

Theoretical analysis of the attoclock offset angle in the hydrogen atom

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We theoretically investigate the physical principles underlying the attoclock method in strong-field ionization, with particular emphasis on the interpretation of the offset angle and the role of the Coulomb potential. Our analysis is based on the numerical solution of the time-dependent Schrödinger equation (TDSE) and several variants of the strong-field approximation (SFA) combined with quantum-orbit theory. Among the investigated models, the best agreement with the TDSE is obtained when the ionization probability and the initial conditions for the electron trajectory are determined using the SFA together with the saddle-point method. The corresponding final electron momentum is then calculated by classical propagation in the combined laser and Coulomb fields.

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I. INTRODUCTION

The attosecond angular streaking or attoclock is an experimental method primarily used for extracting temporal information from the photoelectron momentum distribution (PMD) by employing ultrashort laser pulses with near-circular polarization [1]. The first attoclock was devised with a goal to measure the tunneling delay time in laser-induced ionization of atoms [1–3]. For review of the attoclock setup see Ref. [4].

The principal idea of the attoclock technique is that an electron initially bound by the atomic potential is emitted into the continuum at the instant t_0 when the electric field is at its peak via tunneling through a potential barrier formed by the atomic potential and the laser field. Assuming that electron transverse through potential barrier instantaneously with the zero velocity at the exit point of the barrier, and neglecting its interaction with the residual ion after ionization, the electron dynamics is mainly determined by the ionization time. In particular, the momentum $\mathbf{p}$ corresponding to the maximum probability in the PMD is determined by the relation $\mathbf{p} = -\mathbf{A}(t_0)$, where $\mathbf{A}(t)$ is the laser-field vector potential. These assumptions are valid under the framework of the strong-field approximation (SFA) [5] and the two-step simple man's model [6–10]. However, if tunneling is finite in time, the most electrons will be detected with the momentum $\mathbf{p}_0 = -\mathbf{A}(t_0 + \tau)$, leading to the offset angle θ_0 in the PMD with respect to the direction of $-\mathbf{A}(t_0)$ (see Fig. 1). The time τ is usually interpreted as a tunneling time delay which is related to the offset angle via simple relation, $\theta_0 = \omega\tau$, where ω is the laser-field angular frequency. Since the electron dynamics is characterized by the attosecond timescale, the configuration shown in Fig. 1

is named an attoclock (the name attoclock is due to similarity of the electron momentum vector and the minute hand of a clock). This interpretation is not universally accepted and is a widely contested concept since the initial measurements suggested that tunneling is instantaneous [1–3], but subsequent results appeared to imply that the tunneling process takes a finite time [11,12]. Recent experimental studies have provided strong evidence for the absence of a measurable tunneling time delay [13] and revealed a pronounced dependence of the attoclock offset angle on the ionization potential of the atomic target [14]. Even more, there is no consensus on the definition of the tunneling time delay and numerous approaches to this topic are still debated and disputed (see Refs. [15–26]). Very

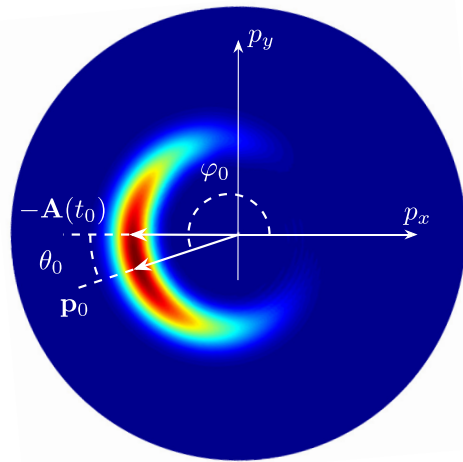


FIG. 1. Characteristic PMD resulting from the strong-field ionization of atomic targets using the attoclock configuration. The vector potential has its peak value pointing in the direction of the x axis, thus the expected direction of the photoelectron is on the opposite side, depicted by the vector $-\mathbf{A}(t_0)$. The peak of the PMD is in the direction depicted by the vector $\mathbf{p}_0 = p(\cos \varphi_0 \hat{\mathbf{e}}_x + \sin \varphi_0 \hat{\mathbf{e}}_y)$ expressed in the polar coordinates. The offset angle $\theta_0 = \varphi_0 - 180^\circ$ is calculated with respect to the vector $-\mathbf{A}(t_0)$.

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recently [27], it was shown, using a simple one-dimensional model, that the attoclock tunneling time may be finite in the vicinity of the tunnel exit while vanishing asymptotically at the detector. This result further emphasizes that the experimentally observed attoclock offset angle should not, in general, be identified directly with a tunneling delay.

