field in the presence of a planar surface, is given by

$$\begin{aligned} \bar{\bar{G}}^s(\mathbf{r}_1, \mathbf{r}_2, \omega) = & \frac{1}{8\pi} \int_0^\infty dk \frac{k^\parallel}{\kappa^\perp} e^{-\kappa^\perp z_+} \left[\begin{pmatrix} J_0(k^\parallel x) + J_2(k^\parallel x) & 0 & 0 \\ 0 & J_0(k^\parallel x) - J_2(k^\parallel x) & 0 \\ 0 & 0 & 0 \end{pmatrix} r_s \right. \\ & \left. + \frac{c^2}{\omega^2} \begin{pmatrix} \kappa^\perp 2[J_0(k^\parallel x) - J_2(k^\parallel x)] & 0 & 2k^\parallel \kappa^\perp J_1(k^\parallel x) \\ 0 & \kappa^{\perp 2}[J_0(k^\parallel x) + J_2(k^\parallel x)] & 0 \\ -2k^\parallel \kappa^\perp J_1(k^\parallel x) & 0 & 2k^{\parallel 2} J_0(k^\parallel x) \end{pmatrix} r_p \right], \quad (\text{A3}) \end{aligned}$$

where $k^{\parallel}, \kappa^{\perp}$ are the wave vectors parallel and perpendicular to the surface, with $\omega^2/c^2 = k^{\parallel 2} - \kappa^{\perp 2}$, $z_+ = z_1 + z_2$, $x = x_1 - x_2$; $J_0(k^{\parallel}x)$ and $J_2(k^{\parallel}x)$ are the cylindrical Bessel function of the zeroth- and second kind, and r_s, r_p are the Fresnel coefficients for s - and p -polarized light.

APPENDIX B: CASIMIR-POLDER POTENTIALS

The interaction between the fluctuating field and the polarization induced by it in the particle leads to the Casimir-Polder potential. The second-order master equation approach used in this paper yields Casimir-Polder potential up to second order in the polarizability $\alpha(\omega)$. The order refers to the number of scattering events taken into account. First order thus means that no scattered fields are considered, whereas second order means that fields that scatter off the particle once are taken into account. The first-order Casimir-Polder potential is given by [41]

$$\mathcal{U}_{\text{CP}}^{(1)} = -\frac{\hbar\mu_0}{2\pi} \int_0^\infty d\omega \alpha(\omega) \omega^2 (2n_{\text{th}}(\omega) + 1) \times \text{Tr}[\text{Im} \bar{\bar{G}}(\mathbf{r}_0, \mathbf{r}_0, \omega)_{pk}]. \quad (\text{B1})$$

The first term in Eq. (9) gives rise to the first-order Casimir-Polder force, defined as the derivative of the above Casimir-Polder potential along the i axis, i.e., $C_i^{(1)} \equiv \partial_i \mathcal{U}_{\text{CP}}^{(1)}$. When the motion of the particle is restricted to the x - y plane, the first derivative of the Casimir-Polder potential is zero because $\mathcal{U}_{\text{CP}}^{(1)}$ only varies in the z direction via the scattering Green tensor (see Appendix A).

The second-order Casimir-Polder potential $\mathcal{U}_{\text{CP}}^{(2)}$ is given by [45]

$$\mathcal{U}_{\text{CP}}^{(2)} = \frac{\hbar\mu_0^2}{2\pi} \sum_{pk=\{x,y,z\}} \int_0^\infty d\omega \omega^4 |\alpha(\omega)|^2 (2n_{\text{th}}(\omega) + 1) \times \text{Tr}[\text{Re} \bar{\bar{G}}(\mathbf{r}_0, \mathbf{r}_0, \omega) \cdot \text{Im} \bar{\bar{G}}(\mathbf{r}_0, \mathbf{r}_0, \omega)]. \quad (\text{B2})$$

It is a second-order potential and therefore represents processes that involve scattering off the particle. The second term in Eq. (14) corresponds to the second derivative of the above potential along i and j axes, i.e., $C_{ij}^{(2)} \equiv \partial_i \mathcal{U}_{\text{CP}}^{(2)} \partial_j$.

APPENDIX C: NOISE AND DISSIPATION KERNELS

1. Noise kernel

In the following, the indices i, j will take the values $i, j = \{x, y\}$ and the indices p, k will take the values $p, k = \{x, y, z\}$:

$$\begin{aligned} \mathcal{N}_{ij}(\tau) &= \frac{1}{4} [\langle \partial_i \hat{\mathbf{P}}(\mathbf{r}_0, t) \cdot \hat{\mathbf{E}}(\mathbf{r}_0, t) + \hat{\mathbf{P}}(\mathbf{r}_0, t) \cdot \partial_i \hat{\mathbf{E}}(\mathbf{r}_0, t) \rangle [\partial_j \hat{\mathbf{P}}(\mathbf{r}_0, t - \tau) \cdot \hat{\mathbf{E}}(\mathbf{r}_0, t - \tau) + \hat{\mathbf{P}}(\mathbf{r}_0, t - \tau) \cdot \partial_j \hat{\mathbf{E}}(\mathbf{r}_0, t - \tau)] \\ &\quad + [\partial_j \hat{\mathbf{P}}(\mathbf{r}_0, t - \tau) \cdot \hat{\mathbf{E}}(\mathbf{r}_0, t - \tau) + \hat{\mathbf{P}}(\mathbf{r}_0, t - \tau) \cdot \partial_j \hat{\mathbf{E}}(\mathbf{r}_0, t - \tau)] [\partial_i \hat{\mathbf{P}}(\mathbf{r}_0, t) \cdot \hat{\mathbf{E}}(\mathbf{r}_0, t) + \hat{\mathbf{P}}(\mathbf{r}_0, t) \cdot \partial_i \hat{\mathbf{E}}(\mathbf{r}_0, t)] \rangle \quad (\text{C1}) \\ &= \frac{1}{4} \sum_{p,k} [\underbrace{\langle \partial_i \hat{P}_p(\mathbf{r}_0, t) \partial_j \hat{P}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IA})} \underbrace{\langle \hat{E}_p(\mathbf{r}_0, t) \hat{E}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IB})} + \underbrace{\langle \hat{P}_p(\mathbf{r}_0, t) \hat{P}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IIA})} \underbrace{\langle \partial_i \hat{E}_p(\mathbf{r}_0, t) \partial_j \hat{E}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IIB})} \\ &\quad + \underbrace{\langle \hat{P}_p(\mathbf{r}_0, t) \hat{E}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IIIA})} \underbrace{\langle \partial_i \hat{E}_p(\mathbf{r}_0, t) \partial_j \hat{P}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IIIB})} + \underbrace{\langle \partial_i \hat{P}_p(\mathbf{r}_0, t) \partial_j \hat{E}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IVA})} \underbrace{\langle \hat{E}_p(\mathbf{r}_0, t) \hat{P}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IVB})} \\ &\quad + \underbrace{\langle \partial_j \hat{P}_p(\mathbf{r}_0, t - \tau) \partial_i \hat{P}_k(\mathbf{r}_0, t) \rangle}_{(\text{VA})} \underbrace{\langle \hat{E}_p(\mathbf{r}_0, t - \tau) \hat{E}_k(\mathbf{r}_0, t) \rangle}_{(\text{VB})} + \underbrace{\langle \hat{P}_p(\mathbf{r}_0, t - \tau) \hat{P}_k(\mathbf{r}_0, t) \rangle}_{(\text{VIA})} \underbrace{\langle \partial_j \hat{E}_p(\mathbf{r}_0, t - \tau) \partial_i \hat{E}_k(\mathbf{r}_0, t) \rangle}_{(\text{VIB})} \\ &\quad + \underbrace{\langle \hat{P}_p(\mathbf{r}_0, t - \tau) \hat{E}_k(\mathbf{r}_0, t) \rangle}_{(\text{VIIA})} \underbrace{\langle \partial_j \hat{E}_p(\mathbf{r}_0, t - \tau) \partial_i \hat{P}_k(\mathbf{r}_0, t) \rangle}_{(\text{VIIB})} + \underbrace{\langle \partial_j \hat{P}_p(\mathbf{r}_0, t - \tau) \partial_i \hat{E}_k(\mathbf{r}_0, t) \rangle}_{(\text{VIIIA})} \underbrace{\langle \hat{E}_p(\mathbf{r}_0, t - \tau) \hat{P}_k(\mathbf{r}_0, t) \rangle}_{(\text{VIIB})}], \quad (\text{C2}) \end{aligned}$$

