Erratum: Multichromatic Floquet engineering of quantum dissipation [Phys. Rev. A 109, 062618 (2024)]

François Impens and David Guéry-Odelin

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We correct the comparison made in the original paper between the kick-operator formulation used there and the time-coarse-graining formalism of Gamel and James [1]. The results derived in the original paper are unchanged. However, the claim that the kick-operator approach gives more accurate predictions than the method of Ref. [1] was incorrect. The two perturbative constructions organize the expansion according to different bookkeeping schemes, but yield the same time-coarse-grained dynamics when the hierarchy of small parameters and the truncation order are treated consistently. The discrepancy shown in Fig. 1 of the original paper resulted from comparing expressions truncated at different effective orders in a regime where the hierarchy of small parameters makes this distinction important.

In the original paper, we derived an effective master equation for the time-coarse-grained dynamics of a closed quantum system under multichromatic driving. The derivation was based on the effective-Hamiltonian and kick-operator formalism commonly used in Floquet theory [2,3], followed by a perturbative expansion and by the elimination of fast frequency components. We then compared the resulting equation with the time-coarse-graining method introduced by Gamel and James [1], which is based on a perturbative expansion of the time-ordered evolution operator.

This comparison was misleading. In particular, the discussion below Fig. 1 of the original paper incorrectly suggested that the method of Ref. [1] fails to reproduce one of the nonunitary contributions obtained in our kick-operator formulation. The origin of this apparent discrepancy is not a difference between the two formalisms, but a mismatch in the truncation order used in the comparison.

Indeed, the two approaches are perturbatively equivalent once the hierarchy of small parameters and the truncation order are treated consistently. In the notation of Ref. [1], the evolution operator is expanded as

$$\hat{U}(t,t_0)=\sum_{n=0}^{\infty}\lambda^n\hat{U}_n(t,t_0),$$

where the auxiliary parameter λ is introduced by replacing $\hat{H}(t)$ with $\lambda\hat{H}(t)$. The coefficients obey the recursion relation

$$i\hbar\,\frac{\partial\hat{U}_n(t,t_0)}{\partial t}=\hat{H}(t)\hat{U}_{n-1}(t,t_0),\quad n\geqslant 1,$$

with

$$\hat{U}_0(t,t_0)=\hat{I},\quad \hat{U}_n(t_0,t_0)=0\quad (n\geqslant 1).$$

In the original paper, the same evolution operator was written in the form [3]

$$\hat{U}(t,t_0)=e^{-i\hat{K}(t)}\mathcal{T}\exp\left[-\frac{i}{\hbar}\int_{t_0}^{t}dt'\hat{H}_{\rm eff}(t')\right]e^{i\hat{K}(t_0)}.$$

The operators $\hat{K}(t)$ and $\hat{H}_{\rm eff}(t)$ were then expanded in powers of Ω/ω, where Ω is a characteristic Rabi frequency associated with the driving Hamiltonian and ω is the fast Floquet frequency. This is not the same bookkeeping as the auxiliary λ expansion used in Ref. [1]. Therefore, the equality between the two methods should not be understood as a term-by-term identity between coefficients defined with respect to the same formal parameter. Rather, both constructions are perturbative representations of the same unitary evolution, organized according to different order-counting schemes. After the time-coarse-graining step, and provided the truncation is performed consistently with the hierarchy of small parameters, they yield the same effective dynamics.

The apparent discrepancy was due to the different organization of the perturbative series. In the kick-operator formulation, the slow secular dynamics is isolated in the effective Hamiltonian, while the fast bounded corrections are encoded in the kick operator. In the Dyson-series formulation of Ref. [1], the same information is distributed among different perturbative orders. A direct comparison of selected low-order terms can therefore be misleading, especially when several small parameters are present.

This is precisely what happened in the driven two-level example considered in Fig. 1 of the original paper. Besides the high-frequency parameter associated with the drive amplitudes, the beat frequency $\Delta\omega$ introduces an additional small scale.

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Consequently, terms that appear at different orders in the primary perturbative expansion may contribute at the same effective order after the timescale separation is performed. In particular, the contribution denoted $\mathcal{L}_2^{FSF}[\rho]$ in the original paper must be compared with a third-order term in the expansion of Ref. [1]. When this term is retained, the Gamel-James formalism reproduces the same contribution.

Thus, the statement in the original paper that the Dyson-series approach of Ref. [1] is restricted, in this comparison, to a less accurate description of the nonunitary time-coarse-grained dynamics should be withdrawn. The results of the original paper, including the effective master equation and the applications to multichromatic Floquet engineering of quantum dissipation, remain valid. The correction concerns only the interpretation of the comparison with Ref. [1]: the kick-operator and Dyson-series approaches are two equivalent perturbative expansions of the same time-coarse-grained dynamics, once the truncation is performed consistently at the same order.

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