

# Relative enhancement of low-mass vector-boson exchange in states with higher angular momentum: Parity nonconservation in hydrogen

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Models of unification predict additional  $Z'$  boson, which contributes to parity non-conservation (PNC) in atoms. If  $Z'$  boson is light, ratio of  $Z'$ -boson contribution to the standard model  $Z$ -boson contribution to atomic PNC increases with decreasing nuclear charge  $Z$  faster than  $1/Z^2$ . This motivated our previous study of PNC in hydrogen and deuterium, where  $s$ - $p$  wave mixing induced by the weak interaction was considered. An even more pronounced relative enhancement occurs when higher-angular-momentum states are involved, such as in the  $p$ - $d$  weak matrix elements. A light boson mediates a long-range interaction, in contrast to the effectively contactlike interaction mediated by the standard model  $Z$  boson. Consequently, higher-angular-momentum states provide a background-free system: Their suppressed overlap with the nucleus strongly reduces the standard model contribution while leaving the long-range  $Z'$  contribution unsuppressed.

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## I. INTRODUCTION

Atomic parity nonconservation (PNC) continues to provide one of the most accurate low-energy tests of the electroweak interaction and a sensitive means of searching for new neutral-current physics; see, for example, the reviews in Refs. [1,2]. The basic mechanism is well known: Weak electron-nucleus interactions admix atomic states of opposite parity and thereby induce electric-dipole amplitudes that are forbidden in the absence of parity violation. Because these amplitudes can be both measured and calculated with high precision, atomic PNC has long served as a probe of the standard model and of possible extensions of it, including additional neutral gauge bosons [3,4]. Limits on the interaction of the finite mass  $Z'$  boson with nucleons and electron have been obtained in Refs. [5,6] using measurements of PNC effects in Cs [7] and Yb [6].

In the case of very close levels of opposite parity, matrix elements of the weak interaction between these states may be measured using interference between the weak interaction and the interaction with oscillating electric field, which also produces this mixing between close levels. This type of experiment has been done in Dy atom [8]. Such measurement may also be performed in hydrogen and deuterium.

Among atomic systems, hydrogen is distinguished by its exceptional theoretical simplicity. In contrast to heavy many-electron atoms, it is free from many-body electronic-structure

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uncertainties and allows one to study parity-violating electron-proton interactions in an especially transparent form. The possibility of observing parity violation in hydrogen was pointed out long ago in Refs. [9–11]. Subsequent work developed the formalism for hydrogen and deuterium in detail, identified the nuclear-spin-independent (NSI) weak coupling constant  $C_{1p}$  and the nuclear-spin-dependent (NSD) weak coupling constant  $C_{2p}$  as the relevant low-energy parameters, and emphasized the important role of metastable states and level crossings in enhancing observable effects [12,13]. More recently, laser-based and quantum-control approaches have renewed interest in hydrogen PNC as a clean source of information on the proton weak charge and on weak neutral-current couplings [14,15].

At the same time, parity-violating electron scattering has emerged as a complementary and conceptually clean probe of semileptonic neutral-current interactions. The proton weak charge is especially sensitive to new physics because its standard model value is accidentally small [16]. The final Qweak result for parity-violating elastic  $ep$  scattering agrees with the standard model and places strong limits on additional parity-violating semileptonic interactions [17]. In the heavy-boson regime, these bounds admit a model-independent effective-field-theory interpretation in terms of contact operators, and Ref. [18] gives a useful summary of the present constraints. Future and ongoing parity-violating electron-scattering programs will extend this sensitivity to extra neutral gauge bosons over a broad mass range [19–21].

The possibility of an additional neutral boson  $Z'$  is therefore of considerable current interest. When the boson is sufficiently heavy, its exchange reduces at atomic energies to an effective local four-fermion interaction. In atomic language, the corresponding effect may then be absorbed into shifts of the conventional weak charge or weak coupling constants. This short-range regime underlies most model-independent discussions of heavy- $Z'$  constraints from parity-violation experiments [4,18]. If, however, the boson mass is small enough that its Compton wavelength becomes comparable to or larger than the atomic length scale, the interaction is no longer local, and one must retain the full Yukawa form of the potential. In that case, the atomic matrix elements depend on the detailed spatial overlap of the electronic wave functions with the finite-range parity-violating potential rather than only on contact densities.

This finite-range regime has already been investigated in our earlier work on heavy atoms and ions [5,22]. Those studies provided the general framework for treating parity-violating interactions generated by a vector boson of arbitrary mass and for connecting the contact and long-range limits in a unified way. We subsequently applied this approach explicitly to hydrogen in Ref. [23], where we considered parity violation induced by a  $Z'$  boson of arbitrary mass for the standard  $s$ - $p$  mixing channels, including both NSI and NSD interactions. In that paper, the new contribution was analyzed in relation to the familiar standard model  $Z$ -boson background.

The motivation of the present work is different. Here, we turn to parity mixing between the  $3d$  and  $3p$  states of hydrogen induced by exchange of a  $Z'$  boson. This case is especially attractive because there is no corresponding standard model  $Z$ -boson contribution since  $p_{3/2}$  and  $d_{3/2,5/2}$  wave functions vanish at  $r \rightarrow 0$ , suppressing matrix elements of the contact  $Z$ -boson-mediated contact electron-nucleus interaction. As a result, any observable parity-violating effect in this channel would provide a direct signature of the new interaction, without the need to separate it from an irreducible standard model background. This feature makes the  $3d$ - $3p$  manifold an unusually clean setting for a search for new neutral-current physics.