where we have used the statistical independence of the electric field $\hat{E}_p(\mathbf{r}_0, t)$ and its spatial derivative $\partial_i \hat{E}_p(\mathbf{r}_0, t)$ as in Eq. (28). The same applies to the particle polarization $\hat{P}_p(\mathbf{r}_0, t)$ ($\langle \hat{P}_p(\mathbf{r}_0, t) \partial_i \hat{P}_k(\mathbf{r}_0, t - \tau) \rangle = 0$).

We simplify each of the above terms as follows:

$$(\text{IA}) = \langle \partial_i \hat{P}_p(\mathbf{r}_0, t) \partial_j \hat{P}_k(\mathbf{r}_0, t - \tau) \rangle \quad (\text{C3})$$

$$\begin{aligned} &= \int d\omega_1 \int d\omega_2 \int d\mathbf{r}_1^3 \int d\mathbf{r}_2^3 \sum_{\lambda_1, \lambda_2} [\alpha(\omega_1) \partial_i (\bar{\bar{G}}_{0,1}(\omega_1) \cdot \hat{\mathbf{f}}_1)_p e^{-i\omega_1 t} + \alpha^*(\omega_1) \partial_j (\hat{\mathbf{f}}_1^\dagger \cdot \bar{\bar{G}}_{0,1}^\dagger(\omega_1))_p e^{i\omega_1 t}] \\ &\quad \times [\alpha(\omega_2) \partial_i (\bar{\bar{G}}_{0,2}(\omega_2) \cdot \hat{\mathbf{f}}_2)_k e^{-i\omega_2(t-\tau)} + \alpha^*(\omega_2) \partial_j (\hat{\mathbf{f}}_2^\dagger \cdot \bar{\bar{G}}_{0,2}^\dagger(\omega_2))_k e^{i\omega_2(t-\tau)}], \quad (\text{C4}) \end{aligned}$$

where we have used the shorthand notation $\bar{\bar{G}}_{0,1}^{(\dagger)}(\omega_1) \equiv \bar{\bar{G}}_{\lambda_1}^{(\dagger)}(\mathbf{r}_0, \mathbf{r}_1, \omega_1)$ and $\hat{\mathbf{f}}_1^{(\dagger)} \equiv \hat{\mathbf{f}}_{\lambda_1}^{(\dagger)}(\mathbf{r}_1, \omega_1)$. To take the average over the thermal density matrix of the EM field bath, we note that $\langle \hat{\mathbf{f}}_1 \hat{\mathbf{f}}_2^\dagger \rangle = (n_{\text{th}}(\omega_1) + 1) \delta(\omega_1 - \omega_2) \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta_{\lambda_1 \lambda_2}$, $\langle \hat{\mathbf{f}}_1^\dagger \hat{\mathbf{f}}_2 \rangle =$

$n_{\text{th}}(\omega_1)\delta(\omega_1 - \omega_2)\delta(\mathbf{r}_1 - \mathbf{r}_2)\delta_{\lambda_1\lambda_2}$, and $\langle \hat{\mathbf{f}}_1 \hat{\mathbf{f}}_2 \rangle = \langle \hat{\mathbf{f}}_1^\dagger \hat{\mathbf{f}}_2^\dagger \rangle = 0$. We thus obtain

$$(IA) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 |\alpha(\omega)|^2 \text{Im}[\partial_i \bar{G}_{0,0}(\omega)_{pk} \partial_j] [(n_{\text{th}}(\omega) + 1)e^{-i\omega\tau} + n_{\text{th}}(\omega)e^{i\omega\tau}], \quad (C5)$$

where we have used the fluctuation-dissipation relation for the Green tensor. Similarly, one can simplify the remaining terms:

$$(IB) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 \text{Im}[\bar{G}_{0,0}(\omega)_{pk}] [(n_{\text{th}}(\omega) + 1)e^{-i\omega\tau} + n_{\text{th}}(\omega)e^{i\omega\tau}], \quad (C6)$$

$$(IIA) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 |\alpha(\omega)|^2 \text{Im}[\bar{G}_{0,0}(\omega)_{pk}] [(n_{\text{th}}(\omega) + 1)e^{-i\omega\tau} + n_{\text{th}}(\omega)e^{i\omega\tau}], \quad (C7)$$

$$(IIB) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 \text{Im}[\partial_i \bar{G}_{0,0}(\omega)_{pk} \partial_j] [(n_{\text{th}}(\omega) + 1)e^{-i\omega\tau} + n_{\text{th}}(\omega)e^{i\omega\tau}], \quad (C8)$$