Numerous theoretical studies have suggested that the attoclock offset angle is primarily caused by the Coulomb potential. Below, we briefly review some of the most representative works. A semiclassical model of above-threshold ionization was employed in Ref. [28] to investigate the influence of the Coulomb potential on the photoelectron momentum distribution in an elliptically polarized laser field. Using the analytical R -matrix method, the authors of Ref. [29] demonstrated that, in single-active-electron systems, the dominant contribution to the attoclock offset angle arises from the attractive Coulomb interaction with the ionic core. In Ref. [30], classical backpropagation of electron trajectories, including the Coulomb interaction, was used to examine the existence of a tunneling time delay, yielding no evidence for a finite tunneling time. In Ref. [31], a simple expression for the attoclock offset angle was derived by combining the classical Rutherford scattering formula with the Keldysh estimate of the tunnel exit width, further supporting the conclusion that the offset angle was predominantly of Coulombic origin. The authors of Ref. [32] showed, using the Bohmian trajectory formalism, that the offset angle was mainly determined by the Coulomb interaction in the vicinity of the classical tunnel exit. More recent theoretical studies employing a variety of approaches reached similar conclusions [33–35]. In this paper, we investigate the influence of the Coulomb potential on the attoclock offset angle by employing numerical solutions of the three-dimensional time-dependent Schrödinger equation (TDSE), as well as several variants of the strong-field approximation (SFA) combined with quantum-orbit theory. Our findings lend further support to the interpretation that the attoclock offset angle is predominantly determined by the long-range Coulomb interaction between the photoelectron and the residual ion, rather than by an intrinsic tunneling time delay. However, the small offset-angle differences between our best model and the TDSE indicate that this important problem is not yet fully resolved.

The paper is organized as follows. In Sec. II we give a brief introduction to the methods used to simulate the attoclock. In Sec. III we present our results and analysis, and in Sec. IV we summarize and conclude our findings.

Atomic units (a.u.; $\hbar = 1$, $4\pi\epsilon_0 = 1$, $e = 1$, and $m_e = 1$) are used throughout the paper unless otherwise stated.

II. METHODS

In this paper we use an elliptically polarized laser field defined in the xy plane by the vector potential $\mathbf{A}(t) = A_x(t)\hat{\mathbf{e}}_x + A_y(t)\hat{\mathbf{e}}_y$ where the vector components are

$$\begin{aligned} A_x(t) &= \frac{A_0}{\sqrt{1+\varepsilon^2}} f(t) \cos(\omega t), \\ A_y(t) &= \frac{\varepsilon A_0}{\sqrt{1+\varepsilon^2}} f(t) \sin(\omega t), \quad t \in [0, T_p]. \end{aligned} \quad (1)$$

Here A_0 is the amplitude, $f(t) = \sin^2[\omega t/(2N_c)]$ is the envelope, $\omega = 2\pi/T$ is the laser frequency, $T_p = N_c T$ is the total pulse duration with N_c the number of optical cycles per pulse, and ε is the ellipticity of the laser field ($\varepsilon = 1$ for circularly polarized field). The amplitude A_0 in a.u. is related to the peak intensity I of the laser field by the relation $I = (\omega A_0)^2$. The vector potential (1) reaches the peak value at $t_0 = T = 2\pi/\omega$ pointing in the direction $\hat{\mathbf{e}}_x$, while the expected direction of the peak in the PMD is in the direction $-\hat{\mathbf{e}}_x$ ($\varphi_p = 180^\circ$). In our simulations we use $N_c = 2$, $\varepsilon = 1$, and wavelength $\lambda = 800$ nm.

Within the SFA framework the liberated electron moves solely under the influence of the laser field. Under this condition, the transition amplitude from the ground state $|\psi_g\rangle$ to the plane-wave state with the momentum $\mathbf{p}$ is given by (see Ref. [5] and references therein)

$$M_{\mathbf{p}}^{\text{SFA}} = -i \int_0^{T_p} dt e^{iS_{I_p\mathbf{p}}(t)} (\mathbf{p} + \mathbf{A}(t)) \cdot \mathbf{E}(t) |\psi_g\rangle, \quad (2)$$

where $\mathbf{E}(t) = -d\mathbf{A}(t)/dt$ is the electric-field vector, and

$$S_{I_p\mathbf{p}}(t) = I_p t + \frac{1}{2} \int^t dt' [\mathbf{p} + \mathbf{A}(t')]^2, \quad (3)$$

is the action in the laser field. The state $|\mathbf{p}\rangle$ denotes a plane-wave state $\Phi_{\mathbf{p}}(\mathbf{r}) = \langle \mathbf{r} | \mathbf{p} \rangle = (2\pi)^{-3/2} \exp(i\mathbf{p} \cdot \mathbf{r})$. In our analysis we use the hydrogen atom in its $1s$ ground state ($I_p = 0.5$ a.u.) as the target so that the dipole matrix element in Eq. (2) is given by

$$\langle \mathbf{k} | \mathbf{r} \cdot \mathbf{E}(t) | \psi_{1s} \rangle = -i 2^{7/2} (2I_p)^{5/4} \frac{\mathbf{k} \cdot \mathbf{E}(t)}{\pi(\mathbf{k}^2 + 2I_p)^3}. \quad (4)$$

