

Robust exceptional point chains and chirality switch in a vast optical parameter spaceChang-Hwan Yi  and Jung-Wan Ryu *Center for Theoretical Physics of Complex Systems, Institute for Basic Science (IBS), Daejeon 34126, Korea*Tom Simon Rodemund  and Martina Hentschel ^{*}*Institute of Physics, Technische Universität Chemnitz, D-09107 Chemnitz, Germany*

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While the special properties of exceptional points (EPs) may improve application relevant features such as hypersensing, often their realization requires a precise fine-tuning of parameters. Here we report the robust formation of EP chains in coupled optical microcavities over intercavity distances D on the order of the cavity size. We find that coupling, or more generally, the availability of an outer (coupling) channel, is sufficient to induce EPs in the parameter space spanned by the refractive index n along with D . This occurs even at D of several wavelengths, yielding a network of EPs in parameter space with a periodicity of the order of the wavelength in the D direction. Upon the increase of n , we observe a sharp transition in the sign of the chirality measured in one of the cavities that further structures the system properties. While the details of the Riemannian surfaces associated with the formation of exceptional points are mode dependent and allow for a variety of single-cavity chiralities, we find the underlying mechanism to be universal, robust, applicable to other and more general coupled systems, and promising for potential applications such as optical measurements of pressure, voltages, or distances.

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Introduction. Photonic microsystems [1] provide a robust platform for application relevant devices ranging from microlasers to sensors, while at the same time their intrinsic openness makes them unique model systems for non-Hermitian physics [2–6]. Their openness originates in the possibility of refractive or evanescent escape, leading to complex eigenfrequencies with the real part $\text{Re}(\Omega) = kR$ of the normalized frequency $\Omega = \omega R/c_0$ (with c_0 being speed of light in vacuum, R being the resonator size parameter, and k being the wave number), while the imaginary part is related to the resonance lifetime $\tau = -1/[2 \text{Im}(\omega)]$. Upon variation of external parameters such as the refractive index n or the length scale R , the eigenfrequencies move in the complex plane and two (or more) of them can form an exceptional point (EP) at which their respective real and imaginary parts coincide and intriguing properties result. Starting from an EP, one can find curves in parameter space where the real (imaginary) parts are degenerate, while the imaginary (real) parts remain separate. This generic phenomenon near the EP is known as avoided resonance crossing [7–11].

Non-Hermitian features in photonic mesoscopic systems have been considered in the context of, e.g., microlasers [12–17], sensors [18–24], encryption [25], topological devices [26–30] and optical gyroscopes [31,32]. Non-Hermitian

properties are not limited to optical systems. They can be realized in a variety of fields, such as acoustic [33,34], optomechanical [35], magnetic [36], and plasmonic devices [37], as well as integrated electrical circuits [38]. A key challenge in exploiting the unique properties of EPs lies in the requirement for extreme fine-tuning of system parameters. Conventional methods, such as employing small scatterers [39], typically operate at deep subwavelength spatial scales and remain highly vulnerable to environmental disturbances such as vibrations or ambient fluctuations. Likewise, approaches based on reshaping cavity boundaries are inherently inflexible: once a cavity is fabricated, its geometry cannot be altered. This not only makes postfabrication tuning impractical but also requires the production of multiple cavity samples to explore parameter space. Even in such efforts, unavoidable variations in material properties may hinder reproducible realization of the desired conditions, despite identical nominal geometries.

Here, we introduce coupled microcavities as a system with a robust formation of chains and networks of EPs over wide ranges in parameter space and identify the intercavity distance D as a useful additional external parameter besides the refractive index. In particular we observe the formation of EPs over large $D \gg \lambda = 2\pi/k$ that are easily accessible in experiments and suitable for applications. The vicinity and the exact setting of an EP can be universally controlled solely via adjusting D , independent of resonator's specific properties. The distance D can be controlled using piezoelectric actuators, temperature change, or mechanical pull (applied to the base carrying the resonators). This opens the path to straightforward experimental verification and implementation in applications ranging from enhanced sensing to nanophotonic light control and chirality engineering that could enable an alternative access