A further advantage is that the  $3p_{3/2}$  and  $3d_{3/2}$  levels are separated by a very small energy interval. Here, the weak mixing amplitude is divided by a small residual energy denominator, which can strongly enhance the observable parity-violating signal. The intervals between all  $3p_{1/2,3/2}$  and  $3d_{3/2,5/2}$  energy levels may be manipulated by an external magnetic field including crossing of the levels. In this respect, the mechanism is similar to the level-crossing enhancement discussed previously in hydrogen PNC [12,13], but in the present case it is combined with the absence of a standard model  $Z$ -boson contribution in the same mixing channel.

Heavy-atom PNC, parity-violating electron scattering, and hydrogen spectroscopy probe related but not identical combinations of couplings [16–18]. Hydrogen remains particularly valuable

because the relevant matrix elements may be derived analytically or semianalytically with controlled accuracy, making the dependence on the boson mass and on the underlying axial-electron/vector-proton and vector-electron/axial-proton coupling structures especially transparent. As emphasized in Ref. [5], the ratio of the low-mass  $Z'$  contribution to the standard model  $Z$ -boson contribution increases rapidly with decreasing nuclear charge, faster than  $1/Z^2$ , which further strengthens the case for light systems.

In this paper, we extend the analysis of Ref. [23] to the  $3d-3p$  manifold in hydrogen. We derive the NSI and NSD parity-violating matrix elements produced by a  $Z'$  boson of arbitrary mass. The absence of a standard model contribution of the same type makes this system a particularly transparent probe of a new parity-violating interaction.

## II. INTERACTION LAGRANGIAN FOR $Z$ AND $Z'$ BOSONS

The neutral-current Lagrangian, including  $Z$ -boson field and its interaction with fermions, is ( $\hbar = c = 1$ )

$$\mathcal{L}_Z = -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu - \frac{g}{2\cos\theta_W} Z_\mu J_Z^\mu, \quad (1)$$

with the fermion current

$$J_Z^\mu = \sum_f \bar{f} \gamma^\mu (g_f^V - g_f^A \gamma_5) f. \quad (2)$$

Here,  $m_Z$  is the  $Z$ -boson mass and  $g$  is the universal interaction constant of the unified electroweak theory where proton electric charge is  $e = g \sin\theta_W$  and  $\theta_W$  is the Weinberg angle. Dimensionless fermion interaction constants  $g_f^V$  and  $g_f^A$  for proton and electron are presented below;  $\gamma^\mu$  and  $\gamma_5$  are the Dirac matrices.

### A. Nuclear-spin-independent PV interaction

The parity-violating nuclear-spin-independent part of the interaction comes from the electron axial current and the proton vector current. For the standard model (SM)  $Z$  boson (without radiative corrections),

$$g_e^A = -\frac{1}{2}, \quad g_p^V = \frac{1}{2} - 2\sin^2\theta_W. \quad (3)$$

Proton weak charge, which determines strength of PNC electron-proton interaction, is equal to

$$Q_W^p = -4g_e^A g_p^V = 1 - 4\sin^2\theta_W. \quad (4)$$

Inclusion of radiative corrections gives  $Q_W^p = 0.0705$  [24]. Corrections produced by the dispersive parity violating interaction may also be included in the definition of the weak charge giving  $Q_W^p = 0.0719$  [25]. The Qweak experimental result is  $Q_W^{p,\text{exp}} = 0.0719(45)$  [17].

Then, we may write the NSI part of the  $Z$ -exchange parity-violating Hamiltonian as

$$V_Z(r) = -\frac{G_F}{2\sqrt{2}} Q_W^p m_Z^2 \Phi(m_Z, r) \gamma_5, \quad (5)$$

where  $G_F$  is the Fermi constant of the weak interaction,

$$\Phi(m, r) = \frac{1}{4\pi} \frac{e^{-mr}}{r}. \quad (6)$$

In the case of the large mediator mass,  $m \rightarrow \infty$ ,

$$\Phi(m, r) = \frac{1}{m^2} \delta^3(r). \quad (7)$$

For a generic parity-violating  $Z'$  boson, we absorb the model-dependent gauge coupling and charges into two effective couplings  $g'_{eA}$  and  $g'_{pV}$  and present NSI interaction potential as

$$V_{Z'}(r) = g'_{eA} g'_{pV} \Phi(m_{Z'}, r) \gamma_5. \quad (8)$$

### B. $Z'$ -boson matrix elements

We write the Dirac orbital as

$$\psi_{n\kappa m}(\mathbf{r}) = \frac{1}{r} \begin{pmatrix} g_{n\kappa}(r) \Omega_{\kappa m}(\hat{\mathbf{r}}) \\ i\alpha f_{n\kappa}(r) \Omega_{-\kappa m}(\hat{\mathbf{r}}) \end{pmatrix}. \quad (9)$$

Here,  $\alpha$  is the fine structure constant. For the  $3p_{3/2}$  and  $3d_{3/2}$  states, one has  $\kappa_{p_{3/2}} = -2$  and  $\kappa_{d_{3/2}} = +2$ . In the nonrelativistic limit,

$$g_{3p}(r') = r' R_{31}(r'), \quad (10)$$

$$g_{3d}(r') = r' R_{32}(r'), \quad (11)$$

$$f_{3p}(r') = \frac{1}{2} \left( \frac{d}{dr'} - \frac{2}{r'} \right) g_{3p}(r'), \quad (12)$$

$$f_{3d}(r') = \frac{1}{2} \left( \frac{d}{dr'} + \frac{2}{r'} \right) g_{3d}(r'), \quad (13)$$

with

$$R_{31}(r') = \frac{4}{81\sqrt{6}} r' (6 - r') e^{-r'/3}, \quad (14)$$

$$R_{32}(r') = \frac{4}{81\sqrt{30}} r'^2 e^{-r'/3}. \quad (15)$$

Here,  $r' = r/a_B$ , where  $a_B = 1/(m_e \alpha)$  is the Bohr radius.