$$(IIIA) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 \text{Im}[\bar{G}_{0,0}(\omega)_{pk}] [\alpha(\omega)(n_{\text{th}}(\omega) + 1)e^{-i\omega\tau} + \alpha^*(\omega)n_{\text{th}}(\omega)e^{i\omega\tau}], \quad (C9)$$

$$(IIIB) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 \text{Im}[\partial_i \bar{G}_{0,0}(\omega)_{pk} \partial_j] [\alpha^*(\omega)(n_{\text{th}}(\omega) + 1)e^{-i\omega\tau} + \alpha(\omega)n_{\text{th}}(\omega)e^{i\omega\tau}], \quad (C10)$$

$$(IVA) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 \text{Im}[\partial_i \bar{G}_{0,0}(\omega)_{pk} \partial_j] [\alpha(\omega)(n_{\text{th}}(\omega) + 1)e^{-i\omega\tau} + \alpha^*(\omega)n_{\text{th}}(\omega)e^{i\omega\tau}], \quad (C11)$$

$$(IVB) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 \text{Im}[\bar{G}_{0,0}(\omega)_{pk}] [\alpha^*(\omega)(n_{\text{th}}(\omega) + 1)e^{-i\omega\tau} + \alpha(\omega)n_{\text{th}}(\omega)e^{i\omega\tau}], \quad (C12)$$

$$(VA) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 |\alpha(\omega)|^2 \text{Im}[\partial_j \bar{G}_{0,0}(\omega)_{pk} \partial_i] [(n_{\text{th}}(\omega) + 1)e^{i\omega\tau} + n_{\text{th}}(\omega)e^{-i\omega\tau}], \quad (C13)$$

$$(VB) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 \text{Im}[\bar{G}_{0,0}(\omega)_{pk}] [(n_{\text{th}}(\omega) + 1)e^{i\omega\tau} + n_{\text{th}}(\omega)e^{-i\omega\tau}], \quad (C14)$$

$$(VIA) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 |\alpha(\omega)|^2 \text{Im}[\bar{G}_{0,0}(\omega)_{pk}] [(n_{\text{th}}(\omega) + 1)e^{i\omega\tau} + n_{\text{th}}(\omega)e^{-i\omega\tau}], \quad (C15)$$

$$(VIB) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 \text{Im}[\partial_j \bar{G}_{0,0}(\omega)_{pk} \partial_i] [(n_{\text{th}}(\omega) + 1)e^{i\omega\tau} + n_{\text{th}}(\omega)e^{-i\omega\tau}], \quad (C16)$$

$$(VIIA) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 \text{Im}[\bar{G}_{0,0}(\omega)_{pk}] [\alpha(\omega)(n_{\text{th}}(\omega) + 1)e^{i\omega\tau} + \alpha^*(\omega)n_{\text{th}}(\omega)e^{-i\omega\tau}], \quad (C17)$$

$$(VIIB) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 \text{Im}[\partial_j \bar{G}_{0,0}(\omega)_{pk} \partial_i] [\alpha^*(\omega)(n_{\text{th}}(\omega) + 1)e^{i\omega\tau} + \alpha(\omega)n_{\text{th}}(\omega)e^{-i\omega\tau}], \quad (C18)$$

$$(VIIIA) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 \text{Im}[\partial_j \bar{G}_{0,0}(\omega)_{pk} \partial_i] [\alpha(\omega)(n_{\text{th}}(\omega) + 1)e^{i\omega\tau} + \alpha^*(\omega)n_{\text{th}}(\omega)e^{-i\omega\tau}], \quad (C19)$$

$$(VIIIB) = \frac{\hbar\mu_0}{\pi} \int d\omega \omega^2 \text{Im}[\bar{G}_{0,0}(\omega)_{pk}] [\alpha^*(\omega)(n_{\text{th}}(\omega) + 1)e^{i\omega\tau} + \alpha(\omega)n_{\text{th}}(\omega)e^{-i\omega\tau}]. \quad (C20)$$

Putting all the terms together, we get the noise kernel:

$$\begin{aligned} \mathcal{N}_{ij}(\tau) = & \frac{\hbar^2 \mu_0^2}{2\pi^2} \sum_{p,k} \int_0^\infty d\omega \omega^2 \int_0^\infty d\omega' \omega'^2 [\text{Im}\{\partial_i \bar{G}_{0,0}(\omega)_{pk} \partial_j\} \text{Im}\{\bar{G}_{0,0}(\omega')_{pk}\} + \text{Im}\{\bar{G}_{0,0}(\omega)_{pk}\} \text{Im}\{\partial_j \bar{G}_{0,0}(\omega')_{pk} \partial_i\}] \\ & \times [|\alpha(\omega)|^2 \{((n_{\text{th}}(\omega) + 1)(n_{\text{th}}(\omega') + 1) + n_{\text{th}}(\omega)n_{\text{th}}(\omega')) \cos((\omega + \omega')\tau) \\ & + ((n_{\text{th}}(\omega) + 1)n_{\text{th}}(\omega') + n_{\text{th}}(\omega)(n_{\text{th}}(\omega') + 1)) \cos((\omega - \omega')\tau)\} \\ & + \{(\alpha(\omega)\alpha^*(\omega')(n_{\text{th}}(\omega) + 1)(n_{\text{th}}(\omega') + 1) + \alpha^*(\omega)\alpha(\omega')n_{\text{th}}(\omega)n_{\text{th}}(\omega'))\} \cos((\omega + \omega')\tau) \\ & + \{(\alpha(\omega)\alpha(\omega')(n_{\text{th}}(\omega) + 1)n_{\text{th}}(\omega') + \alpha^*(\omega)\alpha^*(\omega')n_{\text{th}}(\omega)(n_{\text{th}}(\omega') + 1)) \cos((\omega - \omega')\tau)\}]. \end{aligned} \quad (C21)$$

The decoherence rate as defined in Eq. (17) is obtained by integrating the noise kernel over τ , using $\int_0^\infty d\tau \cos((\omega - \omega')\tau) = \pi \delta(\omega - \omega')$, giving

$$\Lambda_{ij} = \frac{2\mu_0^2}{\pi} \int d\omega \omega^4 |\alpha(\omega)|^2 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{G}_{0,0}(\omega)_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}(\omega)_{pk} \partial_j]. \quad (C22)$$