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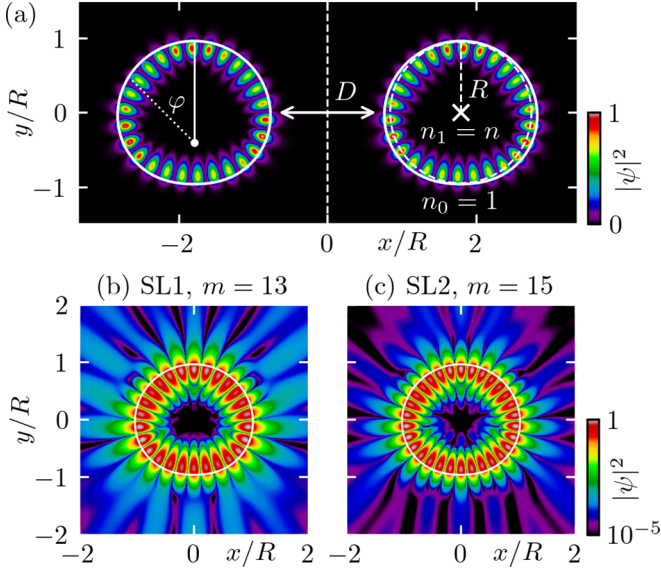


FIG. 1. (a) System geometry of two limaçon-shaped microcavities with refractive index n positioned a distance D apart. The vertical dashed line marks the mirror-symmetric axis and R marks the radius of the dashed circle. (b) Single-cavity mode SL1 with $(m, \Omega) = (13, 6.614\,140 - i0.000\,257)$ and (c) SL2 with $(m, \Omega) = (15, 7.492\,025\,7 - i0.000\,067)$ in logarithmic scale with $n = 2.4$.

to optical gyroscopes. Moreover, we introduce an analytical model that explains all our findings and provides a deeper understanding of the conditions necessary for the formation of an EP. These can be expressed as relations the coupling parameters have to fulfill, and no specific resonator or far-field emission properties enter. This reveals, in turn, that other coupling schemes, such as mirror-based setups, may represent the same non-Hermitian physics and thus widen the range of application relevant devices.

Coupled limaçon system. Our model system consists of two coupled two-dimensional optical microcavities with a face-to-face intercavity distance D , featuring a mirror symmetry about the y axis at $x = 0$, as shown in Fig. 1(a). While we assume deformed microcavities of limaçon shape here, our results are valid in general noncircular geometry consistently. In polar coordinates (ρ, φ) , the limaçon geometry is given as $\rho(\varphi) = R(1 + \varepsilon \cos \varphi)$ with the cavity size parameter R and the shape factor ε , with $\varepsilon = 0.4$ used here.

Eigenmodes of the single limaçon cavity have been studied in detail [40–42]. The lack of rotational symmetry in the limaçon microcavity induces a splitting of orthogonal modes in pairs of even and odd modes with respect to the mirror-symmetric axis of a limaçon shape (see Fig. S1 in Ref. [43]). Two examples of even eigenmodes with different angular momentum m , as identified by the $2m$ azimuthal intensity maxima, are shown in Figs. 1(b) and 1(c) for $m = 13$ (SL1) and $m = 15$ (SL2), respectively. We note here that, unlike in a circular cavity where modes have a well-defined angular momentum m , the broken rotational symmetry of the limaçon cavity leads to modes with distributed angular momentum components. While we still label the limaçon cavity modes based on the dominant m , the distribution plays a crucial role in enabling the emergence of multiple EPs and the nontrivial

chirality behavior discussed below. If the two cavities are mirror-symmetrically positioned, four coupled modes are generated that share the same m . Accordingly, we now distinguish them based on a new mirror-symmetric axis (y axis), classifying them as either bonding or antibonding modes. Each symmetry class includes a single-microcavity-wise pair of even and odd modes that have been previously defined in an individual microcavity. In this system configuration, while the bonding and antibonding modes are mutually orthogonal, the modes previously defined as even and odd in a single cavity are no longer orthogonal within their respective classes; they can now couple with each other. Our focus is on the EPs of the two modes belonging to the bonding mode class.

The parameter space considered in this work is defined by the refractive index n and the dimensionless face-to-face intercavity distance D/λ_0 , where $\lambda_0 = 2\pi k_0^{-1}$ denotes the resonance wavelength of a single cavity in vacuum. The wavelength dependence on n is taken into account by a linear fit approximation: $\lambda_0^{\text{SL1}}(n) \approx (0.387n + 0.019)R$ and $\lambda_0^{\text{SL2}}(n) \approx (0.343n + 0.014)R$.