The above formulas assume a pointlike proton. We have also evaluated numerically the effect of the finite proton size on the weak-interaction matrix elements. We find that the correction is of order  $10^{-4}$  for the  $s$ - $p$  matrix elements in the case of the contact interaction; it is practically zero for the  $p$ - $d$  matrix elements and for the small mediator mass case.

#### 1. NSI $3p_{3/2}$ - $3d_{3/2}$ matrix element

Since  $\kappa_f = -\kappa_i$ , the NSI  $\gamma_5$  matrix element has the same radial structure as in the  $ns_{1/2}$ - $np_{1/2}$  case:

$$\mathcal{M}_{pd}^{Z', \text{NSI}} \equiv \langle 3p_{3/2} m | V_{Z'}^{\text{NSI}} | 3d_{3/2} m \rangle = i g'_{eA} g'_{pV} \alpha \int_0^\infty dr (g_{3p} f_{3d} - f_{3p} g_{3d}) \Phi(m_{Z'}, r). \quad (16)$$

Evaluation of the radial integral gives

$$\mathcal{M}_{pd}^{Z', \text{NSI}} = i \frac{4\sqrt{5}}{135\pi a_B} g'_{eA} g'_{pV} \alpha \frac{15\mu + 2}{(3\mu + 2)^5}, \quad (17)$$

for any  $m = \pm 1/2, \pm 3/2$ . Here,  $\mu = m_{Z'} a_B = m'_{Z'}/(\alpha m_e)$ .

For a heavy SM  $Z$  boson, the contact matrix element between  $3p$  and  $3d$  vanishes for a point proton. It is therefore more useful to compare  $\mathcal{M}_{pd}^{Z', \text{NSI}}$  with the standard model  $2s$ - $2p_{1/2}$  matrix element,

$$\mathcal{M}_2^{Z, \text{NSI}} = i\alpha \frac{G_F Q_W^p \sqrt{3}}{64\pi \sqrt{2} a_B^3}. \quad (18)$$

The ratio is

$$\frac{\mathcal{M}_{pd}^{Z',\text{NSI}}}{\mathcal{M}_2^{Z,\text{NSI}}} = g'_{eA} g'_{pV} \frac{256\sqrt{30}a_B^2}{405 G_F Q_W^p} \frac{15\mu + 2}{(3\mu + 2)^5} \quad (19)$$

with limiting forms

$$\mu \rightarrow 0 : \quad \frac{\mathcal{M}_{pd}^{Z',\text{NSI}}}{\mathcal{M}_2^{Z,\text{NSI}}} \rightarrow g'_{eA} g'_{pV} \frac{16\sqrt{30}a_B^2}{405 G_F Q_W^p}, \quad (20)$$

$$\mu \rightarrow \infty : \quad \frac{\mathcal{M}_{pd}^{Z',\text{NSI}}}{\mathcal{M}_2^{Z,\text{NSI}}} \rightarrow g'_{eA} g'_{pV} \frac{256\sqrt{30}a_B^2}{6561 G_F Q_W^p} \mu^{-4}. \quad (21)$$

Numerically, for  $Q_W^p = 0.0719$ ,

$$\left| \frac{\mathcal{M}_{pd}^{Z',\text{NSI}}}{\mathcal{M}_2^{Z,\text{NSI}}} \right|_{\mu \rightarrow 0} = 1.856 \times 10^{16} |g'_{eA} g'_{pV}|. \quad (22)$$

Note that this value is only  $\sim 4$  times smaller than the corresponding value for the  $s$ - $p$  weak matrix elements [23]. This means that the contribution of a light  $Z'$  boson to the weak mixing of the  $p_{3/2}$  and  $d_{3/2}$  states remains sufficiently large, while the background contribution from the standard model  $Z$  boson is practically absent. This demonstrates the potential advantage of searching for a  $Z'$  boson through PNC transitions involving higher-angular-momentum states.

A low-mass limit on the product of the weak-interaction constants  $|g'_{eA} g'_{pV}| = 1.6 \times 10^{-12}$  was obtained from PNC measurements in Yb [6]. Using this number in Eq. (22), we get

$$\left| \frac{\mathcal{M}_{pd}^{Z',\text{NSI}}}{\mathcal{M}_2^{Z,\text{NSI}}} \right|_{\mu \rightarrow 0} = 3.0 \times 10^4.$$

This means that a measurement of the PNC effects with the sensitivity sufficient to detect  $(2s)$ -( $2p$ ) weak matrix element for the standard model PNC contribution could improve the limit on the ( $Z'$ ) contribution by about 4 orders of magnitude compared with the current limit from Yb.

## 2. Nuclear-spin-dependent interaction

With the current convention  $\bar{f} \gamma^\mu (g_f^V - g_f^A \gamma_5) f$ , the vector-electron-axial-proton part of the finite-range interaction is written as

$$V_{Z'}^{\text{NSD}}(r) = -\frac{g'_{eV} g'_{pA}}{I} \boldsymbol{\alpha} \times \mathbf{I} \Phi(m_{Z'}, r). \quad (23)$$

The overall sign is convention dependent and may equivalently be absorbed into the definition of  $g'_{pA}$ ; it is fixed here so that all NSD amplitudes below are mutually consistent. For comparison, the standard model contact Hamiltonian is

$$H_Z^{\text{NSD}} = \frac{G_F}{\sqrt{2}} \frac{\varkappa_p}{I} \boldsymbol{\alpha} \cdot \mathbf{I} \delta^3(\mathbf{r}), \quad (24)$$

where we use  $\varkappa_p = 0.043$ , the radiatively corrected electron-vector-proton-axial coupling conventionally quoted for hydrogen when the proton anapole contribution is omitted [12].

