

Parity-dependent photon-subtracted two-mode squeezed vacuum states

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The removal of photons from certain quantum light sources produces so-called photon-subtracted states with enhanced mean photon numbers and intricate quantum correlations. Here, we propose an integrated photon-subtraction scheme that, contrary to previous approaches, is not heralded by photon correlation (coincidence) measurements. In this way, our technique exploits the “Welcher Weg” (“which way”) information problem, as it does not provide information about the modes from which the subtracted photons emanated. We show that this lack of information allows the generation of multiphoton states endowed with rich quantum correlations that strongly depend on the parity, evenness or oddness, of the number of subtracted photons.

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I. INTRODUCTION

Sculpting the quantum and statistical properties of light is an essential element for the development of quantum-optical information technology [1]. As a matter of fact, most of such technologies rely on state preparation that requires the engineering of quantum correlations between multiple photons distributed among different modes of the systems in question [2]. Admittedly, the generation of quantum correlated multiphoton wave packets can be achieved using linear optical schemes [3] in combination with nonlinear light sources [4]. In this context, the notion of photon subtraction has emerged as a pragmatic method for the generation of highly correlated multiphoton states [5–8]. Its essence consists in deliberately removing a number of photons from a quantum light source, and counterintuitively, this procedure can transform Gaussian states into non-Gaussian ones endowed with enhanced mean photon numbers and appealing nonclassical properties [9].

In this context, the nonclassicality of zero-photon, single-photon, and multiphoton subtracted states from single-mode radiation fields has been the focus of several investigations [9–11], while the correlation properties of photon-subtracted two-mode entangled states have also been investigated [12]. Concurrently, photon-subtraction techniques have been applied to multimode fields, e.g., multimode thermal states [13], two-mode squeezed thermal states [14], and two-mode squeezed vacuum states (TMSVSs) [15,16]. Interestingly, the application of photon subtraction to TMSVSs has recently yielded to the observation of highly correlated multiphoton states comprising up to ten photons [8]. This latter

achievement, combined with the fact that TMSVSs are the most accessible highly entangled multiphoton states [4], makes photon-subtracted TMSVSs extremely appealing for quantum-enhanced applications, e.g., quantum interferometry [6], teleportation protocols [17], quantum illumination [7], and enhancement of quantum entanglement [18,19].

In practice, the photon-subtraction scheme applied to TMSVSs, $|r\rangle = \sqrt{1 - |r|^2} \sum_{n=0}^{\infty} r^n |n\rangle_a |n\rangle_b$, r being the squeeze parameter, is implemented using two beamsplitters placed in the paths traced out by the modes. Further, the state characterization is done using four photon-number-resolving (PNR) detectors, two for counting the reflected photons, that is, the subtracted photons, and two to register the transmitted photons, i.e., the photons left in the initial beams [8]. In this setting there are two possible photon-subtraction scenarios: (i) the symmetric case when the same number of photons is subtracted from each beam, and (ii) the asymmetric one when a different number of photons is subtracted. Mathematically, both subtraction processes are described by the transformation $\hat{a}^l \hat{b}^m |r\rangle = \mathcal{N}^{-1} \sqrt{1 - |r|^2} \sum_n r^n n! (n - l)! (n - m)!^{-1/2} |n - l\rangle_a |n - m\rangle_b$, where the sum runs from $n = l(n = m)$ if $l > m(m < l)$. Here, $\mathcal{N}$ is the normalizing constant, and $\hat{a}$ and $\hat{b}$ are the bosonic annihilation operators for modes a and b , respectively. However, since the use of two PNR detectors to herald photon-subtracted states provides, in principle, precise information about the number of photons subtracted from each beam, this subtraction scheme yields infinite superpositions of photon-number states. These are of the form $\alpha |m\rangle_a |m\rangle_b$ in the symmetric case and $\beta |m\rangle_a |n\rangle_b$ in the asymmetric case, but not superpositions that combine both types.

In this work we investigate an alternative photon-subtraction scheme that arises when both modes comprised in a TMSVS source become equally coupled to a common channel in such a way that when measuring the number of photons hopping into this channel, we will know the number

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of photons being extracted from the modes, but we will not be able to distinguish from which mode the photons arriving at the detector emanated. This lack of information allows the modes comprised in the photon-subtracted states to be inhabited by infinite superpositions of joint photon-number states of the type $\alpha_1|m\rangle_a|n\rangle_b + \alpha_2|n\rangle_a|m\rangle_b + \alpha_3|m\rangle_a|m\rangle_b$ or $\beta_1|m\rangle_a|n\rangle_b + \beta_2|n\rangle_a|m\rangle_b$, depending on the parity, evenness or oddness, of the number of extracted photons. That is, our scheme will allow the coexistence of superpositions of symmetric and asymmetric multiphoton two-mode states with different degrees of correlation.