Following Appendix A, we can split up the Green tensor into a free-space and a scattering-Green tensor. Doing so leaves us with the free-space and scattering-decoherence coefficients:

$$\Lambda_{ij}^{\text{free}} = \frac{2\mu_0^2}{\pi} \int d\omega \omega^4 |\alpha(\omega)|^2 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{\bar{G}}_{0,0}^f(\omega)_{pk}] \text{Im}[\partial_i \bar{\bar{G}}_{0,0}^f(\omega)_{pk} \partial_j]. \quad (\text{C23})$$

$$\begin{aligned} \Lambda_{ij}^s &= \frac{2\mu_0^2}{\pi} \int d\omega \omega^4 |\alpha(\omega)|^2 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{\bar{G}}_{0,0}^f(\omega)_{pk}] \text{Im}[\partial_i \bar{\bar{G}}_{0,0}^s(\omega)_{pk} \partial_j] \\ &\quad + \frac{2\mu_0^2}{\pi} \int d\omega \omega^4 |\alpha(\omega)|^2 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{\bar{G}}_{0,0}^s(\omega)_{pk}] \text{Im}[\partial_i \bar{\bar{G}}_{0,0}^f(\omega)_{pk} \partial_j] \\ &\quad + \frac{2\mu_0^2}{\pi} \int d\omega \omega^4 |\alpha(\omega)|^2 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{\bar{G}}_{0,0}^s(\omega)_{pk}] \text{Im}[\partial_i \bar{\bar{G}}_{0,0}^s(\omega)_{pk} \partial_j]. \end{aligned} \quad (\text{C24})$$

$\Lambda_{ij}^{\text{free}}$ only depends on the free-space Green tensor and represents decoherence due to processes that do not involve scattering off the surface. The first and second terms of Λ_{ij}^s correspond to the field scattered off the surface interacting with the polarization induced by the free-space field, and vice versa. The last term of Λ_{ij}^s describes processes where the field scattered off the surface interacts with the polarization induced by the scattered field.

To further examine the properties of these coefficients, we note that the ω integral is dominated by the thermal factor $n(\omega)(n(\omega) + 1)$, which is peaked at $\omega \rightarrow 0$ such that the contribution from variation of material responses (e.g., particle polarizability and surface permittivity) is negligible. Thus, assuming a quasistatic response for material responses, the decoherence coefficient is given by

$$\begin{aligned} \Lambda_{ij}^s &\approx \frac{2\mu_0^2}{\pi} \sum_{p,k=\{x,y,z\}} \lim_{\omega \rightarrow 0} \left[|\alpha(\omega)|^2 \omega \text{Im} \partial_i (\bar{\bar{G}}_{0,0}^s(\omega))_{pk} \partial_j \right] \int d\sigma \sigma^3 n_{\text{th}}(\sigma)(n_{\text{th}}(\sigma) + 1) \text{Im}[\bar{\bar{G}}_{0,0}^f(\sigma)_{pk}] \\ &\quad + \frac{2\mu_0^2}{\pi} \sum_{p,k=\{x,y,z\}} \lim_{\omega \rightarrow 0} \left[|\alpha(\omega)|^2 \omega \text{Im} (\bar{\bar{G}}_{0,0}^s(\omega))_{pk} \right] \int d\sigma \sigma^3 n_{\text{th}}(\sigma)(n_{\text{th}}(\sigma) + 1) \text{Im}[\partial_i (\bar{\bar{G}}_{0,0}^f(\sigma))_{pk} \partial_j] \\ &\quad + \frac{2\mu_0^2}{\pi} \sum_{p,k=\{x,y,z\}} \lim_{\omega \rightarrow 0} \left[|\alpha(\omega)|^2 \omega^2 \text{Im} (\bar{\bar{G}}_{0,0}^s(\omega))_{pk} \text{Im} \partial_i (\bar{\bar{G}}_{0,0}^s(\omega))_{pk} \partial_j \right] \int d\sigma \sigma^2 n_{\text{th}}(\sigma)(n_{\text{th}}(\sigma) + 1). \end{aligned} \quad (\text{C25})$$

For a gold surface with permittivity described by the Drude model [Eq. (36)], it further simplifies to Eq. (39).

2. Dissipation kernel

In the following, the indices i, j will take the values $i, j = \{x, y\}$ and the indices p, k will take the values $p, k = \{x, y, z\}$. We can similarly simplify the dissipation kernel as follows:

$$\begin{aligned} \mathcal{D}_{ij}(\tau) &= \frac{1}{4} \langle [\partial_i \hat{\mathbf{P}}(\mathbf{r}_0, t) \cdot \hat{\mathbf{E}}(\mathbf{r}_0, t) + \hat{\mathbf{P}}(\mathbf{r}_0, t) \cdot \partial_i \hat{\mathbf{E}}(\mathbf{r}_0, t)] [\partial_j \hat{\mathbf{P}}(\mathbf{r}_0, t - \tau) \cdot \hat{\mathbf{E}}(\mathbf{r}_0, t - \tau) + \hat{\mathbf{P}}(\mathbf{r}_0, t - \tau) \cdot \partial_j \hat{\mathbf{E}}(\mathbf{r}_0, t - \tau)] \\ &\quad - [\partial_j \hat{\mathbf{P}}(\mathbf{r}_0, t - \tau) \cdot \hat{\mathbf{E}}(\mathbf{r}_0, t - \tau) + \hat{\mathbf{P}}(\mathbf{r}_0, t - \tau) \cdot \partial_j \hat{\mathbf{E}}(\mathbf{r}_0, t - \tau)] [\partial_i \hat{\mathbf{P}}(\mathbf{r}_0, t) \cdot \hat{\mathbf{E}}(\mathbf{r}_0, t) + \hat{\mathbf{P}}(\mathbf{r}_0, t) \cdot \partial_i \hat{\mathbf{E}}(\mathbf{r}_0, t)] \rangle \quad (\text{C26}) \\ &= \frac{1}{4} \sum_{p,k} \underbrace{\langle \partial_i \hat{P}_p(\mathbf{r}_0, t) \partial_j \hat{P}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IA})} \underbrace{\langle \hat{E}_p(\mathbf{r}_0, t) \hat{E}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IB})} \\ &\quad + \underbrace{\langle \hat{P}_p(\mathbf{r}_0, t) \hat{P}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IIA})} \underbrace{\langle \partial_i \hat{E}_p(\mathbf{r}_0, t) \partial_j \hat{E}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IIB})} \\ &\quad + \underbrace{\langle \hat{P}_p(\mathbf{r}_0, t) \hat{E}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IIIA})} \underbrace{\langle \partial_i \hat{E}_p(\mathbf{r}_0, t) \partial_j \hat{P}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IIIB})} \\ &\quad + \underbrace{\langle \partial_i \hat{P}_p(\mathbf{r}_0, t) \partial_j \hat{E}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IVA})} \underbrace{\langle \hat{E}_p(\mathbf{r}_0, t) \hat{P}_k(\mathbf{r}_0, t - \tau) \rangle}_{(\text{IVB})} \\ &\quad - \underbrace{\langle \partial_j \hat{P}_p(\mathbf{r}_0, t - \tau) \partial_i \hat{P}_k(\mathbf{r}_0, t) \rangle}_{(\text{VA})} \underbrace{\langle \hat{E}_p(\mathbf{r}_0, t - \tau) \hat{E}_k(\mathbf{r}_0, t) \rangle}_{(\text{VB})} \\ &\quad - \underbrace{\langle \hat{P}_p(\mathbf{r}_0, t - \tau) \hat{P}_k(\mathbf{r}_0, t) \rangle}_{(\text{VIA})} \underbrace{\langle \partial_j \hat{E}_p(\mathbf{r}_0, t - \tau) \partial_i \hat{E}_k(\mathbf{r}_0, t) \rangle}_{(\text{VIB})} \end{aligned}$$