## 3. NSD $3p_{3/2}$ - $3d_{3/2}$ matrix element

For hydrogen in the stretched hyperfine state  $I = 1/2$ ,  $m_I = 1/2$ , and  $m_j = 3/2$ , one has  $(\boldsymbol{\alpha} \times \mathbf{I})/I \rightarrow \boldsymbol{\alpha}_z$ . The relevant angular factors are

$$\langle \Omega_{-2,3/2} | \sigma_z | \Omega_{-2,3/2} \rangle = 1, \quad \langle \Omega_{2,3/2} | \sigma_z | \Omega_{2,3/2} \rangle = -\frac{3}{5}. \quad (25)$$

Hence,

$$\begin{aligned}\mathcal{M}_{pd,z}^{Z',\text{NSD}} &\equiv \left\langle 3p_{3/2}, m_j = \frac{3}{2} \left| V_{Z'}^{\text{NSD}} \right| 3d_{3/2}, m_j = \frac{3}{2} \right\rangle \\ &= -i g'_{eV} g'_{pA} \alpha \int_0^\infty dr \left( g_{3p} f_{3d} + \frac{3}{5} f_{3p} g_{3d} \right) \Phi(m_{Z'}, r),\end{aligned}\quad (26)$$

which yields

$$\mathcal{M}_{pd,z}^{Z',\text{NSD}} = -i \frac{4\sqrt{5}}{675\pi a_B} g'_{eV} g'_{pA} \alpha \frac{225\mu^2 + 36\mu + 4}{(3\mu + 2)^6} \quad (27)$$

for the  $z$  component in the stretched state.

Comparing with the standard  $2s$ - $2p_{1/2}$  NSD matrix element,

$$\mathcal{M}_2^{Z,\text{NSD}} = -i\alpha \frac{\sqrt{3} G_F \varkappa_p}{32\pi \sqrt{2} a_B^3}, \quad (28)$$

we obtain

$$\frac{\mathcal{M}_{pd,z}^{Z',\text{NSD}}}{\mathcal{M}_2^{Z,\text{NSD}}} = g'_{eV} g'_{pA} \frac{128\sqrt{30} a_B^2}{2025 G_F \varkappa_p} \frac{225\mu^2 + 36\mu + 4}{(3\mu + 2)^6}, \quad (29)$$

with the low-mass limit

$$\left| \frac{\mathcal{M}_{pd,z}^{Z',\text{NSD}}}{\mathcal{M}_2^{Z,\text{NSD}}} \right|_{\mu \rightarrow 0} = 3.103 \times 10^{15} |g'_{eV} g'_{pA}|. \quad (30)$$

The stretched states corresponds to the hyperfine component  $F = 2$ . The  $F = 1$  and  $F = 2$  hyperfine amplitudes are related by (see Appendix)

$$\langle 3p_{3/2}, F = 1, F_z | V_{Z'}^{\text{NSD}} | 3d_{3/2}, F = 1, F_z \rangle = -\frac{5}{3} \langle 3p_{3/2}, F = 2, F_z | V_{Z'}^{\text{NSD}} | 3d_{3/2}, F = 2, F_z \rangle. \quad (31)$$

We will also need matrix elements of NSD operator between  $3p_{3/2}$  and  $3d_{5/2}$  states with  $F = 2$ :

$$\langle 3p_{3/2}, F = 2, F_z | V_{Z'}^{\text{NSD}} | 3d_{5/2}, F = 2, F_z \rangle = -i\alpha \frac{32\sqrt{30}}{675\pi a_B} g'_{eV} g'_{pA} \frac{9\mu + 1}{(3\mu + 2)^6}, \quad (32)$$

and between  $3p_{1/2}$  and  $3d_{3/2}$  states with  $F = 1$ :

$$\langle 3p_{1/2}, F = 1, F_z | V_{Z'}^{\text{NSD}} | 3d_{3/2}, F = 1, F_z \rangle = -i\alpha \frac{16\sqrt{10}}{405\pi a_B} g'_{eV} g'_{pA} \frac{45\mu^2 + 18\mu + 2}{(3\mu + 2)^6}. \quad (33)$$

### C. Parity-violating E1 amplitudes and magnetic-field effect

For a  $3d_j$ , final state mixed with  $3p_{3/2}$ ,  $j = 3/2, 5/2$ , the induced E1 amplitude is

$$E_{\text{PNC}}(E) = \frac{\langle 3d_j | V_{Z'} | 3p_{3/2} \rangle \langle 3p_{3/2} | D_q | n_s s_{1/2} \rangle}{E - E_{3p_{3/2}} + i\Gamma_{3p_{3/2}}/2}, \quad (34)$$

where the excitation energy  $E \approx E_{3d_j}$  [26]. The E1 matrix elements entering this and the following expressions are given in Appendix 2. The energy interval  $E_{3d_j} - E_{3p_{3/2}}$  can be tuned by a magnetic field.

The experimental zero-field interval,

$$\Delta_0(3p_{3/2} - 3d_{3/2}) = 5.5(9) \text{ MHz},$$

is smaller than the widths of the states,

$$\Gamma_{3p_{3/2}}/h = 30.21 \text{ MHz}, \quad (35)$$

$$\Gamma_{3d_{3/2}}/h = 10.30 \text{ MHz}. \quad (36)$$

The resonances  $3p_{3/2}$  and  $3d_{3/2}$  overlap and this makes experiment to measure  $E_{\text{PNC}}$  in the transitions  $1s, 2s - 3d_{3/2}$  practically impossible, since a tail from the strong E1 transition from  $s$  states to  $3p_{3/2}$  state will make observation of the many orders of magnitude weaker direct  $1s, 2s - 3d_{3/2}$  transitions impossible. A more realistic possibility may be to measure interference between electric  $\langle 3p_{3/2} | e\mathbf{Er} | 3d_{3/2} \rangle$  and weak  $\langle 3p_{3/2} | W | 3d_{3/2} \rangle$  amplitudes between the states  $3p_{3/2}$  and  $3d_{3/2}$  as it was done in the Dy experiment [8]. The resonances  $3p_{3/2}$  and  $3d_{5/2}$  do not overlap and here the measurement of NSD  $E_{\text{PNC}}$  in the transitions  $1s, 2s - 3d_{5/2}$  is not excluded.

