

Dual-domain coherent emission: Photon-magnon laser in a hybrid quantum system

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We present a quantum theoretical framework that describes the simultaneous emergence of magnon- and photon lasing in a tripartite hybrid solid-state platform. The system integrates optically pumped nitrogen-vacancy (NV) centers, a magnetic yttrium-iron-garnet (YIG) microsphere, and a microwave (MW) cavity. Through a comprehensive analysis of the system's dynamics, we reveal the cooperative interactions that drive coherent-stimulated emission in both the magnonic (spin-wave) and photonic (electromagnetic) domains. The model outlines a scalable configuration in which NV centers, excited via optical pumping, couple efficiently to the magnon modes of the YIG sphere placed inside the microwave cavity. Numerical simulations based on experimentally accessible parameters confirm the feasibility of dual-domain lasing. This work advances the integration of photonic and magnonic platforms, opening new opportunities for quantum-enhanced sensing, low-power spintronics, and on-chip quantum information processing.

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I. INTRODUCTION

A laser (light amplification by stimulated emission of radiation) generates coherent light by optically exciting a gain medium—incoherently—within an optical resonator. Although conventional lasers are confined to the photonic domain, recent developments have generalized the lasing principle to a variety of physical systems. Notable examples include phonon lasers, which generate coherent vibrational excitations [1,2]; atom lasers, which emit coherent matter waves [3,4]; and plasmon lasers, which sustain nanoscale coherent plasmonic oscillations [5].

Within this broader context, magnon lasers—which coherently amplify spin waves—have surged to prominence, drawing upon decades of magnonics research [6]. In analogy to electric currents, magnons carry quantized units of energy, momentum, and spin angular momentum (spin-1) through magnetic materials, and can be excited either directly via magnetic dipole interactions or indirectly via electric dipole coupling combined with spin-orbit effects [7]. These properties make magnonics a compelling platform for nanoscale information processing, though the efficient transduction of conventional electronic or photonic signals into propagating spin waves remains a key challenge [8–12]. To translate these conceptual advances into working devices and overcome the signal-transduction bottleneck, a magnon laser based on Brillouin scattering in YIG microspheres has been proposed

[13]. Building on this theoretical foundation, experimental breakthroughs have realized nonreciprocal magnon lasers in spinning resonators for one-way spin-wave emission [14–16] and parity-time-symmetric magnon lasers in cavity optomagnonic systems to harness balanced gain and loss for enhanced stability [17]. YIG spheres utilized in these studies are ferromagnetic materials distinguished by their high magnetic quality factor, enabling the support of spin waves with exceptionally low damping and acoustic dissipation rates [18]. These spheres host distinct magnon modes, collectively known as Walker modes. Among these, the Kittel mode—a uniform magnetization precession mode—stands out for its exceptional coherence and low-damping characteristics [19,20]. These attributes make YIG-based systems promising candidates for advancing critical technologies, including quantum memory architectures, ultrasensitive magnetometry, and spin-wave-driven logic devices.

Although the laser was originally conceived in the context of photonics, over recent decades, its underlying principle has been extended to a variety of physical systems, including coherent excitation of phonons, atoms, and plasmons. These advancements expand the scope of stimulated emission beyond light, redefining the laser as a general mechanism for amplifying bosonic excitations. A particularly striking example is the recent realization of simultaneous photon-phonon lasing [21], which demonstrates that distinct excitation types can be coherently amplified within a single physical platform. This result establishes a new paradigm: two-domain lasers that link fundamentally different physical degrees of freedom through a shared coherent process. Building on this conceptual shift, spin-based systems provide a promising route toward realizing such dual-domain lasing. Magnons, the quantized excitations of spin waves, offer strong electromagnetic coupling and are well suited to integration with solid-state quantum devices. Unlike existing magnon lasers, which operate solely within the spin domain, the coherent cogen-eration of magnons and photons remains largely unexplored.

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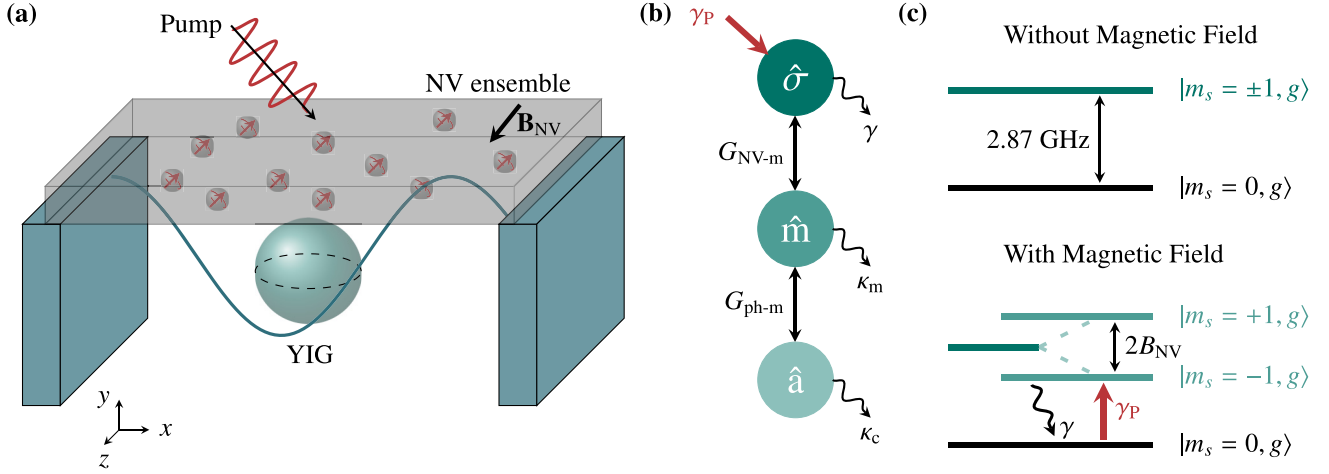


FIG. 1. (a) Schematic diagram of the proposed experimental setup. A spherical YIG micromagnet (teal sphere) is magnetically coupled to an ensemble of NV centers (gray spheres with red arrows), embedded in a diamond slab (not to scale). A static magnetic field $\mathbf{B}_{\text{NV}}$ is applied to tune the NV energy levels. The system is integrated within a coplanar waveguide (CPW) resonator, where a microwave signal excites a resonant mode. The corresponding oscillating magnetic field interacts with both the NV ensemble and the YIG sphere, enabling control of the Kittel mode frequency. The depicted resonator geometry represents a conceptual segment of the CPW layout for clarity. (b) Schematic of the model and the relevant parameters. The NV centers are coupled to a magnetic sphere, which is in turn coupled to a cavity. The lasing effect will be caused by the NV center pumping and the successive couplings between the tripartite system. (c) Simplified energy diagram of the NV⁻ center without and in the presence of a magnetic field B_{NV} , which lifts the degeneracy of the states $m_s = \pm 1$. Optical pumping (red arrow) induces transitions from the ground-state spin-1 triplet to the excited triplet state. The $m_s = 0$ ground state is populated via spontaneous decay.

The present work addresses this challenge by proposing a concrete, physically realizable model of a two-domain laser that coherently links optical and spin excitations—laying the groundwork for integrated quantum technologies that leverage both photonic and magnonic degrees of freedom.

To realize this vision, we propose a tripartite hybrid platform comprising a YIG sphere, nitrogen-vacancy (NV) center spins, and a frequency-tunable microwave (MW) cavity. This architecture is designed to exploit the multimodal coupling capabilities of magnons [22–24]. NV centers, optically pumped to achieve population inversion, act as the gain medium. Their magnetic coupling to both the YIG sphere and the MW cavity enables simultaneous stimulated emission of photons and magnons, realizing the core mechanism of dual-domain lasing in a physically implementable system. We provide a comprehensive theoretical analysis of the system’s performance across several critical metrics: the threshold pump power required to initiate dual-domain lasing, the spectral linewidth and coherence of the emitted modes, and the tunability of lasing frequencies via detuning control. Additionally, we assess the platform’s resilience to temperature fluctuations—an essential factor for real-world applications. These results define the operational regime of the device and suggest strategies for improving quantum coherence and thermal stability. Beyond performance analysis, the platform offers broader utility in quantum technologies: dual-mode emission supports quantum memories with hybrid read-out [25,26], enables precision sensing that exploits distinct magnonic and photonic noise profiles [27], and facilitates integration into reconfigurable on-chip quantum interconnects linking spin-based logic with photonic communication layers [24,28]. Our theoretical framework thus provides both a fundamental understanding and a practical design strategy for

realizing magnon-photon two-domain lasers, establishing a new paradigm for coherent excitation control across disparate quantum subsystems.

The paper is structured as follows: In Sec. II, we introduce the theoretical framework and key components of the tripartite hybrid system. In Sec. III, we provide a thorough characterization of its quantum dynamics. A comprehensive analysis of the system’s dynamics—detailing numerical solutions and key properties that demonstrate lasing—is presented in Sec. IV. Building on these results, Sec. V examines the dual-mode lasing response as a function of system parameters. Finally, Sec. VI offers concluding remarks and broader implications of the work.

II. THEORETICAL MODEL

To describe simultaneous photon-magnon lasing, we consider a hybrid setup [29], as depicted in Fig. 1(a). The three ingredients of our configuration are (1) a magnetic YIG sphere supporting magnons, (2) a cavity of microwave photons, and (3) a gain medium consisting of an ensemble of NV centers embedded in a diamond slab. The NV centers are magnetically coupled to the YIG sphere, which is in turn coupled to the MW cavity, as shown in Fig. 1(b). Below, we present details of these components.

A. YIG micromagnetic sphere

The first element of our hybrid setup is a YIG microsphere. This structure sustains spin waves and magnetostatic modes, which are associated with Walker modes [19,20]. To simplify the analysis, we assume the YIG sphere exhibits a uniform magnetostatic mode known as the Kittel mode [30]. In this mode, all spins within the sphere precess

coherently with identical phase and amplitude. The frequency of such a mode, given by $\omega_m = |\gamma_g|B_z$, is tunable via an external magnetic field B_z . Here, the gyromagnetic ratio is $\gamma_g = -1.76 \times 10^{11} \text{ T}^{-1}\text{s}^{-1}$. The Hamiltonian describing the free energy of the YIG sphere is thus expressed as

$$\hat{H}_{\text{mag}} = \hbar\omega_m \hat{m}^\dagger \hat{m}, \quad (1)$$

where $\hat{m}^\dagger$ ($\hat{m}$) is the creation (annihilation) operator of a magnon quasiparticle.

B. MW cavity

The second component of our hybrid configuration is a one-dimensional MW cavity engineered to host a single photonic mode with resonant frequency ω_c . This cavity acts as a confining structure for electromagnetic waves, with its geometry and material properties tailored to sustain a standing wave pattern along its axis. The quantum-mechanical description of the cavity's electromagnetic field is captured by the bare Hamiltonian,

$$\hat{H}_{\text{MWC}} = \hbar\omega_c \hat{a}^\dagger \hat{a}, \quad (2)$$

where $\hat{a}^\dagger$ ($\hat{a}$) is the creation (annihilation) operator of the photon mode. Additionally, the MW cavity is designed to achieve frequency matching between its photonic mode (ω_c) and the magnonic resonance frequency (ω_m) of the YIG sphere. This condition, $\omega_c \approx \omega_m$, is pivotal for facilitating strong coherent coupling between magnons and photons, a prerequisite for hybrid quantum systems.

