

Exact analytic characterization of the Autler-Townes doublet via the extended semiclassical Rabi model

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We study the resonance-enhanced multiphoton ionization (REMPI) of atoms by intense, short, XUV radiation fields. To characterize Rabi oscillations and the resulting Autler-Townes doublet in the photoelectron spectrum, a minimal-state effective description is utilized, incorporating energy-level shifts, multiphoton couplings, and irreversible losses. Extending the semiclassical Rabi model with such higher-order terms, the complex population amplitudes of the Rabi cycling atom are found in simple analytic forms. This allows for the exact analytic evaluation of the photoelectron spectrum under various simultaneous physical effects. The origins of the spectral characteristics, such as the splitting, shifting, asymmetry, and multipeak pattern, are identified through the found spectrum formula. Furthermore, the dependence of these features on the field intensity, frequency detuning, and interaction time are analyzed in detail. Finally, as the simplicity of the spectrum formula is preserved when further higher-order light-matter interaction terms are added, the presented general framework is suitable for studying arbitrary $(n+m)$ photon REMPI phenomena.

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I. INTRODUCTION

Rabi oscillations (ROs) [1,2]—the time-periodic oscillations of the populations of quantum two-level systems (TLS) under external driving—have been receiving great research interest ever since their discovery in the 1930's. Being one of the most fundamental light-matter interaction phenomena, ROs take place in numerous physical systems, including, e.g., superconducting circuits, cold atoms, Rydberg atoms, quantum dots, and cavity polaritons [3]. From the theoretical point of view, the semiclassical Rabi model (SRM) that describes the time evolution of quantum TLSs under time-periodic classical radiation field driving [1] is among the very few analytically exactly solvable two-state models. This renders the SRM and its various extensions crucially important in describing ROs and related phenomena. In the traditional SRM, the complex population amplitudes of the coupled levels are expressed in terms of the frequency detuning and field amplitude parameters in lowest order. Despite its simplicity, the SRM could provide a powerful testbed for rather involved phenomena, allowing for the transparent illustration of various physical processes.

When particle emission happens during the ROs, the wave-packet amplitudes liberated at different times interfere, leading to the appearance of the Autler-Townes (AT) doublet [2] in the kinetic energy spectrum. The AT doublet and the

underlying ROs have been widely studied in the resonance-enhanced multiphoton ionization (REMPI) of atoms and molecules since the 1980's [4–9]. In recent years, thanks to the availability of intense, short XUV laser pulses [10–14], the investigation of ROs is experiencing its renaissance [15–19], including various aspects such as the generation of electron-ion entanglement [20–24]. The spectral features of the AT doublet—such as the shifting, splitting, asymmetry, and multipeak pattern—are mostly shaped by the resonant Stark effect and contain relevant information on the underlying bound-state electron dynamics. The physical origins and controllability of these spectral characteristics have been the subject of numerous current researches [25–33].

The multipeak pattern of the AT doublet, observed for intense (large-area) laser pulses, was found to be closely intertwined with ROs and dynamic interference, two prominent competing phenomena under the strong dynamical dressing of atomic scale systems [34–40]. It has been demonstrated that under asymmetric envelope functions, for which the electrons are ionized with completely different energies at the distinct Rabi cycles, the multipeak pattern of the AT doublet is preserved [40].

The asymmetry of the AT doublet has been attributed to various reasons, including, e.g., the dynamic Stark shifts (DSS) of the coupled levels [41,42], the photon energy detuning [43], the energy-dependence of the bound-to-continuum couplings [44], the loss of the excited state [45], the phase of the driving field [46], or the interplay between different ionization routes [12,43]. The underlying physical mechanism of the ultrafast-pulse-shaping control of ROs in the XUV, initiated in Ref. [47] and demonstrated experimentally in Ref. [13], could be transparently described within the framework of the selective population of dressed states [48–50]. Another prominent effect that can be identified through the

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asymmetry of the AT doublet using parameter values accessible at seeded free-electron lasers is the stabilization of Rabi cycling dressed atoms in a circularly polarized intense XUV field [51,52]. Furthermore, we mention that counter-rotating circularly polarized pulses have the potential of manipulating the photoemission anisotropy through the competition of different partial waves generated by delayed pulses [53].

Tracing of the ultrafast buildup of ROs in $1 + 1$ photon REMPI allowed for the characterization of the two-photon transition phase in the high-intensity region [54] and the identification of the contributions of different partial waves employing spectrally and angularly resolved electron interferometry [55]. Utilizing an $\omega - 2\omega$ XUV pulse pair, the ultrafast emergence of ROs in the time domain was found to be accompanied by the phase jump between the one- and two-photon transition amplitudes [18]. The energy separation of the two prominent peaks of the AT doublet, which is primarily determined by the Rabi frequency and frequency detuning, could be drastically enhanced with the help of two time-delayed XUV pulses. This dynamically enhanced AT splitting was attributed to the interference of attosecond electron wave packets liberated around the zero value of the ground-state population amplitude of Rabi cycling atoms [56,57].

Since the exact analytic evaluation of the AT doublet for laser pulses with smoothly varying envelope functions is extremely rare and limited [45], considering rectangular shape (flat-top) classical radiation fields, and hence the SRM, is highly relevant. Utilizing flat-top models for studying REMPI has become widely accepted [18,36–38,51], and was found useful for elucidating the observations made by realistic laser pulses [12]. Usually, flat-top models with substantial simplifications are applied upon neglecting small and seemingly unimportant terms. Thus, there is a need for inspecting the spectrum within the extended SRM as accurately as possible. Furthermore, as the spectral features generated by the sine-shape laser pulse can be regarded as resulting from different rectangular-pulse-shape excitations [45], flat-top models can also be useful for studying pulse shape effects despite their constant envelope function.

In this paper, we study REMPI and the resulting AT doublet exactly analytically under rectangular envelope driving, via the SRM. To that end, a minimal-level effective description is utilized, incorporating various physical terms such as multiphoton couplings, energy-level shifts, and irreversible losses. Extending the SRM with such higher-order terms, the complex population amplitudes of the bound and continuum levels are found in simple analytic forms, utilizing elementary functions. The found spectrum formula provides additional insight into the structural buildup of the AT doublet, unfolding the physical origins and controllability of the main spectral features, such as the shifting, splitting, asymmetry, and multi-peak pattern. A great merit of the framework to be presented here is its simplicity and flexibility despite the large number of included parameters of different orders in the electric field amplitude. As the parameters of the extended SRM all have a common (rectangular) temporal profile irrespective of their order, the analytic expressions found here for $1 + 1$ photon REMPI are straightforwardly extended with arbitrary higher-order terms, allowing for the study of $n + m$ photon REMPI processes.

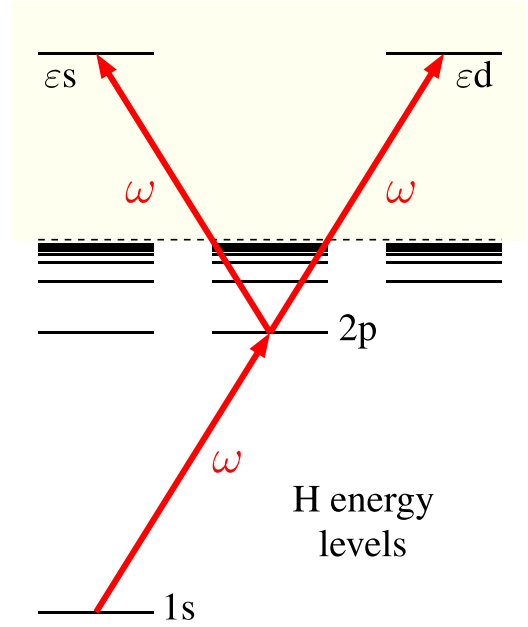


FIG. 1. Energy-level scheme of hydrogen, showing the $1 + 1$ photon resonance-enhanced multiphoton ionization process studied in this work. The dynamics of the (near-)resonantly coupled ($1s$, $2p$) two-level system is probed upon absorption of additional photons from the same field that promote the electron to the s and d continua. The horizontal dashed line indicates the ionization threshold, while the electron continuum is shown by the shaded area.

II. $1 + 1$ PHOTON IONIZATION

To demonstrate the extension of the Rabi model on a concrete example, below we will consider the $1 + 1$ photon REMPI of the hydrogen atom driven by a rectangular laser pulse (see Fig. 1). Working in the essential state formalism, we will derive an effective model that goes beyond the usual minimal description, and is thus able to capture several physical effects in the strong coupling regime. This model will serve as an important basis for the analytic studies in Secs. III and IV. Concretely, we will retain terms up to fourth order in the electric field amplitude. Generalization to higher orders is straightforward and does not affect the considerations regarding the extension of the SRM to be presented in Sec. III.

The hydrogen atom is assumed initially at $t = 0$ in its $1s$ ground state and the photon energy is set to near resonance with the $2p$ excited state. Upon absorption of an additional photon, ionization occurs predominantly to the s and d continua labeled here as ϵs and ϵd , respectively. This $1s - 2p - \text{continuum}$ pathway of H was considered by several groups recently [34,36–40,43,44,54,55]. Throughout this work, linearly polarized electric field is considered along the z direction, in the $t \in [0, T]$ time interval

$$\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega t), \quad (1)$$

where $\mathcal{E}_0$ is the electric field amplitude (related to the field intensity as $I_0 = \mathcal{E}_0^2 / 8\pi\alpha_{fs}$, where α_{fs} is the fine-structure constant), ω is the photon energy, and T is the duration parameter of the rectangular pulse. Atomic units are utilized unless stated otherwise.

The time-dependent wave function of the sole electron is expanded in the basis of the populated ($1s$, $2p$, εl) and non-populated (m) eigenstates of the H_0 Hamiltonian of the free atom, utilizing the ω_k eigenenergies in the phase factors

$$\begin{aligned} \Psi(t) = & c_{1s}(t)|1s\rangle e^{-i\omega_{1s}t} + c_{2p}(t)|2p\rangle e^{-i\omega_{2p}t} \\ & + \sum_m c_m(t)|m\rangle e^{-i\omega_m t} \\ & + \sum_{l=s,d} \int c_{\varepsilon l}(t)|\varepsilon l\rangle e^{-i\omega_{\varepsilon l}t} d\omega_{\varepsilon l}. \end{aligned} \quad (2)$$

In the above expansion, the bound and continuum m states are dipole coupled to $1s$ or $2p$ or to εl . These states are, however, hardly populated because they lie energetically out of one-photon resonance with the populated (essential) states. Nevertheless, it is important to explicitly keep the m states in Eq. (2), since they are responsible for higher-order terms like energy-level shifts and multiphoton couplings (see below). Here, we adopt the length gauge form of the interaction in line with other related studies [4,28,34–36,38,44,50], as it allows a fast convergence of the higher-order interaction terms in REMPI [58,59], to be detailed below. Utilizing the velocity gauge involves similar steps and provides identical observables, but the proper convergence of the higher-order interaction terms would require substantially more high-lying states in that case.

