

Interaction of a Poincaré beam with optically polarized atoms in the presence of a constant magnetic field

S. Ramakrishna¹ and S. Fritzsche²

Helmholtz-Institut Jena, D-07743 Jena, Germany;

GSI Helmholtzzentrum für Schwerionenforschung GmbH, D-64291 Darmstadt, Germany;

and Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität Jena, D-07743 Jena, Germany



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Recent studies have highlighted the frequent applications of structured light modes in optically pumped atomic magnetometers. In this work, we theoretically explore how a Poincaré beam probes an optically polarized atomic medium. Specifically, we consider atoms polarized by a plane wave with linear polarization, immersed in a constant external magnetic field. We analyze how the polarization of the pump and probe light fields, along with the external magnetic field, impact the absorption profile. To this end, we employ a density-matrix approach based on the Liouville–von Neumann equation. Our results reveal that the absorption profile exhibits an asymmetric pattern that depends on the magnetic-field strength and the mutual orientation of the pump and probe light propagation directions relative to the quantization axis. For illustration, we assume the incoming radiation drives an electric dipole transition, $5s^2S_{1/2} (F = 1) \rightarrow 5p^2P_{3/2} (F = 0)$, in rubidium atoms subjected to a magnetic field. These findings may aid in designing future experiments on optically pumped atomic magnetometers utilizing structured light modes.

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I. INTRODUCTION

Optically pumped atomic magnetometers can be used to detect magnetic fields by monitoring properties such as intensity or polarization of the light at room temperatures [1]. This detection scheme has found significant applications in fields such as geophysics [2], medicine [3], and fundamental physics [4]. As a result, their use is steadily increasing compared to superconducting quantum interference devices, which need to be operated at cryogenic temperatures [5]. Additionally, notable progress has been made in developing compact and miniaturized atomic magnetometers [6].

Traditional atomic magnetometers rely on light fields with uniform polarization across their beam cross section. However, modern optical techniques have enabled the generation of light fields with spatially varying polarization profiles [7]. One prominent member of this class of light fields is known as vector light modes, for example, radially and azimuthally polarized beams. These light modes contain spatially variable linear polarization states within their beam cross section [8]. Moreover, vector light beams have been shown to excite locally varying magnetization profiles in atoms, which can be utilized for measuring both static and oscillating magnetic-field components [9–13]. In addition, an evolved version of

vector beams, known as Poincaré beams, can also be generated [14,15]. These beams exhibit a richer polarization texture, meaning more than one state of polarization across their beam cross section [16]. Beyond vector light beams, a recent experiment demonstrated the elimination of dead zones in an atomic magnetometer successfully with the help of a Poincaré beam [17].

Conventional atomic magnetometers operate in either single- or dual-beam configurations, with the latter offering higher sensitivity for magnetic-field measurements [6,18,19]. In atomic magnetometry using structured light, prior studies have employed both configurations. For example, Cai *et al.* [12] experimentally analyzed the effects of a constant magnetic field on the absorption profile of a radially polarized beam interacting with atomic vapor polarized by a linearly polarized plane wave. In their experiment, both pump and probe fields shared the same propagation axis. While coaxial pump-probe configurations facilitate miniaturization, practical challenges arise in filtering pump light prior to probe analysis. A simple solution is to use orthogonally propagating pump and probe beams. Here, we consider a pump beam propagating perpendicular to the probe direction. Specifically, we explore the use of a full-Poincaré beam to probe the response of an atomic medium—polarized by a linearly polarized plane wave—to a constant magnetic field.

We first examine the interaction between a Poincaré beam and an unpolarized atomic target under a constant magnetic field. This foundation is then extended to analyze interactions with polarized atomic targets. Our analysis reveals that the Poincaré beam’s absorption profile exhibits axial asymmetry, contrasting with the symmetric profile observed for vector

*Contact author: shreyas.ramakrishna@uni-jena.de

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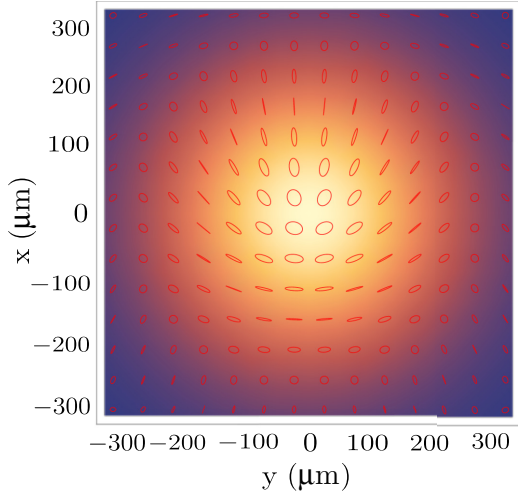


FIG. 1. The intensity and polarization profile of a Poincaré beam constructed using the Bessel light of opening angle $\theta_k = 0.05^\circ$.

beams. Furthermore, this asymmetric absorption depends on the relative orientation between the pump and probe propagation directions and the quantization axis. We also investigate how the asymmetry varies with magnetic-field strength, introducing an asymmetry parameter to quantify this relationship. This parameter provides a means to determine magnetic-field strength. To demonstrate these principles, we model the electric dipole transition $5s\ ^2S_{1/2} (F=1) \rightarrow 5p\ ^2P_{3/2}$

($F=0$) in Rb atoms subjected to a constant magnetic field. These findings offer guidance for future experiments with optically pumped atomic magnetometers using structured light modes.

This paper is structured as follows: A brief mathematical description of a linearly polarized plane wave and Poincaré beam in Bessel basis is provided in Sec. II A. In Sec. II B, we derive the required transition amplitude of the interaction between the pump and probe light field with the atomic target. To determine the effect of the applied magnetic field on the absorption profile of a Poincaré beam, we employ density-matrix theory, the basic formulas of which are briefly reviewed in Sec. II C. In Sec. III A, we first discuss the absorption profile of the Poincaré beams interacting with unpolarized atomic target. As a next step, we discuss the absorption profile of Poincaré beams in the case of a polarized atomic target, in Sec. III B. Finally, in Sec. IV we provide a brief summary and outlook. Atomic units ($\hbar = m_e = e = 1, c = 1/\alpha$) are used throughout the manuscript.