In the linear Zeeman approximation with Landé factors

$$g(3p_{3/2}) = \frac{4}{3}, \quad g(3d_{3/2}) = \frac{4}{5},$$

the crossing field for the stretched  $m_j = 3/2$  components of  $3p_{3/2}$  and  $3d_{3/2}$  states is about

$$B_c(3p_{3/2}, 3d_{3/2}) = \frac{\Delta_0}{\mu_B(g_{3p_{3/2}} - g_{3d_{3/2}})(3/2)} = 4.9 \text{ G}, \quad (37)$$

using  $\mu_B/h = 13.996\,245\,55 \text{ GHz/T}$ . Since the hyperfine splittings are comparable to this very small electronic interval, a prediction for a specified hyperfine component must include the hyperfine Hamiltonian.

The energy intervals between the levels  $3p_{3/2}$  and  $3d_{5/2}$ , and between  $3p_{1/2}$  and  $3d_{3/2}$  may be reduced by magnetic field, which could enhance their mixing and PNC amplitude. However, the magnetic interaction in this case is comparable to the fine structure interval and this leads to a nonlinear dependence of the energy intervals and wave functions on the magnetic field. The calculations indicate that there are several crossing points between  $3p_{1/2}$ - $3d_{3/2}$  levels and between  $3p_{3/2}$ - $3d_{5/2}$  levels that may be used to enhance NSD PV effects.

The  $2s$ - $2p_{1/2}$  interval is

$$\Delta_0(2s_{1/2} - 2p_{1/2}) = 1057.845(3) \text{ MHz},$$

and its crossing occurs near 0.112 T. Using the natural width of the  $2p_{1/2}$  level,

$$\Gamma_{2p_{1/2}}/h \approx 99.7 \text{ MHz},$$

the enhancement factor

$$K_{\text{max}}(2s - 2p_{1/2}) \lesssim \frac{\Delta_0}{\Gamma_{2p_{1/2}}} = 10.$$

The different magnetic-field dependence of the  $2s$ - $2p$  and  $3d$ - $3p$  contributions to  $E_{\text{PNC}}$  may be used to separate them in  $2s$ - $3d$  measurements by varying magnetic field, provided that the full field-dressed Hamiltonian is used.

### III. DEUTERIUM

Deuterium PNC experiments are of independent interest since they may give us interaction constant of  $Z'$  boson with neutron. Energy intervals, widths, and electromagnetic amplitudes in deuterium are very close to those in hydrogen. Therefore, to find results for  $Z'$  in the NSI case it is sufficient to replace the proton interaction constant  $g'_{pV}$  by the deuterium interaction constant  $g'_{dV} = g'_{pV} + g'_{nV}$ .

The NSD rescaling is less direct. The deuteron has spin  $I = 1$ . In a zero approximation, spins of proton and neutron in deuterium are parallel. However, proton and neutron spin expectation values

are modified by the deuteron  $D$ -state admixture. Thus,  $g'_{dA} \simeq g'_{pA} + g'_{nA}$  is only a leading  $S$ -wave estimate. Also, the hyperfine recoupling coefficients are different from those for hydrogen.

Within the standard model, deuterium weak charge  $Q_W^d = -0.982\,07N + 0.071\,918Z = -0.910$  is dominated by the neutron contribution and significantly exceeds the proton weak charge. The standard model value of  $\kappa_d$  is very small since  $\kappa_n \approx -\kappa_p$ . A significant contribution is given by the nuclear anapole moment [27]. However, in the pointlike nucleus approximation this contribution, as well as contribution of  $Z$  boson, vanishes in  $p$ - $d$  matrix elements.

The angular reduction coefficient for deuterium with  $I = 1$  is different from that for hydrogen with  $I = 1/2$ . Here, we give only the resulting matrix elements. All amplitudes below are diagonal in  $F$  and  $F_z$  and are independent of  $F_z$ .

For the NSI  $2s_{1/2}$ - $2p_{1/2}$  matrix element, the allowed hyperfine values are  $F = 1/2, 3/2$ , and

$$\mathcal{M}_{2,d}^{Z',\text{NSI}} = i \frac{\sqrt{3}\alpha}{96\pi a_B} g'_{eA} g'_{dV} \frac{3\mu + 1}{(\mu + 1)^3}. \quad (38)$$

The corresponding standard model contact matrix element is

$$\mathcal{M}_{2,d}^{Z,\text{NSI}} = i \alpha \frac{\sqrt{3} G_F Q_W^d}{64\pi \sqrt{2} a_B^3}. \quad (39)$$

For the NSI  $3p_{3/2}$ - $3d_{3/2}$  matrix element,  $F = 1/2, 3/2, 5/2$ , and

$$\mathcal{M}_{pd,d}^{Z',\text{NSI}} = i \frac{4\sqrt{5}\alpha}{135\pi a_B} g'_{eA} g'_{dV} \frac{15\mu + 2}{(3\mu + 2)^5}. \quad (40)$$

For compactness, we define

$$\mathcal{A}_d \equiv \frac{\alpha}{\pi a_B} g'_{eV} g'_{dA} \quad (41)$$

and

$$X_{33}(\mu) = \frac{225\mu^2 + 36\mu + 4}{(3\mu + 2)^6}, \quad X_{35}(\mu) = \frac{9\mu + 1}{(3\mu + 2)^6}, \quad X_{13}(\mu) = \frac{45\mu^2 + 18\mu + 2}{(3\mu + 2)^6}. \quad (42)$$