Inserting Eq. (2) into the time-dependent Schrödinger equation (TDSE) $i\dot{\Psi} = [H_0 + z\mathcal{E}(t)]\Psi$, one finds a set of first-order coupled differential equations for the population amplitudes. As the nonpopulated (m) states are far from resonance, they rapidly oscillate during the interaction, and their time evolution can be obtained by adiabatic elimination [60]. Integrating the equation of the nonpopulated states by parts and omitting small terms, one finds an explicit expression for $c_m(t)$, which is valid as long as the detuning of the m states is large compared to the pulse bandwidth and the related Stark shifts. Substituting the obtained expression into the equations of c_{1s} , c_{2p} , and $c_{\varepsilon l}$, applying the rotating wave-, the dipole-, and the local approximations [61], one obtains the three-state minimal effective model [39]

$$i\dot{c}_{1s}(t) = \left[S_{1s} - \frac{i}{2}\gamma \right] c_{1s}(t) + \left[\frac{1}{2}\Omega - \frac{i}{2}L \right] e^{-i\Delta t} c_{2p}(t) \quad (3a)$$

$$i\dot{c}_{2p}(t) = \left[\frac{1}{2}\Omega^\dagger - \frac{i}{2}L^\dagger \right] e^{i\Delta t} c_{1s}(t) + \left[S_{2p} - \frac{i}{2}\Gamma \right] c_{2p}(t) \quad (3b)$$

$$\begin{aligned} i\dot{c}_{\varepsilon l}(t) = & \frac{1}{2}\Omega_{\varepsilon l} e^{i[\delta t + \Delta t]} c_{2p}(t) + \frac{1}{4}M_{\varepsilon l} e^{i[\delta t + 2\Delta t]} c_{1s}(t) \\ & + S_{\varepsilon l} c_{\varepsilon l}(t) \quad (l = s, d), \end{aligned} \quad (3c)$$

where the notations for the transition energies $\omega_{kl} = \omega_k - \omega_l$ and for the dipole moments $\mu_{kl} = \langle k|z|l \rangle$ have been introduced. In Eqs. (3a)–(3c), $\Omega = \mathcal{E}_0 \mu_{1s2p}$ is the Rabi frequency for the bound-bound $1s - 2p$ transition, and $\Delta = \omega_{2p1s} - \omega$ is the corresponding detuning. Furthermore, $\Omega_{\varepsilon l} = \mathcal{E}_0 \mu_{\varepsilon l2p}$ is the one-photon Rabi frequency for the transition from the resonant state $|2p\rangle$ to the continuum state $|\varepsilon l\rangle$ (the transition amplitude for ionization from the excited state), and $\delta = \omega_{\varepsilon l2p} - \omega_0$ is the corresponding detuning with $\omega_0 = \omega_{2p1s}$

being the resonance frequency. The S_k quantities are the Stark shifts of the involved levels in the oscillating field ($k = 1s, 2p, \varepsilon l$)

$$S_k = - \sum_{j \neq m} \frac{|\mu_{kj}|^2 \mathcal{E}_0^2}{2} \frac{\omega_{mk}}{\omega_{mk}^2 - \omega^2}. \quad (4)$$

We note that the Stark shift of the continuum is well approximated with the ponderomotive energy $S_{\varepsilon l} = \mathcal{E}_0^2/(4\omega^2)$.

In Eq. (3c), the two-photon Rabi frequency that describes the nonresonant ionization pathway from the ground state (the transition amplitude for ionization from the ground state) is given as

$$M_{\varepsilon l} = - \sum_{j \neq m} \frac{\mu_{m1s} \mu_{\varepsilon l m} \mathcal{E}_0^2}{\omega_{m1s} - \omega}. \quad (5)$$

The so-called leakage-induced complex coupling (LIC coupling) in Eqs. (3a) and (3b) that is an indirect coupling between the resonant $2p$ state and the $1s$ ground state is found as [61]

$$L = 2\pi \left[\frac{1}{8} M_{\varepsilon s}^\dagger \Omega_{\varepsilon s} + \frac{1}{8} M_{\varepsilon d}^\dagger \Omega_{\varepsilon d} \right]. \quad (6)$$

It can be interpreted as follows: the photoelectron emitted from the $1s$ ground state is recaptured by the residual core to produce the resonant $2p$ state; and reversely, the electron emitted from the resonant state can be captured by the residual core, which then becomes the neutral atom in its ground state. The LIC coupling can thus describe the stabilization of the atom that emerges at very strong couplings.

Finally, in Eqs. (3a) and (3b), the ionization rates of the ground (γ) and excited (Γ) states are found in the local approximation [61]

$$\gamma = 2\pi \left[\left| \frac{1}{4} M_{\varepsilon 0s} \right|^2 + \left| \frac{1}{4} M_{\varepsilon 0d} \right|^2 \right], \quad (7)$$

$$\Gamma = 2\pi \left[\left| \frac{1}{2} \Omega_{\varepsilon 0s} \right|^2 + \left| \frac{1}{2} \Omega_{\varepsilon 0d} \right|^2 \right]. \quad (8)$$

The ionization rates in Eqs. (7) and (8) are calculated at the expected energy of the spectrum ($\omega_{\varepsilon 0} = \omega_{1s} + 2\omega_0$), which is indicated by the ε_0 in the indexes.

Equations (3a)–(3c) can be transformed into an alternative yet more instructive form upon redefining the amplitudes as $c_{1s}(t) \rightarrow c_{1s}(t) e^{-\frac{i}{2}[S_{1s}+S_{2p}+\Delta]t}$, $c_{2p}(t) \rightarrow c_{2p}(t) e^{-\frac{i}{2}[S_{1s}+S_{2p}-\Delta]t}$, and $c_{\varepsilon l}(t) \rightarrow c_{\varepsilon l}(t) e^{-\frac{i}{2}[S_{1s}+S_{2p}-2\delta-3\Delta]t}$. Applying these transformations, one finds

$$i\dot{c}_{1s}(t) = \left[-\frac{1}{2}\delta S - \frac{1}{2}\Delta - \frac{i}{2}\gamma \right] c_{1s}(t) + \left[\frac{1}{2}\Omega - \frac{i}{2}L \right] c_{2p}(t) \quad (9a)$$

$$i\dot{c}_{2p}(t) = \left[\frac{1}{2}\Omega^\dagger - \frac{i}{2}L^\dagger \right] c_{1s}(t) + \left[\frac{1}{2}\delta S + \frac{1}{2}\Delta - \frac{i}{2}\Gamma \right] c_{2p}(t) \quad (9b)$$

$$\begin{aligned} i\dot{c}_{\varepsilon l}(t) = & \frac{1}{2}\Omega_{\varepsilon l} c_{2p}(t) + \frac{1}{4}M_{\varepsilon l} c_{1s}(t) \\ & + \left[\delta + \frac{3}{2}\Delta - \frac{1}{2}\Sigma + S_{\varepsilon l} \right] c_{\varepsilon l}(t) \quad (l = s, d), \end{aligned} \quad (9c)$$

where the sum of Stark shifts $\Sigma = S_{1s} + S_{2p}$ and the relative Stark shift $\delta S = S_{2p} - S_{1s}$ parameters have been defined. Since the validity of the applied approximations is well justified, Eqs. (9a)–(9c)—widely utilized in the community [4,34,36,38–40,42,43,61]—can accurately describe the 1 + 1 photon REMPI process under consideration for various laser parameter values. This has also been verified previously by comparison with the full solution of the TDSE [39]. To calculate the $c_{\varepsilon l}(t)$ continuum amplitudes at a later section, we write the formal solution of Eq. (9c) with the initial condition $c_{\varepsilon l}(0) = 0$ as (for more details, see Sec. 1.5 of Ref. [62])

$$c_{\varepsilon l}(t) = -ie^{-i[\delta + \frac{3}{2}\Delta - \frac{1}{2}\Sigma + S_{\varepsilon l}]t} \times \int_0^t \left\{ \frac{1}{2}\Omega_{\varepsilon l}c_{2p}(t') + \frac{1}{4}M_{\varepsilon l}c_{1s}(t') \right\} \times e^{i[\delta + \frac{3}{2}\Delta - \frac{1}{2}\Sigma + S_{\varepsilon l}]t'} dt'. \quad (10)$$

Having the explicit expressions for the $c_{1s}(t)$ and $c_{2p}(t)$ amplitudes, it is possible to evaluate the above integral exactly analytically, as will be demonstrated in Sec. IV. Finally, one can find the exact spectrum formula at the end of the interaction (at $t = T$) as $\sigma(\omega_{\varepsilon l} = \omega_{\varepsilon 0} + \delta) = |c_{\varepsilon l}(T)|^2$, where $\omega_{\varepsilon 0} = \omega_{1s} + 2\omega_0$ is the nominal spectrum position expected in the weak-field limit in the case of resonant driving ($\omega = \omega_0$).

III. EXTENSION OF THE RABI MODEL

In this section, we will treat the two-level problem in general to arrive at the most comprehensive solution for the population amplitudes under rectangular pulse driving. The below general treatment involves as specific cases (1) the traditional SRM and (2) the more advanced perturbative model of Eqs. (9a) and (9b). The connections to these two particular cases, which are useful for practical applications, will be also pointed out.

The general form of the time-dependent Schrödinger equation for two-level systems under rectangular pulse driving [such as given by Eq. (1)] can be written as [51,63–65]

$$i \begin{bmatrix} \dot{a}_{1s}(t) \\ \dot{a}_{2p}(t) \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} a_{1s}(t) \\ a_{2p}(t) \end{bmatrix}. \quad (11)$$

Here, the $h_{kl}(k, l = 1, 2)$ generally complex-valued constant parameters of the system are related to Rabi frequencies, energy level shifts, or loss (ionization) rates, as discussed around Eqs. (9a)–(9c) on a concrete example. For example, in the case of the traditional SRM, these parameters are simply given as $h_{11} = -h_{22} = -\Delta/2$ and $h_{21} = h_{12} = \Omega/2$, and the solutions are [65]

$$b_{1s}(t) = \cos(Wt/2) + i\frac{\Delta}{W} \sin(Wt/2), \quad (12a)$$

$$b_{2p}(t) = -i\frac{\Omega}{W} \sin(Wt/2), \quad (12b)$$

where $W = \sqrt{\Omega^2 + \Delta^2}$ is the Rabi splitting. It is of great interest for practical applications how the solutions of Eq. (11) can be given in the general case when the h_{kl} parameters contain arbitrary higher-order terms in the field amplitude $\mathcal{E}_0$, describing various physical effects. To extend Eqs. (12a) and (12b) to such a general situation, we make use of the

picture of decoupled resonances [34], which is an illustrative way of elucidating REMPI phenomena. Owing to the (near-)resonant coupling with the field, two decaying dressed states are formed, the (complex-valued) quasienergies of which are found upon diagonalizing the Hamiltonian matrix on the right side of Eq. (11):

$$E_{\pm} = \frac{h_{11} + h_{22}}{2} \pm \frac{1}{2}\sqrt{(h_{11} - h_{22})^2 + 4h_{21}h_{12}}. \quad (13)$$

The real parts of $E_{\pm}$ that are shifted in opposite directions [see the $\pm$ sign in Eq. (13)] cover the energy regions of the upper (“+”) and lower (“−”) dressed states. The energy separation $E_+ - E_-$ obtained from Eq. (13) is the generalization of the usual W Rabi splitting, discussed after Eq. (12b). Finally, the imaginary parts of $E_{\pm}$ are related to the decay rates of the emerging dressed states.