The integral over time in Eq. (2) is numerically evaluated using the Gauss-Legendre quadrature with a few hundred points per optical cycle. This integral can also be approximated using the modified saddle-point method (SPM) [5]. In this case, the transition amplitude (2) is given by

$$M_{\mathbf{p}}^{\text{SPM}} = -2^{-1/2} (2I_p)^{5/4} \sum_{s=1}^{N_s} \frac{e^{iS_{I_p\mathbf{p}}(t_s)}}{\mathbf{E}(t_s) \cdot [\mathbf{p} + \mathbf{A}(t_s)]}, \quad (5)$$

where the summation is carried over N_s saddle points t_s which are solutions of the equation

$$\left. \frac{\partial S_{I_p\mathbf{p}}(t)}{\partial t} \right|_{t=t_s} = \frac{1}{2} [\mathbf{p} + \mathbf{A}(t_s)]^2 + I_p = 0. \quad (6)$$

The saddle points t_s are, in general, complex-valued and depend parametrically on the final electron momentum $\mathbf{p}$. Among all solutions of the saddle-point equation (6), only those satisfying the conditions

$$0 \leq \text{Re } t_s \leq T_p, \quad \text{Im } t_s > 0, \quad (7)$$

are relevant. The requirement $\text{Im } t_s > 0$ ensures an exponentially decaying tunneling amplitude, while the constraint on the real part restricts the ionization time to the duration of the laser pulse. The real part $\text{Re } t_s$ can be interpreted as the time at which the electron emerges into the continuum after tunneling.

For a circularly polarized laser field, the number of saddle points within one pulse is given by $N_s = N_c + 1$ [5]. For $N_c = 2$,

the three saddle-point solutions are ordered according to their real parts as

$$\text{Re } t_{1s} < \text{Re } t_{2s} < \text{Re } t_{3s}. \quad (8)$$

Each saddle-point solution defines a so-called quantum orbit [36,37]. The s th quantum orbit is a complex electron trajectory that starts at the complex ionization time t_s from the position $\mathbf{r}(t_s)$, with the velocity $\mathbf{v}(t) = d\mathbf{r}/dt|_{t=t_s} = \mathbf{p} + \mathbf{A}(t_s)$, and which satisfies Newton's equation $\ddot{\mathbf{r}}(t) = -\mathbf{E}(t)$. The real electron trajectory is defined as the projection of the quantum orbit into the real plane for real times $t \geq \text{Re } t_s$:

$$\mathbf{r}_L(t) = (t - \text{Re } t_s)\mathbf{p} + \boldsymbol{\alpha}(t) - \text{Re } \boldsymbol{\alpha}(t_s), \quad (9)$$

$$\mathbf{v}_L(t) = \mathbf{p} + \mathbf{A}(t), \quad (10)$$

where $\boldsymbol{\alpha}(t) = \int^t dt' \mathbf{A}(t')$ denotes the displacement induced by the vector potential.

In the case $N_c = 2$, only the quantum orbit associated with the solution t_{2s} contributes significantly to the PMD. In particular, for the peak of the PMD, this saddle point satisfies

$$\text{Re } t_{2s} \approx T = 2\pi/\omega, \quad (11)$$

indicating that the ionization occurs with the highest probability near the maximum of the electric field [38]. This behavior is consistent with the tunneling picture, according to which the instantaneous ionization rate is maximal when the electric field amplitude reaches its peak value.

A modification to the SFA amplitude (2) was introduced in Ref. [39] by replacing the pure plane-wave with the Coulomb distorted plane wave

$$\Phi_{Cp}^{(-)}(\mathbf{r}) = (2\pi)^{-3/2} e^{i[\mathbf{p} \cdot \mathbf{r} - \gamma \ln(pr + \mathbf{p} \cdot \mathbf{r})]}, \quad \gamma = -1/p. \quad (12)$$

This modification takes into account the Coulomb interaction $V_C = -1/r$ with the residual ion via the Coulomb phase in the final wave function and the corresponding transition amplitude is given by

$$M_{\mathbf{p}}^{\text{CDW}} = -i \int_0^{T_p} dt e^{iS_{ip}(t)} \langle \mathbf{p} | \frac{e^{-i\mathbf{A}(t) \cdot \mathbf{r}} \mathbf{r} \cdot \mathbf{E}(t)}{(pr + \mathbf{p} \cdot \mathbf{r})^{i/p}} | \psi_{1s} \rangle. \quad (13)$$

We call this approximation the Coulomb distorted plane wave (CDW) SFA. The matrix element (13) can be evaluated analytically and the result is [39]

$$M_{\mathbf{p}}^{\text{CDW}} = z(v) \int_0^{T_p} dt e^{iS_{ip}(t)} a^{iv-3} b^{-iv-2} \mathbf{E}(t) \cdot \mathbf{d}, \quad (14)$$

where $v = 1/p$, $z(v) = 2^{5/2} \Gamma(1 - iv)/\pi$, $a = 1 - [\mathbf{A}(t) + \mathbf{p}]^2$, $b = 2[p - i\mathbf{p} \cdot (\mathbf{p} + \mathbf{A}(t))]$, $c = (iv - 1)b - ivpa$, and $\mathbf{d} = va(c - pa)\mathbf{p} - (iv - 1)b(c - b)[\mathbf{A}(t) + \mathbf{p}]$.