C. Ensemble of NV centers

The third element of the hybrid system is a gain medium comprising an ensemble of NV centers [31]. Typically, NV defects are available in neutral (NV^0) and negatively charged (NV^-) forms [32]. Out of these arrangements, the negatively charged state of the defect center is of particular interest due to the presence of its spin triplet states that can be initialized, manipulated, and read out by coherent optical methods. The electronic ground state $|g\rangle$ is split into two magnetic spin levels, the $|m_s = 0, g\rangle$ (bright) and the $|m_s = \pm 1, g\rangle$ (dark) states separated by 2.87 GHz, in the absence of any external field, a consequence of the spin-spin interaction between the electrons [32,33]. A degree of spin polarization in $m_s = 0$ can be achieved by shining the system with optical fields followed by spin-selective intersystem crossing. Once the ground state is prepared, a microwave drive is used for manipulation. The spin-triplet ground-state manifold are eigenstates of the spin operator $\hat{S}_z$, with $\hat{S}_z|i\rangle = i|i\rangle$, and $i = 0, \pm 1$. In the presence of the external magnetic field $\mathbf{B}_{\text{NV}}$ oriented along the NV symmetry axis, the degeneracy of the states $m_s = \pm 1$ are split via Zeeman effect. With proper magnetic fields, we can choose the states $|m_s = 0\rangle$ and $|m_s = -1\rangle$ as a spin qubit, which are selected to be resonant with the magnon mode. Taking $|0\rangle$ as the energy reference, the Hamiltonian of the NV center becomes [34]

$$\hat{H}_{\text{NV}} = \hbar \frac{\omega_{\text{NV}}}{2} \hat{\sigma}_z, \quad (3)$$

where we define the frequency as $\omega_{\text{NV}} = D_0 - |\gamma_g|B_{\text{NV}}$, with zero-field-splitting $D_0 = 2\pi \times 2.87 \text{ GHz}$, and the spin

operator as $\hat{\sigma}_z \equiv |m_s = -1\rangle\langle m_s = -1| - |m_s = 0\rangle\langle m_s = 0|$, see level scheme in Fig. 1(c). The state $|m_s = 1\rangle$ could be excluded if it is off-resonance with the magnon mode. The spin frequency can be designed to match the frequencies of magnons and photons, thus the three frequencies of the individual systems can be equivalent.

III. QUANTUM DYNAMICS

A. Interactions

The coupling dynamics of the hybrid setup, illustrated schematically in Fig. 1(b), involve two key interactions: (1) between the MW cavity and the YIG microsphere, and (2) between the spin ensemble [e.g., NV centers] and the magnonic mode of the sphere. The interaction between the MW cavity photons and the magnons in the YIG sphere under the rotating-wave approximation (RWA) is governed by a beam-splitter-type Hamiltonian,

$$\hat{H}_{\text{MWC-mag}} = -\hbar G_{\text{ph-m}} (\hat{m}^\dagger \hat{a} + \hat{m} \hat{a}^\dagger), \quad (4)$$

where $G_{\text{ph-m}}$ represents photon-magnon coupling. This term describes the coherent exchange of quanta between the photonic and magnonic subsystems, enabling energy transfer at a rate proportional to $G_{\text{ph-m}}$.

The interaction between the spin ensemble (e.g., NV centers) and the magnon mode is similarly derived under the RWA. Here, the magnon mode couples to the spin transitions between the $|0\rangle$ and $|-1\rangle$ states of the NV centers. The Hamiltonian takes the form

$$\hat{H}_{\text{NV-mag}} = -\hbar G_{\text{NV-m}} (\hat{m} \hat{\sigma}^+ + \hat{m}^\dagger \hat{\sigma}^-), \quad (5)$$

where $\hat{\sigma}^\pm$ are the spin raising and lowering operators for the NV centers. This interaction mediates the transfer of excitations between the magnon mode and the spin subsystem, with $G_{\text{NV-m}}$ defining the coupling strength. Both $G_{\text{ph-m}}$ and $G_{\text{NV-m}}$ are highly sensitive to the geometry of the YIG microsphere. By optimizing the sphere's size and its placement relative to the MW cavity and spin ensemble, these coupling strengths can be enhanced to values exceeding 1 MHz. For instance, $G_{\text{ph-m}}$ scales with the overlap between the cavity's magnetic field and the magnon mode's spatial profile, while $G_{\text{NV-m}}$ depends on the local magnetic field gradient generated by the magnon mode at the spin location. Such tunability is critical for achieving strong coupling regimes, where coherent energy exchange dominates over dissipation.

B. Full Hamiltonian

Taking into account free energy and interactions, we can finally write the full Hamiltonian for the dynamical evolution of the hybrid tripartite system, in the frame rotating at the individual spin frequency ω_{NV} , as

$$\begin{aligned} \hat{H} = & \hbar\Delta_1 \hat{m}^\dagger \hat{m} + \hbar\Delta_2 \hat{a}^\dagger \hat{a} - \hbar G_{\text{NV-m}} (\hat{m} \hat{\sigma}^+ + \hat{m}^\dagger \hat{\sigma}^-) \\ & - \hbar G_{\text{ph-m}} (\hat{m}^\dagger \hat{a} + \hat{m} \hat{a}^\dagger), \end{aligned} \quad (6)$$

where the first two terms correspond to the free Hamiltonians with detunings $\Delta_1 = \omega_m - \omega_{\text{NV}}$ and $\Delta_2 = \omega_c - \omega_{\text{NV}}$, and the last two terms describe two interactions: one between the

Kittel mode and the NV center spin, and the other between the Kittel mode and the cavity.

Finally, a full Travis-Cummings Hamiltonian for the entire ensemble of N NV centers with the same frequency ω_{NV} is written as

$$\begin{aligned}\hat{H} &= \hbar\Delta_1\hat{m}^\dagger\hat{m} + \hbar\Delta_2\hat{a}^\dagger\hat{a} \\ &- \sum_{j=1}^N \hbar G_{\text{NV-m}}^j (\hat{m}\hat{\sigma}_j^+ + \hat{m}^\dagger\hat{\sigma}_j^-) \\ &- \hbar G_{\text{ph-m}} (\hat{m}^\dagger\hat{a} + \hat{m}\hat{a}^\dagger).\end{aligned}\quad (7)$$

C. Equations of motion

Despite the dynamical evolution described by the Hamiltonian, it is crucial to include the losses resulting from the environments in the master equation. For that, we consider the spontaneous emission of the NV center spin γ , the decay of the Kittel mode κ_m , and the cavity κ_c . Furthermore, to generate the population inversion, the NV centers are pumped incoherently at a rate γ_p . Thus, the master equation of the total system is expressed as

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \mathcal{L}[\hat{\rho}], \quad (8)$$

where $\hat{\rho}$ is the density operator, and the Liouvillian operators for the NV centers spontaneous decay and incoherent pumping are defined, respectively, as

$$\mathcal{L}[\hat{\sigma}_j^-]\hat{\rho} = \gamma_j(2\hat{\sigma}_j^-\hat{\rho}\hat{\sigma}_j^+ - \hat{\sigma}_j^-\hat{\sigma}_j^+\hat{\rho} - \hat{\rho}\hat{\sigma}_j^+\hat{\sigma}_j^-), \quad (9)$$

$$\mathcal{L}[\hat{\sigma}_j^+]\hat{\rho} = \gamma_p(2\hat{\sigma}_j^+\hat{\rho}\hat{\sigma}_j^- - \hat{\sigma}_j^+\hat{\sigma}_j^-\hat{\rho} - \hat{\rho}\hat{\sigma}_j^-\hat{\sigma}_j^+). \quad (10)$$

Photon losses through the cavity mirrors $\{\hat{O} \equiv \hat{a}, \alpha \equiv c\}$, and magnon decay $\{\hat{O} \equiv \hat{m}, \alpha \equiv m\}$ are described as

$$\begin{aligned}\mathcal{L}[\hat{O}]\hat{\rho} &= \kappa_\alpha(n_{\text{th}}(\omega_\alpha) + 1)(2\hat{O}\hat{\rho}\hat{O}^\dagger - \hat{O}^\dagger\hat{O}\hat{\rho} - \hat{\rho}\hat{O}^\dagger\hat{O}) \\ &+ \kappa_\alpha n_{\text{th}}(\omega_\alpha)(2\hat{O}^\dagger\hat{\rho}\hat{O} - \hat{O}\hat{O}^\dagger\hat{\rho} - \hat{\rho}\hat{O}\hat{O}^\dagger),\end{aligned}\quad (11)$$

defining the thermal occupation number for temperature T as

$$n_{\text{th}}(\omega_j) = \left[\exp\left(\frac{\hbar\omega_j}{k_B T}\right) - 1 \right]^{-1}, \quad (12)$$

with k_B the Boltzmann constant.

Based on the highly efficient julia programming language [35], we have used the quantumoptics.jl package [36] as it includes several tools to solve quantum optics problems. Furthermore, the numerical simulations for the dynamics of our system are performed using the quantumcumulants.jl package, which allows us to perform mean-field calculations to any specified order [37]; for our case we used second and fourth order. Hence, we can derive the equations for all operators to obtain

$$\langle \dot{\hat{a}} \rangle = -(i\Delta_2 + \kappa_c)\langle \hat{a} \rangle + iG_{\text{ph-m}}\langle \hat{m} \rangle + \eta, \quad (13)$$

$$\langle \dot{\hat{\sigma}}_j^- \rangle = -(\gamma + \gamma_p)\langle \hat{\sigma}_j^- \rangle + iG_{\text{NV-m}}^j\langle \hat{m} \rangle - 2iG_{\text{NV-m}}^j\langle \hat{m}\hat{\sigma}_j^{ee} \rangle, \quad (14)$$

$$\langle \dot{\hat{\sigma}}_j^{ee} \rangle = -2(\gamma + \gamma_p)\langle \hat{\sigma}_j^{ee} \rangle + iG_{\text{NV-m}}^j(\langle \hat{m}\hat{\sigma}_j^+ \rangle - \langle \hat{m}^\dagger\hat{\sigma}_j^- \rangle) + 2\gamma_p, \quad (15)$$

$$\langle \dot{\hat{m}} \rangle = -(i\Delta_1 + \kappa_m)\langle \hat{m} \rangle + iG_{\text{ph-m}}\langle \hat{a} \rangle + i\sum_{j=1}^N G_{\text{NV-m}}^j\langle \hat{\sigma}_j^- \rangle. \quad (16)$$

Next, we define the collective operators $\hat{S}$ and $\hat{S}_z$ operators by summing over the corresponding operators for one two-level system

$$\hat{S} = \frac{1}{N}\sum_{j=1}^N \hat{\sigma}_j^-, \quad \hat{S}_z = \frac{1}{N}\sum_{j=1}^N \hat{\sigma}_j^z.$$

This allows us to write governing equations in terms of collective operators

$$\langle \dot{\hat{a}} \rangle = -(i\Delta_2 + \kappa_c)\langle \hat{a} \rangle + iG_{\text{ph-m}}\langle \hat{m} \rangle + \eta, \quad (17)$$

$$\langle \dot{\hat{S}} \rangle = -(\gamma + \gamma_p)\langle \hat{S} \rangle + iG_{\text{NV-m}}\langle \hat{m} \rangle - 2iG_{\text{NV-m}}\langle \hat{m}\hat{S}_{ee} \rangle, \quad (18)$$

$$\begin{aligned}\langle \dot{\hat{S}}_{ee} \rangle &= -2(\gamma + \gamma_p)\langle \hat{S}_{ee} \rangle \\ &+ iG_{\text{NV-m}}(\langle \hat{m}\hat{S}^\dagger \rangle - \langle \hat{m}^\dagger\hat{S} \rangle) + 2\gamma_p,\end{aligned}\quad (19)$$

$$\langle \dot{\hat{m}} \rangle = -(i\Delta_1 + \kappa_m)\langle \hat{m} \rangle + iG_{\text{ph-m}}\langle \hat{a} \rangle + iG_{\text{NV-m}}N\langle \hat{S} \rangle. \quad (20)$$

For the complete set of equations of differential equations see Appendix A.