As the solutions of the traditional SRM in Eqs. (12a) and (12b) are superpositions of the complex exponentials $e^{-iE_{\pm}t}$ (note that $E_{\pm} = \pm W/2$ holds in that case), we also write the general $a_{1s}(t)$ and $a_{2p}(t)$ solutions of Eq. (11) as superpositions of such complex exponentials, but now using Eq. (13). Imposing the initial conditions $a_{1s}(t=0) = 1$, $a_{2p}(t=0) = 0$, the population amplitudes in the general case are written as (see Sec. 3.4 of Ref. [63] or Sec. 3.10 of Ref. [64] for details)

$$a_{1s}(t) = \text{Pe}^{-iE_+t} + \text{Re}^{-iE_-t} \quad (14a)$$

$$a_{2p}(t) = Q[e^{-iE_+t} - e^{-iE_-t}]. \quad (14b)$$

The expressions for P , $R = 1 - P$, and Q can be found upon substituting Eqs. (14a) and (14b) into Eq. (11), after which one finds

$$a_{1s}(t) = \frac{h_{11} - E_-}{E_+ - E_-} e^{-iE_+t} + \frac{E_+ - h_{11}}{E_+ - E_-} e^{-iE_-t} \quad (15a)$$

$$a_{2p}(t) = \frac{h_{21}}{E_+ - E_-} [e^{-iE_+t} - e^{-iE_-t}]. \quad (15b)$$

Equations (15a) and (15b) with Eqs. (11) and (13) are the generalizations of the traditional SRM to the case when the Hamiltonian contains arbitrary higher-order terms. In the concrete example of 1 + 1 photon REMPI discussed in Sec. II, terms up to fourth order in $\mathcal{E}_0$ are included. Specifically, the h_{kl} parameters of the Hamiltonian in Eqs. (9a) and (9b) read as (note that $\Omega^{\dagger} = \Omega$)

$$h_{11} = -\frac{1}{2}\delta S - \frac{1}{2}\Delta - \frac{i}{2}\gamma, \quad (16a)$$

$$h_{22} = +\frac{1}{2}\delta S + \frac{1}{2}\Delta - \frac{i}{2}\Gamma, \quad (16b)$$

$$h_{21} = \frac{1}{2}\Omega - \frac{i}{2}L^{\dagger}, \quad (16c)$$

$$h_{12} = \frac{1}{2}\Omega - \frac{i}{2}L. \quad (16d)$$

Furthermore, the quasienergies are found upon substituting Eqs. (16a)–(16d) into Eq. (13) and doing some algebra,

$$E_{\pm} = \left\{ \pm \frac{1}{2} \sqrt{\frac{\alpha + \sqrt{\alpha^2 + \beta^2}}{2}} \right\} + i \left\{ -\frac{\Gamma + \gamma}{4} \pm \text{sgn}[\beta] \frac{1}{2} \sqrt{\frac{-\alpha + \sqrt{\alpha^2 + \beta^2}}{2}} \right\}. \quad (17)$$

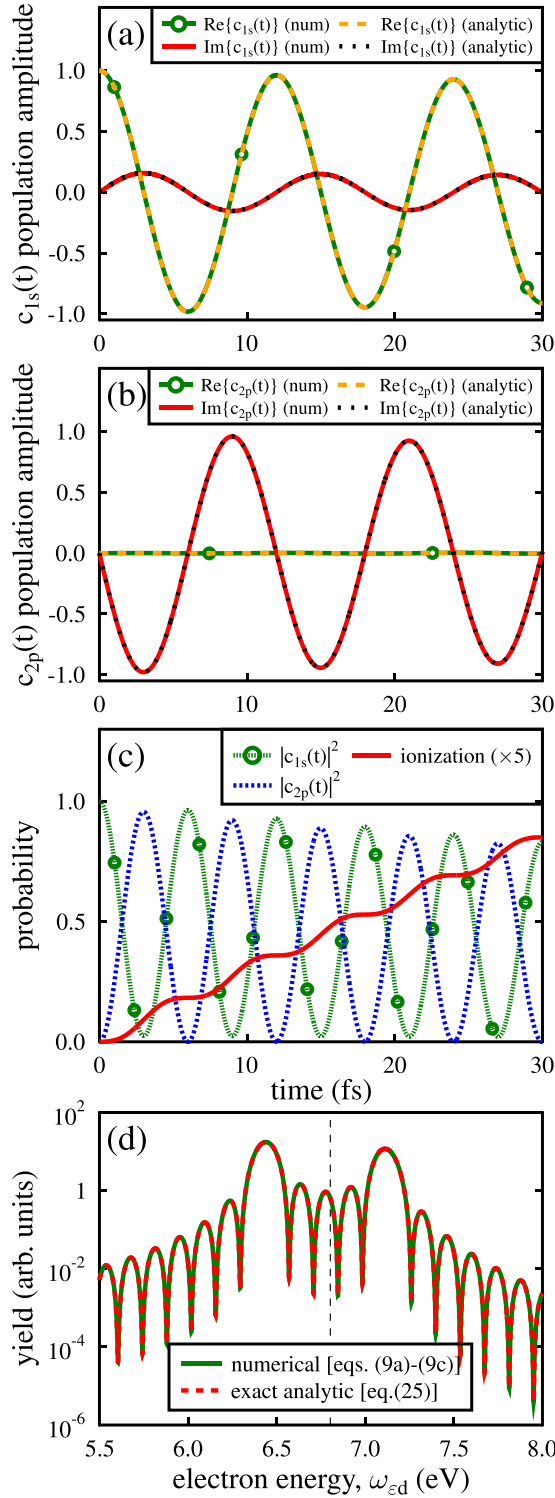


FIG. 2. Time-dependent population amplitudes of the $1s$ ground (a) and $2p$ excited (b) states of hydrogen, obtained numerically via Eqs. (9a)–(9c) and analytically from Eqs. (15a) and (15b). (c) Time-dependent populations of the $1s$ and $2p$ levels, and total ionization probability obtained as $1 - |c_{1s}(t)|^2 - |c_{2p}(t)|^2$ (note that for better visibility the latter is multiplied by five). (d) Photoelectron spectra of H in the d channel, after $1 + 1$ photon resonance-enhanced ionization, obtained numerically via Eqs. (9a)–(9c) and analytically from Eq. (25). The photon energy is set to resonance with the $1s$ - $2p$ transition ($\omega = 0.375$ au), and the remaining laser parameters are chosen such that five Rabi oscillations are generated: $I_0 =$

In Eq. (17), $\alpha = \Omega^2 + (\delta S + \Delta)^2 - |L|^2 - (\Gamma - \gamma)^2/4$, $\beta = -(\delta S + \Delta)(\Gamma - \gamma) - 2\Omega \text{Re}\{L\}$, while $\text{sgn}[\beta] = +1$ for $\beta \geq 0$ and $\text{sgn}[\beta] = -1$ for $\beta < 0$.

To demonstrate the applicability of Eqs. (15a) and (15b), we consider the extended SRM with the Hamiltonian parameters given in Eqs. (16a)–(16d) (see also Table I) and the quasienergies given in Eq. (17). As seen in Figs. 2(a)–2(c), the population amplitudes and populations obtained numerically via Eqs. (9a)–(9c) perfectly agree with their exact analytic counterparts given by Eqs. (15a) and (15b). There the field parameter values are chosen to remain consistent with related studies [18,28,34,36,37,43,44,54,55] such that five Rabi oscillations are generated between the $1s$ and $2p$ levels, $\omega = 0.375$ au, $I_0 = 3.96 \times 10^{13}$ W/cm², and $T = 30$ fs. As the photon energy is set to exact resonance ($\Delta = 0$) and the relative DSS of the coupled levels is rather small, the contrast of these ROs is high [see Fig. 2(c)]. For the considered laser intensity, the strength of the ionization rate from the excited state is substantially larger than that from the ground state ($\Gamma = 2.95 \times 10^{-4}$ au, $\gamma = 3.36 \times 10^{-6}$ au). Therefore, ionization predominantly occurs when the atom is in the excited state. This is demonstrated in Fig. 2(c), where the ionization curve features five ramp-ups around the times when $|c_{2p}(t)|^2$ is maximal. We notice here that the ionization curve in Fig. 2(c) is not completely flat around the times when the atom is in the ground state. Although the small amount of ionization occurring when $|c_{1s}(t)|^2$ is maximal is hardly visible in Fig. 2(c), it will have a clear fingerprint in the photoelectron spectrum, to be discussed in the next section.

Although we considered here single photon transitions and $1 + 1$ photon REMPI as a concrete example, the application of the found generalizations in Eqs. (15a), (15b), (11), and (13) to multiphoton transitions is simple (keeping in mind the appropriate selection rules). For example, to study two-photon transitions and $2 + 1$ photon REMPI within the extended SRM, one just replaces the h_{kl} Hamiltonian matrix elements with the corresponding appropriate expressions [42,66].

Finally, we notice that the simplicity of the found solutions in Eqs. (15a) and (15b) is identical to that of the traditional SRM in Eqs. (12a) and (12b). This will allow us to calculate the $c_{el}(t)$ amplitudes [Eq. (10)], and hence the spectrum, exactly analytically in the next section.

IV. ANALYTIC SPECTRUM RESULTS

Utilizing the explicit integral expression obtained for the $c_{el}(t)$ continuum amplitudes [Eq. (10)], the energy spectrum of photoelectrons at the end of the interaction (at $t = T$) can be calculated according to $\sigma(\omega_{el} = \omega_{e0} + \delta) = |c_{el}(T)|^2$. The temporal integral inside $c_{el}(t)$ can be evaluated exactly only in a very few cases [45]. Usually, some approximate methods are employed to arrive at simple final spectrum formulas that provide a pictorial explanation for the origin of the spectral patterns [40]. As the spectrum shape varies sensitively

←
 3.96×10^{13} W/cm², $T = 30$ fs. The vertical dashed line indicates the nominal spectrum position expected in the weak-field limit $\omega_{e0} = \omega_{1s} + 2\omega_0$.

with the system parameters, different approximations—e.g., neglecting certain terms—might lead to improper interpretations. The simple analytic forms of the $c_{1s}(t)$ and $c_{2p}(t)$ amplitudes found in Sec. III [denoted there generally as $a_{1s}(t)$ and $a_{2p}(t)$] allow one to evaluate Eq. (10) exactly analytically, providing additional insight into the buildup of the Autler-Townes doublet under various simultaneous physical effects. Below, we will carry out such an analysis making use of the results of Secs. II and III.

A. Derivation of the spectrum formula

To start the discussion, in Eq. (10) we introduce the notation

$$K = \delta + \frac{3}{2}\Delta - \frac{1}{2}\Sigma + S_{\varepsilon l}. \quad (18)$$

Physically, the energy factor K describes how the continuum states are shifted due to the frequency detuning of the laser as well as the Stark shifts of the bound and continuum levels.