The ionization probability for emitting a photoelectron with kinetic energy $E_{\mathbf{p}} = \mathbf{p}^2/2$ into the direction specified by the angle $\varphi_{\mathbf{p}}$, corresponding to the momentum $\mathbf{p} = (p_x, p_y, 0) = (p \cos \varphi_{\mathbf{p}}, p \sin \varphi_{\mathbf{p}}, 0)$ in the xy polarization plane, is given by

$$w(E_{\mathbf{p}}, \varphi_{\mathbf{p}}) = p |M_{\mathbf{p}}|^2 |_{\theta_{\mathbf{p}}=\pi/2}. \quad (15)$$

Integrating over the kinetic energy we obtain angular distribution

$$w(\varphi_{\mathbf{p}}) = \int_0^{E_{\mathbf{p}}^{\text{max}}} dE_{\mathbf{p}} w(E_{\mathbf{p}}, \varphi_{\mathbf{p}}), \quad (16)$$

where $E_{\mathbf{p}}^{\text{max}}$ is the upper boundary for the kinetic energy.

Our method for numerically solving the TDSE is based on the expansion of the time-dependent wave function into the basis of B-spline functions and spherical harmonics. We use the dipole approximation for the interaction operator. The PMD in the polarization plane $P(E_{\mathbf{p}}, \varphi_{\mathbf{p}})$ is extracted from the time-dependent wave function $\Psi(\mathbf{r}, t)$ at $t = T_p$ by projecting the $\Psi(\mathbf{r}, t)$ onto the continuum wave function $\Phi_{\mathbf{p}}^{(-)}(\mathbf{r})$ obeying the so-called incoming boundary condition

$$P(E_{\mathbf{p}}, \varphi_{\mathbf{p}}) = p |\langle \Phi_{\mathbf{p}}^{(-)} | \Psi(T_p) \rangle|^2 |_{\theta_{\mathbf{p}}=\pi/2}. \quad (17)$$

The continuum states $\Phi_{\mathbf{p}}^{(-)}(\mathbf{r})$ are normalized on the momentum scale, i.e., $\langle \Phi_{\mathbf{p}'}^{(-)} | \Phi_{\mathbf{p}}^{(-)} \rangle = \delta(\mathbf{p}' - \mathbf{p})$. More details regarding our numerical method for solving the TDSE can be found in Refs. [40–42].

In our analysis, we use the TDSE in combination with the SFA, CDW SFA, and SPM to simulate the photoelectron momentum distribution. These results are then used to extract the attoclock offset angle using the same procedure as in experiments [1,11,13]. Specifically, the PMD (17) is integrated over kinetic energy to obtain the angular distribution

$$P(\varphi_{\mathbf{p}}) = \int_0^{E_{\mathbf{p}}^{\text{max}}} dE_{\mathbf{p}} P(E_{\mathbf{p}}, \varphi_{\mathbf{p}}), \quad (18)$$

which is then fitted with a Gaussian function

$$P(\varphi_{\mathbf{p}}) \propto e^{-(\varphi_{\mathbf{p}} - \varphi_{\text{max}})^2 / \sigma^2}, \quad \phi_0 = \varphi_{\text{max}} - 180^\circ, \quad (19)$$

where φ_{max} determines the peak position of the distribution, σ is its width, and ϕ_0 is the attoclock offset angle. However, the definition of the attoclock offset angle is not unique. For example, the offset angle θ_0 extracted from the momentum distribution shown in Fig. 1 generally differs from the offset angle ϕ_0 obtained from the angular distribution, i.e., $\phi_0 \neq \theta_0$. Therefore, care must be taken when comparing the offset angles reported in the literature, as the extracted values depend on the analysis procedure. Specifically, the offset angle may be determined from the angular distribution, the peak of the energy-resolved angular distribution, or the peak of the momentum-resolved angular distribution, each of which can yield a different value [43,44]. These differences can amount to several degrees. When comparing attoclock offset angles obtained using different theoretical approaches, it is essential to employ the same extraction procedure. Throughout this work, the attoclock offset angles are extracted from the angular distribution given by Eq. (18).

III. RESULTS

In this section, we present our results, which are based on the analysis of the PMD calculated using Eqs. (15) and (17). From these PMDs we extract the offset angle by integrating over the electron energy, Eq. (18) with Eq. (16). Our analysis is restricted to the total pulse duration of two optical cycles, which is in accordance with the experimental attoclock setup [2,13,14]. For such short pulses we have only one peak in the PMD. In all our simulations, we use the hydrogen atom in its ground state as the target.