II. THEORETICAL BACKGROUND

A. Pump and probe light fields

1. Plane waves

In the current work, we consider the pump light field to be a linearly polarized plane wave. The vector potential of this plane wave, which is linearly polarized along the $x^{(\text{pump})}$ axis

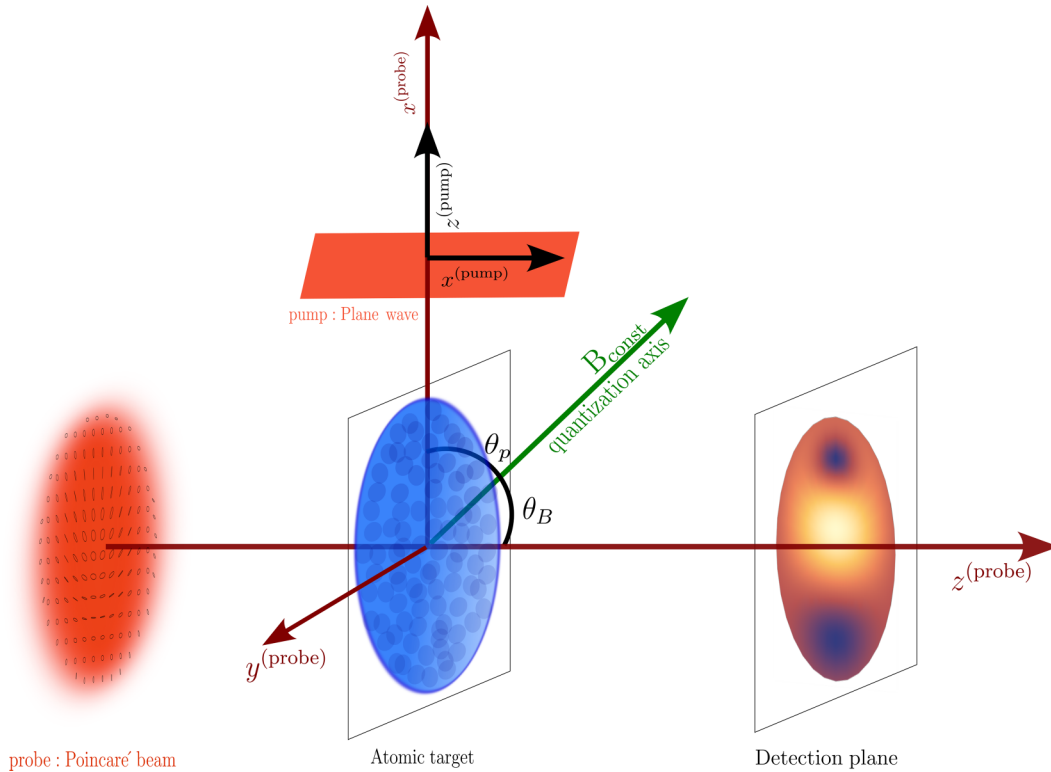


FIG. 2. Geometry of the proposed pump-probe atomic magnetometer setup. The atomic target interacts with a pump and a probe light propagating perpendicular to each other along $z^{(\text{pump})}$ and $z^{(\text{probe})}$, respectively. The pump plane wave is linearly polarized along the $x^{(\text{pump})}$ direction and the probe Poincaré beam has complex polarization texture. The external magnetic field, $\mathbf{B}_{\text{const}}$, is applied at an angle of θ_B with respect to the probe light field. The quantization axis of this system is chosen along the applied constant magnetic field.

(see Fig. 2), can be expressed as

$$\mathbf{A}_x^{(\text{lin})}(\mathbf{r}) = \frac{1}{\sqrt{2}} [\mathbf{A}_{\lambda=+1}^{(\text{circ})}(\mathbf{r}) + \mathbf{A}_{\lambda=-1}^{(\text{circ})}(\mathbf{r})], \quad (1)$$

where $\mathbf{A}_\lambda^{(\text{circ})}(\mathbf{r})$ is the vector potential of a circularly polarized plane wave with helicity $\lambda = \pm 1$, given by

$$\mathbf{A}_\lambda^{(\text{circ})}(\mathbf{r}) = A_0 \mathbf{e}_{k\lambda} e^{i\mathbf{k}\cdot\mathbf{r}}. \quad (2)$$

Here, $\mathbf{k}$ is the propagation vector, and A_0 is the amplitude, the value of which will be specified later.

2. Poincaré beams

We employ a Poincaré beam as the probe light field. Such beams are fundamentally constructed as superpositions of two or more circularly polarized structured light modes. Experimentally, this is typically realized using circularly polarized Laguerre-Gaussian modes. Theoretically, observed physical properties of these paraxial beams near their center can be accurately modeled using Bessel modes [20]. We therefore construct the Poincaré beam in the Bessel basis, with its vector potential expressed as

$$\mathbf{A}^{(\text{poin})}(\mathbf{r}) = \frac{1}{\sqrt{2}} [\mathbf{A}_{m_\gamma=+1, \lambda=+1}^{(\text{B})}(\mathbf{r}) - \mathbf{A}_{m_\gamma=0, \lambda=-1}^{(\text{B})}(\mathbf{r})]. \quad (3)$$

Here, $\mathbf{A}_{m_\gamma, \lambda}^{(\text{B})}(\mathbf{r})$ is the vector potential of a circularly polarized Bessel light field carrying a projection of total angular momentum m_γ onto its propagation axis. Since the theory of Bessel light fields has been frequently discussed in past publications [21, 22], we will limit ourselves to the basic expressions. In particular, the vector potential of a circularly polarized Bessel light field can be written as

$$\mathbf{A}_{m_\gamma, \lambda}^{(\text{B})}(\mathbf{r}) = A_0 \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} a_{\mathcal{Z}m_\gamma}(\mathbf{k}_\perp) \mathbf{e}_{k\lambda} e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (4)$$

where $a_{\mathcal{Z}m_\gamma}(\mathbf{k}_\perp)$ is a weight function given by

$$a_{\mathcal{Z}m_\gamma}(\mathbf{k}_\perp) = \frac{2\pi}{\mathcal{Z}} (-i)^{m_\gamma} e^{im_\gamma \phi_k} \delta(k_\perp - \mathcal{Z}). \quad (5)$$

From the above expressions, one can understand the Bessel light field as a superposition of plane waves in momentum space the wave vectors $\mathbf{k} = (k_\perp, k_z)$ of which lie on the surface of a cone with an opening angle of $\theta_k = \arctan(\mathcal{Z}/k_z)$. By choosing smaller opening angle θ_k , one can obtain paraxial Bessel light fields in which the transverse momentum is much weaker than its longitudinal counterpart, that is, $\mathcal{Z} \ll k_z$ (see Ref. [22]). By using this condition, one can approximate the

vector potential of a Poincaré beam (3) as

$$\begin{aligned} \mathbf{A}^{(\text{poin})}(\mathbf{r}, t) \approx & A_0 [-i \{J_0(\mathcal{Z}r_\perp) + J_1(\mathcal{Z}r_\perp) e^{i\phi_r}\} \mathbf{e}_x \\ & + \{J_0(\mathcal{Z}r_\perp) - J_1(\mathcal{Z}r_\perp) e^{i\phi_r}\} \mathbf{e}_y] e^{ik_z z} e^{i\omega t}. \end{aligned} \quad (6)$$

The electric field of the Poincaré beam is derived from its vector potential [Eq. (6)] using $\mathbf{E}(\mathbf{r}, t) = -\partial_t \mathbf{A}(\mathbf{r}, t)$, enabling characterization of transverse intensity and polarization profiles. Although the intensity distribution resembles a Gaussian (Fig. 1), the beam exhibits a complex polarization structure: The local state evolves continuously from pure circular polarization at the beam center to elliptical with position-dependent ellipticity radially outward. Crucially, while the ellipticity becomes small far from the center, the polarization never collapses into a perfectly linear state. This residual ellipticity may appear linear in Fig. 1 due to scale limitations. Our implementation combines structured light modes with orbital angular momentum projections $m_\ell = 0$ ($m_\gamma = +1$) and $m_\ell = 1$ ($m_\gamma = 0$), though other combinations are possible. This configuration—termed a full-Poincaré beam—generates a lemon-shaped polarization pattern [14].