To proceed further, we write the P , $R = 1 - P$, and Q factors of the $c_{1s}(t)$ and $c_{2p}(t)$ amplitudes [explicitly given in Eqs. (14a), (14b), (15a), and (15b)] in Euler form:

$$P = \frac{h_{11} - E_-}{E_+ - E_-} = |P|e^{i\tau_P}, \quad (19a)$$

$$R = \frac{E_+ - h_{11}}{E_+ - E_-} = |R|e^{i\tau_R}, \quad (19b)$$

$$Q = \frac{h_{21}}{E_+ - E_-} = |Q|e^{i\tau_Q}, \quad (19c)$$

where the $\tau_z = \arg\{z\}$ ($z = P, R, Q$) phases have been introduced. Furthermore, we also write the bound-to-continuum couplings in Euler form $\Omega_{\varepsilon l} = |\Omega_{\varepsilon l}|e^{i\varphi_{\varepsilon l}}$ and $M_{\varepsilon l} = |M_{\varepsilon l}|e^{i\phi_{\varepsilon l}}$, denoting the corresponding phases as $\varphi_{\varepsilon l} = \arg\{\Omega_{\varepsilon l}\}$ and $\phi_{\varepsilon l} = \arg\{M_{\varepsilon l}\}$, respectively. Incorporating the above notations in Eq. (10), the $c_{\varepsilon l}(t)$ continuum amplitude is written as

$$\begin{aligned} c_{\varepsilon l}(t) = & -ie^{-iKt} \int_0^t \left[\left\{ \frac{1}{2} |\Omega_{\varepsilon l}| |Q| e^{i(\varphi_{\varepsilon l} + \tau_Q)} \right. \right. \\ & + \frac{1}{4} |M_{\varepsilon l}| |P| e^{i(\phi_{\varepsilon l} + \tau_P)} \left. \right\} e^{i(K-E_+)t'} \\ & + \left\{ -\frac{1}{2} |\Omega_{\varepsilon l}| |Q| e^{i(\varphi_{\varepsilon l} + \tau_Q)} \right. \\ & + \frac{1}{4} |M_{\varepsilon l}| |R| e^{i(\phi_{\varepsilon l} + \tau_R)} \left. \right\} e^{i(K-E_-)t'} \left. \right] dt'. \quad (20) \end{aligned}$$

For the sake of brevity and generality, we introduce the notation for the complex quantities inside the braces:

$$\lambda_+ = \frac{1}{2} |\Omega_{\varepsilon l}| |Q| e^{i(\varphi_{\varepsilon l} + \tau_Q)} + \frac{1}{4} |M_{\varepsilon l}| |P| e^{i(\phi_{\varepsilon l} + \tau_P)}, \quad (21a)$$

$$\lambda_- = -\frac{1}{2} |\Omega_{\varepsilon l}| |Q| e^{i(\varphi_{\varepsilon l} + \tau_Q)} + \frac{1}{4} |M_{\varepsilon l}| |R| e^{i(\phi_{\varepsilon l} + \tau_R)}. \quad (21b)$$

These $\lambda_{\pm}$ factors—which are related to the spectral contributions of the upper (+) and lower (−) dressed states—will also be written in Euler form at a later stage $\lambda_{\pm} = |\lambda_{\pm}|e^{i\eta_{\pm}}$, denoting the corresponding phases as $\eta_{\pm} = \arg\{\lambda_{\pm}\}$. As will be demonstrated below, the spectrum heights at the lower (ω_-) and upper (ω_+) dressed-state energies in the AT doublet are

related to the $|\lambda_-|^2$ and $|\lambda_+|^2$ quantities, respectively. Since the $\lambda_{\pm}$ factors are given as the coherent sums of the one- and two-photon contributions [see Eqs. (21a) and (21b)], not only the magnitudes but also the τ_P, τ_R, τ_Q phases of the bound terms, and the $\varphi_{\varepsilon l}, \phi_{\varepsilon l}$ phases of the bound-to-continuum terms are expected to play an important role in the actual values of $|\lambda_{\pm}|^2$, and hence in the electron yields at the two prominent peaks of the AT doublet. This allows one to capture the fingerprint of the competition of resonant and nonresonant ionization pathways via the asymmetry of the AT doublet, as will be clarified below.

Evaluating the simple integrals in Eq. (20), one finds

$$c_{\varepsilon l}(t) = -ie^{-iKt} \left[\lambda_+ \frac{e^{i(K-E_+)t} - 1}{i(K-E_+)} + \lambda_- \frac{e^{i(K-E_-)t} - 1}{i(K-E_-)} \right]. \quad (22)$$

Next, we introduce the notations for the real $\mathcal{R}_{\pm} = \text{Re}\{E_{\pm}\}$ and imaginary $\mathcal{J}_{\pm} = \text{Im}\{E_{\pm}\}$ parts of the complex quasienergies $E_{\pm} = \mathcal{R}_{\pm} + i\mathcal{J}_{\pm}$ and write the $K - E_{\pm}$ energy terms in the denominators of Eq. (22) in Euler forms

$$K - E_{\pm} = A_{\pm} e^{i\Theta_{\pm}}, \quad (23a)$$

$$A_{\pm} = \sqrt{(K - \mathcal{R}_{\pm})^2 + (-\mathcal{J}_{\pm})^2}, \quad (23b)$$

$$\Theta_{\pm} = \text{atan2}(-\mathcal{J}_{\pm}, K - \mathcal{R}_{\pm}). \quad (23c)$$

Here, the $\text{atan2}(y, x)$ function returns the phase of the complex number $z = x + iy$ in the interval $[-\pi, +\pi]$. As the real parts of Eq. (23a) $K - \mathcal{R}_{\pm}$ change sign at the $\delta_{\pm} = \mathcal{R}_{\pm} - \frac{3}{2}\Delta + \frac{1}{2}\Sigma - S_{\varepsilon l}$ energy positions [see Eq. (18) for the definition of $\tilde{K}$], or alternatively, at the $\omega_{\pm}$ dressed-state energies in the continuum

$$\omega_{\pm} = \omega_{\varepsilon 0} + \delta_{\pm} = \omega_{\varepsilon 0} + \mathcal{R}_{\pm} - \frac{3}{2}\Delta + \frac{1}{2}\Sigma - S_{\varepsilon l}, \quad (24)$$

the $\Theta_{\pm}$ phases in Eq. (23c) steeply drop down from $\sim\pi$ to ~ 0 when the increasing δ passes through $\delta_{\pm}$ (see also Appendix B). This renders the $\text{atan2}(y, x)$ function more appropriate than the usual $\text{atan}(y/x)$, which returns phases inside $[-\pi/2, +\pi/2]$.

Finally, taking the absolute square of Eq. (22) at the end of the interaction $t = T$ and doing some simple algebra, the spectrum in the l angular momentum channel $\sigma(\omega_{\varepsilon l} = \omega_{\varepsilon 0} + \delta) = |c_{\varepsilon l}(T)|^2$ is found to be the sum of three contributions according to the three braces

$$\begin{aligned} \sigma(\omega_{\varepsilon l}) = & \sigma_+(\omega_{\varepsilon l}) + \sigma_-(\omega_{\varepsilon l}) + \sigma_{+-}(\omega_{\varepsilon l}) \\ = & \frac{|\lambda_+|^2}{A_+^2} \{e^{2\mathcal{J}_+ T} - 2e^{\mathcal{J}_+ T} \cos[(K - \mathcal{R}_+)T] + 1\} \\ & + \frac{|\lambda_-|^2}{A_-^2} \{e^{2\mathcal{J}_- T} - 2e^{\mathcal{J}_- T} \cos[(K - \mathcal{R}_-)T] + 1\} \\ & + \frac{2|\lambda_+||\lambda_-|}{A_+ A_-} \{e^{(\mathcal{J}_+ + \mathcal{J}_-)T} \cos[\Delta\eta - \Delta\Theta - \Delta\mathcal{R}T] \\ & - e^{\mathcal{J}_+ T} \cos[\Delta\eta - \Delta\Theta + (K - \mathcal{R}_+)T] \\ & - e^{\mathcal{J}_- T} \cos[\Delta\eta - \Delta\Theta - (K - \mathcal{R}_-)T] \\ & + \cos[\Delta\eta - \Delta\Theta]\}. \quad (25) \end{aligned}$$

TABLE I. Parameter values of the two-level model of hydrogen, presented in Eqs. (9a)–(9c) (without the $\mathcal{E}_0$ dependence). The parameter values were obtained with the transition energies and dipoles taken from Ref. [39], setting the laser frequency to exact resonance for the $1s \rightarrow 2p$ transition ($\omega = 0.375$ au ≈ 10.204 eV). Γ , γ , and L were evaluated within the local approximation.

Parameter	Value (au)
$\Omega/\mathcal{E}_0 = \mu_{1s2p}$	0.74494
$\Omega_{es}/\mathcal{E}_0 = \mu_{es2p}$	$0.14703 - i \times 0.02960$
$\Omega_{ed}/\mathcal{E}_0 = \mu_{ed2p}$	$-0.07482 - i \times 0.37199$
$M_{es}/\mathcal{E}_0^2$	$-1.04442 + i \times 0.21029$
$M_{ed}/\mathcal{E}_0^2$	$0.46567 + i \times 2.31511$
$L/\mathcal{E}_0^3$	$-0.82925 - i \times 5.96 \times 10^{-8}$
$S_{1s}/\mathcal{E}_0^2$	-0.85488
$S_{2p}/\mathcal{E}_0^2$	2.73562
$S_{el}/\mathcal{E}_0^2$	1.77778
$\gamma/\mathcal{E}_0^4$	2.63564
$\Gamma/\mathcal{E}_0^2$	0.26149

Here, the phase differences $\Delta\Theta = \Theta_+ - \Theta_-$ and $\Delta\eta = \eta_+ - \eta_-$ as well as the real part of the generalized Rabi splitting $\Delta\mathcal{R} = \mathcal{R}_+ - \mathcal{R}_-$ have been introduced. In Eq. (25), the first (second) term $[\sigma_+(\sigma_-)]$ describes the spectral contribution of the upper (lower) dressed state. Furthermore, the last term σ_{+-} can be interpreted as the interference of the upper and lower dressed-state contributions. As seen in Fig. 2(d), the exact analytic spectrum formula Eq. (25) perfectly reproduces the electron yield through several orders of magnitude in a wide energy region, provided by the numerical solution of Eqs. (9a)–(9c). Throughout the paper, spectra in the d channel are calculated. The shapes of the spectra in the s channel are very similar (not shown here) but their intensities are much lower due to the much weaker coupling to the s continua than to the d continua (see Table I). Furthermore, for the sake of clarity, the energy dependence of the Ω_{el} and M_{el} bound-to-continuum couplings inside the $\lambda_{\pm}$ factors will be neglected, and the values of these quantities at the nominal spectrum position $\omega_{e0} = \omega_{1s} + 2\omega_0$ will be taken, as indicated in Table I. The energy dependence of these quantities in the continuum leads to further asymmetry of the AT doublet, as demonstrated in Appendix A.