To place our findings on solid ground, we propose an experimental feasible scheme based on an integrated waveguide trimer, which can be implemented using current technology, e.g., silicon nanowires [20], or femtosecond-laser-writing techniques [21]. In this latter technique the required couplings can be accomplished by judiciously varying the waveguide separation distance, where, in the weak-coupling regime, the coupling coefficients vary exponentially with the separation distance, while the propagation constants can be tuned by varying the waveguide writing velocity; see Ref. [21] for details. The waveguide trimer configuration has previously been studied as a versatile platform for the generation and manipulation of multipartite quantum states, including multimode W states [22,23] and three-mode squeezed states [24].

Having the waveguide trimer configuration in mind, in what follows we explore the multiphoton quantum correlations exhibited by the emerging photon-subtracted states, and we analyze how the parity of the subtracted photons influences such correlations.

II. QUANTUM INTERFERENCE IN A WAVEGUIDE TRIMER

Consider a trimer formed by three coupled single-mode waveguides with identical propagation constant β . The dynamics of the quantized electromagnetic field in such a system is governed by the Heisenberg equations of motion,

$$i \frac{d}{dz} \begin{pmatrix} \hat{a}^\dagger(z) \\ \hat{b}^\dagger(z) \\ \hat{c}^\dagger(z) \end{pmatrix} = - \begin{pmatrix} \beta & \kappa & 0 \\ \kappa & \beta & \kappa \\ 0 & \kappa & \beta \end{pmatrix} \begin{pmatrix} \hat{a}^\dagger(z) \\ \hat{b}^\dagger(z) \\ \hat{c}^\dagger(z) \end{pmatrix}, \quad (1)$$

where κ is the coupling constant between adjacent waveguides, z is the propagation coordinate, and $\hat{a}^\dagger$, $\hat{b}^\dagger$, and $\hat{c}^\dagger$ are the bosonic creation operators for waveguides a , b , and c , respectively. Integration of Eq. (1) yields the input-output relation $|\Psi(z)\rangle = U(z)|\Psi(0)\rangle$, with $|\Psi(z)\rangle = [\hat{a}^\dagger(z), \hat{b}^\dagger(z), \hat{c}^\dagger(z)]^T$, and the evolution matrix

$$U(z) = \begin{pmatrix} \frac{1}{2} + \frac{1}{2} \cos \Theta & -\frac{i \sin \Theta}{\sqrt{2}} & -\frac{1}{2} + \frac{1}{2} \cos \Theta \\ -\frac{i \sin \Theta}{\sqrt{2}} & \cos \Theta & -\frac{i \sin \Theta}{\sqrt{2}} \\ -\frac{1}{2} + \frac{1}{2} \cos \Theta & -\frac{i \sin \Theta}{\sqrt{2}} & \frac{1}{2} + \frac{1}{2} \cos \Theta \end{pmatrix}, \quad (2)$$

where $\Theta = \sqrt{2}\kappa z$. We now assume that the modes from a TMSVS source are launched into the two outermost waveguides, while the central waveguide remains in the vacuum state as illustrated in Fig. 1. That is, the initial state is $|\psi_{\text{in}}\rangle = (1 - |r|^2)^{\frac{1}{2}} \sum_{l=0}^{\infty} r^l |l\rangle_a |0\rangle_b |l\rangle_c$, and the photon subtraction is

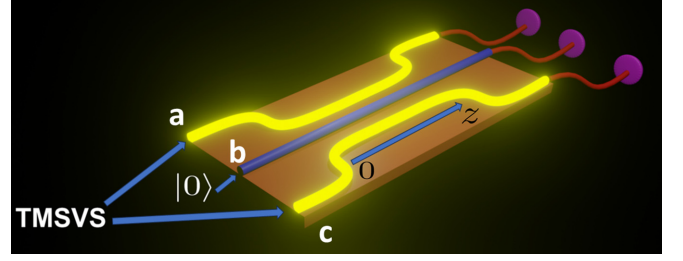


FIG. 1. The schematic setup of an integrated waveguide trimer proposed for generating photon-subtracted TMSVS. The modes comprised in the TMSVS source are coupled into the outermost waveguides, while photons are subtracted from the central port b using a PNR detector.

achieved by monitoring the number of photons hopping into the central waveguide. Further, to maximize light transmission at the output of the outermost waveguides while maintaining a feasible probability of photon hopping to the central waveguide, we choose the optimized length of the coupling section such that the ratio between the intensity in the central waveguide (I_{cen}) and the intensity in waveguides a and c (I_{out}) is $I_{\text{cen}}/I_{\text{out}} = 10/90$, that is, $z = z_f \approx 0.23/\kappa$. For the details of the calculations, we refer to Appendix A.