$$\begin{aligned}
& - \underbrace{\langle \hat{P}_p(\mathbf{r}_0, t - \tau) \hat{E}_k(\mathbf{r}_0, t) \rangle}_{(\text{VIIA})} \underbrace{\langle \partial_j \hat{E}_p(\mathbf{r}_0, t - \tau) \partial_i \hat{P}_k(\mathbf{r}_0, t) \rangle}_{(\text{VIIB})} \\
& - \underbrace{\langle \partial_j \hat{P}_p(\mathbf{r}_0, t - \tau) \partial_i \hat{E}_k(\mathbf{r}_0, t) \rangle}_{(\text{VIIIA})} \underbrace{\langle \hat{E}_p(\mathbf{r}_0, t - \tau) \hat{P}_k(\mathbf{r}_0, t) \rangle}_{(\text{VIIB})}].
\end{aligned} \tag{C27}$$

Substituting Eqs. (C5)–(C20) in the above, we obtain the dissipation kernel as

$$\begin{aligned}
\mathcal{D}_{ij}(\tau) = & \frac{\hbar^2 \mu_0^2}{2\pi^2} \sum_{p,k} \int_0^\infty d\omega \omega^2 \int_0^\infty d\omega' \omega'^2 [\text{Im}\{\partial_i \bar{G}_{0,0}(\omega)_{pk} \partial_j\} \text{Im}\{\bar{G}_{0,0}(\omega')_{pk}\} + \text{Im}\{\bar{G}_{0,0}(\omega)_{pk}\} \text{Im}\{\partial_j \bar{G}_{0,0}(\omega')_{pk} \partial_i\}] \\
& \times [|\alpha(\omega)|^2 \{((n_{\text{th}}(\omega) + 1)(n_{\text{th}}(\omega') + 1) - n_{\text{th}}(\omega)n_{\text{th}}(\omega')) \sin((\omega + \omega')\tau) \\
& + ((n_{\text{th}}(\omega) + 1)n_{\text{th}}(\omega') - n_{\text{th}}(\omega)(n_{\text{th}}(\omega') + 1)) \sin((\omega - \omega')\tau)\} \\
& + \{(\alpha(\omega)\alpha^*(\omega')(n_{\text{th}}(\omega) + 1)(n_{\text{th}}(\omega') + 1) - \alpha^*(\omega)\alpha(\omega')n_{\text{th}}(\omega)n_{\text{th}}(\omega'))\} \sin((\omega + \omega')\tau) \\
& + \{(\alpha(\omega)\alpha(\omega')(n_{\text{th}}(\omega) + 1)n_{\text{th}}(\omega') - \alpha^*(\omega)\alpha^*(\omega')n_{\text{th}}(\omega)(n_{\text{th}}(\omega') + 1)) \sin((\omega - \omega')\tau)\}].
\end{aligned} \tag{C28}$$

The dissipation coefficient Γ_{ij} can be thus calculated using Eq. (16). Noting that $\int_0^\infty d\tau \tau \sin((\omega - \omega')\tau) = -\pi \frac{\partial}{\partial \omega} \delta(\omega - \omega')$ and using integration by parts, we get Eq. (30). Similar to the decoherence coefficient, we can split up the Green tensor into its free and scattering parts to give

$$\Gamma_{ij}^{\text{free}} = \frac{\hbar^2 \mu_0^2}{M\pi k_B T} \int_0^\infty d\omega \omega^4 |\alpha(\omega)|^2 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{G}_{0,0}^{\text{f}}(\omega)_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}^{\text{f}}(\omega)_{pk} \partial_j], \tag{C29}$$

$$\begin{aligned}
\Gamma_{ij}^{\text{s}} = & \frac{\hbar^2 \mu_0^2}{M\pi k_B T} \int_0^\infty d\omega \omega^4 |\alpha(\omega)|^2 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{G}_{0,0}^{\text{f}}(\omega)_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}^{\text{s}}(\omega)_{pk} \partial_j] \\
& + \frac{\hbar^2 \mu_0^2}{M\pi k_B T} \int_0^\infty d\omega \omega^4 |\alpha(\omega)|^2 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{G}_{0,0}^{\text{s}}(\omega)_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}^{\text{f}}(\omega)_{pk} \partial_j] \\
& + \frac{\hbar^2 \mu_0^2}{M\pi k_B T} \int_0^\infty d\omega \omega^4 |\alpha(\omega)|^2 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{G}_{0,0}^{\text{s}}(\omega)_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}^{\text{s}}(\omega)_{pk} \partial_j],
\end{aligned} \tag{C30}$$

where the interpretation of the terms follows the same line of reasoning as for the decoherence coefficient. Note that each line in the above expression contains distinct combinations of free-space $[\bar{G}_{0,0}^{\text{f}}(\omega)]$ and scattered Green tensors, $\bar{G}_{0,0}^{\text{s}}(\omega)$. Again, noting that the integrand is peaked around $\omega \rightarrow 0$, owing to the thermal occupation factors, the dissipation coefficient can be simplified to

$$\begin{aligned}
\Gamma_{ij}^{\text{s}} \approx & \frac{\hbar^2 \mu_0^2}{M\pi k_B T} \sum_{p,k=\{x,y,z\}} \lim_{\omega \rightarrow 0} [|\alpha(\omega)|^2 \omega \text{Im} \partial_i (\bar{G}_{0,0}^{\text{s}}(\omega))_{pk} \partial_j] \int d\sigma \sigma^3 n_{\text{th}}(\sigma)(n_{\text{th}}(\sigma) + 1) \text{Im}[(\bar{G}^{\text{f}}(\sigma))_{pk}] \\
& + \frac{\hbar^2 \mu_0^2}{M\pi k_B T} \sum_{p,k=\{x,y,z\}} \lim_{\omega \rightarrow 0} [|\alpha(\omega)|^2 \omega \text{Im} (\bar{G}_{0,0}^{\text{s}}(\omega))_{pk}] \int d\sigma \sigma^3 n_{\text{th}}(\sigma)(n_{\text{th}}(\sigma) + 1) \text{Im}[\partial_i (\bar{G}_{0,0}^{\text{f}}(\sigma))_{pk} \partial_j] \\
& + \frac{\hbar^2 \mu_0^2}{M\pi k_B T} \sum_{p,k=\{x,y,z\}} \lim_{\omega \rightarrow 0} [|\alpha(\omega)|^2 \omega^2 \text{Im} (\bar{G}_{0,0}^{\text{s}}(\omega))_{pk} \text{Im} \partial_i (\bar{G}_{0,0}^{\text{s}}(\omega))_{pk} \partial_j] \int d\sigma \sigma^2 n_{\text{th}}(\sigma)(n_{\text{th}}(\sigma) + 1).
\end{aligned} \tag{C31}$$