B. Analysis of the buildup of the spectrum

To gain deeper insight into the buildup of the spectrum, we analyze the impact of the different spectral components σ_+ , σ_- , and σ_{+-} in Fig. 3, where the same laser parameter values are utilized as in Fig. 2. We notice here that in Eq. (25) the K , $A_{\pm}$, and $\Delta\Theta$ factors explicitly depend on δ , while the implicit δ dependence of the $\lambda_{\pm}$ and $\Delta\eta$ factors is neglected as mentioned above (see Appendix A for their impact on the spectral shape). All three spectral components σ_+ , σ_- , and σ_{+-} contain the product of a δ -dependent oscillatory term (the brace terms) and a corresponding δ -dependent prefactor. Let us start with the analysis of the prefactors that primarily determine the weights of the σ_k components. As

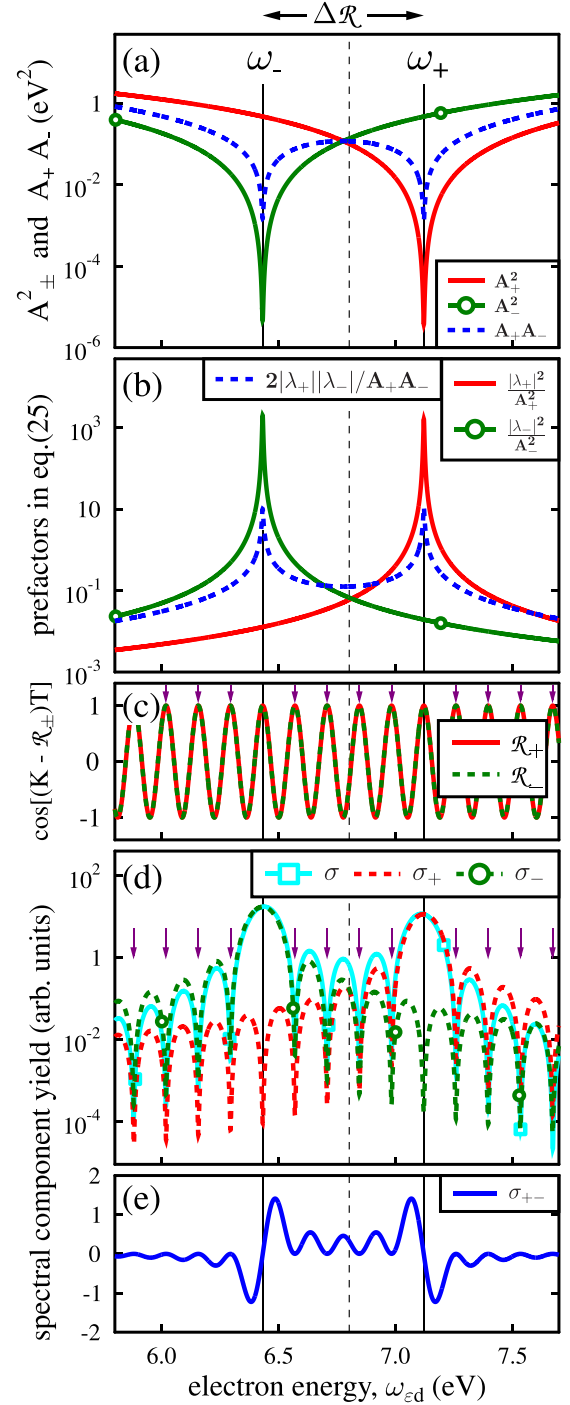


FIG. 3. Analysis of the energy dependence of the three spectral components σ_+ , σ_- (d) and σ_{+-} (e) of the exact spectrum formula in the d channel [Eq. (25)]. (b) Prefactors of these spectral components, and their denominators (a), exhibiting sharp peaks around the dressed-state energies $\omega_{\pm}$ indicated by the vertical solid lines. (c) Oscillatory cos terms of the σ_+ and σ_- components. The vertical arrows indicate the energy positions where the cos terms are maximal and the σ_+ and σ_- components are minimal (the two main peaks of the AT doublet at $\omega_{\pm}$ are exceptions, see text for details). The applied laser parameter values are the same as in Fig. 2. The vertical dashed lines indicate the nominal spectrum position $\omega_{e0} = \omega_{1s} + 2\omega_0$. $\Delta\mathcal{R}$ is the Rabi splitting.

TABLE II. Relevant parameter values appearing—either explicitly or implicitly—in the exact spectrum formula Eq. (25). The presented values along with those in Table I are useful supplements for the spectrum analysis presented in Fig. 3. The applied laser parameter values are $\omega = 0.375$ au ≈ 10.204 eV, and $I_0 = 3.96 \times 10^{13}$ W/cm². The presented values of $|\lambda_{\pm}|$ and $\eta_{\pm}$ are calculated for the d channel.

Parameter	Value	Parameter	Value
$ P $	0.420	ω_{e_0}	6.803 eV
$ R $	0.580	ω_+	7.122 eV
$ Q $	0.494	ω_-	6.432 eV
τ_P	0.006 90 rad	$\mathcal{R}_{\pm}$	± 0.345 eV
τ_R	-0.005 00 rad	$\mathcal{J}_+$	-0.001 93 eV
τ_Q	0.000 95 rad	$\mathcal{J}_-$	-0.002 14 eV
φ_{es}	-0.199 rad	$ \lambda_+ $	0.077 98 eV
φ_{ed}	-1.769 rad	$ \lambda_- $	0.096 11 eV
ϕ_{es}	2.943 rad	η_+	-1.769 rad
ϕ_{ed}	1.372 rad	η_-	1.373 rad

seen in Fig. 3(b), these prefactors exhibit sharp peaks at the dressed-state energies $\omega_{\pm}$, which is a consequence of the fact that their denominators possess sharp negative peaks at those energies [see Fig. 3(a)]. The values of these denominators at $\omega_{\pm}$ are related to the ionization rates of the dressed states $A_{\pm}(\delta_{\pm}) = |\mathcal{J}_{\pm}|$, which are usually small (see Table II). As a result, the prefactor of $\sigma_+(\sigma_-)$ becomes dominant at the $\omega_+(\omega_-)$ position; furthermore, that of σ_{+-} remains more than two orders of magnitude smaller at $\omega_{\pm}$ [Fig. 3(b)]. Thus, σ_{+-} is expected to have the largest impact in between and outside the two prominent peaks of the AT doublet, but not at the exact peak positions. Inserting the concrete system parameter values from Table II, one finds that the prefactor of $\sigma_-(\omega_-)$ is larger than that of $\sigma_+(\omega_+)$, $|\lambda_-|^2/\mathcal{J}_-^2 > |\lambda_+|^2/\mathcal{J}_+^2$. This suggests that the left peak of the AT doublet is higher than the right one, which is indeed the case as seen in Fig. 3(d) (better seen on a linear vertical scale in Fig. 6 of Appendix A).

The height of the spectrum is further modified by the oscillatory brace terms inside σ_+ , σ_- , and σ_{+-} . The values of these spectral contributions at the prominent AT peak positions $\omega_{\pm}$ can be found from Eq. (25) upon invoking the definition of $\Delta\mathcal{R}$ and considering that $K - \mathcal{R}_{\pm} = 0$ holds at $\omega_{\pm}$; furthermore, $\Delta\mathcal{R}T = 5 \times 2\pi$, as an integer number of Rabi cycles is completed. With these, one finds the spectral contributions at the main AT peaks

$$\sigma_+(\omega_+) = \frac{|\lambda_+|^2}{\mathcal{J}_+^2} \{1 - e^{\mathcal{J}_+T}\}^2, \quad (26a)$$

$$\sigma_-(\omega_-) = \frac{|\lambda_-|^2}{\mathcal{J}_-^2} \{1 - e^{\mathcal{J}_-T}\}^2, \quad (26b)$$

$$\begin{aligned} \sigma_{+-}(\omega_{\pm}) &= \frac{2|\lambda_+||\lambda_-|}{|\mathcal{J}_+||\mathcal{J}_-|} \cos[\Delta\eta(\omega_{\pm}) - \Delta\Theta(\omega_{\pm})] \\ &\times \{e^{(\mathcal{J}_+ + \mathcal{J}_-)T} - e^{\mathcal{J}_+T} - e^{\mathcal{J}_-T} + 1\}, \end{aligned} \quad (26c)$$

where $\Delta\Theta(\omega_{\pm}) \approx \pi/2$, and $\Delta\eta(\omega_{\pm}) \approx -\pi$ as demonstrated in Appendix B. With these phase difference values, the contributions $\sigma_{+-}(\omega_{\pm})$ are practically zero [see also Fig. 3(e)], showing that the AT peak heights are solely determined by $\lambda_{\pm}$ and $\mathcal{J}_{\pm}$, via the $\sigma_+(\omega_+)$ and $\sigma_-(\omega_-)$ factors in Eqs. (26a) and (26b). This is also apparent from Fig. 3(d), where the σ_+ and σ_- contributions almost perfectly overlap with the total spectrum σ (solid line with open squares) around ω_+ and ω_- , respectively.

We notice here that the small σ_{+-} contribution oscillates with a phase difference $\Delta\eta - \Delta\Theta$ compared to σ_+ and σ_- . Due to the sudden jumps of $\Delta\eta - \Delta\Theta$ around $\omega_{\pm}$ (see Appendix B), σ_{+-} changes sign at the dressed-state energies, increasing the electron yield in between the main AT peaks, and decreasing it outside [see Fig. 3(e)]. In the case of non-integer completed Rabi cycles, Eq. (26c) is not applicable, suggesting that the σ_{+-} term has some nonzero contribution to the main AT peaks at $\omega_{\pm}$. As will be clear from Figs. 4 and 5 below, this causes some very slight modification in the

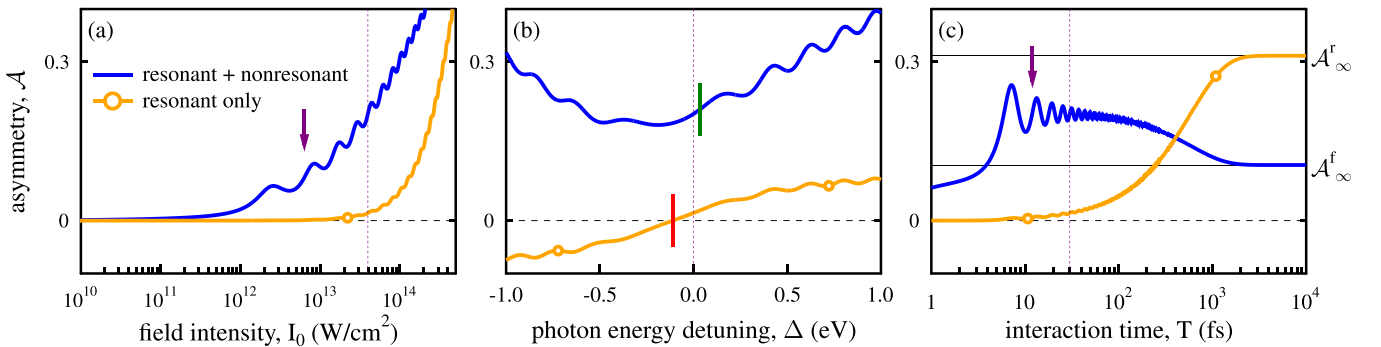


FIG. 4. Asymmetry of the Autler-Townes doublet calculated according to Eq. (28) for different field intensities (a), photon energies (b), and interaction times (c). The solid blue lines are obtained from Eq. (25) as it stands, incorporating the interference of resonant and nonresonant pathways. The solid orange lines with open circles are obtained from Eq. (25) upon omitting the nonresonant pathway ($\gamma = M_{el} = L = 0$). The oscillations of the curves are caused by the σ_{+-} interference term of Eq. (25). The vertical dashed lines indicate the parameter values applied in Figs. 2 and 3. The vertical arrows in (a) and (c) indicate the parameter values where two Rabi oscillations are completed, while the vertical bars in (b) show the detunings at which $\mathcal{J}_+ = \mathcal{J}_-$ (the red one for the resonant case and the green one for the resonant + nonresonant case). The horizontal solid lines in (c) show the asymptotic ($T \rightarrow \infty$) asymmetry values predicted by Eqs. (26a) and (26b). In each panel, the remaining fixed laser parameter values are the same as in Figs. 2 and 3.