IV. NUMERIC RESULTS

A. Realistic experimental parameters

In our proposal, the coupling strength $G_{\text{NV-m}}^j$ between the individual NV center spin j and the Kittel mode in the YIG sphere depends not only on the distance d_j between them and the radius R of the sphere, but also on the orientation in space. In our model, we adopted a uniform coupling constant $G_{\text{NV-m}}$, based on the reasonable approximation that the NV centers are close to the surface of the YIG sphere, within a region where the Kittel mode's magnetic field exhibits minimal spatial variation. Under these conditions, the coupling strength experienced by different NV centers varies only slightly, making the homogeneous coupling approximation a reliable estimate [24,38,39]. Thus, we can consider the problem in terms of collective operators $\hat{S}$. Having this in mind, we consider a $N = 1 \times 10^4$ of spins to which $G_{\text{NV-m}}^j \equiv G_{\text{NV-m}}$, see Fig. 1(a). The magnetic YIG microspheres can be manufactured to have a radius ranging from approximately 10 nm to 10 μm [40]. For small magnetic spheres (a few nanometer-sized), the coupling between Kittel modes and cavity photons is of the order of $G_{\text{ph-m}} \sim 2\pi \times 0.2$ MHz [41], where for quality factor of the coplanar waveguide (CPW) resonator of $Q \sim 10^4$, the decay rate of the cavity $\kappa_c \sim 0.4$ MHz [42,43]. Regarding the corresponding dissipation rates, the Kittel magnon linewidth is on the order $\gamma_m \sim 2\pi \times 2.6$ MHz for YIG [38,40,44]. While recent experiments have demonstrated exceptionally low magnon decay rates, down to $2\pi \times 0.45$ MHz in high-purity YIG spheres [45], in this work we intentionally adopt a higher decay rate to reflect a more conservative and controllable experimental regime [46]. Importantly, the magnon linewidth can be engineered through surface treatment [47],

material composition, or controlled introduction of inhomogeneities [48–50], allowing experimentalists to increase magnetic losses when needed. This flexibility is particularly valuable near the lasing threshold, where ensuring that the gain overcomes the losses is critical. Our parameter choice thus ensures that lasing can be robustly achieved and studied under practical and tunable conditions.

For MW cavities, the photon frequency can be designed to match the frequencies of magnons and spins. This could be achieved via (1) magnetic field gradient engineering—by using patterned micromagnets or spatially varying bias coils, we can enable local control over the field experienced by the YIG and the NV ensemble separately, despite their proximity [42,51,52]; (2) Zeeman splitting of NV centers can be tuned by adjusting the angle between the applied magnetic field and the NV axis, reducing the required magnetic field strength for resonance matching [32]; and (3) strain tuning of NV centers [53,54] or temperature-based tuning of magnon resonance frequencies in YIG may offer additional alignment capabilities [55]. For proper geometrical size and distance between the YIG sphere and NV centers, one can tune the coupling strength between magnons and spins to be of the order of $G_{\text{NV-m}} \sim 2\pi \times 0.015$ MHz [41], with typical decay rates for the NV centers of the order of $\gamma \sim 2\pi \times 20$ kHz [56]. Furthermore, our suggested experiment will be performed at very low temperatures (of the order $T = 100$ mK), as $\omega_m \sim \omega_c$, we consider the same thermal population $n_{\text{th}} \approx 0.04$, except when stated otherwise.

B. Lasing threshold

One of the laser characteristics is the emergence of a threshold across which system shows a transition from thermal to coherent behavior. Lasers require a minimum pump rate to establish this transition, commonly referred to as the threshold condition. Above threshold, when gain dominates over losses, the stimulated emission in the system gets coherently amplified. In other words, the characteristic mean occupation grows up and a lasing action is said to be achieved. To find the lasing threshold, we use the first-order cumulant expansion along with large N limit to neglect the two-operator correlations. This allows us to write equations for the averages in the following form:

$$\dot{\alpha} = -(i\Delta_2 + \kappa_c)\alpha + iG_{\text{ph-m}}\mu + \eta, \quad (21)$$

$$\dot{s} = -(\gamma + \gamma_p)s + iG_{\text{NV-m}}\mu - 2iG_{\text{NV-m}}\mu s_{ee}, \quad (22)$$

$$\dot{s}_{ee} = -2(\gamma + \gamma_p)s_{ee} + iG_{\text{NV-m}}(\mu s^* - \mu^* s) + 2\gamma_p, \quad (23)$$

$$\dot{\mu} = -(i\Delta_1 + \kappa_m)\mu + iG_{\text{ph-m}}\alpha + iG_{\text{NV-m}}Ns, \quad (24)$$

where we have introduced the averages as $\langle \hat{a} \rangle = \alpha$, $\langle \hat{S} \rangle = s$, $\langle \hat{S}_{ee} \rangle = s_{ee}$, and $\langle \hat{m} \rangle = \mu$. The steady state of the system can be governed by setting the time derivatives equal to zero in the above equations. The corresponding steady-state solutions for the mean magnon and photon number, excited-state

population, and dipole coherence can be written in the following form, respectively:

$$|\mu|^2 = \frac{1}{2} \left(\frac{(\gamma_p - \gamma)N\kappa_c}{\kappa_c\kappa_m + G_{\text{ph-m}}^2} - \frac{(\gamma + \gamma_p)^2}{G_{\text{NV-m}}^2} \right), \quad (25)$$

$$|\alpha|^2 = \frac{G_{\text{ph-m}}^2}{\kappa_c^2} |\mu|^2, \quad (26)$$

$$s_{ee} = \frac{1}{2} \left(1 + \frac{(\gamma + \gamma_p)(\kappa_c\kappa_m + G_{\text{ph-m}}^2)}{NG_{\text{NV-m}}^2\kappa_c} \right), \quad (27)$$

$$\frac{|s|}{|\alpha|} = \frac{\kappa_m\kappa_c + G_{\text{ph-m}}^2}{G_{\text{NV-m}}G_{\text{ph-m}}N}, \quad (28)$$

where for simplification we set $\Delta_{1,2} = 0$. The lasing steady state exists only above the threshold where the mean magnon number takes positive values, as is evident from the above equations. Ensuring the positivity of $|\mu|^2$ in the lasing regime, we obtain the upper and lower pumping thresholds, written as

$$\gamma_p^{\pm} = \frac{NG_{\text{NV-m}}^2\kappa_c}{2(\kappa_c\kappa_m + G_{\text{ph-m}}^2)} - \gamma \pm \frac{G_{\text{NV-m}}\sqrt{N\kappa_c}}{2(\kappa_c\kappa_m + G_{\text{ph-m}}^2)} \times \sqrt{g^2\kappa_cN - 8\gamma(\kappa_c\kappa_m + G_{\text{ph-m}}^2)}. \quad (29)$$

Here, the lower threshold $\gamma_p^- \equiv \gamma_{\text{thr}}$ marks the onset of population inversion, while the upper threshold $\gamma_p^+ \equiv \gamma_p^{\text{max}}$ corresponds to a regime where coherent amplification is suppressed. Within this interval, the average magnon (and photon) number exhibits a quadratic dependence on the pump rate, increasing with γ_p until reaching a maximum and subsequently declining beyond γ_p^{max} . For instance, with $\Delta_{1,2} = 0$, lasing occurs between $\gamma_{\text{thr}} \approx 2\pi \times 20$ kHz and $\gamma_p^{\text{max}} \approx 2\pi \times 0.77$ MHz. This nonmonotonic response indicates the onset of self-quenching, a phenomenon analogous to that found in single-atom lasers [57]. As the pump strength increases, the NV ensemble can become overdriven: the population inversion begins to degrade due to enhanced damping induced by the combined effects of incoherent pumping and spontaneous emission. Once the inversion is significantly suppressed, the gain medium can no longer support stimulated emission, causing the intracavity fields—and hence the lasing output—to collapse. This defines a fundamental limit for sustained lasing and highlights the importance of optimizing pump rates to maintain coherence.