B. Transition amplitudes

Following the discussion of the mathematical formulation of pump and probe light fields, we now proceed to derive transition amplitude for the light atom interaction process. In particular, we will consider atomic transition from the initial $|\alpha_g F_g M_g\rangle$ to final $|\alpha_e F_e M_e\rangle$ atomic state driven by the incoming laser in the presence of external magnetic field $\mathbf{B}_{(\text{const})} = (0, 0, B_{(\text{const})})$. Moreover, this external constant magnetic field is chosen as the quantization axis of the total system throughout this paper. Then, one could write the first-order transition matrix element as

$$V_{eg} = \frac{1}{\alpha} \langle \alpha_e F_e M_e | \sum_q \boldsymbol{\alpha}_q \cdot \mathbf{A}(\mathbf{r}_q) | \alpha_g F_g M_g \rangle, \quad (7)$$

where α is the fine-structure constant; $\mathbf{F} = \mathbf{I} + \mathbf{J}$, where $\mathbf{I}$ and $\mathbf{J}$ are the nuclear and electron angular momenta, respectively; M is the projection of $\mathbf{F}$ on the quantization axis; $\alpha_g(\alpha_e)$ denotes all additional quantum numbers required to specify the ground (and excited) state uniquely. Moreover, q runs over all electrons in a target atom and $\boldsymbol{\alpha}_q$ denotes the vector of Dirac matrices for the q th particle [23]. In the above transition amplitude (7), $\mathbf{A}(\mathbf{r})$ denotes the vector potential of either pump or probe light field. Before proceeding further, it can be noted that the transition matrix elements for the interaction process between a linearly polarized plane wave and a Poincaré beam with an atom can be constructed in a similar way to their respective vector potentials. The transition amplitude for a circularly polarized plane wave of helicity $\lambda = \pm 1$ is given by

$$\begin{aligned} V_{eg}^{(\text{circ})}(\lambda) = & \frac{A_0}{\alpha} \sqrt{2\pi} \sum_{L,p} i^L [L, F_g]^{1/2} (i\lambda)^p D_{\Delta M, \lambda}^L(-\pi, \theta_p, -\pi) \langle F_g M_g, L \Delta M | F_e M_e \rangle \\ & \times (-1)^{I+F_g+L+J_e} \begin{Bmatrix} F_e & F_g & L \\ J_g & J_e & I \end{Bmatrix} \langle \alpha_e J_e || \sum_q \boldsymbol{\alpha}_q \cdot \mathbf{a}_{L,q}^{(p)} || \alpha_g J_g \rangle, \end{aligned} \quad (8)$$

and the transition amplitude for a Bessel light field with a given m_γ and $\lambda = \pm 1$ can be written as

$$V_{eg}^{(B)}(m_\gamma, \lambda) = \frac{A_0}{\alpha} \sqrt{2\pi} \sum_M i^{L+M} [L, F_g]^{1/2} (i\lambda)^p (-1)^{m_\gamma} e^{i(m_\gamma - M)\phi_b} J_{m_\gamma - M}(\alpha b) d_{M, \lambda}^L(\theta_k) \\ \times D_{\Delta M, M}^L(\pi, \theta_B, \pi) \langle F_g M_g, L \Delta M | F_e M_e \rangle (-1)^{I+F_g+L+J_e} \begin{Bmatrix} F_e & F_g & L \\ J_g & J_e & I \end{Bmatrix} \langle \alpha_e J_e || \sum_q \alpha_q \cdot \mathbf{a}_{L,q}^{(p)} || \alpha_g J_g \rangle. \quad (9)$$

In the above expressions, $\Delta M = M_e - M_g$, $[L, F_g] = (2L+1)(2F_g+1)$, $p = 1$ (or 0) denotes electric (or magnetic) atomic transition with a multipolarity L , $\langle \alpha_e J_e || \sum_q \alpha_q \cdot \mathbf{a}_{L,q}^{(p)} || \alpha_g J_g \rangle$ is the reduced matrix elements, and $D_{\Delta M, \lambda}^L(\pi, \theta_B, \pi)$ is the Wigner D functions. Here, the arguments of the Wigner D functions are Euler angles that characterize the rotation from the atomic frame with the quantization axis along the magnetic field $\mathbf{B}_{\text{const}}$ to the photon frame with the quantization axis along the respective wave vectors $\mathbf{k}$ [24]. Moreover, the angle $\theta_p = \pi/2 - \theta_B$ ensures that at all the given times the pump and probe light fields are propagating perpendicular to each other, as shown in Fig. 2. Furthermore, taking into account the fact that the Poincaré beam has a complex spatial structure, we have introduced impact parameter $\mathbf{b} = (b \cos \phi_b, b \sin \phi_b, 0)$ in (9) which will characterize the position of the atom within its beam cross section [22].

One can write the transition amplitude for a linearly polarized pump light field interacting with an atomic target as

$$V_{eg,x}^{(\text{lin})} = \frac{1}{\sqrt{2}} [V_{eg}^{(\text{circ})}(\lambda = +1) + V_{eg}^{(\text{circ})}(\lambda = -1)]. \quad (10)$$

Similarly, the transition amplitude for the probe Poincaré beam can be written as

$$V_{eg}^{(\text{poin})} = \frac{1}{\sqrt{2}} [V_{eg}^{(B)}(m_\gamma = +1, \lambda = +1) - V_{eg}^{(B)}(m_\gamma = 0, \lambda = -1)]. \quad (11)$$

It should be noted that the vector potential of (nonparaxial) Bessel light (4) was used to derive the transition amplitude for the Poincaré beam atom interaction and this holds true for any arbitrary opening angle θ_k . That being said, we will restrict ourselves to smaller θ_k values in our calculation to remain within the paraxial regime.

C. Density-matrix formalism

In this section, we model atomic state dynamics interacting with light fields using the Liouville–von Neumann equations [25]. As this formalism was comprehensively detailed in our previous work [13], we present only essential expressions here. The atomic state is described by the density operator $\hat{\rho}(t)$, with its time evolution governed by

$$\frac{d}{dt} \hat{\rho}(t) = -i[\hat{H}(t), \hat{\rho}(t)] + \hat{R}(t). \quad (12)$$

In this expression, $\hat{H}(t)$ represents the total Hamiltonian of the system, encompassing interactions between the atomic system and the external magnetic field, pump field, and probe field. The relaxation operator $\hat{R}(t)$ phenomenologically models spontaneous decay from excited states to ground states [26].