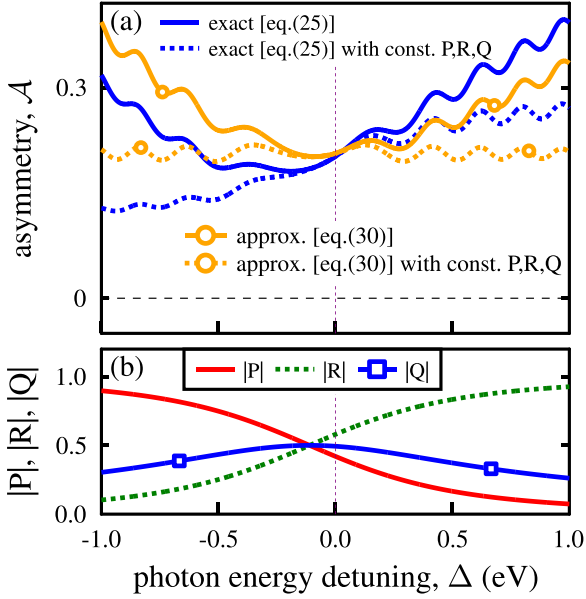


FIG. 5. (a) Photon energy dependence of the asymmetry [Eq. (28)] of the Autler-Townes doublet, considering the interference of the resonant and nonresonant pathways. Values obtained with the exact spectrum formula [Eq. (25)] are shown by blue. Values obtained with the approximate spectrum formula [Eq. (30)]—that neglects J_+ and J_- —are shown by orange. The broken curves show asymmetry values that are obtained with constant P , R , and Q bound terms inside $\lambda_{\pm}$, taking their values at $\Delta = 0$. The oscillations of the curves are caused by the σ_{+-} interference term of the upper and lower dressed-state contributions. (b) Photon energy dependence of the $|P|$, $|R|$, and $|Q|$ bound term magnitudes inside $\lambda_{\pm}$ of the spectrum formulas (the corresponding phases are very small and almost independent of Δ). The laser intensity and duration values are the same as in Figs. 2 and 3.

asymmetry of the AT doublet when the number of completed ROs is noninteger.

C. The multipeak pattern of the AT doublet

Now, let us inspect the multipeak pattern of the spectrum, seen in Fig. 3(d). As is clear from Eq. (25), the oscillations of the spectrum intensity are governed by the $\cos[(K - \mathcal{R}_{\pm})T]$ functions inside the brace terms of σ_+ and σ_- (note that σ_{+-} is small). For the actual values of the laser parameters, the phase difference between these $\cos[(K - \mathcal{R}_{\pm})T]$ functions is an integer multiple of 2π [$\Delta\mathcal{R}T = 5 \times 2\pi$]. Thus, these functions completely overlap [see also Fig. 3(c)], and the corresponding σ_+ and σ_- contributions oscillate in sync with each other [see Fig. 3(d)]. The minimal-intensity energy positions in the spectrum are expected when the $\cos[(K - \mathcal{R}_{\pm})T]$ terms are maximal, making the braces of σ_+ and σ_- minimal [67] (see the vertical arrows in Fig. 3). Here, the dressed-state energies are exceptions, as the prefactors of σ_+ and σ_- exhibit sharp peaks around ω_+ and ω_- , respectively [see above and Fig. 3(b)]. These large prefactor values $|\lambda_{\pm}|^2/J_{\pm}^2$ turn the minima of the brace terms into maxima at $\omega_{\pm}$. As a result, the width of the two prominent AT peaks ($\frac{4\pi}{T}$) becomes two times larger than that of the other spectral peaks ($\frac{2\pi}{T}$).

As the arguments of the $\cos[(K - \mathcal{R}_{\pm})T]$ functions accumulate a total phase of $(\delta_+ - \delta_-)T = \Delta\mathcal{R}T = 5 \times 2\pi$ in the interval $\delta \in [\delta_-, \delta_+]$, the AT doublet is expected to possess five oscillations of its intensity between ω_- and ω_+ . Because of the above-mentioned modifying effect of the spiky prefactors of σ_+ and σ_- around ω_+ and ω_- , the five intensity oscillations of the AT doublet occur in the slightly modified interval $[\delta_- - \frac{2\pi}{T}, \delta_+ + \frac{2\pi}{T}]$, see Fig. 3(d).

The number of Rabi oscillations completed by the electron can be found from the $2p$ population via Eq. (14b), keeping only the oscillatory part and omitting the small damping part $|c_{2p}(t)|^2 \approx 4|Q|^2 \sin^2(\frac{\Delta\mathcal{R}t}{2})$. During the interaction with the field $t \in [0, T]$, the argument of the $\sin^2(\frac{\Delta\mathcal{R}t}{2})$ function accumulates a phase of $\frac{\Delta\mathcal{R}T}{2} = 5 \times \pi$, revealing that the electron executes five Rabi floppings. This number happens to be equal to the number of spectral peaks in the doublet found above. Therefore, as a general rule, the number of peaks within the AT doublet equals the number of completed Rabi oscillations under the resonant or near-resonant field.

Finally, we notice that for continuous wave (or monochromatic) driving ($T \rightarrow \infty$), in Eq. (25) the exponential decay terms vanish. As a result, the energy-dependent oscillatory functions (the $\cos$ functions containing K) are eliminated and Eq. (25) is simplified to

$$\sigma^{cw}(\omega_{el}) = \frac{|\lambda_+|^2}{A_+^2} + \frac{|\lambda_-|^2}{A_-^2} + \frac{2|\lambda_+||\lambda_-|}{A_+A_-} \cos[\Delta\eta - \Delta\Theta], \quad (27)$$

which is a simple doublet without any additional multipeak pattern. It is thus obvious that the multipeak spectral pattern of the AT doublet, a clear manifestation of the completed ROs, is a direct consequence of the finite nature of the driving radiation field.

D. Shifting and splitting of the AT doublet

The energy positions of the main AT peaks in the electron continuum are given by Eq. (24), which allows one to quantify the energy splitting and shifting of the AT doublet. In view of the real parts of $E_{\pm}$ in Eq. (17), the $\Delta\mathcal{R} = \mathcal{R}_+ - \mathcal{R}_-$ energy splitting is primarily determined by the Ω Rabi frequency and the Δ detuning, which is slightly modified by the $\delta\mathcal{S}$ relative Stark shift (note that γ , Γ , and L are rather small). Furthermore, the shifting of the AT doublet relative to the nominal spectrum position ω_{e0} is determined by Δ , as well as by the bound and continuum Stark shifts via the Σ and S_{el} terms, respectively, as seen in Eq. (24). For the actual values of the field parameters applied in Fig. 3, $\Delta\mathcal{R} \approx 0.69$ eV splitting and $-\frac{3}{2}\Delta + \frac{1}{2}\Sigma - S_{el} \approx -0.026$ eV shifting values are found. This latter explains why the center of the doublet $\omega_{\text{center}} = (\omega_+ + \omega_-)/2$ (not indicated explicitly here) is positioned slightly below $\omega_{e0} \approx 6.803$ eV (see the vertical lines in Fig. 3). According to the above discussion, the shifting and splitting of the AT doublet are efficiently controlled by the field intensity and detuning parameters. Furthermore, the intensity and detuning dependence of these spectral features allows for the determination of the bound and continuum level shifts, as suggested by the explicit simple formulas Eqs. (24) and (17).

E. Asymmetry of the AT doublet

Numerous recent studies have demonstrated that various physical effects can give rise to the asymmetry of the AT doublet [12,42–46,51]. The exact formula Eq. (25) can provide a simple and unified explanation for the spectrum asymmetry, unfolding its physical origins and controllability. To quantify the asymmetry of the AT doublet, we introduce the asymmetry parameter using Eq. (25):

$$\mathcal{A} = \frac{\sigma(\omega_-) - \sigma(\omega_+)}{\sigma(\omega_-) + \sigma(\omega_+)}. \quad (28)$$

Positive (negative) values of the $\mathcal{A}$ parameter imply that the electron yield at ω_- (ω_+) is larger than at ω_+ (ω_-). Furthermore, $\mathcal{A} = 0$ indicates a completely symmetric AT doublet.

According to the discussion in Sec. IV B, the spectral intensities at the main AT peak positions ω_- and ω_+ can be excellently approximated as $\sigma(\omega_-) \approx \sigma_-(\omega_-)$ and $\sigma(\omega_+) \approx \sigma_+(\omega_+)$, respectively [see Eqs. (26a) and (26b) and their discussion]. We emphasize here that (a) the $\sigma_{+-}(\omega_{\pm})$ interference contributions are rather small and even vanish when the number of completed ROs is integer, and (b) the cross contributions $\sigma_-(\omega_+)$ and $\sigma_+(\omega_-)$ are also negligible (see Fig. 3). Therefore, as suggested by Eqs. (26a) and (26b), the main AT peak heights are predominantly determined by the $\lambda_{\pm}$ interference- and $\mathcal{J}_{\pm}$ decay rate factors. Let us start by inspecting the role of $\lambda_{\pm}$ played in the spectrum asymmetry.

As seen in Eqs. (21a) and (21b), the $\lambda_{\pm}$ factors contain the coherent sums of the one- and two-photon bound-to-continuum Rabi frequencies, multiplied by the P , R , and Q complex quantities of the bound state population amplitudes [Eqs. (19a)–(19c)]. Thus, physically, $\lambda_{\pm}$ describe the quantum interference of the electron amplitudes emitted resonantly from the $2p$ excited state and those ejected non-resonantly from the $1s$ ground state. Such interference was often neglected in earlier investigations [4,34,36,38,40,44,45]. As demonstrated in Fig. 4, the interference of the resonant and nonresonant ionization pathways plays a crucial role in the asymmetry of the AT doublet, which is consistent with recent findings [12,43]. Except for long interaction times when the depletion becomes substantial, the asymmetry of the AT doublet calculated exclusively from the resonant pathway contribution upon setting $\gamma = M_{el} = L = 0$ in Eq. (25) remains below the exact asymmetry given by the interplay of the resonant and nonresonant contributions (see Fig. 4). This is explained as follows. Since the phases of the Ω_{el} and M_{el} ionization amplitudes differ by π (see Appendix A), and the τ_P , τ_R , and τ_Q phases of the bound terms are practically zero (see Table II), the one- and two-photon ionization pathways interfere constructively inside λ_- , and they interfere destructively inside λ_+ . As a result, the electron yield is greatly enhanced at ω_- and suppressed at ω_+ . This leads to an increased value of $\mathcal{A}$, compared to the case when only the resonant contribution is considered inside $\lambda_{\pm}$ and thus no interference is happening. Despite the small value of $|M_{el}|/4$ as compared to $|\Omega_{el}|/2$ [see Fig. 6(a) in Appendix A] and the corresponding small ionization probability from $1s$ [see Fig. 2(c)], the impact of the nonresonant pathway on the spectrum asymmetry is significant. The above enhancement of the spectrum asymmetry is expected to be more pronounced at

stronger light intensities, as the value of $|M_{el}|/4$ gets comparable to that of $|\Omega_{el}|/2$, allowing one to capture the fingerprint of the competition between different ionization pathways. This kind of competition is more favorable to observe, e.g., for helium, as the critical field intensity to satisfy $|M_{el}|/4 = |\Omega_{el}|/2$ is lower than for hydrogen and does not violate the two-level approximation [51].