For the NSD  $3p_{3/2}$ - $3d_{3/2}$  matrix elements, the results for  $F = 1/2, 3/2, 5/2$  are

$$\begin{aligned} \mathcal{M}_{33,d}^{(F=1/2)} &= +i \frac{4\sqrt{5}}{405} \mathcal{A}_d X_{33}, \\ \mathcal{M}_{33,d}^{(F=3/2)} &= +i \frac{8\sqrt{5}}{2025} \mathcal{A}_d X_{33}, \\ \mathcal{M}_{33,d}^{(F=5/2)} &= -i \frac{4\sqrt{5}}{675} \mathcal{A}_d X_{33}. \end{aligned} \quad (43)$$

For the NSD  $3p_{3/2}$ - $3d_{5/2}$  matrix elements, the common hyperfine values are  $F = 3/2, 5/2$ , and

$$\begin{aligned} \mathcal{M}_{35,d}^{(F=3/2)} &= -i \frac{16\sqrt{5}}{225} \mathcal{A}_d X_{35}, \\ \mathcal{M}_{35,d}^{(F=5/2)} &= -i \frac{16\sqrt{70}}{675} \mathcal{A}_d X_{35}. \end{aligned} \quad (44)$$

Finally, for the NSD  $3p_{1/2}$ - $3d_{3/2}$  matrix elements for  $F = 1/2, 3/2$ ,

$$\begin{aligned} \mathcal{M}_{13,d}^{(F=1/2)} &= -i \frac{8\sqrt{10}}{405} \mathcal{A}_d X_{13}, \\ \mathcal{M}_{13,d}^{(F=3/2)} &= -i \frac{8}{81} \mathcal{A}_d X_{13}. \end{aligned} \quad (45)$$

#### IV. CONCLUSION

We have investigated nuclear-spin-independent and nuclear-spin-dependent parity nonconserving interactions between the  $3p$  and  $3d$  states of hydrogen and deuterium mediated by a  $Z'$  boson of arbitrary mass. Analytic expressions were obtained for the relevant matrix elements. Because the leading contact contribution from standard model  $Z$ -boson exchange is practically absent in these higher-angular-momentum channels, the  $3p$ - $3d$  mixings provide a particularly clean probe of a finite-range parity-violating interaction. The relative contribution of a light  $Z'$  is therefore strongly enhanced compared with the conventional  $s$ - $p$  channels. Therefore, study of PNC effects in hydrogen and deuterium offers a theoretically clean and experimentally accessible route to searching for neutral-current interactions beyond the standard model.

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#### DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

#### APPENDIX

##### 1. $3p_{3/2}$ - $3d_{3/2}$ NSD matrix elements in the hyperfine basis

For the nuclear-spin-dependent interaction, it is convenient to pass from the uncoupled basis to the hyperfine basis

$$|nk; F F_z\rangle = \sum_{m_j, m_I} C_{jm_j, Im_I}^{FF_z} |nk jm_j\rangle |Im_I\rangle, \quad I = \frac{1}{2}, \quad (\text{A1})$$

where  $\mathbf{F} = \mathbf{j} + \mathbf{I}$ . Since

$$V_{Z'}^{\text{NSD}}(r) = \frac{g'_{eV} g'_{pA}}{I} \boldsymbol{\alpha} \times \mathbf{I} \Phi(m_{Z'}, r) \quad (\text{A2})$$

is a scalar in the total angular-momentum space, its matrix elements are diagonal in  $F$  and  $F_z$  and do not depend on  $F_z$ .

Introducing the electronic rank-1 operator

$$T_q^{(1)} = g'_{eV} g'_{pA} \alpha_q \Phi(m_{Z'}, r), \quad (\text{A3})$$

one may write

$$V_{Z'}^{\text{NSD}} = \frac{1}{I} \sum_q (-1)^q T_q^{(1)} I_{-q}. \quad (\text{A4})$$

The hyperfine matrix element is then obtained from the electronic reduced matrix element as

$$\mathcal{M}_{ab}^{(F)} = (-1)^{F+j_b+I} \begin{Bmatrix} j_a & j_b & 1 \\ I & I & F \end{Bmatrix} \frac{\langle I \| I \| I \rangle}{I} \langle a j_a \| T^{(1)} \| b j_b \rangle, \quad (\text{A5})$$

with

$$\frac{\langle I \| I \| I \rangle}{I} = \sqrt{6} \quad \left( I = \frac{1}{2} \right). \quad (\text{A6})$$

For the  $3p_{3/2}$ - $3d_{3/2}$  channel, the electronic reduced matrix element derived above is

$$\langle 3p_{3/2} \| T^{(1)} \| 3d_{3/2} \rangle = -i \frac{8\sqrt{3}}{405\pi a_B} g'_{eV} g'_{pA} \alpha \frac{225\mu^2 + 36\mu + 4}{(3\mu + 2)^6}. \quad (\text{A7})$$

For  $F = 2$ , one finds

$$(-1)^{F+j_b+I} \begin{Bmatrix} \frac{3}{2} & \frac{3}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 2 \end{Bmatrix} \sqrt{6} = \frac{\sqrt{15}}{10}. \quad (\text{A8})$$

Therefore, for any  $F_z = -2, -1, 0, 1, 2$ ,

$$\langle 3p_{3/2}, F = 2, F_z | V_{Z'}^{\text{NSD}} | 3d_{3/2}, F = 2, F_z \rangle = -i\alpha \frac{4\sqrt{5}}{675\pi a_B} g'_{eV} g'_{pA} \frac{225\mu^2 + 36\mu + 4}{(3\mu + 2)^6}, \quad (\text{A9})$$

which coincides with the stretched-state result, as expected.