EP chains, chirality, and phase rigidity. We compute the complex eigenfrequencies Ω and the corresponding wave functions ψ of the eigenmodes in the coupled microcavity system by solving the Helmholtz equation,

$$-\nabla^2 \psi(\mathbf{r}) = n^2(\mathbf{r}) \Omega^2 \psi(\mathbf{r}), \quad (1)$$

over a vast parameter space $(D/\lambda_0, n) \in [0, 8] \times [1.5, 3.5]$. The boundary conditions applied are transverse-magnetic polarization at the cavity boundary, where both the wave function ψ and its normal derivative $\partial_n \psi$ are continuous, and the pure-outgoing wave condition at infinity. To numerically solve Eq. (1) for the nonintegrable limaçon shape, the boundary element method (BEM) is employed [44]. We examine the EPs of eigenmodes, CL1 and CL2, in the coupled microcavities, which correspond to the single limaçon cavity modes, SL1 and SL2. Obtaining the eigenmodes, the chirality α and the phase rigidity r are analyzed concurrently.

The chirality α of a coupled cavity mode is defined as the intracavity wave propagation direction in the right cavity,

$$\alpha = \frac{\int |\phi_{+m}|^2 dm - \int |\phi_{-m}|^2 dm}{\int |\phi_{+m}|^2 dm + \int |\phi_{-m}|^2 dm}, \quad (2)$$

where $\phi_{\pm m}$ denote the amplitudes of the positive and negative angular momentum components, obtained by applying the Fourier transform to the boundary wave function $\psi(s)$, with $s \in [0, L]$, where L represents the perimeter of the cavity boundary. The boundary wave function is directly obtained in computing eigenmodes using the BEM. The chirality α quantifies the imbalance between the two counterpropagating components in the right cavity. Note that, due to the mirror symmetry, the left cavity possesses the opposite chirality: $-\alpha$.

Another intriguing quantity that is prominently peaked at EPs is the phase rigidity r . It is formally defined as the inner-product between the left and right eigenvectors of mode i , i.e., $\langle \psi_i^L | \psi_i^R \rangle$. However, due to the symmetry of the Helmholtz wave equation, the relation $\langle \psi_i^L | = \langle (\psi_i^R)^* |$ holds, allowing for the simplification [45–48]

$$r_i = |\langle \psi_i^L | \psi_i^R \rangle| = |\langle \psi_i^* | \psi_i \rangle|, \quad (3)$$

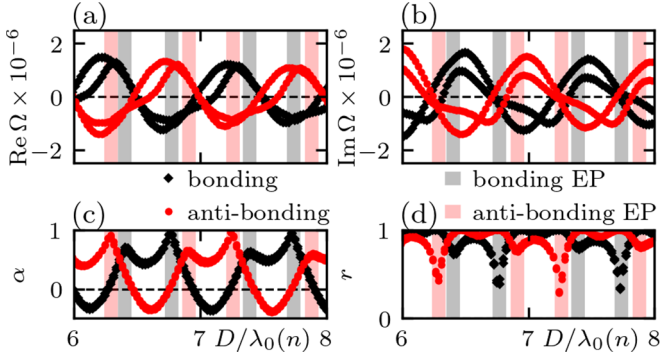


FIG. 2. Intercavity distance-dependent eigenfrequency of CL1 modes with $n = 2.9$ in the weak-coupling regime. Shown are the (a) real and (b) imaginary components of the eigenfrequencies Ω relative to the SL1 frequency Ω_0 , (c) chirality α , and (d) phase rigidity r . Black diamonds (red circles) correspond to bonding (antibonding) modes. The light gray (red) vertical bands indicate proximity to an EP of the bonding (antibonding) modes in parameter space.

with the normalization condition $\langle \psi_i^L | \psi_i^L \rangle = 1$ or $\langle \psi_i^R | \psi_i^R \rangle = 1$. In this work, we compute r_i using the boundary wave function, which has been found to be sufficient for capturing the essential non-Hermitian mode properties [49].

The eigenfrequency, chirality, and phase rigidity for the bonding and antibonding CL1 modes are shown in Fig. 2. In the weak-coupling regime of $D > \lambda_0$, bonding and antibonding eigenfrequencies oscillate with an almost identical period, the only prominent difference being a shift in D of $\lambda_0/2$. The real and imaginary components of the frequencies in Figs. 2(a) and 2(b) oscillate around the SL1 mode. This oscillation is a universal feature of coupled cavities [50,51].