To show the existence of lasing threshold, we resort to numerical simulations. In Figs. 2(a)–2(c), we show the time evolution of the average magnon number $\langle \hat{n}_m \rangle \equiv \langle \hat{m}^\dagger \hat{m} \rangle$, average photon number $\langle \hat{n}_{\text{ph}} \rangle \equiv \langle \hat{a}^\dagger \hat{a} \rangle$, and average excited-state population $\langle \hat{S}_{ee} \rangle$, for different pumpings $\gamma_p = 5\gamma$, 10γ , and 20γ (corresponding to $\gamma_p = 2\pi \times 0.10, 0.25$, and 0.40 MHz). For all cases, the average initial states are trivial, but grow with time, reaching the steady state under continuous pumping of the NV centers. For both magnon and photon modes, the steady-state populations significantly exceed their initial values, confirming the successful generation of coherent excitations. This growth occurs as the NV centers act as an energy reservoir, converting pump energy into ordered magnonic and photonic states through stimulated

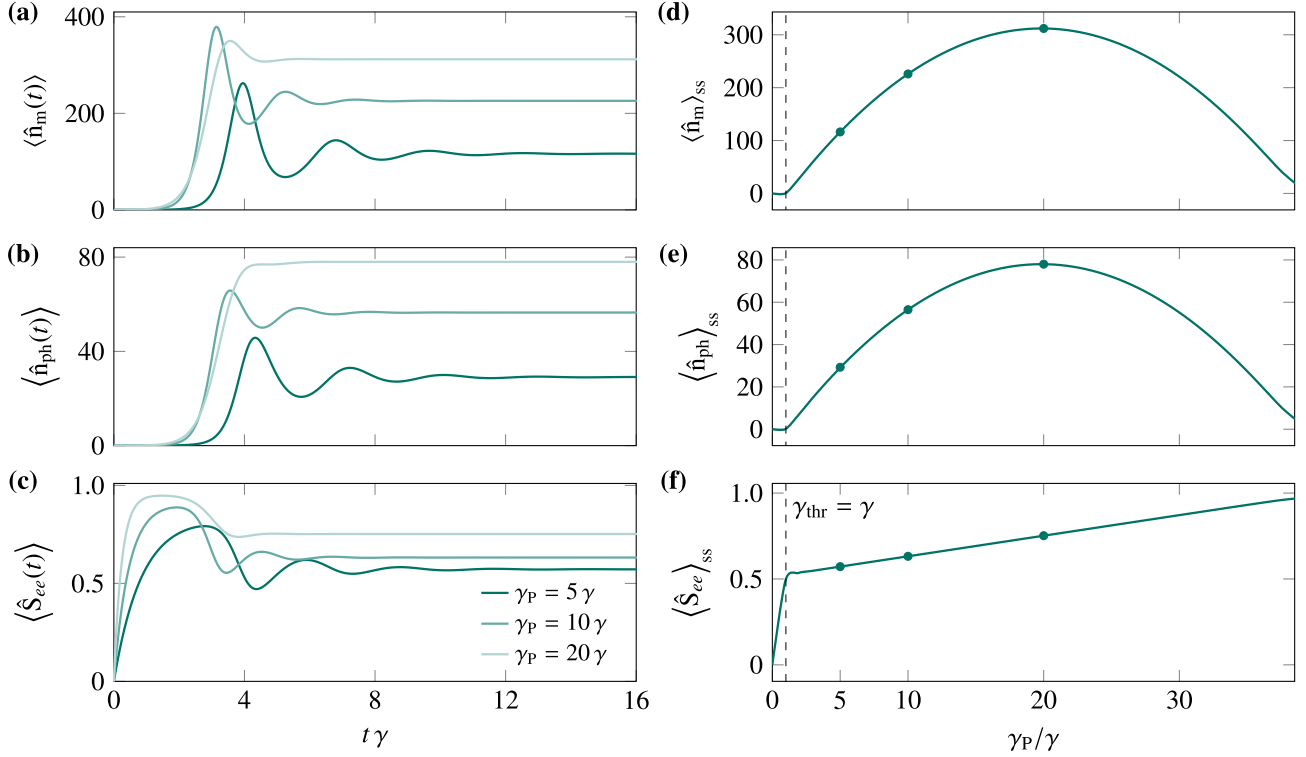


FIG. 2. Time evolution of the (a) average magnon number $\langle \hat{n}_m(t) \rangle$, (b) average photon number $\langle \hat{n}_{ph}(t) \rangle$, and (c) average excited-state population $\langle \hat{S}_{ee}(t) \rangle$ for different pumping of the NV centers where $\gamma_p = 5\gamma$, 10γ , and 20γ (corresponding to $\gamma_p/2\pi = 0.1, 0.2$, and 0.4 MHz). Evolution of the average steady state of the (d) magnon number $\langle \hat{n}_m \rangle_{ss}$, (e) photon number $\langle \hat{n}_{ph} \rangle_{ss}$, and (f) excited-state population $\langle \hat{S}_{ee} \rangle_{ss}$ with pumping γ_p . We have taken γ_p within the lasing regime, $\gamma_p \leq \gamma_p^{\max}$, where we highlighted with a vertical line the pumping threshold γ_{thr} . We marked the three values for $\gamma_p/2\pi = 0.1, 0.2$, and 0.4 MHz. For each simulation, we considered the numerical parameters given in Table I.

emission. The system's transition from thermal equilibrium to an active, excited regime highlights the critical role of sustained pumping in overcoming intrinsic dissipation. To illustrate the lasing threshold, in Figs. 2(d)–2(f), we show the steady-state evolution of the magnon number, photon number, and excited-state population as a function of the pumping rate. As shown in Figs. 2(d)–2(f), stronger pump intensities correlate with higher magnon and photon densities, reflecting the direct energy transfer from the NV centers to the hybrid system. Below the threshold ($\gamma_p < \gamma_{thr}$), no quanta are observed, as losses dominate over gain. Once the threshold is crossed ($\gamma_p > \gamma_{thr}$), populations surge rapidly, peaking near

$\gamma_p^{\max}/2$. The sharp threshold behavior and tunable steady-state populations underscore the potential for engineering coherent states in our quantum hybrid system.

C. Laser spectrum

Another important characteristic of a laser is the linewidth narrowing above the threshold of the emission spectrum. The laser spectrum can be calculated as the Fourier transform of the first-order correlation function $\mathcal{G}^{(1)}(t, t + \tau)$. For a single-mode field, the first-order correlation function can be written

TABLE I. Parameters used for the numerical simulations.

Number of NV ⁻ centers	N	1×10^4
Detuning 1	Δ_1	0
Detuning 2	Δ_2	0
Coupling between NVCs and magnons	G_{NV-m}	$2\pi \times 0.015$ MHz
Coupling between photons and magnons	G_{ph-m}	$2\pi \times 0.2$ MHz
Cavity loss	κ_c	$2\pi \times 0.4$ MHz
Kittel model decay	κ_m	$2\pi \times 2.6$ MHz
NV center decay	γ	$2\pi \times 0.02$ MHz
Temperature $T = 100$ mK	n_{th}	0.04
Temperature $T = 500$ mK	n_{th}	1.09
Temperature $T = 1$ K	n_{th}	2.60
Temperature $T = 5$ K	n_{th}	14.88

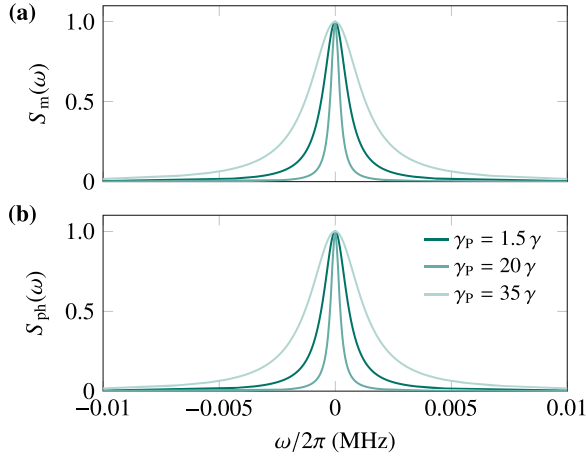


FIG. 3. Normalized spectrum for the (a) magnon and (b) photon modes as a function of frequency for different pumping of the NV centers, where $\gamma_p = 1.5\gamma$, 20γ , and 35γ (corresponding to $\gamma_p = 2\pi \times 0.03$, 0.40 , and 0.70 MHz). For each simulation, we considered the numerical parameters given in Table I.

in terms of creation and annihilation operators as

$$\mathcal{G}_O^{(1)}(t, t + \tau) \propto \langle \hat{O}^\dagger(t) \hat{O}(t + \tau) \rangle,$$

where $\hat{O} \equiv \hat{m}, \hat{a}$ for correlation functions of magnons or photons, respectively. These correlations can be easily found using julia's correlationfunctions.jl package. The spectrum can be expressed in terms of a first-order correlation function as

$$S_O(\omega) = 2\text{Re} \left[\int_0^\infty dt e^{-i\omega t} \langle \hat{O}^\dagger(t + \tau) \hat{O}(\tau) \rangle \right]. \quad (30)$$

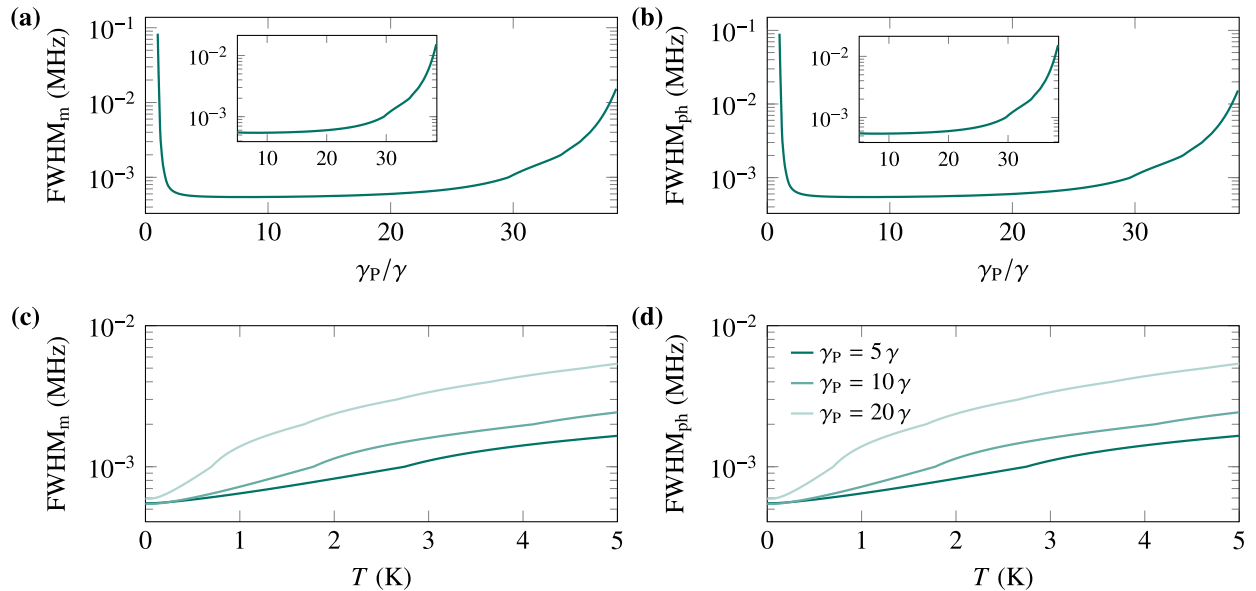


FIG. 4. FWHM of the normalized spectrum as a function of pumping γ_p of the NV centers, for (a) magnons and (b) photon modes, with $T = 0.1$ mK. FWHM of the normalized spectrum as a function of temperature T for (c) magnons and (d) photon modes, where we took $\gamma_p = 5\gamma$, 10γ , and 20γ (corresponding to $\gamma_p/2\pi = 0.1$, 0.2 , and 0.4 MHz). For each simulation, we considered the numerical parameters given in Table I.

If we consider the system to be in a steady state, taking $\tau = 0$, the correlation function $\langle \hat{m}^\dagger(t) \hat{m}(0) \rangle$ can be found by solving equations given in Appendix B.