Using the input-output relation, we obtain the output state $|\psi_{\text{out}}\rangle = U(z_f)|\psi_{\text{in}}\rangle$, given by

$$|\psi_{\text{out}}\rangle = (1 - |r|^2)^{\frac{1}{2}} \sum_{l=0}^{\infty} \sum_{p_1, p_2, p_3=0} \sum_{v_1, v_2, v_3=0} K |v_1\rangle_a |v_2\rangle_b |v_3\rangle_c, \quad (3)$$

where

$$K = \frac{r^l}{p_1! p_2! p_3! (v_1 - p_1)! (v_2 - p_2)! (v_3 - p_3)!} \times U_{11}^{p_1+v_3-p_3} U_{12}^{v_2} U_{13}^{p_3+v_1-p_1} \sqrt{v_1! v_2! v_3!} \delta_{p_1+p_2+p_3, l} \times \delta_{v_1+v_2+v_3, 2l}, \quad (4)$$

which in turn yields the photon-subtracted state $\hat{\rho}_{\text{sub}} = \text{Tr}_b(\hat{P}_b |\psi_{\text{out}}\rangle \langle \psi_{\text{out}}|)$, where $\text{Tr}_b(\cdot)$ is the partial trace with respect to mode b , and $\hat{P}_b = \frac{\eta \hat{n}_b}{N!} e^{-\eta \hat{n}_b}$ is the positive operator-valued measure (POVM) operator representing a PNR detector with a quantum efficiency η . Notice that $\hat{P}_b$ represents the detection process of N photons in waveguide b , the symbol $::$ implies the normal ordering prescription [25], and $\hat{n}_b$ is the photon-number operator acting on mode b . In what follows we assume perfect PNR detectors ($\eta = 1$), and at the end we lift this restriction to explore the influence of minor imperfections in the detection process ($\eta < 1$). For the details of the derivation of the analytical expression for $\hat{\rho}_{\text{sub}}$, we refer to Appendix B.

To analyze the quantum correlation of the photon-subtracted states, we compute the joint photon-number probability distribution

$$\mathcal{P}_{m,n} = \left\langle : \frac{(\eta \hat{n}_a)^m}{m!} e^{-\eta \hat{n}_a} \otimes \frac{(\eta \hat{n}_c)^n}{n!} e^{-\eta \hat{n}_c} : \right\rangle, \quad (5)$$

which characterizes the probability of detecting m photons in waveguide a and n photons in waveguide c , when N photons

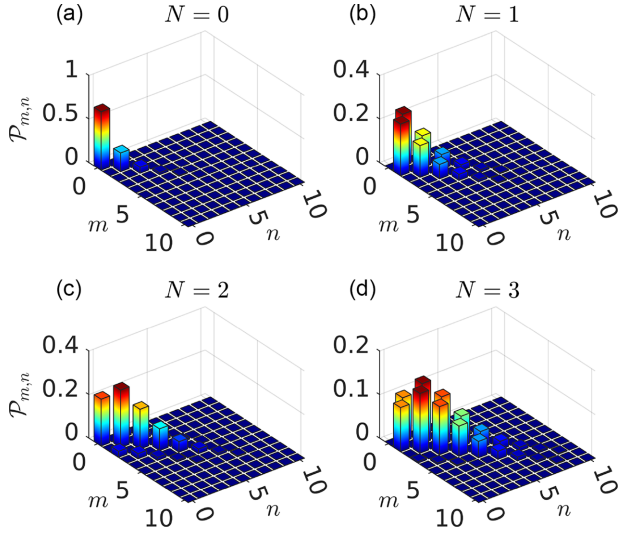


FIG. 2. Joint photon-number probability distributions for photon-subtracted TMSVS with $|r| = 0.6$, $z = z_f$, and detector efficiency $\eta = 1$. Figures (a), (b), (c), and (d) illustrate the distributions for subtracting zero ($N = 0$), one ($N = 1$), two ($N = 2$), and three ($N = 3$) photons from the central port, respectively.

are detected (subtracted) at waveguide *b*. Figure 2 depicts the computed joint photon-number distribution, wherein zero to three photons have been detected in the central waveguide. For the trivial case of zero photon subtraction ($N = 0$), the photon-number distribution contains only diagonal elements, indicating that most of the time no photons will emerge from the outermost waveguides, Fig. 2(a). More interesting scenarios occur when even and odd numbers of photons are subtracted.

For instances of even photon subtraction (N being even), the resultant state is a superposition of multiphoton states of the form $\alpha_1(|m\rangle_a|n\rangle_b + |n\rangle_a|m\rangle_b)$, and $\alpha_2|m\rangle_a|m\rangle_b$. The former states result from asymmetric photon subtraction, where a different number of photons is subtracted from the outermost waveguides, while the latter states are due to symmetric photon subtraction in which the same number of photons is being subtracted. In other words, detecting an even number of photons in the central waveguide heralds a scenario in which photons are simultaneously subtracted symmetrically and asymmetrically from the outermost modes. However, a close inspection of the joint photon-number distribution reveals that even photon-subtracted states exhibit strong (weak) photon (anti-)correlations, as witnessed by the prominent diagonal (small off-diagonal) matrix elements, e.g., Fig. 2(c).