The total asymmetry is also influenced by the depletion of the atom, as demonstrated in Fig. 4(c) for different T interaction times. As the $\lambda_{\pm}$ interference terms and the $\mathcal{J}_{\pm}$ decay rates do not depend on T , the asymmetry change seen in Fig. 4(c) (apart from the small wiggles) is purely caused by the depletion that is described by the exponential decay factors inside the braces of Eqs. (26a) and (26b). For long interaction times ($T \rightarrow \infty$), the exponential decay terms vanish, and Eqs. (26a) and (26b) predict the asymmetry value

$$\mathcal{A}_{\infty}^{f/r} = \frac{\mathcal{J}_+^2 |\lambda_-|^2 - \mathcal{J}_-^2 |\lambda_+|^2}{\mathcal{J}_+^2 |\lambda_-|^2 + \mathcal{J}_-^2 |\lambda_+|^2}, \quad (29)$$

where “ f ” refers to the full solution, namely, when both the resonant and nonresonant routes are considered, while “ r ” refers to the case when only the resonant pathway is accounted for in $\lambda_{\pm}$. The estimates $\mathcal{A}_{\infty}^{f/r}$ are reached by the exact spectra already for $T \approx 2$ ps [Fig. 4(c)]. Obviously, the competition between the depletion and interference contributions to the total asymmetry, shown in Fig. 4(c) by the solid blue line, cannot be captured considering only the resonant route (orange line with symbols), often done in related studies.

We notice that the magnitudes and phases of the P , R , and Q quantities do not depend on T and have a weak dependence on I_0 in Fig. 4(a). However, the magnitudes $|P|$, $|R|$, and $|Q|$ (but not the corresponding phases) exhibit substantial dependence on the photon energy, as shown in Fig. 5(b). Thus, these bound-state quantities also affect the above-discussed interference induced asymmetry of the AT doublet (via $\lambda_{\pm}$), as presented in Fig. 4(b). Furthermore, as $\mathcal{J}_{\pm}$ also depend on Δ , it is possible to find from Eq. (17) an optimal value $\Delta_{eq} = -2\Omega_R \text{Re}\{L\}/(\Gamma - \gamma) - \delta S$, for which the decay rates of the upper and lower dressed states become equal ($\mathcal{J}_+ = \mathcal{J}_-$). For such detuning value, the asymmetry is exclusively caused by the interference contribution [see the red and green vertical bars in Fig. 4(b)]. This explains why the spectrum becomes symmetric at Δ_{eq} , when only the resonant path is accounted for [see the red vertical bar in Fig. 4(b)].

Finally, we mention that the small wiggles of the asymmetry curves in Fig. 4 (also in Fig. 5 below) are caused by the small σ_{+-} terms. These oscillations are the clear manifestations of the variation of the number of completed ROs. Whenever an additional RO is completed by the electron, a new subpeak is developed in the AT doublet, leading to the slight reshuffling of the electron yields at the main AT peaks. This gives rise to the small fluctuations of the asymmetry of the AT doublet, reflected by the oscillations of the asymmetry curves.

Although the impact of the $\sigma_{+-}(\omega_{\pm})$ interference contributions and the $\sigma_-(\omega_+)$, $\sigma_+(\omega_-)$ cross contributions were found to be small here, these factors are expected to have a larger effect on the spectrum shape at weaker couplings, below the onset of ROs. In such a case, the spectrum contains

a single peak, and $\Delta\mathcal{R}$ is small. As a result, the contrast of the spiky prefactors seen in Fig. 3(b) is suppressed in the interval $[\omega_-, \omega_+]$. Thus, the above contributions may be necessary for an accurate characterization of, e.g., the birth of the AT doublet.

F. Approximate spectrum formula

In many physical situations, e.g., in that considered here, the ionization rates and the LIC coupling are small ($\Gamma, \gamma, |L| \ll |\Omega|$, see Table I.), and hence the magnitudes of the $\mathcal{J}_\pm$ imaginary parts of the quasienergies are also small. This allows for great simplifications in the spectrum formula, which was often exploited in related studies on the analytical treatment of the AT doublet. In fact, these ionization rates were often considered equal, or completely omitted [34,36–38,40]. Upon omitting the Γ, γ, L terms in the two-level model Eqs. (9a) and (9b), the imaginary parts of the dressed-state energies are eliminated ($\mathcal{J}_\pm = 0$), and $E_\pm$ become real-valued. Repeating the derivation steps of Sec. IV A, but now using $E_\pm = \mathcal{R}_\pm$, the $\text{sinc}(x) = \sin(x)/x$ sine cardinal functions are identified in each spectral contribution, leading to the approximate spectrum formula

$$\begin{aligned} \sigma^{\text{appr}}(\omega_{\text{el}}) &= \sigma_+^{\text{appr}}(\omega_{\text{el}}) + \sigma_-^{\text{appr}}(\omega_{\text{el}}) + \sigma_{+-}^{\text{appr}}(\omega_{\text{el}}) \\ &= |\lambda_+|^2 T^2 \text{sinc}^2\left(\left[K - \mathcal{R}_+\right] \frac{T}{2}\right) \\ &\quad + |\lambda_-|^2 T^2 \text{sinc}^2\left(\left[K - \mathcal{R}_-\right] \frac{T}{2}\right) \\ &\quad + 2|\lambda_+||\lambda_-| T^2 \text{sinc}\left(\left[K - \mathcal{R}_+\right] \frac{T}{2}\right) \\ &\quad \times \text{sinc}\left(\left[K - \mathcal{R}_-\right] \frac{T}{2}\right) \\ &\quad \times \cos\left(\Delta\eta - \Delta\mathcal{R} \frac{T}{2}\right). \end{aligned} \quad (30)$$

The definitions of the quantities appearing in Eq. (30) are identical to those provided in Sec. IV A, with the only exception that now the P, R , and Q quantities inside $\lambda_\pm$ are real-valued ($\tau_P = \tau_R = \tau_Q = 0$). The approximate formula Eq. (30) can reasonably well capture most of the spectral features of the AT doublet—shifting, splitting, multipeak pattern—provided by the exact expression Eq. (25). However, it is unable to accurately describe the spectrum asymmetry when the ionization is driven off-resonantly due to either the emerging Stark shifts ($\delta\mathcal{S} \neq 0$) or the detuned photon frequency ($\Delta \neq 0$). This is demonstrated in Fig. 5(a), where the difference of the solid curves reveals the important role of the $\mathcal{J}_+$ and $\mathcal{J}_-$ imaginary parts played in the asymmetry of the AT doublet.

As seen in Eq. (17), the magnitudes of $\mathcal{J}_\pm$ sensitively depend on the actual value of Δ via the β term. Furthermore, as the $\text{sgn}[\beta]$ factor changes sign at $\Delta_{\text{eq}} (\approx 0.036 \text{ eV}$, see above), the impacts of $\mathcal{J}_\pm$ on the spectrum asymmetry are opposite below and beyond Δ_{eq} . This explains why in Fig. 5(a) the exact asymmetry is underestimated on one side of Δ_{eq} and overestimated on the other side by the approximate spectrum Eq. (30) that neglects $\mathcal{J}_\pm$. This kind of behavior is also observed when the detuning dependence

of the P, R , and Q bound terms inside $\lambda_\pm$ is neglected [see the broken curves in Fig. 5(a)]. Another main drawback of the approximate spectrum formula Eq. (30) when it assumes constant (parameter-independent) P, R , and Q bound terms—often considered in the literature [34,36,38,40]—is that it is completely unable to describe any asymmetry when the radiation is not resonant [dashed orange line with symbols in Fig. 5(a)].

Finally, we emphasize that the asymmetry effect attributed to the $\mathcal{J}_\pm$ terms here is not fully equivalent with the depletion effect discussed in Sec. IV E. Seeing, e.g., Eqs. (26a)–(26c), it is evident that the effects of $\mathcal{J}_\pm$ are twofold: they modify the spectrum height (1) via the exponential decay factors (depletion) and also (2) directly, through the denominators of the prefactors of the spectral contributions σ_+, σ_- , and σ_{+-} . Thus, despite the factors $|\mathcal{J}_\pm|$ being rather small, they have to be accounted for in order to obtain an accurate quantitative description of the spectrum asymmetry, and Eq. (30) should be treated with caution.

V. SUMMARY AND CONCLUSIONS

Extending the traditional semiclassical Rabi model with higher-order terms, atomic resonance-enhanced multiphoton ionization has been investigated theoretically. The simple analytic forms found for the population amplitudes of the (off-)resonantly coupled bound states allowed us to calculate the photoelectron spectrum exactly analytically [Eq. (25)]. Analyzing the contributions of the emerging dressed states in the continuum, a complete characterization of the Autler-Townes doublet has been provided, unfolding the physical origins of its shifting, splitting, multipeak pattern, and asymmetry.

It has been demonstrated that the asymmetry—which received wide interest recently—is caused by three main factors. Besides the energy dependence of the bound-to-continuum Rabi frequencies (see Appendixes), these are the decay rates of the dressed states and the interference of the different competing ionization pathways that primarily govern the spectrum asymmetry (Sec. IV E).

Dependence of the found spectral features on the field intensity, photon frequency, and interaction time, as well as their controllability, has been demonstrated. These findings contribute to our understanding of how bound-state electron dynamics can be manipulated and probed via the energy spectrum of free electrons ionized by intense, short, (off-)resonant XUV radiation fields.

Finally, we notice that as the parameters of the Rabi model all have the same (rectangular shape) temporal profile irrespective of their order, the presented framework is straightforwardly extended with further higher-order terms, without introducing additional complexity to the found spectrum formula. This allows for the characterization of arbitrary $(n + m)$ photon REMPI processes, provided that the minimal-state description is justified.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: ENERGY DEPENDENCE OF THE BOUND-TO-CONTINUUM COUPLINGS OF HYDROGEN

In this Appendix, we discuss the energy dependence of the one- and two-photon bound-to-continuum couplings of hydrogen and their impact on the spectrum shape. As seen in Figs. 6(a) and 6(b), the $\Omega_{el}/2$ and $M_{el}/4$ complex couplings that appear in the continuum amplitudes [Eq. (22)] and in the spectrum formula [Eq. (25)] via the $\lambda_{\pm}$ factors [Eqs. (21a) and (21b)] exhibit a smooth energy dependence in the region of the photoelectron spectra considered in this work. In particular, the strength of these couplings monotonically decreases with increasing electron energy, and in general, the couplings to the d channel are substantially stronger than their s -channel counterparts [see Fig. 6(a)]. Furthermore, $|\Omega_{ed}/2|$ exhibits the strongest dependence on the electron energy. For the laser intensity $I_0 = 3.96 \times 10^{13} \text{ W/cm}^2$ that is utilized throughout the paper, the magnitudes of the one-photon couplings dominate over those of the two-photon couplings. This dominance gradually vanishes for stronger laser intensities, allowing for the competition of resonant and nonresonant ionization pathways.