For a planar metallic surface described by the Drude model, this leaves us with

$$\Gamma^{\text{s}} = \frac{|\alpha(0)|^2 \pi^3 \gamma}{105 M \epsilon_0^2 c^5 \omega_p^2 \hbar z^3} \left(\frac{k_B T}{\hbar} \right)^6 + \frac{|\alpha(0)|^2 \pi \gamma}{30 M \epsilon_0^2 c^3 \omega_p^2 \hbar z^5} \left(\frac{k_B T}{\hbar} \right)^4 + \frac{3|\alpha(0)|^2 \gamma^2}{128 M \omega_p^4 \epsilon_0^2 \pi \hbar z^8} \left(\frac{k_B T}{\hbar} \right)^2. \tag{C32}$$

It is worth pointing out that the temperature scaling of Γ^{s} differs from that of Λ^{s} by a factor of T^{-1} .

APPENDIX D: MOMENTUM FLUCTUATIONS

In the following, the indices i, j will take the values $i, j = \{x, y\}$ and the indices p, k will take the values $p, k = \{x, y, z\}$:

$$\begin{aligned}
\langle \Delta p_i \Delta p_j \rangle = & \frac{1}{4} \int_0^{\Delta t} d\tau_1 \int_0^{\Delta t} d\tau_2 [\partial_i \hat{\mathbf{P}}(\mathbf{r}_0, \tau_1) \cdot \hat{\mathbf{E}}(\mathbf{r}_0, \tau_1) + \hat{\mathbf{P}}(\mathbf{r}_0, \tau_1) \cdot \partial_i \hat{\mathbf{E}}(\mathbf{r}_0, \tau_1)] \\
& \times [\partial_j \hat{\mathbf{P}}(\mathbf{r}_0, \tau_2) \cdot \hat{\mathbf{E}}(\mathbf{r}_0, \tau_2) + \hat{\mathbf{P}}(\mathbf{r}_0, \tau_2) \cdot \partial_j \hat{\mathbf{E}}(\mathbf{r}_0, \tau_2)].
\end{aligned} \tag{D1}$$

Using the statistical independence of $\{\hat{E}_j, \hat{P}_j\}$ and their spatial derivatives $\{\partial_i \hat{E}_j, \partial_i \hat{P}_j\}$:

$$\begin{aligned}
 \langle \Delta p_i \Delta p_j \rangle &= \underbrace{\frac{1}{4} \int_0^{\Delta t} d\tau_1 \int_0^{\Delta t} d\tau_2 \sum_{p,k} \langle \partial_i \hat{P}_p(\mathbf{r}_0, \tau_1) \partial_j \hat{P}_k(\mathbf{r}_0, \tau_2) \rangle \langle \hat{E}_p(\mathbf{r}_0, \tau_1) \hat{E}_k(\mathbf{r}_0, \tau_2) \rangle}_{(I)} \\
 &+ \underbrace{\frac{1}{4} \int_0^{\Delta t} d\tau_1 \int_0^{\Delta t} d\tau_2 \sum_{p,k} \langle \hat{P}_p(\mathbf{r}_0, \tau_1) \hat{P}_k(\mathbf{r}_0, \tau_2) \rangle \langle \partial_i \hat{E}_p(\mathbf{r}_0, \tau_1) \partial_j \hat{E}_k(\mathbf{r}_0, \tau_2) \rangle}_{(II)} \\
 &+ \underbrace{\frac{1}{4} \int_0^{\Delta t} d\tau_1 \int_0^{\Delta t} d\tau_2 \sum_{p,k} \langle \hat{P}_p(\mathbf{r}_0, \tau_1) \hat{E}_k(\mathbf{r}_0, \tau_2) \rangle \langle \partial_i \hat{E}_p(\mathbf{r}_0, \tau_1) \partial_j \hat{P}_k(\mathbf{r}_0, \tau_2) \rangle}_{(III)} \\
 &+ \underbrace{\frac{1}{4} \int_0^{\Delta t} d\tau_1 \int_0^{\Delta t} d\tau_2 \sum_{p,k} \langle \partial_i \hat{P}_p(\mathbf{r}_0, \tau_1) \partial_j \hat{E}_k(\mathbf{r}_0, \tau_2) \rangle \langle \hat{E}_p(\mathbf{r}_0, \tau_1) \hat{P}_k(\mathbf{r}_0, \tau_2) \rangle}_{(IV)}. \tag{D2}
 \end{aligned}$$

Let us consider the first term in the above equation more carefully:

$$\begin{aligned}
 (I) &= \frac{\hbar^2 \mu_0^2}{4\pi^2} \int_0^\infty d\omega \omega^2 \int_0^\infty d\omega' \omega'^2 \int_0^{\Delta t} d\tau_1 \int_0^{\Delta t} d\tau_2 \sum_{p,k} |\alpha(\omega)|^2 \sum_{p,k} \text{Im}[\bar{G}_{0,0}(\omega')_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}(\omega)_{pk} \partial_j] \\
 &\times [(n_{\text{th}}(\omega) + 1)e^{-i\omega(\tau_1 - \tau_2)} + n_{\text{th}}(\omega)e^{i\omega(\tau_1 - \tau_2)}][(n_{\text{th}}(\omega') + 1)e^{-i\omega'(\tau_1 - \tau_2)} + n_{\text{th}}(\omega')e^{i\omega'(\tau_1 - \tau_2)}] \tag{D3}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\hbar^2 \mu_0^2}{4\pi^2} \int_0^\infty d\omega \omega^2 \int_0^\infty d\omega' \omega'^2 \int_0^{\Delta t} d\tau_1 \int_0^{\Delta t} d\tau_2 \sum_{j,k} |\alpha(\omega)|^2 \sum_{p,k} \text{Im}[\bar{G}_{0,0}(\omega')_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}(\omega)_{pk} \partial_j] \\
 &\times [(n_{\text{th}}(\omega) + 1)(n_{\text{th}}(\omega') + 1)e^{-i(\omega + \omega')(\tau_1 - \tau_2)} + n_{\text{th}}(\omega)n_{\text{th}}(\omega')e^{i(\omega + \omega')(\tau_1 - \tau_2)} \\
 &+ (n_{\text{th}}(\omega) + 1)n_{\text{th}}(\omega')e^{-i(\omega - \omega')(\tau_1 - \tau_2)} + n_{\text{th}}(\omega)(n_{\text{th}}(\omega') + 1)e^{i(\omega - \omega')(\tau_1 - \tau_2)}] \tag{D4}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\hbar^2 \mu_0^2}{\pi^2} \int_0^\infty d\omega \omega^2 \int_0^\infty d\omega' \omega'^2 \sum_{p,k} |\alpha(\omega)|^2 \sum_{p,k} \text{Im}[\bar{G}_{0,0}(\omega')_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}(\omega)_{pk} \partial_j] \\
 &\times \left[\{(n_{\text{th}}(\omega) + 1)(n_{\text{th}}(\omega') + 1) + n_{\text{th}}(\omega)n_{\text{th}}(\omega')\} \frac{\sin^2((\omega + \omega')\Delta t/2)}{(\omega + \omega')^2} \right. \\
 &\left. + \{(n_{\text{th}}(\omega) + 1)n_{\text{th}}(\omega') + n_{\text{th}}(\omega)(n_{\text{th}}(\omega') + 1)\} \frac{\sin^2((\omega - \omega')\Delta t/2)}{(\omega - \omega')^2} \right]. \tag{D5}
 \end{aligned}$$

We note that in the limit where Δt is small, the term proportional to $\sim \frac{\sin^2((\omega + \omega')\Delta t/2)}{(\omega + \omega')^2}$ vanishes, and $\int_0^\infty d\omega' \frac{\sin^2((\omega - \omega')\Delta t/2)}{(\omega - \omega')^2} \approx \frac{\pi \Delta t}{2}$. This yields

$$(I) = \frac{\hbar^2 \mu_0^2 \Delta t}{\pi} \int_0^\infty d\omega \omega^4 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) |\alpha(\omega)|^2 \sum_{p,k} \text{Im}[\bar{G}_{0,0}(\omega)_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}(\omega)_{pk} \partial_j]. \tag{D6}$$

Similarly, we can simplify the remaining terms to find $(I) = (II) = (III) = (IV)$, which gives us the momentum diffusion constant:

$$D_{ij} = \frac{4\hbar^2 \mu_0^2}{\pi} \int_0^\infty d\omega \omega^4 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) |\alpha(\omega)|^2 \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{G}_{0,0}(\omega)_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}(\omega)_{pk} \partial_j], \tag{D7}$$

which we can once again split up into free- and scattering parts:

$$\begin{aligned}
 D_{ij}^{\text{free}} &= \frac{4\hbar^2 \mu_0^2}{\pi} \int_0^\infty d\omega \omega^4 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) |\alpha(\omega)|^2 \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{G}_{0,0}^{\text{f}}(\omega)_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}^{\text{f}}(\omega)_{pk} \partial_j], \tag{D8} \\
 D_{ij}^{\text{s}} &= \frac{4\hbar^2 \mu_0^2}{\pi} \int_0^\infty d\omega \omega^4 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) |\alpha(\omega)|^2 \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{G}_{0,0}^{\text{s}}(\omega)_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}^{\text{s}}(\omega)_{pk} \partial_j]
 \end{aligned}$$

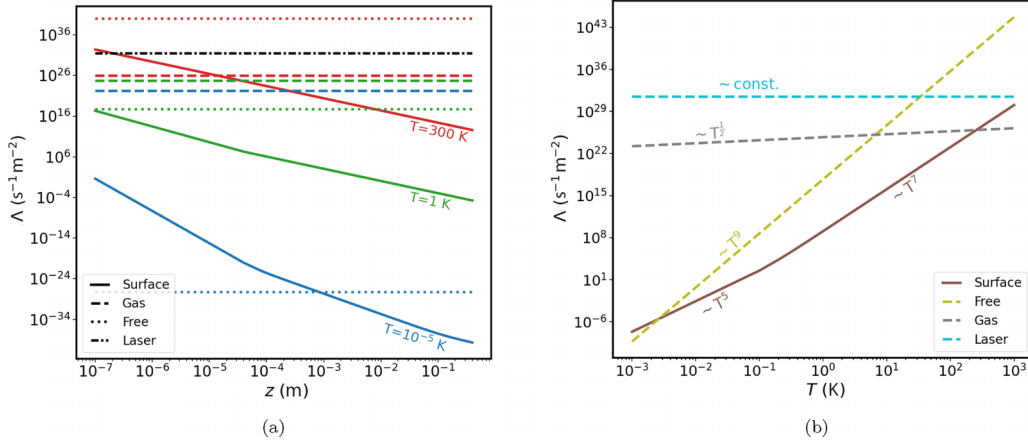


FIG. 5. (a) The decoherence rate of a dielectric polarizable particle near a dielectric surface as a function of distance z from the surface. The solid lines represent the surface-induced decoherence at three different temperatures, red: $T = 300$ K, green: $T = 1$ K, and blue: $T = 10^{-5}$ K. For comparison we also plot the decoherence rate due to three standard sources for each temperature: (1) free-space thermal photon scattering (dotted curves), (2) background gas scattering (dashed curves) at pressure $P = 10^{-8}$ mbar, and (3) laser scattering (dashed-dotted line) with a wavelength of 1064 nm and a power of 0.5 W. (b) The decoherence rate of a dielectric polarizable particle at distance $z = 10 \mu\text{m}$ from a dielectric surface as a function of temperature T . For comparison we plot the decoherence rate due to blackbody radiation, gas scattering at $P = 10^{-8}$ mbar, and laser scattering with a wavelength of 1064 nm and a power of 0.5 W. The labels above the lines indicate the scaling with temperature of the various decoherence mechanisms.

$$\begin{aligned}
& + \frac{4\hbar^2\mu_0^2}{\pi} \int_0^\infty d\omega \omega^4 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) |\alpha(\omega)|^2 \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{G}_{0,0}^s(\omega)_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}^f(\omega)_{pk} \partial_j] \\
& + \frac{4\hbar^2\mu_0^2}{\pi} \int_0^\infty d\omega \omega^4 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) |\alpha(\omega)|^2 \sum_{p,k=\{x,y,z\}} \text{Im}[\bar{G}_{0,0}^s(\omega)_{pk}] \text{Im}[\partial_i \bar{G}_{0,0}^f(\omega)_{pk} \partial_j]. \quad (\text{D9})
\end{aligned}$$