Conversely, for odd photon subtraction (N being odd), the resulting states comprise a superposition of states of the form $\alpha_1(|m\rangle_a|n\rangle_b + |n\rangle_a|m\rangle_b)$, rendering a photon-number distribution with only off-diagonal elements. This occurs solely due to the asymmetric photon subtraction, again without knowledge about which waveguide the photons are taken from. The absence of diagonal elements, or equivalently, the lack of states $|m\rangle_a|m\rangle_b$, gives rise to a central nodal line in the distribution with two peaks located on either side of this line, see Figs. 2(b) and 2(d). Significantly, the peaks of the distributions for even

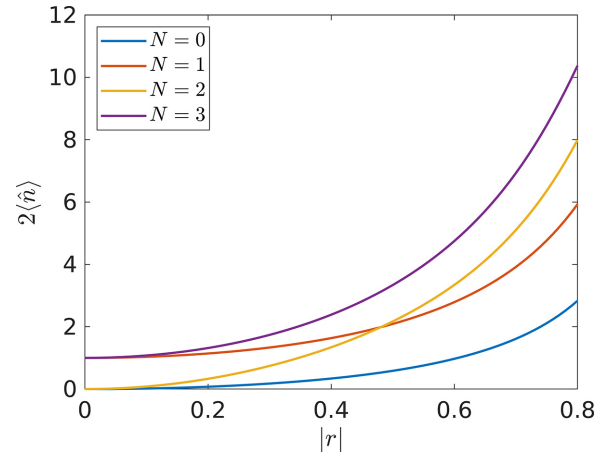


FIG. 3. The total mean photon number of a photon-subtracted TMSVS, $2\langle\hat{n}\rangle$, as a function of $|r|$ for $N = 0, 1, 2$, and 3 at a propagation distance of $z = z_f$, assuming a quantum efficiency of $\eta = 1$.

and odd photon subtraction show a noticeable shift toward higher photon numbers as the number of subtracted photons N increases; this indicates an overall increase in the average photon number. This increment is qualitatively verified by computing the average photon number in one of the modes, $\langle\hat{n}\rangle = \sum_{m,n=0}^{\infty} n\mathcal{P}_{m,n}$, which, due to the symmetric nature of the joint photon-number distribution gives a total mean photon number $2\langle\hat{n}\rangle$. In Fig. 3 we present the total average photon number $2\langle\hat{n}\rangle$ versus the squeeze parameter $|r|$ for different subtracted numbers of photons N . Interestingly, for squeezing values $|r| < 0.5$, the average photon number corresponding to odd photon-subtracted states exceeds the one obtained from even photon-subtracted states. Beyond $|r| = 0.5$, the mean photon number for $N = 3$ yields a higher mean photon number than all the cases presented here. We would like to point out that this behavior is reminiscent to the one observed for Schrödinger-cat states as discussed in [26]. Importantly, both symmetric and asymmetric photon subtraction can be advantageous, depending on the target quantum protocol. For instance, Ref. [7] shows that asymmetric photon subtraction from a TMSVS enhances quantum illumination more effectively than the symmetric approach. Likewise, Ref. [19] reports that asymmetric subtraction results in a greater entanglement gain compared to symmetric subtraction.

As demonstrated in previous investigations, photon-subtracted TMSVSs feature stronger quantum correlations than their TMSVS “parents” [8]. A natural approach to explore such correlations is through the analysis of the matrix of moments [27]. Indeed, theoretical discussions [28] and experimental demonstrations utilizing PNR detectors [8] and click detectors [29] have highlighted nonclassical correlations using the framework of the matrix of moments. To perform such analysis in the present context, we define the operator $\hat{m}_{a(c)} = \eta\hat{n}_{a(c)}$, such that the joint moments are given by $\langle:\hat{m}_a^u \otimes \hat{m}_c^v:\rangle = \sum_{m,n=0}^{\infty} \mathcal{P}_{m,n}m(m-1)\dots(m-u+1)n(n-1)\dots(n-v+1)$, where u and v are positive integers. Importantly, this latter expression allows us to construct a matrix of moments of any order using the joint photon-number distribution $\mathcal{P}_{m,n}$. However, as previously shown [8], the second-order matrix of moments suffices to shed light onto

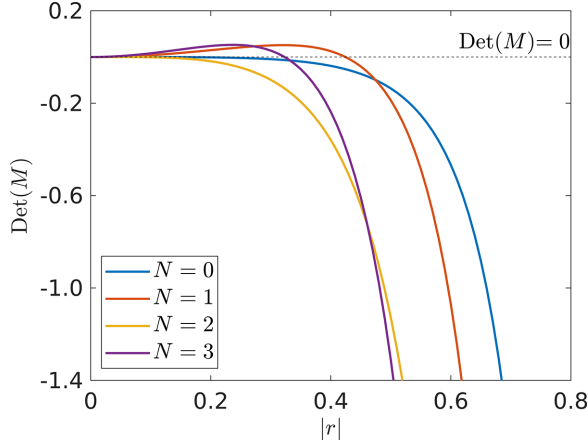


FIG. 4. The determinant of the second-order matrix of moments M is examined as a function of $|r|$ to quantify the correlations between the two output modes of a photon-subtracted TMSVS. This analysis is conducted for values of $N = 0, 1, 2$, and 3 at a propagation distance of $z = z_f$, with the assumption of a quantum efficiency of $\eta = 1$.