For  $F = 1$ , the recoupling coefficient is

$$(-1)^{F+j_b+I} \begin{Bmatrix} \frac{3}{2} & \frac{3}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{Bmatrix} \sqrt{6} = -\frac{\sqrt{15}}{6}. \quad (\text{A10})$$

Hence, for any  $F_z = -1, 0, 1$ ,

$$\langle 3p_{3/2}, F = 1, F_z | V_{Z'}^{\text{NSD}} | 3d_{3/2}, F = 1, F_z \rangle = +i\alpha \frac{4\sqrt{5}}{405\pi a_B} g'_{eV} g'_{pA} \frac{225\mu^2 + 36\mu + 4}{(3\mu + 2)^6}, \quad (\text{A11})$$

which is also independent of  $F_z$ .

Thus, the  $F = 1$  and  $F = 2$  hyperfine amplitudes are related by

$$\langle 3p_{3/2}, F = 1, F_z | V_{Z'}^{\text{NSD}} | 3d_{3/2}, F = 1, F_z \rangle = -\frac{5}{3} \langle 3p_{3/2}, F = 2, F_z | V_{Z'}^{\text{NSD}} | 3d_{3/2}, F = 2, F_z \rangle. \quad (\text{A12})$$

As rank-0 reduced matrix elements in the hyperfine space, one has

$$\langle 3p_{3/2}, F = 2 \| V_{Z'}^{\text{NSD}} \| 3d_{3/2}, F = 2 \rangle = -i\alpha \frac{4}{135\pi a_B} g'_{eV} g'_{pA} \frac{225\mu^2 + 36\mu + 4}{(3\mu + 2)^6}, \quad (\text{A13})$$

$$\langle 3p_{3/2}, F = 1 \| V_{Z'}^{\text{NSD}} \| 3d_{3/2}, F = 1 \rangle = +i\alpha \frac{4\sqrt{15}}{405\pi a_B} g'_{eV} g'_{pA} \frac{225\mu^2 + 36\mu + 4}{(3\mu + 2)^6}. \quad (\text{A14})$$

We have checked explicitly by direct summation over the uncoupled basis  $|jm_j\rangle|Im_I\rangle$  that the matrix elements in Eqs. (A9) and (A11) are independent of  $F_z$ , as required for a scalar operator in the hyperfine basis.

For the  $3p_{3/2}$  and  $3d_{5/2}$  states, one has  $\kappa_{p_{3/2}} = -2$  and  $\kappa_{d_{5/2}} = -3$ . In the nonrelativistic limit, the upper radial functions are again given by

$$g_{3p}(r') = r' R_{31}(r'), \quad g_{3d}(r') = r' R_{32}(r'), \quad (\text{A15})$$

while the small component of the  $3p_{3/2}$  state is

$$f_{3p_{3/2}}(r') = \frac{1}{2} \left( \frac{d}{dr'} - \frac{2}{r'} \right) g_{3p}(r'). \quad (\text{A16})$$

In this channel, only the lower-upper term contributes, since  $l(-\kappa_{p_{3/2}}) = 2 = l(d_{5/2})$ , whereas  $l(p_{3/2}) = 1 \neq l(-\kappa_{d_{5/2}}) = 3$ . For example, for  $m_j = 3/2$  one finds

$$\langle \Omega_{2,3/2} | \sigma_0 | \Omega_{-3,3/2} \rangle = -\frac{4}{5}. \quad (\text{A17})$$

The relevant radial integral is

$$\int_0^\infty dr f_{3p_{3/2}}(r) g_{3d}(r) \Phi(m_{Z'}, r) = -\frac{8\sqrt{5}(9\mu+1)}{135\pi a_B (3\mu+2)^6}. \quad (\text{A18})$$

Using the Wigner-Eckart theorem for the electronic operator,

$$\langle 3p_{3/2} \| T^{(1)} \| 3d_{5/2} \rangle = -i\alpha \frac{32\sqrt{3}}{135\pi a_B} g'_{eV} g'_{pA} \frac{9\mu+1}{(3\mu+2)^6} \quad (\text{A19})$$

and Eq. (A5) gives

$$(-1)^{F+j_b+I} \left\{ \begin{matrix} \frac{3}{2} & \frac{5}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 2 \end{matrix} \right\} \sqrt{6} = \frac{\sqrt{10}}{5}. \quad (\text{A20})$$

Therefore, for any  $F_z = -2, -1, 0, 1, 2$ ,

$$\langle 3p_{3/2}, F=2, F_z | V_{Z'}^{\text{NSD}} | 3d_{5/2}, F=2, F_z \rangle = -i\alpha \frac{32\sqrt{30}}{675\pi a_B} g'_{eV} g'_{pA} \frac{9\mu+1}{(3\mu+2)^6}, \quad (\text{A21})$$

which is manifestly independent of  $F_z$ . As a rank-0 reduced matrix element in the hyperfine space,

$$\langle 3p_{3/2}, F=2 \| V_{Z'}^{\text{NSD}} \| 3d_{5/2}, F=2 \rangle = -i\alpha \frac{32\sqrt{6}}{135\pi a_B} g'_{eV} g'_{pA} \frac{9\mu+1}{(3\mu+2)^6}. \quad (\text{A22})$$

$$\mathbf{3}p_{1/2}\text{-}\mathbf{3}d_{3/2}, F=1.$$

For the  $3p_{1/2}$  and  $3d_{3/2}$  states, one has  $\kappa_{p_{1/2}} = +1$  and  $\kappa_{d_{3/2}} = +2$ . In this case, only the upper-lower term contributes, since  $l(p_{1/2}) = 1 = l(-\kappa_{d_{3/2}})$ , whereas  $l(-\kappa_{p_{1/2}}) = 0 \neq l(d_{3/2}) = 2$ . The small component of the  $3d_{3/2}$  state is

$$f_{3d_{3/2}}(r') = \frac{1}{2} \left( \frac{d}{dr'} + \frac{2}{r'} \right) g_{3d}(r'). \quad (\text{A23})$$