A typical PMD generated by a circularly polarized laser field is shown in Fig. 2. The spectra are calculated using the

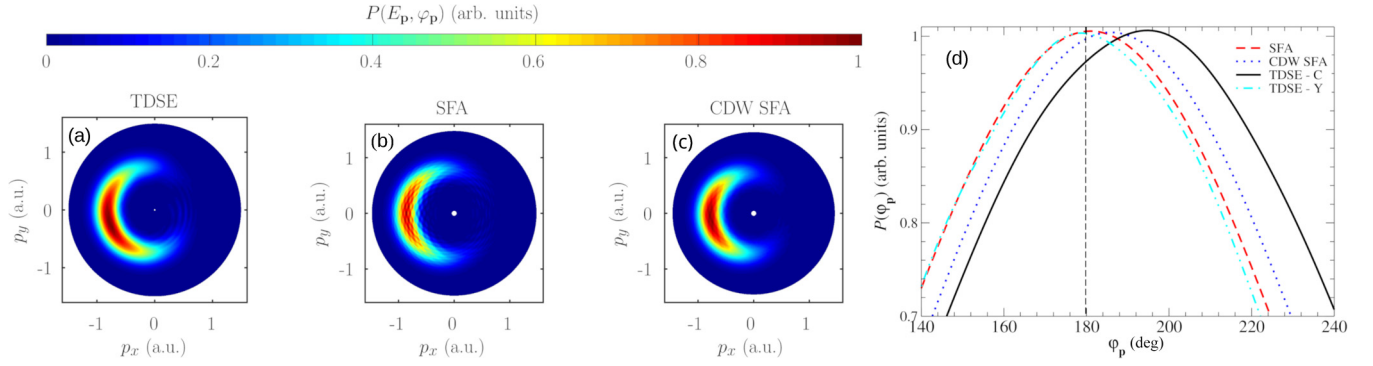


FIG. 2. Normalized PMDs in the circularly polarized laser field with the peak intensity $I = 10^{14}$ W/cm², wavelength $\lambda = 800$ nm, and $N_c = 2$, calculated (a) using numerical solution of the TDSE, (b) using SFA, and (c) using CDW SFA. Panel (d) depicts the corresponding normalized angular distributions.

vector potential defined by Eq. (1), with the peak intensity of $I = 10^{14}$ W/cm², wavelength $\lambda = 800$ nm, and the total pulse duration of $N_c = 2$ optical cycles. These parameters correspond to a standard attoclock configuration. Figure 2(a) displays the PMD obtained using the numerical solution of the TDSE. Figure 2(b) shows the results obtained within the SFA, while Fig. 2(c) presents the PMD calculated using the CDW SFA.

In the TDSE spectrum, the maximum of the PMD is clearly rotated with respect to the direction $-\hat{\mathbf{e}}_x$, which corresponds to the direction of the negative vector potential at the time of ionization in the absence of the Coulomb interaction. This rotation defines the attoclock offset angle. A similar, although smaller, offset is visible in the CDW SFA result. In contrast, the SFA spectrum exhibits no offset angle, with the PMD maximum aligned along $-\hat{\mathbf{e}}_x$. Figure 2(d) shows the corresponding angular distributions. The red dashed line represents the SFA result, the blue dotted line corresponds to the CDW SFA, and the black solid line shows the TDSE result. The cyan dash-dot line depicts the angular distribution obtained by solving the TDSE for a short-range Yukawa-type potential of the form

$$V_Y(r) = -\frac{V_0}{r} e^{-\kappa r}, \quad (20)$$

where the parameters $V_0 = 1.645$ and $\kappa = 0.7$ are chosen such that the ionization potential of the $1s$ ground state is $I_p = 0.5$ a.u. and the corresponding bound-state wave function closely resembles the hydrogenic $1s$ state.

Importantly, both the SFA calculation and the TDSE result obtained with the Yukawa potential show a negligible offset angle. Since the Yukawa potential is short-ranged and lacks the long-range Coulomb tail, this comparison demonstrates that short-range binding potentials do not generate a measurable offset in the PMD [13,29]. This strongly indicates that the observed attoclock offset originates from the long-range nature of the Coulomb potential, which induces a momentum shift of the outgoing electron during its propagation in the continuum.

To further test this hypothesis, we perform an additional analysis using the SPM and quantum-orbit theory. By solving the saddle-point equation (6), we determine the complex ionization time t_s whose real part $\text{Re } t_s$ defines the instant at which the electron emerges into the continuum. After ionization, the

electron propagates in the combined laser and Coulomb fields. Immediately upon entering the continuum, the strong laser field drives the electron away from the atomic center. Consequently, the influence of the Coulomb field rapidly decreases as the electron recedes from the origin. During this short initial stage of propagation, however, the Coulomb interaction induces a shift to the electron momentum. Since the laser field dominates the overall acceleration, this Coulomb-induced momentum shift can be treated as a perturbation with respect to the laser-driven momentum [45]. In general, for the atomic potential $V_i(r)$, the momentum shift can be calculated as

$$\Delta \mathbf{p}_i = - \int_{\text{Re } t_s}^{T_p} dt \frac{dV_i(r)}{dr} \frac{\mathbf{r}}{r}, \quad i = C, Y, \quad (21)$$

where $\mathbf{r} \equiv \mathbf{r}_L(t)$ is the laser-driven quantum orbit defined by Eq. (9), T_p denotes the end of the laser pulse, and $i = C$ (Y) stands for the Coulomb (Yukawa) potential. In the case of Coulomb potential, the integral accounts for the Coulomb force acting along the unperturbed quantum orbit. The final asymptotic momentum is then given by $\mathbf{p}_f = \mathbf{p} + \Delta \mathbf{p}_i = (p_f \cos \varphi_p, p_f \sin \varphi_p)$.