Following our previous approach [13], we construct the system's density matrix with dimensions $(2F_g + 2F_e + 2) \times (2F_g + 2F_e + 2)$. This formulation models transitions between atomic ground and excited states with multiple magnetic sublevels. Crucially, this formalism applies exclusively to two-level atomic systems. For such systems, the density-matrix elements are expressed as

$$\rho_{gg'}(t) = \langle \alpha_g F_g M_g | \hat{\rho}(t) | \alpha_g F_g M_g' \rangle, \quad (13a)$$

$$\rho_{ee'}(t) = \langle \alpha_e F_e M_e | \hat{\rho}(t) | \alpha_e F_e M_e' \rangle, \quad (13b)$$

$$\rho_{ge}(t) = \langle \alpha_g F_g M_g | \hat{\rho}(t) | \alpha_e F_e M_e \rangle, \quad (13c)$$

$$\rho_{eg}(t) = \langle \alpha_e F_e M_e | \hat{\rho}(t) | \alpha_g F_g M_g \rangle. \quad (13d)$$

Because the Liouville–von Neumann equations adopt a matrix representation in our formalism, the density-matrix elements can be explicitly expressed within the rotating-wave approximation [26,27] as

$$\frac{d}{dt} \tilde{\rho}_{gg'}(t; M_g, M_e) = -i\Omega_g^{(L)} [M_g - M_g'] \tilde{\rho}_{gg'}(t) - \frac{i}{2} \left[\sum_{M_e} V_{eg}^* \tilde{\rho}_{eg'}(t) - \sum_{M_e} V_{eg'} \tilde{\rho}_{ge}(t) \right] + R_{gg'}(t), \quad (14a)$$

$$\frac{d}{dt} \tilde{\rho}_{ee'}(t; M_g, M_e) = -i\Omega_e^{(L)} [M_e - M_e'] \tilde{\rho}_{ee'}(t) - \frac{i}{2} \left[\sum_{M_g} V_{eg} \tilde{\rho}_{ge'}(t) - \sum_{M_g} V_{e'g}^* \tilde{\rho}_{eg}(t) \right] + R_{ee'}(t), \quad (14b)$$

$$\frac{d}{dt} \tilde{\rho}_{ge}(t; M_g, M_e) = -i\Delta \tilde{\rho}_{ge}(t) + i[\Omega_e^{(L)} M_e - \Omega_g^{(L)} M_g] \tilde{\rho}_{ge}(t) - \frac{i}{2} \left[\sum_{M_e'} V_{e'g}^* \tilde{\rho}_{e'e}(t) - \sum_{M_g} V_{eg}^* \tilde{\rho}_{gg'}(t) \right] + R_{ge}(t), \quad (14c)$$

$$\frac{d}{dt} \tilde{\rho}_{eg}(t; M_g, M_e) = i\Delta \tilde{\rho}_{eg}(t) - i[\Omega_e^{(L)} M_e - \Omega_g^{(L)} M_g] \tilde{\rho}_{eg}(t) - \frac{i}{2} \left[\sum_{M_g} V_{eg'} \tilde{\rho}_{g'g}(t) - \sum_{M_e'} V_{e'g} \tilde{\rho}_{ee'}(t) \right] + R_{eg}(t). \quad (14d)$$

Here, Δ denotes the light frequency detuning from resonance, and $\Omega^{(L)} = g_F \mu_B B_{\text{const}} / \hbar$ is the Larmor frequency. We also assume the pump and probe fields share the same angular frequency, thereby driving transitions between identical hyperfine levels. The total transition amplitude V_{eg} is given by $V_{eg} = V_{eg}^{(\text{pump})} + V_{eg}^{(\text{probe})}$, representing the sum of matrix elements for pump-atom and probe-atom interactions.

Finally, the spontaneous decay $R(t)$ terms required to solve the above Liouville–von Neumann equations are expressed as

$$R_{gg'}(t) = \Gamma[F_g, J_e] \left\{ \begin{matrix} F_g & F_g & L \\ J_g & J_e & I \end{matrix} \right\}^2 \sum_{M_e, M'_e, M} \langle F_g M_g L M | F_e M_e \rangle \times \tilde{\rho}_{ee'}(t) \langle F_g M'_g L M | F_e M'_e \rangle, \quad (15a)$$

$$R_{ee'}(t) = -\Gamma[F_g, J_e] \left\{ \begin{matrix} F_g & F_g & L \\ J_g & J_e & I \end{matrix} \right\}^2 \tilde{\rho}_{ee'}(t), \quad (15b)$$

$$R_{ge}(t) = -\frac{1}{2} \Gamma[F_g, J_e] \left\{ \begin{matrix} F_g & F_g & L \\ J_g & J_e & I \end{matrix} \right\}^2 \tilde{\rho}_{ge}(t), \quad (15c)$$

$$R_{eg}(t) = -\frac{1}{2} \Gamma[F_g, J_e] \left\{ \begin{matrix} F_g & F_g & L \\ J_g & J_e & I \end{matrix} \right\}^2 \tilde{\rho}_{eg}(t), \quad (15d)$$

where Γ is the decay rate of the upper level $|\alpha_e J_e\rangle$ [13,28,29].

III. RESULTS AND DISCUSSION

In the present work, we investigate the interaction between a Poincaré beam and atoms in the presence of an external magnetic field. In experiments related to atomic magnetometers, this interaction between the probe light and atoms is quantified by detecting the absorption profile of the transmitted light [9,12]. Specifically, the absorption profile reveals which parts of the beam cross section were absorbed by the interacting atoms. Theoretically, this can be examined by monitoring the populations of the excited and ground states of the atoms across the beam cross section. This approach was outlined in our earlier publication [13] and is equivalent to calculating the imaginary part of the refractive index of the polarized atomic medium.

The theoretical framework developed in the previous section applies to atomic transitions between any two hyperfine levels. Here, we consider a rubidium ensemble with the ground state $5S_{1/2}$ ($F_g = 1$) and excited state $5P_{3/2}$ ($F_e = 0$), focusing on the $F_g = 1 \rightarrow F_e = 0$ transition at the resonance frequency $\omega_0 = 2\pi \times 384$ THz. We neglect thermal motion and Doppler broadening, which would otherwise require taking into account all excited-state hyperfine levels. Both the pump and probe fields are assumed to drive this transition simultaneously. Furthermore, interactions between the two lasers—such as electromagnetically induced transparency—are neglected in our analysis. This is because the probe light couples multiple ground-state magnetic sublevels ($m_g = -1, 0, +1$) to the excited state. As a result, the pump and probe do not selectively address two distinct sublevels, and the system does not fulfill the necessary conditions for coherent interactions between the two lasers.

Field amplitudes are set to $A_0^{(\text{pump})} = 1.28 \times 10^{-13}$ and $A_0^{(\text{probe})} = 2.02 \times 10^{-14}$ with a Bessel mode opening angle $\theta_k = 0.05^\circ$. These parameters emulate experimental

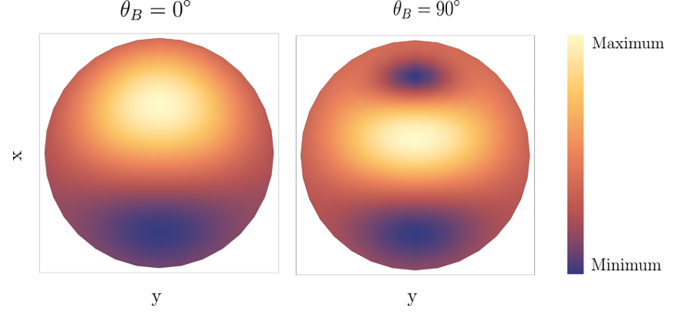


FIG. 3. The asymmetrical absorption profile of the Poincaré beam interacting with an unpolarized rubidium atomic target immersed in an external constant magnetic field of strength $B_{(\text{const})} = 1$ G. The absorption profile is shown for the Poincaré beam propagating at an angle of (left) $\theta_B = 0^\circ$ and (right) 90° with respect to the quantization axis. In these absorption profiles, the bright orange color corresponds to the region in the beam cross section where the Poincaré beam is maximally absorbed by the atoms, while the dark purple color represents the region with minimal absorption.