For $n = 2.9$, several EPs are passed in parameter space for both the bonding and antibonding modes, which is evident from the high chirality and low phase rigidity in Figs. 2(c) and 2(d).

The EPs appear periodically as a function of the intercavity distance D [see the light gray (red) vertical bands in Fig. 2]. This periodicity corresponds to the wavelength of the eigenmodes [50], given by $\lambda = 2\pi/\Omega \approx 0.95$, where we use $\Omega = 6.61$ as in Fig. 1(b). Within each wavelength interval of D , approximately two EPs are encountered for both the bonding and antibonding modes. Namely, the intercavity coupling is periodically modulated with period λ , leading to the recurrent satisfaction of the EP condition. Due to the similar behavior of the bonding and antibonding CL modes, we focus only on the bonding modes without loss of generality.

The chirality and phase rigidity for the CL1 and CL2 modes are shown in Fig. 3. The full structure of the frequency splitting $\Delta\Omega$ for the coupled cavity modes in $(D/\lambda_0, n)$ space, exemplified for the bonding CL1 mode, is presented in Fig. S4 [43]. The EPs occur when both the real and imaginary parts of $\Delta\Omega$ vanish, and, in Fig. 3, it is revealed that they are distributed across the entire parameter space, either scattered or, more commonly, forming chains. The positions of these EPs coincide with regions of extremal chirality α and vanishing phase rigidity r , as shown in Fig. 3. The two key findings from our results are as follows. First, a robust series of EP chains are discovered, forming networks interconnected by real (solid curve) and imaginary (dashed curve) branch cuts. Second, a sharp transition in the overall chirality is observed, with its sign changing from negative (blue) to positive (orange) as the refractive index n increases. Notably, EPs are concentrated at this transition.

Construction of effective Hamiltonian. To analytically model the coupled limaçon microcavity system, we begin by