Within the SPM framework, we assume that the interaction with the atomic core modifies only the continuum propagation, while the ionization probability itself remains unchanged. In other words, the atomic potential is taken to alter the final electron momentum but not the tunneling amplitude. Therefore, the ionization amplitude given by Eq. (5) is evaluated at momentum $\mathbf{p}$ and subsequently mapped to the shifted momentum $\mathbf{p}_f$. This procedure allows us to isolate the effect of Coulomb-induced momentum transfer on the photoelectron momentum distribution and, in particular, on the attoclock offset angle.

Instead of evaluating the Coulomb-induced momentum shift via Eq. (21), the final electron momentum can be obtained by numerically propagating the classical trajectory (CT) according to the Newton's equation of motion in the combined laser and Coulomb fields

$$\ddot{\mathbf{r}}(t) = -\mathbf{E}(t) - \mathbf{r}(t)/r^3. \quad (22)$$

The initial conditions for the CT are supplied by quantum-orbit theory through the saddle-point solution [see Eqs. (9) and (10)], ensuring consistency with the strong-field

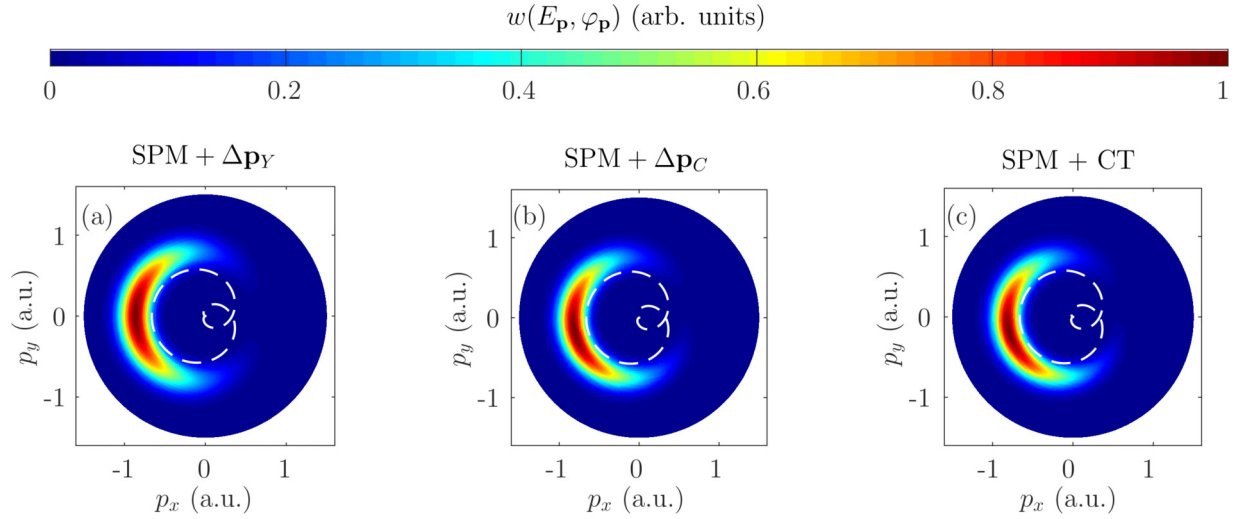


FIG. 3. Normalized PMDs in the circularly polarized laser obtained with the saddle-point method. Panel (a) shows results with included Yukawa-potential-induced momentum shift $\Delta \mathbf{p}_Y$ and panel (b) shows results with included Coulomb-potential-induced momentum shift $\Delta \mathbf{p}_C$. Panel (c) represents results calculated by including the final momentum obtained from the CT in combined Coulomb and laser fields. The white dashed line represent the reference momentum $\mathbf{p} = -\mathbf{A}(t)$. The laser field parameters are the same as in Fig. 2.

ionization step. In this approach, the ionization probability associated with each quantum orbit is preserved, while the mapping from ionization time to the final momentum is modified by the subsequent interaction with the Coulomb and laser fields.

In Fig. 3, we present the PMDs calculated within the SPM, including only the contribution from the saddle-point solutions t_{2s} and incorporating the corresponding momentum shift arising from the interaction with the atomic core. Figure 3(a) displays the results obtained with the Yukawa potential, while Fig. 3(b) shows those calculated using the Coulomb potential. Figure 3(c) presents the results derived from the CT obtained from the numerical solution of the Newton's equation of motion (22). The white dashed line indicates the reference momentum $\mathbf{p} = -\mathbf{A}(t)$. The laser parameters are identical to those used in Fig. 2.

As we can notice, for the Yukawa potential the offset angle is found to be approximately zero. In contrast, employing the Coulomb potential leads to the clear emergence of a finite offset angle. This behavior is consistently observed whether the Coulomb influence is incorporated through Eq. (21) or via CT using the Newton's equation (22). The comparison demonstrates that the nonzero offset angle observed in the attoclock configuration originates from the momentum shift induced by the long-range character of the Coulomb interaction during propagation in the continuum.