In Fig. 3, we show the numerical results for the emission spectrum of the magnons and photons, for different pumping of the NV centers, $\gamma_p = 1.5\gamma$, 20γ , and 35γ (corresponding to $\gamma_p = 2\pi \times 0.03$, 0.40 , and 0.70 MHz). As is evident, the emission spectrum is Lorentzian in the frequency domain. Notably, the linewidth narrows significantly as the pumping rate transitions from below to above the lasing threshold. This narrowing reflects the suppression of phase diffusion noise due to stimulated emission dominating over spontaneous emission in the lasing regime. The laser conditions, such as pumping strength, interactions of the lasing medium, and temperature, have an impact on the width of the spectral line. To characterize the linewidth narrowing, we can numerically find the full width at half maximum. These results are depicted in Figs. 4(a) and 4(b) as a function of pumping strength γ_p and in Figs. 4(c) and 4(d) as a function of temperature T . The physical significance of spectral linewidth is closely related to the coherence and temporal properties of the laser emission. In the lasing regime, emitted light is monochromatic (i.e., single-wavelength emission). A narrower spectral linewidth corresponds to greater monochromaticity and coherence of the emitted modes. Rapid linewidth narrowing further indicates fast stabilization of the system. As anticipated, elevated temperatures broaden the laser linewidth, driven by thermal broadening and temperature-dependent modifications to the gain medium.

D. Laser statistics

To confirm the emission of coherent photons and magnons in our setup, we employ steady-state lasing statistics of the system in terms of normalized second-order correlation

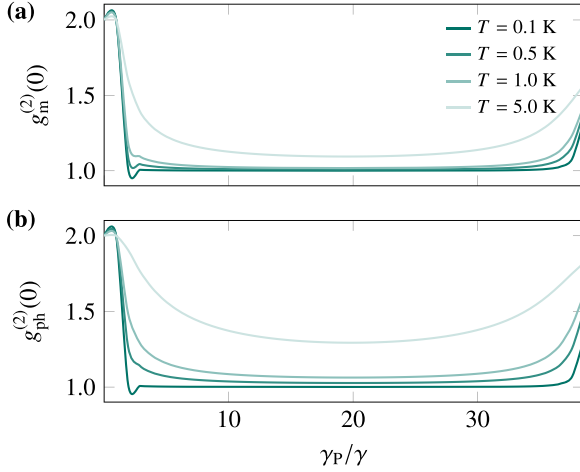


FIG. 5. Second-order correlation function for (a) magnons $g_m^{(2)}(0)$ and (b) photons $g_{ph}^{(2)}(0)$ as a function of pumping for different temperatures $T = 0.1$ K, 0.5 K, 1.0 K and $5T = 0.1$ K, 0.5 K, 1.0 K. For each simulation, we considered the numerical parameters given in Table I.

function

$$g_O^{(2)}(0) = \frac{\langle \hat{O}^\dagger(0) \hat{O}^\dagger(0) \hat{O}(0) \hat{O}(0) \rangle}{\langle \hat{O}^\dagger(0) \hat{O}(0) \rangle^2}, \quad (31)$$

where $\hat{O} \equiv \hat{m}, \hat{a}$ for correlations functions of magnons or photons, respectively. If $g_O^{(2)}(0) \rightarrow 1$, then our source is coherent.

For $\gamma_p < \gamma$ (below threshold), the thermal statistics of the system is confirmed in Figs. 5(a) and 5(b), where zero-delay second-order correlation function is $g_O^{(2)}(0) \approx 2$. However, when the pumping rate exceeds the threshold, the correlation function transitions sharply to $g_O^{(2)}(0) \rightarrow 1$, signaling the onset of coherent emission. This Poissonian statistics confirms the emergence of a classical-like coherent state, where quanta (photons or magnons) are emitted independently, suppressing intensity fluctuations. The transition to $g_O^{(2)}(0) = 1$ far above threshold underscores the dominance of stimulated emission over spontaneous processes, a hallmark of lasing action in our system.

Thermal noise profoundly impacts the coherence of emitted quanta, revealing a striking asymmetry between magnons and photons at elevated temperatures (0.5 , 1.0 , and 5.0 K). The YIG sphere's magnon modes exhibit remarkable resilience to thermal decoherence, maintaining near-Poissonian statistics [$g_O^{(2)}(0) \approx 1$] even at 5.0 K (Fig. 5), due to the strong collective coupling of spins in the Kittel mode, which suppresses individual spin fluctuations. In contrast, cavity photons show significant thermal bunching [$g_O^{(2)}(0) > 1$] at higher temperatures, as their coherence degrades from increased cavity losses (κ_c) and thermal occupation of higher-order modes susceptible to environmental noise. This dichotomy highlights the superior thermal stability of magnonic excitations over photonic ones in hybrid systems, a critical advantage for applications in high-temperature quantum magnonics and fault-tolerant spintronic devices.

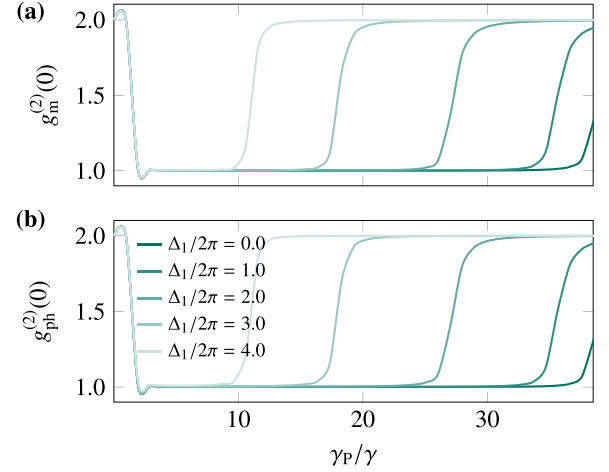


FIG. 6. Second-order correlation function for (a) magnons $g_m^{(2)}(0)$ and (b) photons $g_{ph}^{(2)}(0)$ as a function of pumping for different detunings $\Delta_1 = \omega_m - \omega_{NV} = 0.0, 1.0, 2.0, 3.0, 4.0$ MHz, while keeping $\Delta_2 = 0$. For each simulation, we considered the numerical parameters given in Table I.

V. DISCUSSION

The two-mode lasing dynamics in the hybrid system exhibit strong sensitivity to frequency detunings $\Delta_1 = \omega_m - \omega_{NV}$ (magnon-spin detuning) and $\Delta_2 = \omega_c - \omega_{NV}$ (cavity-spin detuning). When $\Delta_1 \neq 0$ while maintaining $\Delta_2 = 0$, lasing persists but operates within a narrowed parameter regime due to a reduced upper pumping threshold $\gamma_p^{\max}$, see Fig. 6. This reduction arises from the energy mismatch between the magnon mode and the spin-transition frequency, which weakens the effective spin-magnon coupling required for population inversion. Despite this, the system retains lasing capability, though the emission frequency shifts to higher values, enabling spectral tuning of the output—a feature critical for applications such as frequency-agile photonic sources.

The persistence of lasing under detuned conditions can be explained through an effective Hamiltonian derived in Appendix C. By leveraging the strong dissipation of the magnon Kittel mode ($\kappa_m \gg G_{NV-m}, G_{ph-m}, \kappa_c, \gamma$), the magnon degree of freedom is adiabatically eliminated. This approximation condenses the tripartite system (spins, magnons, photons) into a bipartite framework, where the photonic mode directly couples to the spin ensemble via magnon-mediated interactions. The resulting effective Hamiltonian is

$$\begin{aligned} \hat{H}_{\text{eff}} = & \hbar \beta^2 \Delta_1 \hat{a}^\dagger \hat{a} - \hbar \sum_{j=1}^N \Delta_1 \alpha_\sigma \hat{\sigma}_j^z \\ & + \hbar \sum_{j=1}^N \beta G_{NV-m}^j (\hat{a} \hat{\sigma}_j^+ + \hat{a}^\dagger \hat{\sigma}_j^-), \end{aligned} \quad (32)$$

where the renormalized parameters

$$\begin{aligned} \alpha_\sigma &= \frac{(G_{NV-m}^j)^2}{\Delta_1^2 + \kappa_m^2}, \\ \beta &= \frac{G_{ph-m}}{\sqrt{\Delta_1^2 + \kappa_m^2}} \end{aligned}$$

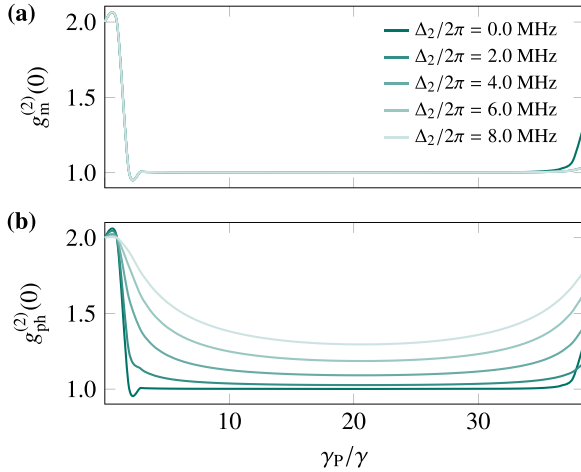


FIG. 7. Second-order correlation function for (a) magnons $g_m^{(2)}(0)$ and (b) photons $g_{ph}^{(2)}(0)$ as a function of pumping for different detunings $\Delta_2 = \omega_c - \omega_{NV} = 0.0, 2.0, 4.0, 6.0, 8.0$ MHz, while keeping $\Delta_1 = 0$. For each simulation, we considered the numerical parameters given in Table I.

quantify the magnon-mediated spin-photon interactions. The effective coupling ($G_{\text{eff}} = \beta G_{NV-m}^I$), cavity loss ($\kappa_{c,\text{eff}} = \kappa_c + \beta^2 \Delta_1$), and spin decay ($\gamma_{\text{eff}} = \gamma + \kappa_m \alpha_\sigma$) collectively govern the narrowed lasing regime and frequency shift. For $\Delta_2 = 0$, this effective model reveals that driving the NV centers directly pumps the photonic mode, bypassing the magnon subsystem.

In contrast, detuning the cavity mode ($\Delta_2 \neq 0$) reveals starkly different behavior. For small Δ_2 , lasing persists with a frequency-shifted output, as the cavity remains near resonance with the spin transition. However, at large Δ_2 (with $\Delta_1 = 0$), photon lasing ceases entirely, as the cavity frequency deviates significantly from the spin resonance, suppressing stimulated emission. This breakdown is accompanied by a rise in the zero-delay second-order photon correlation function $g_{ph}^{(2)}(0)$ (see Fig. 7), reflecting a transition from coherent [$g_{ph}^{(2)}(0) \approx 1$] toward thermal statistics [$g_{ph}^{(2)}(0) > 1$] due to the dominance of spontaneous emission.

It is important to emphasize that in experimental configurations where NV centers are embedded within the microwave cavity [24,29,58], they couple simultaneously to both the magnetic sphere's magnon modes and the cavity's photonic modes. While this dual coupling does not disrupt the coexistence of magnon and photon lasing, it introduces a notable feature: the emitted spectrum's peak frequency shifts dynamically with variations in the pumping strength γ_p . This dependence enables pumping-tunable laser operation, where γ_p acts as a control parameter for real-time spectral control. For instance, increasing γ_p red shifts the emission peak, while decreasing it blue shifts the output—a behavior arising from nonlinear magnon-photon interactions mediated by the spin ensemble (see Appendix D for analytical derivations and numerical simulations). Such tunability is advantageous for reconfigurable photonic devices requiring adaptive frequency operation.