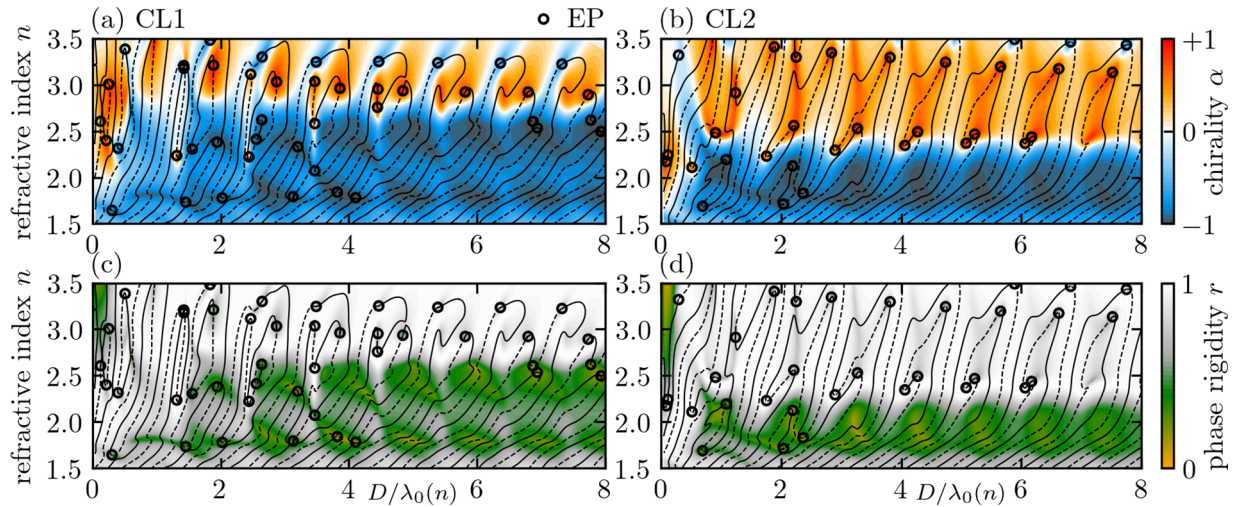


FIG. 3. (a) and (b) Chirality in the cavity at $x > 0$. (c) and (d) Phase rigidity of coupled resonators modes originating from the bonding [panels (a) and (c)] CL1 and [panels (b) and (d)] CL2 modes illustrated over a vast range of parameter space $(D/\lambda_0, n)$. Solid and dashed curves show branch cuts of $\text{Re}(\Delta\Omega) = 0$ and $\text{Im}(\Delta\Omega) = 0$, respectively. The circles mark the position of EPs, which occur when $\Delta\Omega = 0$. The junction points where the two curves (solid-dashed) connect indicate EPs, (cf. Fig. S1 in Ref. [43]). The presence of EPs leads to locally high chirality and phase rigidity near 0. Note a global switch in chirality for the CL1 mode from negative to positive values around a refractive index of $n \approx 2.7$ for all D . The overall behavior for CL2 is similar, but the switch in chirality occurs now at lower refractive indices of $n \approx 2.3$, indicating that mode-specific properties impact the quantitative details of the generic transition.

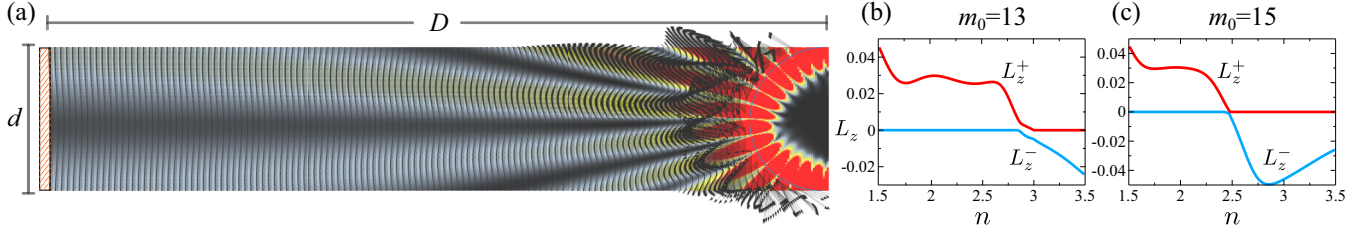


FIG. 4. Refractive index-dependent far-field angular momentum, deduced from the Poynting vector of the far field of a single limaçon-shaped microdisk, measured at a distance of $D = 50R$ along the y direction over a range of $d = 2R$ [red shaded box to the left of panel (a)]. Panel (a) shows the system's geometric configuration. Panels (b) and (c) present the effective angular momentum L_z of modes with $m_0 = 13$ and 15, extracted from the n -dependent far-field patterns.

examining the coupled circular disk system. The eigenmodes of this system can be determined analytically using the two-dimensional generalized Lorenz-Mie theory [52,53]. This approach exploits the rotational symmetry of the circular disks, allowing us to express the solutions of Eq. (1) in a single disk using Bessel and Hankel functions. These functions form a complete basis in the radial coordinate, along with rotational harmonics $e^{\pm im\theta}$ in the azimuthal direction.

While we refer the reader to the Supplemental Material [43] for further details, we illustrate the result for the two-disk case, where we assume a single angular momentum component $|m_0|$ in each disk, i.e., $m = \pm m_0$. In this case, the transfer matrix $\mathbf{T}$ simplifies, and the unknown coefficient vector $\mathbf{b}$ to be computed is obtained from the emission condition $\mathbf{T}\mathbf{b} = 0$, which is expressed as follows:

$$\mathbf{T}\mathbf{b} = \begin{pmatrix} 1 & 0 & t_{11} & t_{12} \\ 0 & 1 & t_{21} & t_{22} \\ \tau_{11} & \tau_{12} & 1 & 0 \\ \tau_{21} & \tau_{22} & 0 & 1 \end{pmatrix} \begin{pmatrix} b_{1,-m_0} \\ b_{1,+m_0} \\ b_{2,-m_0} \\ b_{2,+m_0} \end{pmatrix} = 0. \quad (4)$$

Equation (4) implies that the amplitudes of the angular momentum components ($\pm m_0$) in the second disk (index 2) are transferred to those in the first disk (index 1) through the transfer strength t_{ij} , and vice versa through τ_{ij} .