As seen in Fig. 6(b), the phases of the one- and two-photon couplings [$\varphi_{el} = \arg\{\Omega_{el}/2\}$ and $\phi_{el} = \arg\{M_{el}/4\}$, respectively] differ by π for any energy. Owing to this sign difference of the Ω_{el} and M_{el} couplings, the one- and two-photon contributions inside λ_+ add up destructively, while inside λ_- they interfere constructively [see Eqs. (21a) and (21b) and note that the magnitudes of the τ_P , τ_R , and τ_Q phases are very small ($<0.007 \text{ rad}$), as indicated in Table II]. As a result, the $|\lambda_-|^2$ factor becomes greater than $|\lambda_+|^2$, already for energy-independent couplings, as discussed in Sec. IV [see also the horizontal dashed lines with symbols in Fig. 6(c)]. In the case of energy-dependent couplings, $|\lambda_-|^2$ is further enhanced for $\omega_e < \omega_{e0}$, while $|\lambda_+|^2$ gets further suppressed for $\omega_e > \omega_{e0}$, as shown by the solid lines in Fig. 6(c). This leads to an increased (decreased) electron yield on the lower (higher) energy side of the spectrum, further modifying the asymmetry, as demonstrated in Fig. 6(d). We notice here that the spectrum asymmetry is hardly modified by the small cross term σ_{+-} in Eq. (25) [see Fig. 3(e)]. Consequently, the energy dependence of the $|\lambda_+||\lambda_-|$ term inside σ_{+-} also hardly modifies the spectrum asymmetry.

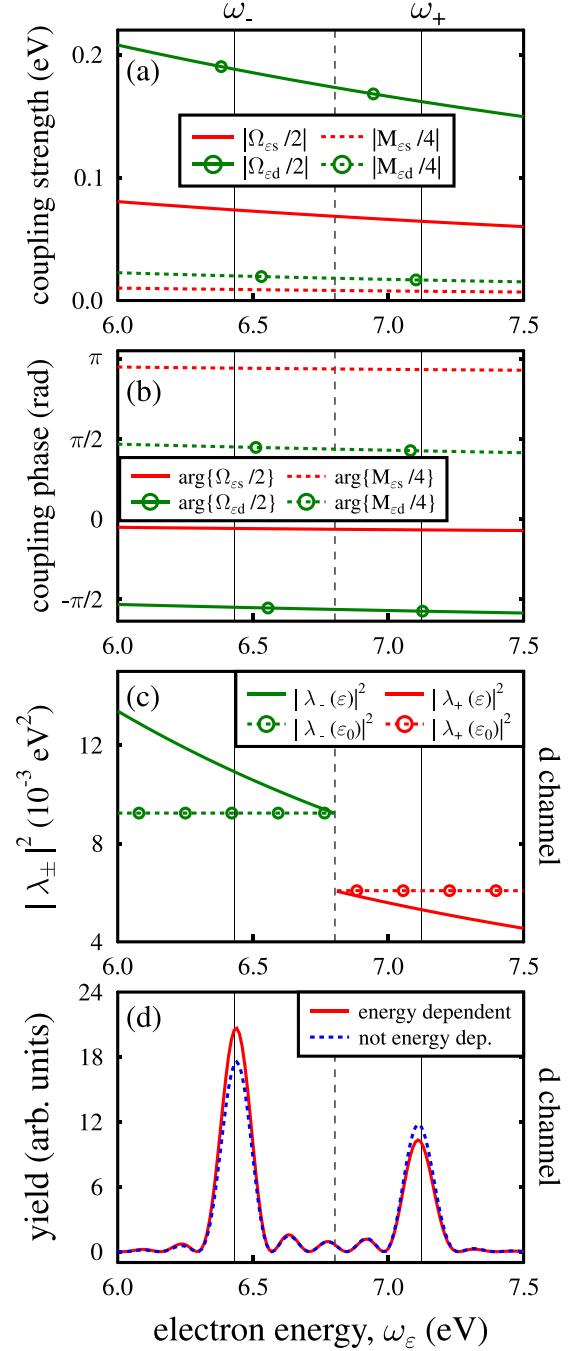


FIG. 6. Energy dependence of the magnitudes (a) and phases (b) of the one- and two-photon bound-to-continuum couplings of hydrogen in the s and d channels. (c) Energy dependence of the $|\lambda_{\pm}|^2$ factors in the d channel [see Eqs. (21a) and (21b)], shown in the energy regions, where they become dominant upon division by the $A_{\pm}^2$ factors in the spectrum formula Eq. (25). (d) Photoelectron spectra of H in the d channel, either considering or omitting the energy dependence of the bound-to-continuum couplings. The applied laser parameter values are the same as in Figs. 2 and 3. The vertical dashed lines indicate the nominal spectrum position $\omega_{e0} = \omega_{1s} + 2\omega_0$. The vertical solid lines denote the dressed-state energies in the continuum $\omega_{\pm}$.

Finally, we mention that the spectrum shown by the dashed line in Fig. 6(d) on a linear scale is identical to the spectra shown in Figs. 2(d) or 3(d) (there the one shown by solid line with open squares) on logarithmic scales.

APPENDIX B: ENERGY DEPENDENCE OF THE $\Delta\eta$ AND $\Delta\Theta$ PHASE DIFFERENCES

In this Appendix, we discuss the energy dependence of the $\Delta\eta = \eta_+ - \eta_-$ and $\Delta\Theta = \Theta_+ - \Theta_-$ phase differences that appear in the σ_{+-} component of the exact spectrum formula in Eq. (25). As the contribution σ_{+-} is small, the impact of these phase differences on the spectrum shape is also expected to be rather small. We notice that the $\Theta_{\pm}$ phases explicitly depend on δ [see Eq. (23c)], while $\eta_{\pm}$ have an implicit δ -dependence via the Ω_{el} and M_{el} couplings inside $\lambda_{\pm}$.

As seen in Fig. 7(a), the $\Theta_{\pm}$ phases suddenly drop down from $\sim\pi$ to ~ 0 at the dressed-state energies $\omega_{\pm}$ (or alternatively at $\delta_{\pm}$). This is a direct consequence of the fact that the real parts of $K - E_{\pm}$ change sign at $\delta_{\pm}$ [see the discussion around Eq. (24) and note that $\mathcal{I}_{\pm} < 0$]. On the other hand, the energy dependence of $\eta_{\pm}$ [shown in Fig. 7(a) for the d channel] is rather weak. To explain this, we notice that the $\lambda_{\pm}$ factors in Eqs. (21a) and (21b) are practically built up as the coherent sums of the $\Omega_{ed}/2$ and $M_{ed}/4$ coupling terms (note that $\tau_P \approx \tau_R \approx \tau_Q \approx 0$ and $|P| \approx |R| \approx |Q| \approx 0.5$ hold approximately). Since $\frac{1}{2}|\Omega_{ed}||Q| \gg \frac{1}{4}|M_{ed}||P|$ holds in Eqs. (21a) and (21b) and the relative phase of these terms remains π at any energy [see Fig. 6(b)], λ_+ inherits the phase of $\Omega_{ed}/2$ [$\eta_+ \approx \varphi_{ed}$], while λ_- inherits the phase of $M_{ed}/4$ [$\eta_- \approx \phi_{ed}$]. These phase relations are slightly modified by the very small τ_P , τ_R , and τ_Q phases of the P , R , and Q bound terms. We notice that for stronger couplings when $M_{ed}/4$ dominates over $\Omega_{ed}/2$, the above phase inheritances can be modified.

According to the above discussions, $\Delta\eta = -\pi$ holds at any energy [see Fig. 7(b)]. Furthermore, as $\Delta\Theta$ jumps between 0 and π around $\omega_{\pm}$, the $\Delta\eta - \Delta\Theta$ term changes between $-\pi$ and -2π , as shown in Fig. 7(c). Thus, the $\Delta\eta - \Delta\Theta$ term does not influence the σ_{+-} component in between the main AT peaks, while it changes its sign outside those peaks. Since at $\omega_{\pm}$ the value of $\Delta\eta - \Delta\Theta$ is $\approx -3\pi/2$, this term does not influence the electron yield at the main AT peaks when an integer number of ROs are completed, as discussed around Eq. (26c).

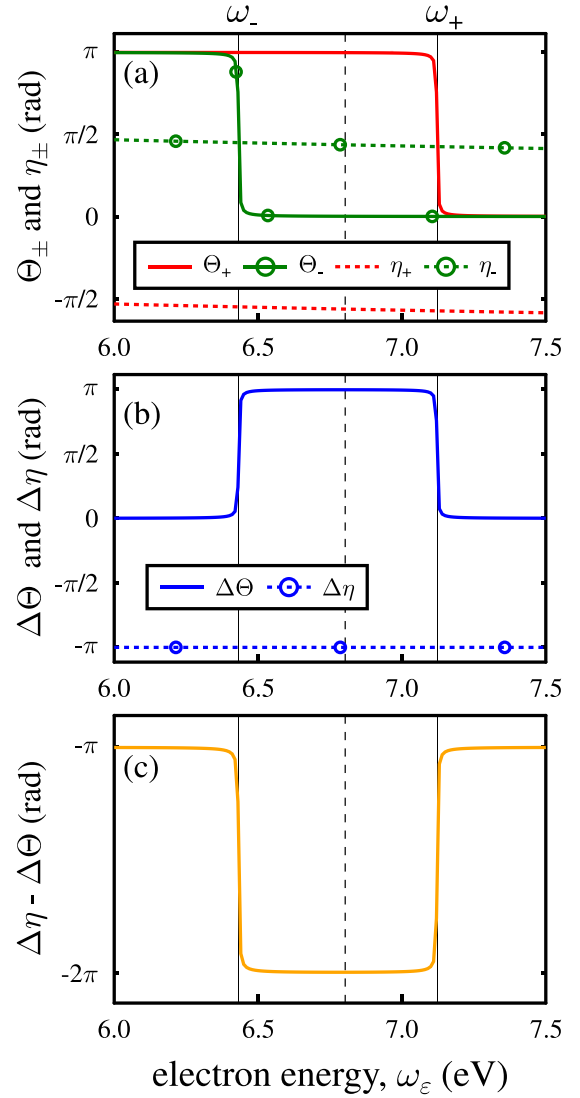


FIG. 7. Energy dependence of the $\Theta_{\pm}$ and $\eta_{\pm}$ phases (a), the $\Delta\eta = \eta_+ - \eta_-$ and $\Delta\Theta = \Theta_+ - \Theta_-$ phase differences (b), and the $\Delta\eta - \Delta\Theta$ phase term (c) appearing in the spectral contribution σ_{+-} in Eq. (25). The applied laser parameter values are the same as in Figs. 2 and 3. The $\eta_{\pm}$ and $\Delta\eta$ terms are shown for the d channel. The vertical dashed lines indicate the nominal spectrum position $\omega_{e0} = \omega_{1s} + 2\omega_0$. The vertical solid lines denote the dressed-state energies in the continuum $\omega_{\pm}$.

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