Laguerre-Gaussian conditions: a 300- μm beam waist for both Gaussian (pump) and Poincaré (probe) beams. The pump power exceeds the probe by a factor of 10, consistent with standard magnetometer configurations [12].

Along with these beam parameters, we require the spontaneous decay rate Γ and the reduced matrix element $\langle \alpha_e J_e || \sum_q \alpha_q \cdot \mathbf{a}_{L,q}^{(p)} || \alpha_g J_g \rangle$. The decay rate $\Gamma = 4.042 \times 10^7 \text{ s}^{-1}$ is obtained from the Jena Atomic Calculator code [30], which also enables calculation of the reduced matrix element. These parameters determine the transition amplitude V_{eg} for both pump and probe interactions with the rubidium target.

The steady-state absorption profile of the Poincaré beam is obtained by computing the excited-state population distribution across its beam cross section. This requires solving the Liouville–von Neumann equations (14) in steady state ($\partial \hat{\rho} / \partial t = 0$), resulting in a system of 16 coupled linear equations. Due to linear dependence among these equations, only ten independent equations need solving, which we implement using the computer algebra system MATHEMATICA.

A. Unpolarized atomic target

Let us begin our discussion by considering the interaction of a Poincaré beam with an unpolarized atomic target, meaning the pump light field is turned off. In this simplified scenario, all three magnetic sublevels in the ground state are assumed to be equally populated. For this case, we examine the population of the excited atomic state across the beam cross section under steady-state conditions to construct the absorption profile. Moreover, we assume the strength of the external magnetic field to be $B_{(\text{const})} = 1$ G. For clarity, we focus on two cases: a Poincaré probe field propagating (a) parallel ($\theta_B = 0^\circ$) and (b) perpendicular (90°) to the quantization axis. As shown in Fig. 3, the absorption profile exhibits asymmetry along the y axis in both cases. In the following, we will provide a detailed explanation to understand these absorption profiles.

1. For $\theta_B = 0^\circ$

When $\theta_B = 0^\circ$, Fig. 3 reveals stronger light absorption by atoms in the upper beam cross section, indicating enhanced interaction with the Poincaré beam. Conversely, atoms in the lower cross section exhibit weaker interaction. This position-dependent variation reflects the beam's inhomogeneous polarization pattern, which manifests in the transition amplitude through the Bessel function $J_{m_\gamma - M}(\kappa b)$. To understand the asymmetric absorption profile in Fig. 3, we analyze the local light-atom transition amplitude.

In Fig. 3, as one moves away from the beam center along the positive x axis, the population of atoms in the excited state increases. Specifically, the excited-state population reaches a maximum at $b = 200 \mu\text{m}$, $\phi_b = 0^\circ$. In contrast, moving in the opposite direction (negative x axis), the excited-state population decreases rapidly, attaining a minimum at $b = 200 \mu\text{m}$, $\phi_b = 180^\circ$. This behavior can be explained in terms of the local polarization structure of the Poincaré beam and the corresponding transition amplitude V_{eg} . As shown in Fig. 1, the local polarization evolves from being purely circular at the center to elliptical as the radial distance increases. In simpler terms, the ellipticity of the polarization ellipse depends on the impact parameter b . For instance, atoms located at $b = 200 \mu\text{m}$, $\phi_b = 0^\circ$ experience a locally elliptically polarized light field, as shown in Fig. 1, where the major axis of the ellipse is aligned along the x axis. On the other hand, atoms at $b = 200 \mu\text{m}$, $\phi_b = 180^\circ$ encounter an elliptically polarized field with the major axis oriented along the y axis, as illustrated in Fig. 1. It is important to note that the quantization axis in both cases remains perpendicular to the local polarization plane. As a result, atoms at these positions can undergo transitions from all three magnetic sublevels in the ground state ($M_g = 0, \pm 1$) to the excited state. However, the strengths of these individual transition amplitudes are not identical and depend sensitively on the atom's position within the beam cross section.

In principle, the strength of the local transition amplitude $|V_{eg}|$ for atomic transitions between $M_g = \pm 1$ and $M_e = 0$ is found to be axially symmetric for any $b \neq 0$. As an example, Fig. 4(c) shows a polar plot of $|V_{eg}^{(\text{point})}|$ at a fixed radius of $b = 200 \mu\text{m}$. From this plot, we observe that the transition amplitude corresponding to $\Delta M = \pm 1$ ($M_g = \pm 1 \rightarrow M_e = 0$) is indeed axially symmetric. However, the transition amplitude for $M_g = 0 \rightarrow M_e = 0$ ($\Delta M = 0$) does not exhibit axial symmetry. This is shown in Fig. 4(b), where we present a polar plot of the absolute value of the $\Delta M = 0$ transition amplitude for $b = 200 \mu\text{m}$ as a function of the azimuthal angle ϕ_b . These plots clearly reveal that the $\Delta M = 0$ transition amplitude lacks axial symmetry. This asymmetry arises from the inhomogeneous polarization profile of the Poincaré beam and is strongly position dependent. In particular, the local transition amplitude for the $\Delta M = 0$ case attains its maximum at $b = 200 \mu\text{m}$, $\phi_b = 0^\circ$ (upper half of the beam), and reaches a minimum at $b = 200 \mu\text{m}$, $\phi_b = 180^\circ$ (lower half of the beam). As a result, atoms located in the lower part of the Poincaré beam cross section exhibit weaker coupling with the light field compared to those in the upper part. This leads to the observed asymmetry in the absorption profile, where the atomic excited-state