A. Experimental feasibility

To test and confirm magnon lasing, two key signatures could be looked for. First, linewidth narrowing of the magnon mode can be directly measured via Brillouin light scattering (BLS) spectroscopy. As the pumping intensity increases and reaches the lasing threshold, one expects the magnon spectral linewidth to sharply decrease, Fig. 4. Microfocused BLS setups are capable of resolving subgigahertz magnon linewidth changes and have already been successfully used to study both thermal and parametrically driven magnon populations [59–61].

Second, measuring the second-order correlation function $g^{(2)}(\tau)$ of the magnon field provides a direct test for the transition to coherent statistics. Adapted from photon-based Hanbury Brown–Twiss experiments, recent proposals and experimental efforts suggest that time-resolved BLS can access magnon number fluctuations and distinguish coherent (laser-like) from thermal or chaotic states. While not yet demonstrated for magnons, analogous measurements in exciton-polariton [62,63] and phonon systems [64,65] indicate feasibility. Thus, implementing BLS-based spectroscopy to track linewidth narrowing, combined with correlation-statistics measurements, presents a conceptually straightforward and experimentally reachable pathway to verify magnon lasing. These suggestions are presented as guidance for future experimental validation and do not entail any modifications to the current theoretical framework.

VI. CONCLUSIONS

In this work, we introduced a pioneering theoretical framework for a hybrid two-domain laser capable of simultaneously generating coherent photons and magnons within a tripartite solid-state system. By optically pumping an ensemble of NV centers integrated with a microwave cavity and a magnetic sphere, we demonstrated stimulated emission in both photonic and magnonic domains. Critical lasing characteristics—including threshold behavior, spectral linewidth narrowing, laser statistics, and dynamic frequency control—were rigorously analyzed, revealing that robust lasing regimes for photons and magnons emerge under elevated pumping conditions. This achievement underscores the viability of coherent energy exchange in hybrid quantum systems, bridging photonic and spin-wave excitations.

The simultaneous generation of both photons and magnons could prove to be highly advantageous and provide novel capabilities. Many potential applications could be envisaged, particularly in areas where the combination of photons and magnons could yield distinct functions, such as quantum communication, spintronics, signal processing, and quantum information. Our proposal opens the door to the exploration of the coupling between photons and magnons in a coherent manner. For instance, the energy transfer between photons and magnons could be harnessed for the effective modulation or amplification of signals in both domains. Photon-magnon interactions could also enable the development of hybrid communication systems that incorporate optical and spin-based signals. Moreover, it can be shown that in our proposal one can achieve the frequency laser control by adjusting

the external magnetic field. The magnon-photon interactions could allow for precise control of the frequency and coherence of emitted light, which could be applied in microwave imaging, spectroscopy, and sensing applications where both types of excitations are useful.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are openly available [66].

APPENDIX A: SOME RELEVANT EQUATIONS

Starting from the full Travis-Cummings Hamiltonian for our system given by

$$\begin{aligned} \hat{H} = & \hbar\Delta_1\hat{m}^\dagger\hat{m} + \hbar\Delta_2\hat{a}^\dagger\hat{a} - \sum_{j=1}^N \hbar G_{\text{NV-m}}^j (\hat{m}\hat{\sigma}_j^+ + \hat{m}^\dagger\hat{\sigma}_j^-) \\ & - \hbar G_{\text{ph-m}} (\hat{m}^\dagger\hat{a} + \hat{m}\hat{a}^\dagger) \end{aligned} \quad (\text{A1})$$

and related decay channels of each operator specified in the main text, the time evolution for operators are obtained as

$$\langle \dot{\hat{a}} \rangle = - (i\Delta_2 + \kappa_c) \langle \hat{a} \rangle + iG_{\text{ph-m}} \langle \hat{m} \rangle + \eta, \quad (\text{A2})$$

$$\langle \dot{\hat{\sigma}}_j^- \rangle = - (\gamma + \gamma_p) \langle \hat{\sigma}_j^- \rangle + iG_{\text{NV-m}}^j \langle \hat{m} \rangle - 2iG_{\text{NV-m}}^j \langle \hat{m}\hat{\sigma}_j^{ee} \rangle, \quad (\text{A3})$$

$$\begin{aligned} \langle \dot{\hat{\sigma}}_j^{ee} \rangle = & -2(\gamma + \gamma_p) \langle \hat{\sigma}_j^{ee} \rangle \\ & + iG_{\text{NV-m}}^j (\langle \hat{m}\hat{\sigma}_j^+ \rangle - \langle \hat{m}^\dagger\hat{\sigma}_j^- \rangle) + 2\gamma_p, \end{aligned} \quad (\text{A4})$$

$$\langle \dot{\hat{m}} \rangle = - (i\Delta_1 + \kappa_m) \langle \hat{m} \rangle + iG_{\text{ph-m}} \langle \hat{a} \rangle + i \sum_{j=1}^N G_{\text{NV-m}}^j \langle \hat{\sigma}_j^- \rangle. \quad (\text{A5})$$

Assuming that all NV centers couple equally to the cavity and magnon mode, this allows us to move to a collective basis where $\hat{S}$ and $\hat{S}_z$ operators are defined by summing over the corresponding operators for one two-level system

$$\hat{S} = \frac{1}{N} \sum_{j=1}^N \hat{\sigma}_j^-, \quad (\text{A6})$$

$$\hat{S}_z = \frac{1}{N} \sum_{j=1}^N \hat{\sigma}_j^z. \quad (\text{A7})$$

To get a closed set of equations, we need to complete the equation system. Assuming that all NV centers behave identically, then the excited-state population is equal for all, and the dynamics of the system can be solved by solving the complete set of ordinary differential equations, using the julia package `differentialequations.jl`.

APPENDIX B: EQUATIONS FOR SPECTRUM

For simplicity, let us consider the procedure to find the spectrum for the magnon emission. To do so, we use the Laplace transform of the two-time correlation function obtain as

$$\begin{aligned} \frac{d}{dt} \langle \hat{m}^\dagger(t) \hat{m}(0) \rangle = & (i\Delta_1 - \kappa_m) \langle \hat{m}^\dagger(t) \hat{m}(0) \rangle \\ & - iG_{\text{ph-m}} \langle \hat{a}^\dagger(t) \hat{m}(0) \rangle \\ & - iNG_{\text{NV-m}} \langle \hat{S}^+(t) \hat{m}(0) \rangle, \end{aligned} \quad (\text{B1})$$

$$\begin{aligned} \frac{d}{dt} \langle \hat{a}^\dagger(t) \hat{m}(0) \rangle = & (i\Delta_2 - \kappa_c) \langle \hat{a}^\dagger(t) \hat{m}(0) \rangle \\ & - iG_{\text{ph-m}} \langle \hat{m}^\dagger(t) \hat{m}(0) \rangle, \end{aligned} \quad (\text{B2})$$

$$\begin{aligned} \frac{d}{dt} \langle \hat{S}^+(t) \hat{m}(0) \rangle = & -(\gamma + \gamma_p) \langle \hat{S}^+(t) \hat{m}(0) \rangle \\ & - iG_{\text{NV-m}} \langle \hat{m}^\dagger(t) \hat{m}(0) \rangle \\ & + 2iG_{\text{NV-m}} \langle \hat{S}_{ee} \rangle_{ss} \langle \hat{m}^\dagger(t) \hat{m}(0) \rangle. \end{aligned} \quad (\text{B3})$$

These equations can be written in the form [37]

$$\frac{d}{d\tau} \mathbf{y}(\tau) = \mathbf{M} \mathbf{y}(\tau) \quad (\text{B4})$$

with, for the magnon case, $\mathbf{y}(\tau) = (\langle \hat{m}^\dagger(\tau) \hat{m}(0) \rangle, \langle \hat{a}^\dagger(\tau) \hat{m}(0) \rangle, \langle \hat{S}^+(\tau) \hat{m}(0) \rangle)^T$ and

$$\mathbf{M} = \begin{pmatrix} i\Delta_1 - \kappa_m & -iG_{\text{ph-m}} & iNG_{\text{NV-m}} \\ -iG_{\text{ph-m}} & i\Delta_2 - \kappa_c & 0 \\ -iG_{\text{NV-m}} + 2iG_{\text{NV-m}} \langle \hat{S}_{ee} \rangle_{ss} & 0 & -\gamma - \gamma_p \end{pmatrix}. \quad (\text{B5})$$

We define $\mathbf{x}(s) = \mathcal{L}\{\mathbf{y}(\tau)\}$, where $\mathcal{L}$ denotes the Laplace transform regarding τ . Instead of computing the time evolution of the correlation function, we can directly compute the spectrum by solving the linear equation

$$\mathbf{x} = (i\omega - \mathbf{M})^{-1} \mathbf{y}(0). \quad (\text{B6})$$

A similar procedure can be used to find $\langle \hat{a}^\dagger(t) \hat{a}(0) \rangle$, needed to find the photon emission spectrum,

$$\begin{aligned} \frac{d}{dt} \langle \hat{a}^\dagger(t) \hat{a}(0) \rangle = & (i\Delta_2 - \kappa_c) \langle \hat{a}^\dagger(t) \hat{a}(0) \rangle \\ & - iG_{\text{ph-m}} \langle \hat{m}^\dagger(t) \hat{a}(0) \rangle, \end{aligned} \quad (\text{B7})$$

$$\begin{aligned} \frac{d}{dt} \langle \hat{m}^\dagger(t) \hat{a}(0) \rangle = & (i\Delta_1 - \kappa_m) \langle \hat{m}^\dagger(t) \hat{a}(0) \rangle \\ & - iG_{\text{ph-m}} \langle \hat{a}^\dagger(t) \hat{a}(0) \rangle \\ & - iNG_{\text{NV-m}} \langle \hat{S}^+(t) \hat{a}(0) \rangle, \end{aligned} \quad (\text{B8})$$

$$\begin{aligned} \frac{d}{dt} \langle \hat{S}^+(t) \hat{a}(0) \rangle = & -(\gamma + \gamma_p) \langle \hat{S}^+(t) \hat{a}(0) \rangle \\ & - iG_{\text{NV-m}} \langle \hat{m}^\dagger(t) \hat{a}(0) \rangle \\ & + 2iG_{\text{NV-m}} \langle \hat{S}_{ee} \rangle_{ss} \langle \hat{m}^\dagger(t) \hat{a}(0) \rangle. \end{aligned} \quad (\text{B9})$$