the quantum correlations of the generated states:

$$M = \begin{pmatrix} \langle \hat{m}_a^0 \hat{m}_c^0 \rangle & \langle \hat{m}_a^1 \hat{m}_c^0 \rangle & \langle \hat{m}_a^0 \hat{m}_c^1 \rangle \\ \langle \hat{m}_a^1 \hat{m}_c^0 \rangle & \langle \hat{m}_a^2 \hat{m}_c^0 \rangle & \langle \hat{m}_a^1 \hat{m}_c^1 \rangle \\ \langle \hat{m}_a^0 \hat{m}_c^1 \rangle & \langle \hat{m}_a^1 \hat{m}_c^1 \rangle & \langle \hat{m}_a^0 \hat{m}_c^2 \rangle \end{pmatrix}. \quad (6)$$

Notice the presence of nonclassical correlations is witnessed whenever $\text{Det}(M) < 0$. Conversely, $\text{Det}(M) > 0$ implies classical correlations. We provide details on computing the second-order moment matrix in Appendix D. In Fig. 4, $\text{Det}(M)$ is plotted against the squeeze parameter $|r|$ for different values of N . Notably, the uncertainty regarding which beam the photons are extracted from leads to intriguing observations about the nature of the correlations. For $N = 1$ our results indicate the existence of classical correlations in the interval $|r| \leq 0.5$. Interestingly, this classical interval is reduced as a larger odd number of photons is subtracted, as clearly seen for the case $N = 3$, which yields a shorter classical interval $|r| \leq 0.4$. On the other hand, for N being even, nonclassical correlations emerge in a larger interval $|r| \in [0.2, 1]$. These examples suggest that for small squeezing, odd photon subtraction destroys the quantum correlations of the initial TMSVS [30]. In both scenarios, however, the combination of a TMSVS source with large $|r|$ and high photon subtraction facilitates the production of multiphoton states with enhanced nonclassical correlations. The behavior observed for photon-subtracted TMSVSs with a small squeezing parameter is reminiscent to the one reported for two-mode Schrödinger cat states [31,32], in which considering even and odd two-mode coherent states with low coherent amplitudes exhibit distinct nonclassical characteristics [32]. To fully characterize the system, we analyze the entanglement of the photon-subtracted TMSVS. Photon subtraction is well known to enhance the entanglement of the parent TMSVSs [7,19]. To quantify this enhancement, we compute the participation ratio $\xi = 1 / \sum_i \lambda_i^2$ [33], where λ_i are the Schmidt coefficients of the bipartite photon-subtracted state. These coefficients

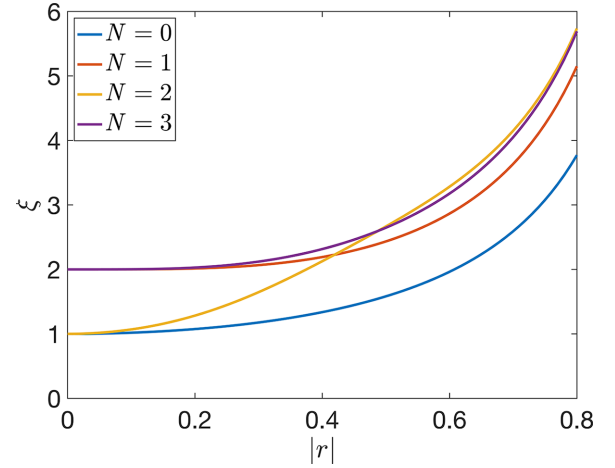


FIG. 5. The participation ratio ξ is examined as a function of $|r|$ to quantify the degree of entanglement of the photon-subtracted TMSVS. This analysis is performed for photon-subtraction numbers $N = 0, 1, 2$, and 3 , at a fixed propagation distance $z = z_f$, assuming ideal detection with quantum efficiency $\eta = 1$.

are derived from the reduced density matrix of one subsystem [subsystem (a)], obtained by partially tracing over the other [subsystem (c)] $\rho_a = \text{Tr}_c[|\psi_{\text{out}}\rangle\langle\psi_{\text{out}}|] = \sum_i \lambda_i^2 |u_i\rangle\langle u_i|$. A separable state yields a minimal participation ratio of ($\xi = 1$), while ($\xi > 1$) directly indicates the presence of entanglement [33].