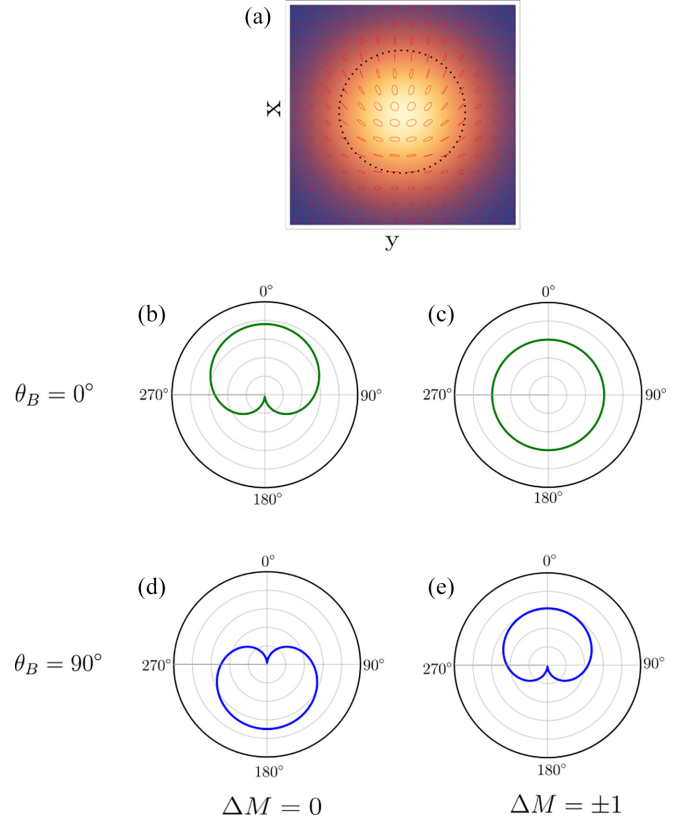


FIG. 4. Polar plots of the absolute value of the transition amplitude $|V_{eg}^{(\text{point})}|$ for the rubidium atom interacting with a Poincaré beam. (a) Here, $|V_{eg}^{(\text{point})}|$ is calculated for a rubidium atom located on a circle with a radius of $b = 200 \mu\text{m}$, depicted by the dotted line on the Poincaré beam's cross section. Along this circumference, the local polarization is found to be elliptical. In this case, the Poincaré beam propagates at an angle of $\theta_B = 0^\circ$ and interacts with the atoms, coupling all three magnetic sublevels in the ground state to the excited state. The polar plot of the local $|V_{eg}^{(\text{point})}|$, which corresponds to the circle of radius b , is asymmetrical for the transition (b) $M_g = 0 \rightarrow M_e = 0$, and symmetrical for the transition (c) $M_g = \pm 1 \rightarrow M_e = 0$. When the Poincaré beam propagates at an angle of $\theta_B = 90^\circ$, the polar plot of the local $|V_{eg}^{(\text{point})}|$ corresponding to the radius b is asymmetrical for both transitions: (d) $M_g = 0 \rightarrow M_e = 0$ and (e) $M_g = \pm 1 \rightarrow M_e = 0$. Here, the transition amplitude is calculated by setting the following values for the beam parameters: $\theta_k = 0.05^\circ$, $A^{(\text{probe})} = 2.02 \times 10^{-14}$.

population is minimized in the lower half of the beam cross section.

From Fig. 3, we observe that the atoms located at the center of the beam absorb relatively minimum light in comparison to those in the upper part of the beam cross section. To understand this, we turn our attention to the local transition amplitude V_{eg} . Since the angle $\theta_B = 0^\circ$, the Wigner D function in the transition amplitude expression (9) simplifies to a delta function: $D_{\Delta M, M}^L(\pi, 0, \pi) = \delta_{\Delta M, M}$. Additionally, for atoms at the center ($b = 0$), the Bessel function in the transition amplitude (9) reduces to $J_{m_\gamma - M}(0) = \delta_{m_\gamma - M, 0}$. These simplifications yield the condition $\Delta M = M$, and the transition amplitude is nonzero only when $m_\gamma = M$. In our case, this implies that the transition amplitude is nonzero for $M = m_\gamma = +1$ and 0. This

is evident from the simplified expression for the transition amplitude $V_{eg}^{(\text{poin})}$:

$$V_{eg}^{(\text{poin})} \approx d_{m_\gamma, 1}^1(\theta_k) \langle 1M_g, 1m_\gamma | 00 \rangle - d_{m_\gamma, -1}^1(\theta_k) \langle 1M_g, 1m_\gamma | 00 \rangle, \quad (16)$$

where we have omitted other factors from the transition amplitude expression (9) as they do not influence the result. Consequently, two transitions are allowed: $M_g = -1 \rightarrow M_e = 0$ and $0 \rightarrow M_e = 0$. However, since the local polarization of the Poincaré beam is circular at the center, the amplitude for the $M_g = -1 \rightarrow M_e = 0$ transition is found to be stronger than that for $M_g = 0 \rightarrow M_e = 0$. As the system evolves to a steady state, the population in the $M_g = -1$ sublevel depletes significantly compared to the other ground-state sublevels ($M_g = 0, +1$). In contrast, the $M_g = +1$ sublevel remains uncoupled to the light field and thus retains the highest population. Ultimately, the atom interacts only weakly with the light field via the residual $M_g = 0 \rightarrow M_e = 0$ transition. This results in a relatively lower excited-state population at the center of the beam cross section compared to the upper part of the absorption profile, as shown in Fig. 3.

2. For $\theta_B = 90^\circ$

Let us now consider the case where the Poincaré beam propagates perpendicular to the quantization axis, i.e., $\theta_B = 90^\circ$. In this scenario, the external magnetic field (which defines the quantization axis) lies along the x axis. As discussed earlier, atoms located at $b = 200 \mu\text{m}$ and $\phi_b = 0^\circ$ experience a local elliptically polarized light field, as shown in Fig. 1(a). However, in this case, the major axis of the polarization ellipse is aligned with the quantization axis. Conversely, atoms located in the lower part of the beam, at $b = 200 \mu\text{m}$ and $\phi_b = 180^\circ$, encounter elliptically polarized light the minor axis of which is aligned with the quantization axis. The calculation of the local transition amplitude in this configuration reveals that the absolute value of $|V_{eg}^{(\text{poin})}|$ is asymmetric for both $\Delta M = 0$ and ± 1 transitions, as illustrated in Figs. 4(d) and 4(e). Specifically, the transition amplitude $|V_{eg}^{(\text{poin})}|$ for the $M_g = 0 \rightarrow M_e = 0$ transition attains its minimum in the upper part of the beam, particularly at $b = 200 \mu\text{m}$, $\phi_b = 0^\circ$. In contrast, the transition amplitude $|V_{eg}^{(\text{poin})}|$ for $M_g = \pm 1 \rightarrow M_e = 0$ reaches its minimum in the lower part of the beam, especially at $b = 200 \mu\text{m}$, $\phi_b = 180^\circ$. As a result, atoms located in both the upper and lower halves of the Poincaré beam cross section interact weakly with the light field. Notably, the transition amplitude $|V_{eg}^{(\text{poin})}|$ is relatively smaller for atoms at $b = 200 \mu\text{m}$, $\phi_b = 180^\circ$ compared to those at $b = 200 \mu\text{m}$, $\phi_b = 0^\circ$. In conclusion, atoms positioned in the lower part of the beam cross section absorb less light relative to those in the upper part. This leads to an asymmetric absorption profile featuring two dark lobes, as shown in Fig. 3.

In this scenario, with $\theta_B = 90^\circ$, atoms located at the beam center interact strongly with the incoming Poincaré beam—unlike the previous case with $\theta_B = 0^\circ$. Since $\mathbf{b} = 0$, the Bessel function in Eq. (9) simplifies to a delta function: $J_{m_\gamma - M}(0) = \delta_{m_\gamma - M, 0}$. However, the Wigner D function is no longer a delta function in this configuration, and therefore $\Delta M \neq M$ in the general expression for the transition amplitude (9). As

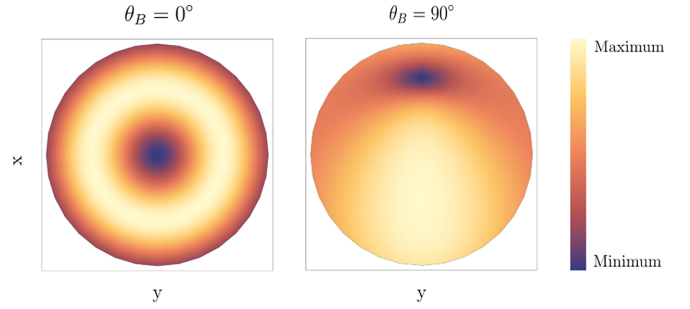


FIG. 5. The absorption profile of the Poincaré beam interacting with a polarized rubidium atomic target immersed in an external constant magnetic field of strength $B_{(\text{const})} = 1 \text{ G}$. Here, the absorption profile is shown for the Poincaré beam propagating at an angle of (left) $\theta_B = 0^\circ$ and (right) 90° with respect to the quantization axis. In these absorption profiles, the bright orange color corresponds to regions in the beam cross section where the Poincaré beam is absorbed maximally by the atoms, and the dark purple color represents regions with minimal absorption.

a result, transitions from all three magnetic sublevels of the ground state, $M_g = 0, \pm 1$, to the excited state $M_e = 0$ become allowed. Consequently, atoms at the center of the beam cross section experience strong coupling to the light field. This leads to a relatively higher population of the excited state at $\mathbf{b} = 0$, compared to other regions of the beam cross section.