Next, we analyze the dependence of the attoclock offset angle on the laser peak intensity. To this end, we perform a systematic set of simulations using the SPM, the CDW SFA, and full numerical solutions of the TDSE. Our calculations shows that the SPM with included Yukawa-potential-induced momentum shift gives the offset angle close to zero regardless of the laser peak intensity. In Fig. 4 we show the offset angle ϕ_0 as a function of the laser peak intensity obtained by the TDSE, the CDW SFA, and by incorporating the Coulomb-induced momentum shift $\Delta \mathbf{p}_C$ into the SPM, and by using the final momentum obtained from the numerical solution of the

Newton's equation of motion in the combined Coulomb and laser fields.

The CDW SFA results (blue dotted line) display a clear intensity dependence. The offset angle decreases with increasing intensity, from $\phi_0 = 6.68^\circ$ at $I = 0.7 \times 10^{14} \text{ W/cm}^2$ to $\phi_0 = 4.11^\circ$ at $I = 3 \times 10^{14} \text{ W/cm}^2$. This trend indicates that the inclusion of the Coulomb phase modifies the electron's final momentum in an intensity-dependent manner. The TDSE results (black solid line) predict the largest offset angles over the considered intensity range. Specifically, ϕ_0 decreases

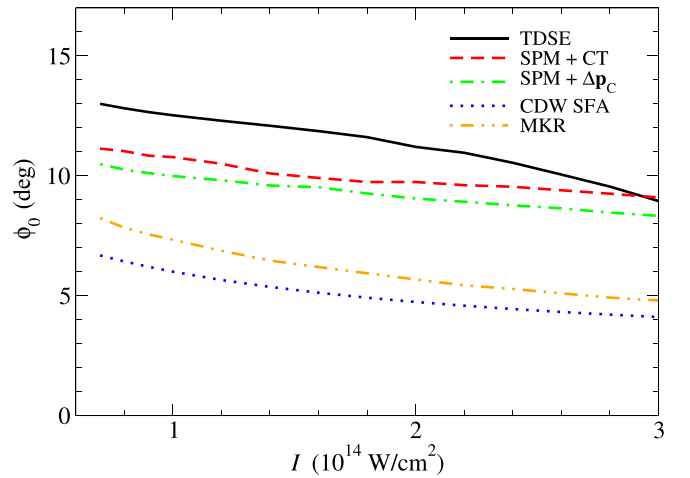


FIG. 4. The attoclock offset angle ϕ_0 plotted as a function of the laser field intensity I for the wavelength $\lambda = 800 \text{ nm}$. The black solid line (blue dotted line) depicts the results obtained with the TDSE (CDW SFA). The green dot-dashed line shows predictions of the SPM with included Coulomb-induced momentum shift. The red dashed line depicts the results obtained with the SPM with the final momentum calculated from the CT in the combined Coulomb and laser fields. The orange dash-double-dotted line depicts results obtained using MKR model.

from 13.00° at $I = 0.7 \times 10^{14}$ W/cm² to 8.95° at $I = 3 \times 10^{14}$ W/cm². Although the decrease is more pronounced than in the CDW SFA, the overall downward trend with increasing intensity is clearly visible. Importantly, the TDSE curve $\phi_0(I)$ is vertically shifted with respect to the CDW SFA results, yielding systematically larger offset angles. This quantitative discrepancy highlights the role of the full long-range Coulomb interaction and nonperturbative electron dynamics, which are only partially accounted for in approximate models. Nevertheless, both the TDSE and CDW SFA calculations consistently predict a decreasing offset angle with increasing intensity. The key result of these calculations is that incorporating the Coulomb phase into the final-state wave function yields an intensity-dependent offset angle in qualitative agreement with the TDSE, despite a systematic vertical shift between the two sets of results.

However, the saddle-point analysis with included Coulomb-induced momentum shift $\Delta \mathbf{p}_C$, depicted with the green dashed-dot line, predicts offset angles close to those obtained from the TDSE results. The red dashed line depicts the results obtained with the SPM by including the final momentum obtained from the CT obtained from the solution of the Newton's equation of motion. As we can notice, these results are slightly below those obtained from the TDSE calculations. The difference between the offset angles obtained from the TDSE and the SPM with included Coulomb field effect is less than two degrees at the lowest laser peak intensity. As the intensity increases, this discrepancy becomes smaller.