Figure 5 clearly shows how the participation ratio ξ (our entanglement quantifier) depends on the squeezing parameter $|r|$ for different photon-subtraction numbers ($N = 0, 1, 2, 3$). A key finding is the significant influence of the parity of N . Notably, within the low squeezing regime ($|r| \lesssim 0.4$), odd photon subtraction (specifically, $N = 1$ and $N = 3$) consistently leads to higher ξ values compared to even cases ($N = 0$ and $N = 2$). This suggests that subtracting an odd number of photons proves more efficient for enhancing entanglement when the initial state shows weak squeezing.

At this point it is worth emphasizing that our theoretical model hinges on the assumption of ideal waveguide propagation, where propagation constants are equal and coupling coefficients are uniform along the propagation coordinate, as described by Eq. (1). This is a reasonable premise given the sophistication of current fabrication technologies, such as femtosecond-laser written waveguide platforms, which exhibit low propagation losses of the order < 0.3 dB/cm [34,35] and allow for precise control over coupling coefficients and propagation constants. These high-quality characteristics lead us to conclude that any imperfections introduced during the state generation process are negligible for our analysis, and any imperfection in the detection scheme can be modeled by assuming detectors with a quantum efficiency parameter $\eta < 1$ [8]. Hence, before closing we comment on the effects of using imperfect PNR detectors to herald photon-subtracted states.

To evaluate the effect of nonideal detection, we analyze the determinant of the second-order matrix of moments for $N = 0, 1, 2$, and 3 , considering various values of the parameter $|r|$, and we assume a detector efficiency $\eta = 0.8$. From

TABLE I. The determinant of the second-order matrix of moments M is calculated for various values of $|r|$ to examine the impact of imperfections stemming from the PNR detection process. These calculations are conducted for $z = z_f$ and $\eta = 0.8$.

$ r $	$N = 0$	$N = 1$	$N = 2$	$N = 3$
0.2	-0.62	1.71	-7.49	8.48
0.3	-3.79	9.35	-40.67	8.18

the results presented in Table I, we note that the distinctive correlation patterns between even and odd photon-subtracted states persist despite imperfections in the detection process.

III. CONCLUSION

To conclude, we have proposed and analyzed a waveguide trimer to engineer photon-subtracted TMSVSs, effectively reducing the number of required PNR detectors to three, compared to the four PNR detectors typically needed in the conventional photon-subtraction process from twin beams [8,15,16]. Moreover, our scheme requires monitoring only a single output port for photon subtraction, eliminating the need for coincidence detection typically employed in standard twin-beam configurations. Notably, we observe distinct characteristics in the photon-subtracted state when retrieving an odd or even number of photons from the central waveguide. Specifically, we find that the nature of correlations exhibits a clear dependence on the parity of the extracted photons, particularly at low values of the squeeze parameter, resembling the behavior of two-mode cat states. We have considered the effect of small imperfections in PNR detection on the correlation properties of the photon-subtracted TMSVSs. However, we refrain from a detailed analysis of other experimental imperfections, as such effects are highly dependent on the specific implementation [8,29], waveguide parameters [21,36], and PNR detector characteristics [37].

Looking forward, the proposed photon-subtraction scheme can be applied to other interesting two-mode quantum states, e.g., NOON states. Our protocol could potentially open up new avenues for investigating multiphoton processes in integrated photonics platforms [38]. Introducing detuning might also provide a valuable way to study more nuanced quantum correlations in such systems [39]. In a broader context, this work represents a step toward scalable and stable quantum-optical implementations within integrated photonics, where such approaches may support the development of future quantum technologies [40,41]. The feasibility of our approach is further reinforced by recent technological advances in integrated quantum photonics, including quantum-state purification protocols [42] and simplified continuous-variable quantum key distribution implementations [43]. In this context, our work holds practical significance in various correlation-based schemes. For instance, the creation of multiphoton states with controlled correlations through photon subtraction could notably enhance applications in quantum illumination [7] and improve robustness of boson sampling in integrated photonics circuits [44]. Finally, as the unveiled quantum correlations exhibited by the present multiphoton

photon-subtracted states has not been discussed in previous investigations [6–8,15,17,18], our results may open the door to new explorations on the generation of more complex multiphoton light states.

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The authors declare no conflicts of interest.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: CALCULATING THE PROPAGATION DISTANCE z_f

In this Appendix we determine the propagation distance $z = z_f$ required to establish a specific ratio of light intensity between the output ports of the two outermost waveguides, denoted by a and c , compared to that of the central waveguide b (see Fig. 1). To begin, we envision a scenario where both input ports a and c of the waveguide trimer, as depicted in Fig. 1, receive simultaneous excitation by photons. In this context the input state can be represented as

$$|\phi(0)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}. \quad (\text{A1})$$

To trace the evolution of the system, we find the output state at a propagation distance z within the trimer, which is computed using the evolution operator in Eq. (2):

$$|\phi(z)\rangle = \hat{U}(z) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{\cos(\sqrt{2}z\kappa)}{\sqrt{2}} \\ -i \sin \sqrt{2}z\kappa \\ \frac{\cos(\sqrt{2}z\kappa)}{\sqrt{2}} \end{pmatrix}. \quad (\text{A2})$$