Based on these transfer strengths, we can now construct the effective Hamiltonian that describes the coupled limaçon-shaped microcavities by assuming the coupling elements to be proportional to the transfer strengths. Explicitly, we have

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{V} = \begin{pmatrix} k_0 & v & 0 & 0 \\ v & k_0 & 0 & 0 \\ 0 & 0 & k_0 & v \\ 0 & 0 & v & k_0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & V_{11} & V_{12} \\ 0 & 0 & V_{21} & V_{22} \\ W_{11} & W_{12} & 0 & 0 \\ W_{21} & W_{22} & 0 & 0 \end{pmatrix},$$

where k_0 denotes the wave number of the degenerate $\pm m_0$ traveling basis modes in a single circular disk, while v represents the coupling strength between these modes, induced by the boundary deformation into a limaçon shape. The elements of the coupling matrix $\mathbf{V}$ are deduced from the scaled transfer coefficients as $V_{ij} = \eta \xi_{ij} t_{ij}$ ($W_{ij} = \eta \xi_{ij} \tau_{ij}$), where η represents the overall scaling coefficient, and ξ_{ij} denotes the angular momentum component-wise scaling coefficients.

Accordingly, the model Hamiltonian for coupled limaçon-shaped microcavities, separated by a distance D , is constructed as follows:

$$\mathbf{H} = \begin{pmatrix} k_0 & \gamma^+ & V_0 & V_{-2m_0} \\ \gamma^- & k_0 & V_{2m_0} & \beta V_0 \\ \beta V_0 & V_{-2m_0} & k_0 & \gamma^- \\ V_{2m_0} & V_0 & \gamma^+ & k_0 \end{pmatrix}, \quad (5)$$

where each element is defined as

$$k_0 = (k_e^s + k_o^s)/2, \quad v = (k_e^s - k_o^s)/2,$$

$$\gamma^+ = v(1 + \alpha|L_z^+|), \quad \gamma^- = v(1 + \alpha|L_z^-|),$$

$$V_0 = \eta H_0^{(1)}(kD) s_{m_0}(k_0), \quad V_{\pm 2m_0} = \eta H_{\pm 2m_0}^{(1)}(kD) s_{m_0}(k_0),$$

and k_e^s and k_o^s are the wavenumbers of the even (e) and odd (o) eigenmodes in a single limaçon-shaped microcavity, respectively. An overall scaling parameter for the intercavity coupling is set as $\eta = 1/2n$. In this model, two important asymmetry parameters are included: $\beta = 0.1$ for intercavity couplings, and $\alpha|L_z^\pm|$ for intracavity couplings, with $\alpha = 5.0$. Here, $|L_z^\pm|$, the effective angular momentum of the far-field, is computed from modes in the single limaçon-shaped microcavity. The chirality is subsequently deduced from the eigenstate of Eq. (5). We obtain $|L_z^\pm|$ by calculating the Poynting vector $\vec{j} = (j_x, j_y)$ at a distance D within the range $y \in [y_{\min}, y_{\max}]$, where $y_{\max} - y_{\min} = 2R$ (cf. Fig. 4). Namely, after computing the position-dependent Poynting vector,

$$\vec{j}(x = D, y) = \text{Im}[\psi^*(D, y) \vec{\nabla} \psi(D, y)], \quad (6)$$

we find $L_z(x, y) = x j_y - y j_x$ and compute the expectation value $L_z^+ = \int_{y_{\min}}^{y_{\max}} L_z(D, y) |\psi(D, y)|^2 dy$ for $L_z(D, y) > 0$, and L_z^- for $L_z(D, y) < 0$, with a normalization condition of $\int_{y_{\min}}^{y_{\max}} |\psi(D, y)|^2 dy = 1$. We calculate the refractive index-dependent $L_z^\pm$ in the limaçon-shaped microcavity by averaging the even-odd paired modes (see Fig. 4). As we can see in Figs. 4(b) and 4(c), at a specific refractive index n_s , a transition occurs from the L_z^+ -dominant region to the L_z^- -dominant region: $n_s \approx 2.8$ for $m_0 = 13$ and $n_s \approx 2.4$ for $m_0 = 15$. This transition is the origin of the global chirality switch in the modes of coupled limaçon-shaped microcavities.