In Ref. [31], the so-called Keldysh-Rutherford (KR) model was introduced to estimate the offset angle induced by the Coulomb potential. Within this model, the offset angle is attributed to Coulomb scattering in the presence of the laser field. Accordingly, the trajectory of the emitted photoelectron is approximated by that of a classical particle undergoing elastic Rutherford scattering in the Coulomb field of the residual ion. The angular shift, $\Delta\varphi$, is taken as one-half of the classical Rutherford scattering angle, with the impact parameter b approximated by the Keldysh-theory expression for the tunnel-exit coordinate $|\mathbf{r}_0|$, i.e., $b \approx |\mathbf{r}_0| = I_p/E_{\text{max}}$, where E_{max} denotes the peak electric-field strength. Instead of that, in our modified KR (MKR) model, we assign a specific angular shift $\Delta\varphi$ to each quantum orbit, and the corresponding tunnel-exit coordinate is determined using Eq. (9). This procedure allows the Coulomb-induced rotation to be evaluated for each quantum orbit individually. Therefore, in our modified Keldysh-Rutherford model, the final emission angle associated with each quantum orbit is $\varphi_{\mathbf{p}_f} = \varphi_{\mathbf{p}} + \Delta\varphi$ and the angular shift is calculated as

$$\Delta\varphi = \arctan\left(\frac{1}{|\mathbf{r}_0|p^2}\right), \quad (23)$$

where $\mathbf{r}_0$ is evaluated from Eq. (9) for $t = \text{Re } t_s$. Using this approach we calculate the PMD using SPM and the corresponding angular distribution, from which the attoclock offset angle ϕ_0 is extracted. Figure 4 shows the attoclock offset angle obtained using the MKR model, represented by the orange dash-double-dotted curve, as a function of the peak laser intensity. As can be seen, the attoclock offset angle predicted

by the MKR model is in good agreement with that obtained from the CDW SFA calculations. This agreement suggests that incorporating the Coulomb phase as a distortion of the plane-wave phase is effectively equivalent to a partial rotation of the quantum orbits due to the Coulomb field.

Overall, the presented results show qualitative agreement with the TDSE calculations and reproduce almost the same intensity-dependent trend. In particular, the offset angle derived from the Coulomb-induced momentum shift decreases with increasing laser peak intensity. An equivalent trend is obtained when the final momentum is determined through CT in the combined Coulomb and laser fields. With increasing laser field strength, the TDSE and the modified SPM predictions exhibit systematic convergence, indicating improved consistency between the two approaches. This behavior can be understood from the underlying electron dynamics: stronger laser fields accelerate the electron away from the ionic core more efficiently immediately after ionization, thereby reducing both the interaction time and the effective strength of the Coulomb potential during continuum motion. Consequently, the Coulomb-induced momentum shift becomes smaller, leading to a reduction of the offset angle. However, for peak intensities exceeding $I = 2 \times 10^{14}$ W/cm², the TDSE results display a more pronounced decrease of the offset angle. This observation suggests that additional physical mechanisms may become relevant in this intensity regime, which is neglected in the SPM.

Our results reinforce the interpretation that the origin of the attoclock offset angle is attributed to the long-range nature of the Coulomb interaction. The small offset-angle differences between the TDSE and our models indicate that additional physical mechanism may influence the offset angle so this important problem is not yet fully resolved.

IV. CONCLUSION

In this work, we investigated the origin and intensity dependence of the attoclock offset angle by systematically comparing the SPM, the CDW SFA, so-called KR model with modifications and full TDSE simulations. By isolating the role of the ionic potential, we demonstrated that a short-range Yukawa interaction lead to a vanishing offset angle, whereas the inclusion of the long-range Coulomb potential resulted in a clearly finite angular shift. We further demonstrated that incorporating the Coulomb phase as a distortion of the final plane-wave phase produced a rotation of the photoelectron spectrum that was nearly identical to that obtained by rotating the quantum orbits according to the angular shift predicted by the MKR model. This comparison unambiguously identifies the Coulomb-induced momentum shift as the primary mechanism responsible for the nonzero offset angle observed in the attoclock configuration.

The analysis of the intensity dependence of the offset angle further supports this interpretation. The TDSE, CDW SFA, and MKR calculations exhibit a decreasing offset angle with increasing laser peak intensity. This trend can be understood from the reduced influence of the Coulomb field at higher intensities, where the electron is driven away from the ionic core more rapidly, limiting the duration and strength of the Coulomb interaction during continuum propagation.

Our findings reinforce the conclusion from previous theoretical studies [28–35] that the attoclock offset angle is predominantly governed by postionization Coulomb effects rather than by an intrinsic tunneling delay. The close agreement between the Coulomb-modified SPM and TDSE results demonstrates that the essential physics of the offset angle can be captured within a semiclassical framework, provided that the long-range nature of the Coulomb interaction is properly taken into account.

In conclusion, we summarize the performance of the different theoretical models used in this work to determine the attoclock offset angle. Both the CDW SFA and the modified KR model substantially underestimate the offset angle across the entire intensity range considered, indicating that these approaches do not capture the dominant Coulomb-driven corrections. Much better agreement with the TDSE results is obtained from the remaining two models. In particular, the

SPM + CT model where the Coulomb interaction is incorporated through postionization classical-trajectory propagation with initial conditions determined from the SFA saddle-point equation without the Coulomb potential provides the closest match to the TDSE. The small residual discrepancy between the SPM + CT and TDSE results suggests that the Coulomb field should also be included during the subbarrier electron motion in complex time. This influence becomes less significant at higher intensities, where the TDSE and SPM + CT predictions converge, consistent with the reduced role of the Coulomb potential in stronger fields.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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