This leads us to derive the output intensity in the two outermost waveguides I_{out} and the central waveguide I_{cen} , expressed as

$$I_{\text{out}} = \cos^2 \sqrt{2}z\kappa, \quad (\text{A3})$$

$$I_{\text{cen}} = \sin^2 \sqrt{2}z\kappa. \quad (\text{A4})$$

With this in hand, we can deduce the ratio of intensity at the output ports of the outermost waveguides to that of the central

waveguide, which is given by

$$\frac{I_{\text{cen}}}{I_{\text{out}}} = \tan^2(\sqrt{2}z\kappa). \quad (\text{A5})$$

By establishing the target ratio $I_{\text{cen}}/I_{\text{out}}$ as 10/90, we can determine the associated propagation distance $z = z_f$ through Eq. (A5), yielding an approximate value of $z_f \approx \frac{0.23}{\kappa}$.

APPENDIX B: ANALYTICAL EXPRESSION OF PHOTON-SUBTRACTED TMSVS

In this Appendix we provide a derivation of the analytical expression for the photon-subtracted TMSVS. We initiate our analysis via the input state

$$\begin{aligned} |\psi_{\text{in}}\rangle &= (1 - |r|^2)^{\frac{1}{2}} \sum_{n=0}^{\infty} r^n |n\rangle_a |0\rangle_b |n\rangle_c \\ &= (1 - |r|^2)^{\frac{1}{2}} \sum_{n=0}^{\infty} r^n \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}} \frac{(\hat{c}^\dagger)^n}{\sqrt{n!}} |0\rangle_a |0\rangle_b |0\rangle_c, \end{aligned} \quad (\text{B1})$$

where $\hat{a}^\dagger$ and $\hat{c}^\dagger$ are creation operators acting on mode a and c , respectively. Next, we proceed by employing the bosonic operator transformation to compute the output state $|\psi_{\text{out}}\rangle$:

$$\begin{aligned} |\psi_{\text{out}}\rangle &= (1 - |r|^2)^{\frac{1}{2}} \sum_{l=0}^{\infty} \frac{r^l}{l!} (U_{11}\hat{a}^\dagger + U_{12}\hat{b}^\dagger + U_{13}\hat{c}^\dagger)^l \\ &\quad \times (U_{31}\hat{a}^\dagger + U_{32}\hat{b}^\dagger + U_{33}\hat{c}^\dagger)^l |0\rangle_a |0\rangle_b |0\rangle_c. \end{aligned} \quad (\text{B2})$$

Utilizing the multinomial theorem, we obtain

$$|\psi_{\text{out}}\rangle = (1 - |r|^2)^{\frac{1}{2}} \sum_{l=0}^{\infty} \sum_{p_1, p_2, p_3=0} \sum_{v_1, v_2, v_3=0} K |v_1\rangle_a |v_2\rangle_b |v_3\rangle_c, \quad (\text{B3})$$

where

$$\begin{aligned} K &= \frac{r^l}{p_1! p_2! p_3!} \frac{l!}{(v_1 - p_1)!(v_2 - p_2)!(v_3 - p_3)!} U_{11}^{p_1+v_3-p_3} \\ &\quad \times U_{12}^{v_2} U_{13}^{p_3+v_1-p_1} \sqrt{v_1! v_2! v_3!} \delta_{p_1+p_2+p_3, l} \delta_{v_1+v_2+v_3, 2l}. \end{aligned} \quad (\text{B4})$$

It is important to note that we utilize the Hermitian nature of the evolution matrix in Eq. (2), where $U_{11} = U_{33}$ and $U_{12} = U_{32}$. Moving forward, the photon-subtracted state for detecting N photons in the central waveguide by a photon-number resolving (PNR) detector with efficiency η is computed as

$$\hat{\rho}_{\text{sub}} = \text{Tr}_b(\hat{P}_b |\psi_{\text{out}}\rangle \langle \psi_{\text{out}}|), \quad (\text{B5})$$

where $\hat{P}_b$ is known as the POVM operator for a PNR detector, given by

$$\begin{aligned} \hat{P}_b &= : \frac{(\eta \hat{n}_b)^N}{N!} e^{-\eta \hat{n}_b} : \\ &= \sum_{j=N}^{\infty} \binom{j}{N} \eta^N (1 - \eta)^{j-N} |j\rangle_b \langle j|_b. \end{aligned} \quad (\text{B6})$$