As before, we assume that the integrand is sharply peaked around $\omega \rightarrow 0$ such that

$$\begin{aligned}
D_{ij}^s & \approx \frac{4\hbar^2\mu_0^2}{\pi} \sum_{p,k=\{x,y,z\}} \lim_{\omega \rightarrow 0} \left[|\alpha(\omega)|^2 \omega \text{Im} \partial_i (\bar{G}_{0,0}^s(\omega))_{pk} \partial_j \right] \int d\omega \omega^3 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) \text{Im}[(\bar{G}^f(\omega))_{pk}] \\
& + \frac{4\hbar^2\mu_0^2}{\pi} \sum_{p,k=\{x,y,z\}} \lim_{\omega \rightarrow 0} \left[|\alpha(\omega)|^2 \omega \text{Im} (\bar{G}_{0,0}^s(\omega))_{pk} \right] \int d\omega \omega^3 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1) \text{Im}[\partial_i (\bar{G}_{0,0}^f(\omega))_{pk} \partial_j] \\
& + \frac{4\hbar^2\mu_0^2}{\pi} \sum_{p,k=\{x,y,z\}} \lim_{\omega \rightarrow 0} \left[|\alpha(\omega)|^2 \omega^2 \text{Im} (\bar{G}_{0,0}^s(\omega))_{pk} \text{Im} \partial_i (\bar{G}_{0,0}^s(\omega))_{pk} \partial_j \right] \int d\omega \omega^2 n_{\text{th}}(\omega)(n_{\text{th}}(\omega) + 1). \quad (\text{D10})
\end{aligned}$$

Once again, for a planar metallic surface described by the Drude model, this leaves us with:

$$D^s = \frac{4|\alpha(0)|^2 \pi^3 \gamma \hbar^2}{105 \epsilon_0^2 c^5 \omega_p^2} \frac{1}{z^3} \left(\frac{k_B T}{\hbar} \right)^7 + \frac{2|\alpha(0)|^2 \pi \gamma \hbar^2}{15 \epsilon_0^2 c^3 \omega_p^2} \frac{1}{z^5} \left(\frac{k_B T}{\hbar} \right)^5 + \frac{3|\alpha(0)|^2 \gamma^2 \hbar^2}{32 \omega_p^4 \epsilon_0^2 \pi} \frac{1}{z^8} \left(\frac{k_B T}{\hbar} \right)^3. \quad (\text{D11})$$

APPENDIX E: DECOHERENCE NEAR PLANAR DIELECTRIC SURFACES

Here, we present the decoherence results for a dielectric nanosphere in the vicinity of a dielectric surface. Dielectric surfaces play an important role when nanospheres are coupled to photonic crystals [13] or levitated near membranes [12]. The analysis is the same as in Sec. III D, but we now consider a silica surface rather than a gold surface. Thus, the permittivity of the surface is given by Eq. (38). The decoherence rate is then

$$\Lambda_D^s = \frac{2|\alpha(0)|^2 \pi^3 K}{105 \epsilon_0^2 c^5} \frac{1}{z^3} \left(\frac{k_B T}{\hbar} \right)^7 + \frac{|\alpha(0)|^2 \pi K}{15 \epsilon_0^2 c^3} \frac{1}{z^5} \left(\frac{k_B T}{\hbar} \right)^5 + \frac{3|\alpha(0)|^2 K^2}{64 \epsilon_0^2 \pi} \frac{1}{z^8} \left(\frac{k_B T}{\hbar} \right)^3. \quad (\text{E1})$$

Here, K is given by

$$K = \frac{2(\gamma_2 \omega_{p_2}^2 \omega_{t_1}^4 + \gamma_1 \omega_{p_1}^2 \omega_{t_2}^4)}{(\omega_{p_2}^2 \omega_{t_1}^2 + (\omega_{p_1}^2 + 2\omega_{t_1}^2) \omega_{t_2}^2)^2}, \quad (\text{E2})$$

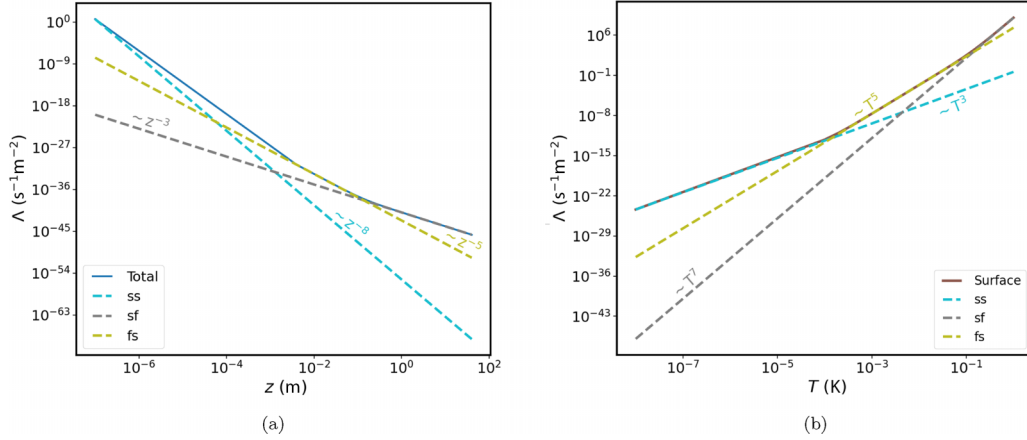


FIG. 6. (a) The different contributions to the distance scaling of Λ_D^s at $T = 10^{-5}$ K. As in the case of a gold surface, the three terms in Eq. (E1) are labeled as ss, sf, and fs according to their origin as terms depending on $\bar{G}^s - \bar{G}^s$, $\bar{G}^s - \bar{G}^f$, and $\bar{G}^f - \bar{G}^s$ in Eq. (E1), where $\bar{G}^f$ and $\bar{G}^s$ are defined in Appendix A. (b) The different contributions to the temperature scaling of Λ_D^s .

with the parameters as defined in the main text. The decoherence rate for a silica surface as a function of distance from the surface is shown in Fig. 5(a) and as a function of temperature in Fig. 5(b). It is worthwhile pointing out that the larger loss parameters of silica, as compared to gold, cause the decoherence induced by a silica surface to be greater than that of a gold surface. Figure 6 shows the three surface contributions to the decoherence coefficient as a function of distance and temperature for a dielectric surface.

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