The refractive-index-dependent reversal of L_z arises from the complex modal structure of the chaotic limaçon cavity that forms from a broad distribution of angular momentum components. As the refractive index varies, the critical angle for total internal reflection shifts, altering the angular momentum

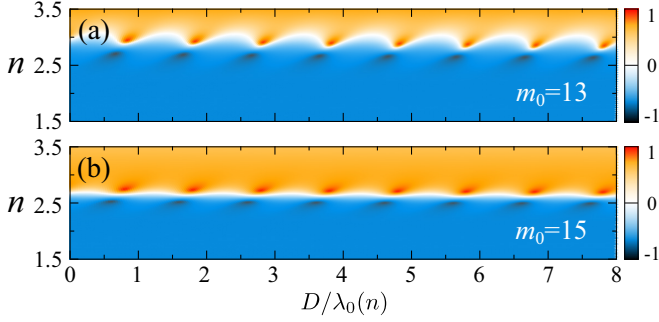


FIG. 5. Chirality (color scale) of eigenstates of the model Hamiltonian [Eq. (5)] according to Eq. (7), plotted over parameter space $(D/\lambda_0, n)$ for (a) $m_0 = 13$ and (b) $m_0 = 15$. We use $(\eta, \alpha, \beta) = (1/2n, 5, 0.1)$, while k_0 and $|L_z^\pm|$ are deduced from the single cavity results (see Fig. 4 and Fig. S7 in Ref. [43]).

components that will dominate the far-field emission. Such a behavior can be observed in all coupled systems of broken rotational symmetry cavities with a single mirror-symmetric axis within each cavity, together with a mirror-symmetric arrangement in the coupled configuration. These symmetry conditions, which enable spontaneous formation of chiral modes, asymmetric backscattering, and chirality transitions, are not unique to the limaçon geometry.

When one disk is placed at $D \gg d$, it behaves as a Rayleigh scatterer [39], with the dominant backscattering direction switching from $L_z^+ \rightarrow L_z^-$ to $L_z^- \rightarrow L_z^+$ around the transition refractive index n_s . This effect is incorporated with a scaling parameter α in the off-diagonal elements of the upper-left block-diagonal matrix in Eq. (5), while in the opposite disk, the effect is mirrored in the lower-right block-diagonal matrix.

By plugging in the obtained $L_z^\pm$, we can compute the eigenstate of the model Hamiltonian in Eq. (5) as $\vec{\psi} = (b_R^+ \ b_R^- \ b_L^+ \ b_L^-)^T$. The chirality of the eigenstate in the right disk (R) is then computed using its definition:

$$\text{Chirality R} := \frac{|b_R^+|^2 - |b_R^-|^2}{|b_R^+|^2 + |b_R^-|^2}. \quad (7)$$

Note that the chirality of the left disk (L) is the negative of that of the right disk. We obtain the eigenstate chirality of the model Hamiltonian for the right disk, as shown in Fig. 5. We confirm good agreement between the eigenstate chirality obtained from our model Hamiltonian in Eq. (5) and the full numerical results presented in Figs. 3(a) and 3(b), including the sequential appearance of the chirality maxima in the D direction, associated with the exceptional points, as well as the global chirality change in the n direction.

Summary and conclusion. We have introduced and demonstrated the robust formation of an exceptional point network built from chains of EPs in extensive parameter space. This is achieved by coupling of modes of a single cavity to an outer channel of size comparable to that of the cavity, for example, by coupling to a neighboring cavity. We have illustrated this using two coupled limaçon cavities and identified chains of EPs in the parameter space spanned by the refractive index n and the inter-cavity distance D . Most notably, the formation of EPs is not limited to small distances of the order of few wavelengths in the strong-coupling regime [54]. Rather, we find EPs to be robustly formed over D of dozens of wavelengths λ . By relaxation of the requirements of very close cavity contact (small D) and precise parameter control we introduce a flexible and robust scheme that opens new opportunities for using the fascinating properties of EPs in a broad range of sensing devices and other applications.

For example, mounting the system on a piezoelectric element and controlling the distance D via an applied voltage would induce a D -dependent output signal in a fiber coupled to one of the cavities, and the traveling direction of this signal could be controlled via an electrical voltage, thereby exploiting the square-root sensing properties in the vicinity of an EP [18,19]. The sudden change in chirality reported here provides another tool to sensitively detect the state of the system. If the change in the distance D is realized or caused by mechanical pressure, the presence of an EP network enables one to measure pressure via an optical signal. Another scheme for experimental verification uses an optical fiber to couple some fixed light into one cavity and then change D . The induced (sudden) change in chirality can be detected via an output fiber attached to the same or the other cavity as variation in the output ratio of the two fiber traveling directions. With a suitable calibration procedure, it can be related to the distance D or the refractive index n that could, e.g., record a variation in environmental conditions.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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