Using Eqs. (B3), (B5), and (B6), we derive the analytical expression for the photon-subtracted TMSVS:

$$\begin{aligned} \hat{\rho}_{\text{sub}} &= (1 - |r|^2) \sum_{l=0}^{\infty} \sum_{l'=0}^{\infty} \sum_{p_1, p_2, p_3=0} \sum_{p'_1, p'_2, p'_3=0} \sum_{v_1, v_3=0} \sum_{v'_1, v'_3=0} \sum_{j=N} \binom{j}{N} \eta^N (1 - \eta)^{j-N} S |v_1\rangle_a |v_3\rangle_c \\ &\quad \times {}_c \langle v'_3| {}_a \langle v'_1| \delta_{p_1+p_2+p_3, l} \delta_{p'_1+p'_2+p'_3, l} \delta_{v_1+j+v_3, 2l} \delta_{v'_1+j+v'_3, 2l'}, \end{aligned} \quad (\text{B7})$$

where

$$\begin{aligned} S &= \frac{r^l}{p_1! p_2! p_3!} \frac{l!}{(v_1 - p_1)!(j - p_2)!(v_3 - p_3)!} \frac{r^{l'}}{p'_1! p'_2! p'_3!} \frac{l'!}{(v'_1 - p'_1)!(j - p'_2)!(v'_3 - p'_3)!} \\ &\quad \times U_{11}^{p_1+v_3-p_3} (U_{11}^*)^{p'_1+v'_3-p'_3} |U_{12}|^{2j} U_{13}^{p_3+v_1-p_1} (U_{13}^*)^{p'_3+v'_1-p'_1} j! \sqrt{v_1! v_3! v'_1! v'_3!}. \end{aligned} \quad (\text{B8})$$

APPENDIX C: PHOTON-NUMBER DISTRIBUTION

Here, we derive the photon-number distribution of the photon-subtracted TMSVS measured in ports a and c . The probability to measure m and n photons in ports a and c , respectively, using PNR detectors of quantum efficiency η is given by

$$\begin{aligned} \mathcal{P}_{m,n} &= \left\langle : \frac{(\eta \hat{n}_a)^m}{m!} e^{-\eta \hat{n}_a} \otimes \frac{(\eta \hat{n}_c)^n}{n!} e^{-\eta \hat{n}_c} : \right\rangle \\ &= \frac{P_N(m, n)}{\sum_{m,n=0} P_N(m, n)}, \end{aligned} \quad (\text{C1})$$

where

$$\begin{aligned} P_N(m, n) &= (1 - |r|^2) \sum_{l=0}^{\infty} \sum_{p_1, p_2, p_3=0} \sum_{p'_1, p'_2, p'_3=0} \sum_{j=N} \binom{j}{N} \eta^N (1 - \eta)^{j-N} \sum_{i=m} \binom{i}{m} \eta^m (1 - \eta)^{i-m} \sum_{k=n} \binom{k}{n} \\ &\quad \times \eta^n (1 - \eta)^{k-n} Q \delta_{p_1+p_2+p_3, l} \delta_{p'_1+p'_2+p'_3, l} \delta_{v_1+v_2+v_3, 2l}, \end{aligned} \quad (\text{C2})$$

and

$$Q = \frac{r^{2l}}{p_1!p_2!p_3!(v_1-p_1)!(j-p_2)!(v_3-p_3)!} \frac{(l!)^2}{p'_1!p'_2!p'_3!(v_1-p'_1)!(j-p'_2)!(v_3-p'_3)!} \frac{1}{U_{11}^{p_1+v_3-p_3}(U_{11}^*)^{p'_1+v_3-p'_3}} \frac{1}{|U_{12}|^{2j}U_{13}^{p_3+v_1-p_1}(U_{13}^*)^{p'_3+v_1-p'_1}} j!v_1!v_3. \quad (C3)$$

APPENDIX D: SECOND-ORDER MATRIX OF MOMENTS

In this Appendix, we calculate the second-order matrix of moments to measure the correlation. Each element in the matrix in Eq. (6) can be expressed in terms of joint photon-number distribution $\mathcal{P}_{m,n}$ as follows:

$$\langle \hat{m}_a^1 \hat{m}_c^0 \rangle = \langle \hat{m}_a^1 \rangle = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} m \mathcal{P}_{m,n}, \quad (D1)$$

$$\langle \hat{m}_a^0 \hat{m}_c^1 \rangle = \langle \hat{m}_c^1 \rangle = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} n \mathcal{P}_{m,n}, \quad (D2)$$

$$\langle \hat{m}_a^1 \hat{m}_a^1 \rangle = \langle \hat{m}_a^2 \rangle = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} m(m-1) \mathcal{P}_{m,n}, \quad (D3)$$

$$\langle \hat{m}_c^1 \hat{m}_c^1 \rangle = \langle \hat{m}_c^2 \rangle = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} n(n-1) \mathcal{P}_{m,n}, \quad (D4)$$

$$\langle \hat{m}_a^1 \hat{m}_c^1 \rangle = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} mn \mathcal{P}_{m,n}. \quad (D5)$$

To obtain the matrix of moments expressed in relation to the joint photon-number distribution, we direct readers to the supplementary section of Ref. [8].

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