APPENDIX C: ADIABATIC ELIMINATION OF THE MAGNON MODE

Picking the full Hamiltonian from Eq. (7),

$$\hat{H} = \hbar\Delta_1\hat{m}^\dagger\hat{m} + \hbar\Delta_2\hat{a}^\dagger\hat{a} - \hbar\sum_j G_{\text{NV-m}}^j(\hat{m}\hat{\sigma}_j^+ + \hat{m}^\dagger\hat{\sigma}_j^-) - \hbar G_{\text{ph-m}}(\hat{m}^\dagger\hat{a} + \hat{m}\hat{a}^\dagger), \quad (\text{C1})$$

where $\Delta_1 = \omega_m - \omega_{\text{NV}}$ and $\Delta_2 = \omega_c - \omega_{\text{NV}}$. In order to have photon lasing, the NV centers need to be in resonance with the cavity mode, thus $\Delta_2 = 0$. The quantum Langevin equations are given by

$$\dot{\hat{m}} = (-i\Delta_1 - \kappa_m)\hat{m} + iG_{\text{ph-m}}\hat{a} + i\sum_j G_{\text{NV-m}}^j\hat{\sigma}_j^-, \quad (\text{C2})$$

$$\dot{\hat{a}} = -\kappa_c\hat{a} + iG_{\text{ph-m}}\hat{m}, \quad (\text{C3})$$

$$\dot{\hat{\sigma}}_j^- = (-\gamma - \gamma_p)\hat{\sigma}_j^- - iG_{\text{NV-m}}^j\hat{\sigma}_j^z\hat{m}, \quad (\text{C4})$$

$$\dot{\hat{\sigma}}_j^z = 2(-\gamma - \gamma_p)\hat{\sigma}_j^z - 2iG_{\text{NV-m}}^j(\hat{m}^\dagger\hat{\sigma}_j^- - \hat{m}\hat{\sigma}_j^+) - 2(\gamma - \gamma_p), \quad (\text{C5})$$

where κ_m , κ_c , and γ are the decay rates of the Kittel mode, cavity mode and the emitter. γ_p is the pumping of the NV center. These equations can be formally integrated as

$$\hat{m}(t) = \hat{m}(0)\exp(-i\Delta_1 t - \kappa_m t) + \exp(-i\Delta_1 t - \kappa_m t) \int_0^t \left[iG_{\text{ph-m}}\hat{a}(\tau) + i\sum_j G_{\text{NV-m}}^j\hat{\sigma}_j^-(\tau) \right] \exp(i\Delta_1 \tau + \kappa_m \tau) d\tau, \quad (\text{C6})$$

$$\hat{a}(t) = \hat{a}(0)\exp(-\kappa_c t) + \exp(-\kappa_c t) \int_0^t iG_{\text{ph-m}}\hat{m}(\tau)\exp(\kappa_c \tau) d\tau, \quad (\text{C7})$$

$$\hat{\sigma}_j^-(t) = \hat{\sigma}_j^-(0)\exp(-\gamma t - \gamma_p t) + \exp(-\gamma t - \gamma_p t) \int_0^t [-iG_{\text{NV-m}}^j\hat{\sigma}_j^z(\tau)\hat{m}(\tau)] \exp(\gamma \tau + \gamma_p \tau) d\tau. \quad (\text{C8})$$

In our system, the magnon mode is highly dissipative, $\kappa_m \gg G_{\text{NV-m}}$, $G_{\text{ph-m}}$, κ_c , γ ; thus, we can eliminate the magnon from the dynamics of the system. By neglecting the effect of highly dissipative mode, we obtain the approximated equations,

$$\hat{a}(t) \simeq \hat{a}(0)\exp(-\kappa_c t), \quad (\text{C9})$$

$$\hat{\sigma}_j^-(t) \simeq \hat{\sigma}_j^-(0)\exp(-\gamma t - \gamma_p t). \quad (\text{C10})$$

By plugging in the equation for $\hat{m}(t)$, we obtain ($\kappa_m \gg \kappa_c$, $\kappa_m \gg \gamma + \gamma_p$)

$$\begin{aligned} \hat{m}(t) &\simeq \hat{m}(0)\exp(-i\Delta_1 t - \kappa_m t) + \exp(-i\Delta_1 t - \kappa_m t) \\ &\times \int_0^t \left[iG_{\text{ph-m}}\hat{a}(0)\exp(-\kappa_c \tau) + i\sum_j G_{\text{NV-m}}^j\hat{\sigma}_j^-(0)\exp(-\gamma \tau - \gamma_p \tau) \right] \exp(i\Delta_1 \tau + \kappa_m \tau) d\tau \\ &\simeq \hat{m}(0)\exp(-i\Delta_1 t - \kappa_m t) + i\frac{G_{\text{ph-m}}}{i\Delta_1 + \kappa_m}\hat{a}(t) + i\sum_j \frac{G_{\text{NV-m}}^j}{i\Delta_1 + \kappa_m}\hat{\sigma}_j^-(t) \end{aligned}$$

Since κ_m is large, the term containing $\exp(-\kappa_m t)$ is a fast-decaying term and, therefore, can be neglected to find

$$\hat{m}(t) \simeq i\frac{G_{\text{ph-m}}}{i\Delta_1 + \kappa_m}\hat{a}(t) + i\sum_j \frac{G_{\text{NV-m}}^j}{i\Delta_1 + \kappa_m}\hat{\sigma}_j^-(t).$$

Replacing the quantum Langevin equations and using $\hat{\sigma}_z\hat{\sigma}^- = (|e\rangle\langle e| - |g\rangle\langle g|)|g\rangle\langle e| = -\hat{\sigma}^-$,

$$\dot{\hat{a}}(t) = [-i\Delta_1\alpha_a - (\kappa_c + \alpha_a)]\hat{a}(t) + i\beta e^{i\theta_a}G_{\text{NV-m}}^j\hat{\sigma}_j^-(t). \quad (\text{C11})$$

As a complex number $a + ib \equiv re^{i\theta}$, $r = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}(b/a)$, where we have defined

$$\alpha_a = \frac{G_{\text{ph-m}}^2}{\Delta_1^2 + \kappa_m^2}, \quad (\text{C12})$$

$$\beta = \frac{G_{\text{ph-m}}}{\sqrt{\Delta_1^2 + \kappa_m^2}}, \quad (\text{C13})$$

$$\theta = \tan^{-1}\left(\frac{\kappa_m}{\Delta_1}\right); \quad (\text{C14})$$

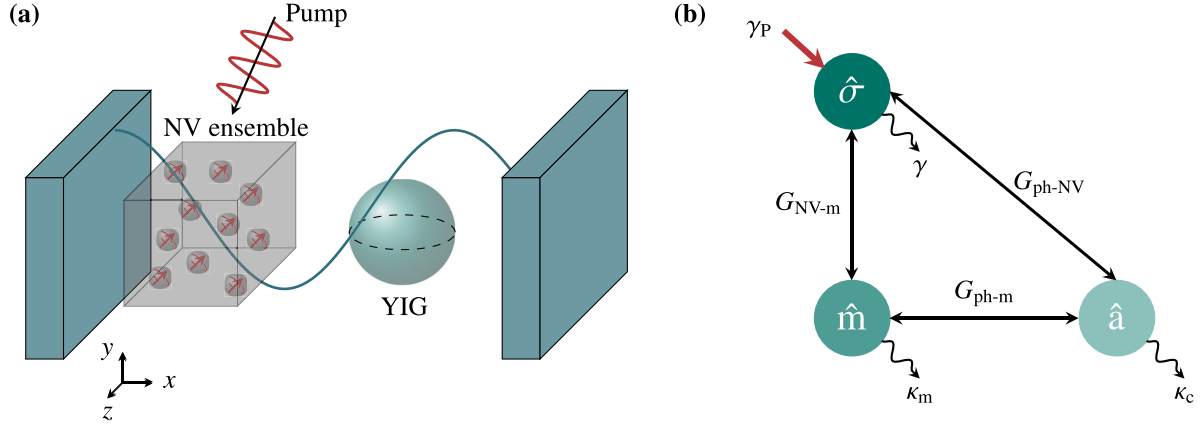


FIG. 8. (a) Schematic diagram of the proposed experimental setup. A spherical micromagnet (teal sphere) with radius R is coupled to an NV centers ensemble (gray spheres with red arrows), contained in a diamond slab (not in scale). (b) Schematic of the model and the relevant parameters. The NV centers are simultaneously coupled to a magnetic sphere and cavity modes. The lasing effect will be caused by the NV center pumping.

similarly for $\hat{\sigma}_j^-(t)$, one finds

$$\begin{aligned} \dot{\hat{\sigma}}_j^-(t) = & [i\Delta_1\alpha_\sigma - (\gamma + \gamma_P + \kappa_m\alpha_\sigma)]\hat{\sigma}_j^-(t) \\ & - i\beta e^{i\theta} G_{NV-m}^j \hat{\sigma}_j^z(t) \hat{a}(t) \end{aligned} \quad (C15)$$

with

$$\alpha_\sigma = \frac{(G_{NV-m}^j)^2}{\Delta_1^2 + \kappa_m^2}, \quad (C16)$$

and $\alpha_a \equiv \beta^2 \Delta_1$. Here, α_σ and β are real numbers, and the phase θ can be absorbed into the operators. Then, we can obtain the effective system Hamiltonian

$$\begin{aligned} \hat{H}_{\text{eff}} = & \hbar \beta^2 \Delta_1 \hat{a}^\dagger \hat{a} - \hbar \sum_j \Delta_1 \alpha_\sigma \hat{\sigma}_j^z \\ & + \hbar \sum_j \beta G_{NV-m}^j (\hat{a} \hat{\sigma}_j^+ + \hat{a}^\dagger \hat{\sigma}_j^-), \end{aligned} \quad (C17)$$

and we can define the following effective parameters:

$$G_{\text{eff}} = \beta G_{NV-m}^j, \quad (C18)$$

$$\kappa_{c,\text{eff}} = \kappa_c + \beta^2 \Delta_1, \quad (C19)$$

$$\gamma_{\text{eff}} = \gamma + \kappa_m \alpha_\sigma. \quad (C20)$$

APPENDIX D: TURNING ON COUPLING BETWEEN NV CENTERS AND PHOTONS

In this section, we will focus on experimental setups where the diamond slab containing NV centers is positioned inside a MW cavity, as depicted in Fig. 8(a). As is schematically shown in Fig. 8(b), the NV centers are now simultaneously coupled to a magnetic sphere and cavity modes. The full

Travis-Cummings Hamiltonian is written as

$$\begin{aligned} \hat{H} = & \hbar \Delta_1 \hat{m}^\dagger \hat{m} + \hbar \Delta_2 \hat{a}^\dagger \hat{a} + \sum_{j=1}^N \hbar G_{ph-NV}^j (\hat{a} \hat{\sigma}_j^+ + \hat{a}^\dagger \hat{\sigma}_j^-) \\ & - \sum_{j=1}^N \hbar G_{NV-m}^j (\hat{m} \hat{\sigma}_j^+ + \hat{m}^\dagger \hat{\sigma}_j^-) - \hbar G_{ph-m} (\hat{m}^\dagger \hat{a} + \hat{m} \hat{a}^\dagger). \end{aligned} \quad (D1)$$

The steady-state solutions for the mean magnon number, excited-state population, and dipole coherence can also be found, and the following relationships are retrieved:

$$\frac{|\alpha|^2}{|\mu|^2} = \frac{G_{ph-m}^2 G_{NV-m}^2 + G_{ph-NV}^2 \kappa_m^2}{G_{ph-NV}^2 G_{ph-m}^2 + G_{NV-m}^2 \kappa_c^2}, \quad (D2)$$

$$\frac{|s|^2}{|\mu|^2} = \frac{(\kappa_c \kappa_m + G_{ph-m}^2)^2}{N^2 (G_{NV-m}^2 \kappa_c^2 + G_{ph-NV}^2 G_{ph-m}^2)}, \quad (D3)$$

where for simplification we set $\Delta_{1,2} = 0$. From our numerical simulations, we observe lasing will be achieved between $\gamma_{\text{thr}} \approx \gamma = 2\pi \times 20$ kHz and $\gamma_P^{\text{max}} \approx 2\pi \times 1.32$ MHz, for $\Delta_{1,2} = 0$.