B. Polarized atomic target

We now examine the interaction between the incoming Poincaré beam and an optically polarized ensemble of rubidium atoms subjected to a constant magnetic field of strength $B_{(\text{const})} = 1 \text{ G}$. As in the previous case, we consider the Poincaré beam to propagate either parallel ($\theta_B = 0^\circ$) or perpendicular ($\theta_B = 90^\circ$) to the quantization axis. When $\theta_B = 0^\circ$, the pump light field propagates perpendicular to the quantization axis, resulting in the polarization of the pump field being aligned parallel to the quantization axis. In this configuration, the pump selectively drives atomic transitions satisfying the angular momentum selection rule $\Delta M = 0$ [see Fig. 6(a)]. As a consequence, the atomic medium becomes optically polarized, with the ground-state sublevel $M_g = 0$ being depopulated and the atomic population redistributed equally between the $M_g = \pm 1$ sublevels.

1. For $\theta_B = 0^\circ$

In Fig. 5, we present the steady-state absorption profile for the case where the Poincaré beam propagates at an angle $\theta_B = 0^\circ$. Under this configuration, the absorption pattern exhibits a characteristic *donutlike* shape. Compared to the unpolarized case, two distinct features emerge: (a) The *axial symmetry* of the absorption profile is restored, and (b) the excited-state population at the center of the beam cross section ($\mathbf{b} = 0$) vanishes. In other words, atoms located at the beam center do not interact with the incoming Poincaré beam. To understand these features, we revisit the local transition amplitude $|V_{eg}^{(\text{poin})}|$. As discussed earlier, for $\theta_B = 0^\circ$, the transition amplitude between the magnetic sublevels $M_g = 0$ and $M_e = 0$ is not axially symmetric [see Fig. 4(b)].

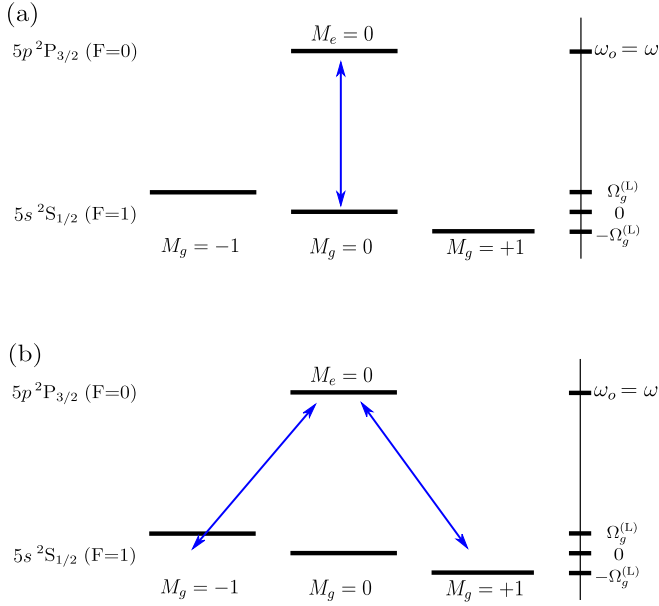


FIG. 6. The transition between $5s\ ^2S_{1/2}$ ($F = 1$) – $5p\ ^2P_{3/2}$ ($F = 0$) in ^{87}Rb driven by the pump linearly polarized plane wave for (a) $\theta_B = 0^\circ$ and (b) $\theta_B = 90^\circ$. The lower sublevels are split by the energy $\hbar\Omega_g^{(L)}$ as given by the Larmor frequency of the atom in the magnetic field.

However, since the atomic medium is polarized such that the population in the $M_g = 0$ ground-state sublevel is zero, transitions with $\Delta M = 0$ are effectively forbidden. Consequently, the Poincaré beam primarily induces transitions between the ground-state sublevels $M_g = \pm 1$ and the excited state $M_e = 0$ across the entire beam cross section. Moreover, the absolute value of the transition amplitude $|V_{eg}^{(\text{poin})}|$ for these $\Delta M = \pm 1$ transitions is axially symmetric, which explains the restored symmetry in the absorption profile. Therefore, the interaction of the Poincaré beam with a polarized atomic target at $\theta_B = 0^\circ$ results in an axially symmetric absorption profile.

Additionally, the absorption profile shows zero excited-state population for atoms located at the beam center, as shown in Fig. 5. From our earlier discussion, we know that at the beam center, atoms experience circularly polarized light, and the transition amplitude is nonzero as long as $m_\gamma = M = 0$ or ± 1 . That is, for $b = 0$, transitions can occur between the magnetic sublevels $M_g = -1, 0$ and $M_e = 0$. However, in the present case of a polarized atomic target, the Poincaré beam drives only the transition from $M_g = -1$ to $M_e = 0$. As the system reaches steady state, only the magnetic sublevel $M_g = +1$ remains populated, while the other two sublevels are depleted. Consequently, the excited-state population at the beam center remains zero.

Thus, the incoming Poincaré beam is absorbed by atoms across the beam profile except those located at $b = 0$, resulting in the observed donutlike absorption profile in Fig. 5.

2. For $\theta_B = 90^\circ$

Now, we consider the case where the Poincaré beam propagates at an angle of $\theta_B = 90^\circ$ with respect to the quantization axis. In this configuration, the pump—a linearly polarized

plane wave—propagates along the quantization axis, which lies in the transverse plane of the probe light field. Under these conditions, the pump light drives atomic transitions in rubidium that satisfy the selection rule $\Delta M = \pm 1$, as illustrated in Fig. 6(b). Consequently, the population in the ground sublevel $M_g = 0$ exceeds that of the other two sublevels. As a result, the incoming Poincaré beam predominantly drives transitions between the ground-state sublevel $M_g = 0$ and the excited state $M_e = 0$. The corresponding steady-state absorption profile of the Poincaré beam is shown in Fig. 5. In contrast to the previous case with $\theta_B = 0^\circ$, this absorption profile exhibits an asymmetrical pattern with respect to the y axis. This behavior can be understood by examining the local polarization and transition amplitudes. For instance, the excited-state population vanishes in the upper part of the absorption profile, particularly at $b = 200\ \mu\text{m}$ and $\phi_b = 0^\circ$. From our earlier discussion on the unpolarized atomic target, atoms located at these coordinates experience locally elliptically polarized light. Moreover, atoms in this region are eligible for transitions between all three magnetic sublevels in the ground state and the excited state. However, in the current polarized scenario, the Poincaré beam can only drive the transition between $M_g = 0$ and $M_e = 0$. From Fig. 4(d), we observe that this particular transition exhibits asymmetry across the beam cross section, with lower values of $|V_{eg}^{(\text{poin})}|$ in the upper half. Consequently, atoms in the upper part of the beam cross section interact weakly with the Poincaré beam, leading to an *axial asymmetry* in the absorption profile.