In Figs. 9(a)–9(c), we show the numerical results obtained for the time evolution of the average magnon number $\langle \hat{n}_m \rangle \equiv \langle \hat{m}^\dagger \hat{m} \rangle$, average photon number $\langle \hat{n}_{ph} \rangle \equiv \langle \hat{a}^\dagger \hat{a} \rangle$, and average excited-state population $\langle \hat{S}_{ee} \rangle$, for different pumpings $\gamma_P = 5\gamma$, 30γ , and 50γ (corresponding to $\gamma_P/2\pi = 0.1, 0.6$, and 1.0 MHz). In Figs. 9(d)–9(f), we show the steady-state evolution of the magnon number, photon number, and excited-state population as a function of the pumping rate. In this setup, we still obtain simultaneous lasing in magnon and photon modes, where we observe that the stronger the pump beam is, the greater will be the average number of photons and magnons produced.

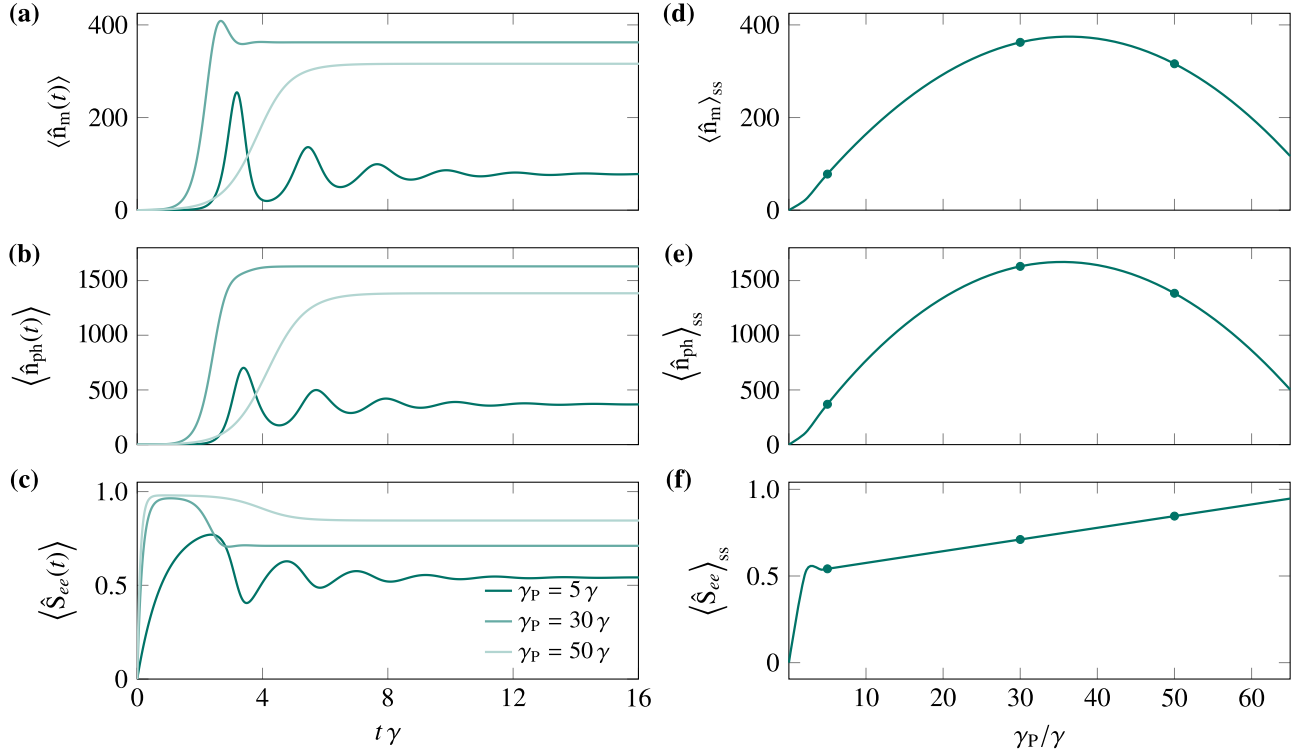


FIG. 9. Time evolution of the (a) average magnon number $\langle \hat{n}_m(t) \rangle$, (b) average photon number $\langle \hat{n}_{ph}(t) \rangle$, and (c) average excited-state population $\langle \hat{S}_{ee}(t) \rangle$ for different pumping of the NV centers where $\gamma_p = 5\gamma$, 30γ , and 50γ (corresponding to $\gamma_p/2\pi = 0.1, 0.6$, and 1.0 MHz). Evolution of the average steady state of the (d) magnon number $\langle \hat{n}_m \rangle_{ss}$, (e) photon number $\langle \hat{n}_{ph} \rangle_{ss}$, and (f) excited-state population $\langle \hat{S}_{ee} \rangle_{ss}$ with pumping γ_p . We have taken γ_p within the lasing regime, $\gamma_p \leq \gamma_p^{\max}$. We marked the three values for $\gamma_p/2\pi = 0.1, 0.6$, and 1.0 MHz. For each simulation, we considered the numerical parameters given in Table I and $G_{ph-NV} = 0.005$ MHz.

However, an interesting feature observed is that the spectrum peak shifts with changes in pumping strength. In Fig. 10, we show the numerical results for the emission spectrum of the magnons and photons, for different pumping of the NV centers, $\gamma_p = 2\gamma$, 30γ , and 65γ (corresponding to $\gamma_p/2\pi = 0.04, 0.6$, and 1.3 MHz). The emission spectrum is Lorentzian in the frequency domain, and it becomes narrower as the pumping rate is tuned from below to above threshold, typical of lasing. Nevertheless, it is clear that there is a shift to higher

frequencies with pumping strength, see Fig. 11. This is due to factors related to the interaction between the pumping strength and the laser medium. As the pumping strength increases, the population inversion in the laser gain medium also increases. In our system, the laser medium reaches a state of gain saturation at relatively small pumping, as the rate of stimulated emission becomes limited by the available population of excited states. This fact causes the laser wavelength to shift as we observe.

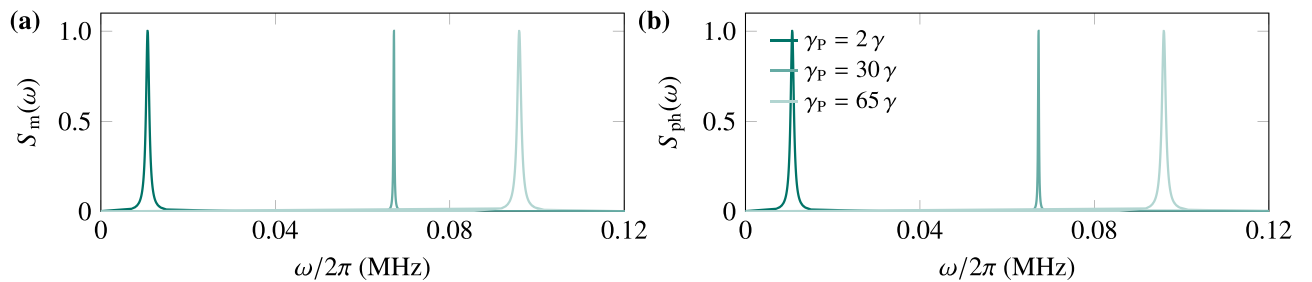


FIG. 10. Normalized spectrum for the (a) magnon and (b) photon modes as a function of frequency for different pumping of the NV centers, where $\gamma_p = 2\gamma$, 30γ , and 65γ (corresponding to $\gamma_p/2\pi = 0.04, 0.6$, and 1.3 MHz). For each simulation, we considered the numerical parameters given in Table I, with $G_{ph-NV} = 0.005$ MHz.

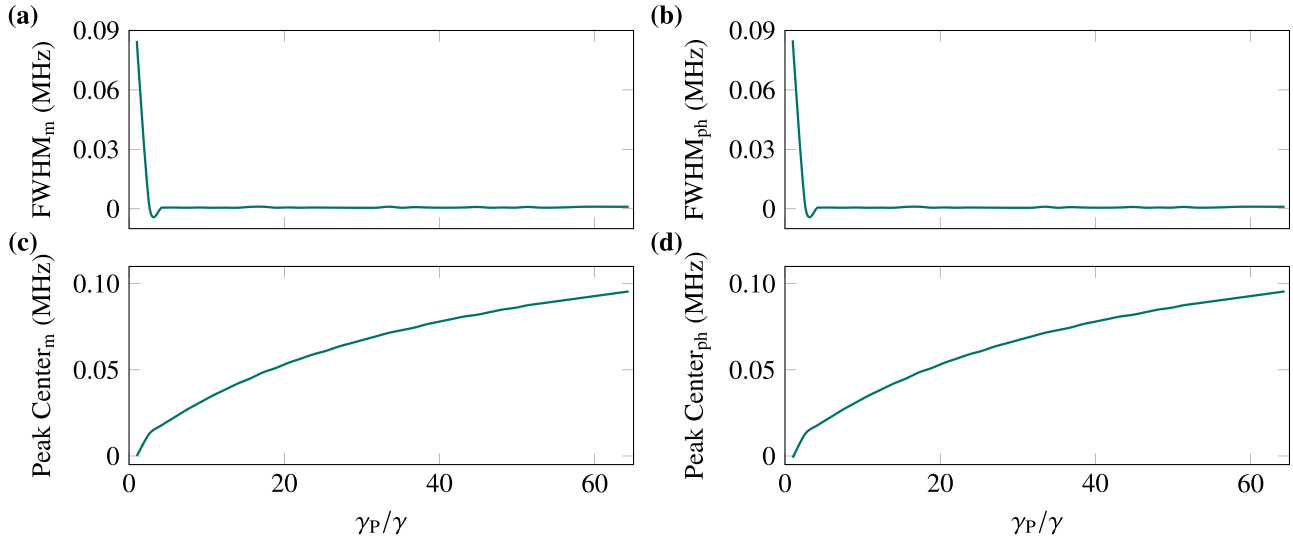


FIG. 11. FWHM of the normalized spectrum for (a) magnons and (b) photons as a function of pumping γ_P of the NV centers, for magnons and photon modes. Spectrum peak center of the normalized spectrum for (c) magnons and (d) photons. For each simulation, we considered the numerical parameters given in Table I, with $G_{\text{ph-NV}} = 0.005$ MHz.

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