Unlike the case for $\theta_B = 0^\circ$, atoms at the center of the beam cross section interact strongly with the Poincaré beam when $\theta_B = 90^\circ$. As discussed earlier, at $b = 0$, atoms can undergo transitions from all three ground-state magnetic sublevels to the excited state. However, in the current scenario, the Poincaré beam drives only the $M_g = 0 \rightarrow M_e = 0$ transition. Furthermore, the pump field maintains a high population in the $M_g = 0$ sublevel. Therefore, as the system reaches steady state, the excited-state population attains a maximal and constant value at the beam center, indicating strong interaction of atoms at $b = 0$ with the Poincaré probe beam.

3. Dependence of absorption profile on the strength of the magnetic field

Next, we examine how the absorption profile of the Poincaré beam interacting with polarized rubidium atoms depends on the strength of the applied magnetic field $B_{(\text{const})}$. Figure 7 presents the absorption profiles alongside their corresponding polar plots, which exhibit a characteristic *applelike* pattern. This pattern represents the absorption intensity for atoms located at a fixed radial distance of $b = 200\ \mu\text{m}$, marked by a solid circle on the absorption profile. Our analysis focuses on the case $\theta_B = 90^\circ$, as this configuration produces an asymmetric absorption profile for the probe light.

The polar plots in Fig. 7 illustrate that, as the strength of the magnetic field increases, the applelike pattern expands in size while maintaining its characteristic asymmetry. This behavior suggests a promising method for quantitatively detecting the magnitude of the constant magnetic field by measuring the asymmetry in the absorption profile of the probe light after interaction with the polarized atomic ensemble.

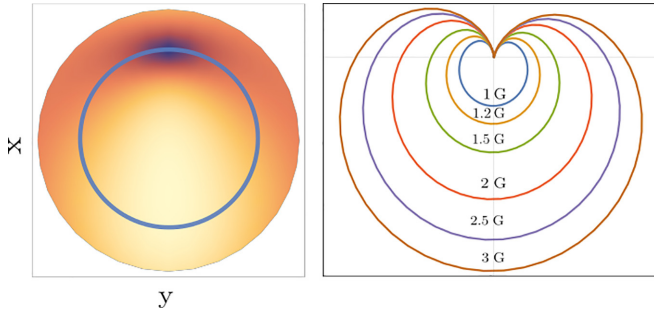


FIG. 7. Left: Absorption profile of the Poincaré beam propagating at $\theta_B = 90^\circ$ interacting with a polarized rubidium atomic target immersed in an external constant magnetic field of $B_{(\text{const})} = 1$ G. Right: Polar plots of the absorption intensity corresponding to the radius $b = 200 \mu\text{m}$ of the circle shown in the adjacent absorption profile. These polar plots are shown for increasing magnetic-field strengths.

It is well known that any periodic function can be expanded in a Fourier cosine series [31]. Following this analogy, we expand the density-matrix element ρ_{ee} , which depends on the azimuthal coordinate ϕ_b at a fixed radial distance $b = 200 \mu\text{m}$, in terms of cosine functions. This expansion allows us to quantify the asymmetry observed in the excited-state population across the beam cross section by defining an asymmetry parameter as

$$\mathcal{A}_{\text{abs}} = \frac{|a_1|}{a_0}, \quad (17)$$

where a_0 and a_1 are the zeroth and first Fourier coefficients, respectively. We consider only the first harmonic in the Fourier series because the absorption profile exhibits a pronounced dipolar asymmetry (see Fig. 5), and the first harmonic coefficient effectively captures this asymmetry. Figure 8 shows the variation of $\mathcal{A}_{\text{abs}}$ as a function of the magnetic-field strength B_{const} . The asymmetry parameter ranges between 0 and 1, with 0 indicating perfect symmetry and values closer to 1 indicating stronger asymmetry. For a weak magnetic field of 0.1 G, $\mathcal{A}_{\text{abs}} \approx 0.17$, indicating low asymmetry. As the field strength increases, $\mathcal{A}_{\text{abs}}$ rises, reaching a maximum of approximately 0.42 near 3.5 G. Beyond this point, the asymmetry parameter saturates, remaining nearly constant.

Physically, this behavior indicates that atoms in the lower half of the beam cross section interact more strongly with the Poincaré beam and absorb more light compared to those in the upper half. Moreover, the increasing asymmetry with magnetic-field strength (also visible in the polar plots of Fig. 7) suggests a direct method to infer the strength of the external magnetic field by analyzing the absorption profile asymmetry of the Poincaré beam interacting with the polarized atomic ensemble.

IV. SUMMARY AND OUTLOOK

In this work, we analyzed the interaction between Poincaré beams and an optically polarized atomic target in the

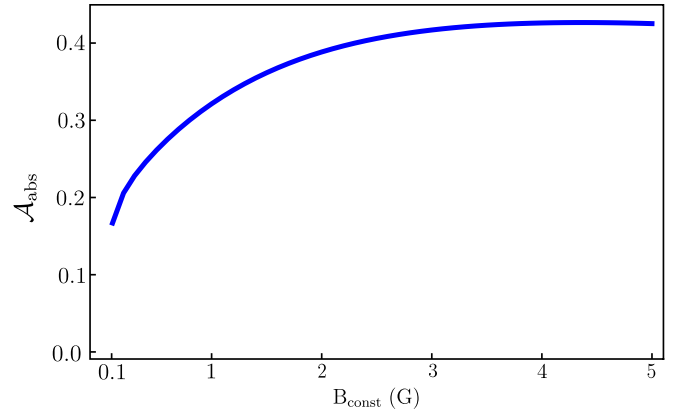


FIG. 8. Asymmetry index for the absorption profile of the Poincaré beam propagating at an angle of $\theta_B = 90^\circ$ and interacting with a polarized rubidium atomic ensemble. The asymmetry in the absorption of the light by the atoms is observed to increase with the strength of the applied constant magnetic field in the range of 0.1–5 G.

presence of constant magnetic field. To conduct this analysis, we used a linearly polarized plane wave as the pump light and Poincaré beam as the probe light. Particularly, we used (paraxial) Bessel modes to construct the vector potential and the corresponding transition amplitude of the Poincaré beam interacting with the target atoms. This interaction process was studied with the help of Liouville–von Neumann equations.

We paid special attention to the population of the excited atomic state, which was subsequently used to plot the absorption profile of the probe Poincaré beam. In particular, we examined this absorption profile both in the absence and presence of the pump light field for $\theta_B = 0^\circ$ and 90° , respectively. Our analysis shows that the axial asymmetry present in the polarization profile of the Poincaré beam influences the local transition amplitude, thereby affecting the absorption profiles. Furthermore, we explored the potential application of using the proposed scheme to detect magnetic-field strengths by analyzing the asymmetry in the absorption profile of the Poincaré beam.

To simplify our theoretical calculations, we made the following assumptions: (a) Thermal motion of atoms in the medium was neglected, (b) the pump and probe light fields were assumed to have same frequency, and (c) a one-dimensional constant magnetic field was considered. In the forthcoming publication, we aim to extend this analysis of detecting a constant magnetic field by incorporating the effects of atomic thermal motion and time-dependent magnetic fields.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [32], embargo periods may apply.

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