For  $m_j = 1/2$ , one finds

$$\langle \Omega_{1,1/2} | \sigma_0 | \Omega_{-2,1/2} \rangle = -\frac{2\sqrt{2}}{3}. \quad (\text{A24})$$

The radial integral is

$$\int_0^\infty dr g_{3p}(r) f_{3d_{3/2}}(r) \Phi(m_{Z'}, r) = \frac{4\sqrt{5}}{135\pi a_B} \frac{45\mu^2 + 18\mu + 2}{(3\mu+2)^6}. \quad (\text{A25})$$

Hence, the electronic reduced matrix element is

$$\langle 3p_{1/2} \| T^{(1)} \| 3d_{3/2} \rangle = -i\alpha \frac{16\sqrt{15}}{405\pi a_B} g'_{eV} g'_{pA} \frac{45\mu^2 + 18\mu + 2}{(3\mu+2)^6}, \quad (\text{A26})$$

and for the hyperfine matrix element one has

$$(-1)^{F+j_b+I} \left\{ \begin{matrix} \frac{1}{2} & \frac{3}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{matrix} \right\} \sqrt{6} = \frac{\sqrt{6}}{3}. \quad (\text{A27})$$

Therefore, for any  $F_z = -1, 0, 1$ ,

$$\langle 3p_{1/2}, F=1, F_z | V_{Z'}^{\text{NSD}} | 3d_{3/2}, F=1, F_z \rangle = -i\alpha \frac{16\sqrt{10}}{405\pi a_B} g'_{eV} g'_{pA} \frac{45\mu^2 + 18\mu + 2}{(3\mu+2)^6}, \quad (\text{A28})$$

which again does not depend on  $F_z$ . The corresponding reduced matrix element in the hyperfine space is

$$\langle 3p_{1/2}, F = 1 \| V_Z^{\text{NSD}} \| 3d_{3/2}, F = 1 \rangle = -i\alpha \frac{16\sqrt{30}}{405\pi a_B} g'_{eV} g'_{pA} \frac{45\mu^2 + 18\mu + 2}{(3\mu + 2)^6}. \quad (\text{A29})$$

We have checked explicitly by direct summation over the uncoupled states  $|jm_j\rangle|Im_I\rangle$  that Eqs. (A21) and (A28) are independent of  $F_z$ , as required for a scalar operator in the hyperfine basis.

## 2. Electric-dipole and electric-quadrupole matrix elements

The PNC amplitudes  $E1_{\text{PNC}}$  in Sec. II C are proportional to the weak mixing coefficient between opposite parity states times ordinary E1 amplitudes. One also needs E1 amplitudes to find mixing of opposite parity states by an electric field and background E2 amplitudes between  $s$  and  $d$  states. The E1 and E2 amplitudes presented below are standard hydrogenic results [28], which we collect here for completeness.

The standard hydrogenic radial integrals are quoted below in atomic units; i.e., the E1 matrix elements are given in units of  $a_B$  and the E2 matrix elements in units of  $a_B^2$ . The numerical coefficients are therefore unchanged from the atomic-unit formulas. The standard hydrogenic radial integrals are

$$\int_0^\infty R_{10}(r) r R_{31}(r) r^2 dr = \frac{27\sqrt{6}}{128}, \quad (\text{A30})$$

$$\int_0^\infty R_{20}(r) r R_{31}(r) r^2 dr = \frac{27\,648\sqrt{3}}{15\,625}, \quad (\text{A31})$$

$$\int_0^\infty R_{10}(r) r^2 R_{32}(r) r^2 dr = \frac{81\sqrt{30}}{256}, \quad (\text{A32})$$

$$\int_0^\infty R_{20}(r) r^2 R_{32}(r) r^2 dr = -\frac{5\,308\,416\sqrt{15}}{390\,625}. \quad (\text{A33})$$

From these one finds the reduced E1 matrix elements

$$\langle 3p_{3/2} \| rC^{(1)} \| 1s_{1/2} \rangle = \frac{27\sqrt{2}}{64}, \quad (\text{A34})$$

$$\langle 3p_{3/2} \| rC^{(1)} \| 2s_{1/2} \rangle = \frac{55\,296}{15\,625}. \quad (\text{A35})$$

For the  $q = 0$  ( $z$ -polarized) E1 components, one has

$$\left\langle 3p_{3/2}, \frac{1}{2} \left| z \right| 1s_{1/2}, \frac{1}{2} \right\rangle = \frac{9\sqrt{3}}{64}, \quad (\text{A36})$$

$$\left\langle 3p_{3/2}, \frac{1}{2} \left| z \right| 2s_{1/2}, \frac{1}{2} \right\rangle = \frac{9216\sqrt{6}}{15\,625}. \quad (\text{A37})$$

For the stretched  $m_j = 3/2$  final state, however, the relevant E1 operator is  $q = +1$ , not  $q = 0$ , because an  $s_{1/2}$  initial state cannot reach  $m_j = 3/2$  with  $\Delta m = 0$ . The needed circular components are

$$\left\langle 3p_{3/2}, \frac{3}{2} \left| r_{+1} \right| 1s_{1/2}, \frac{1}{2} \right\rangle = \frac{27\sqrt{2}}{128}, \quad (\text{A38})$$

$$\left\langle 3p_{3/2}, \frac{3}{2} \left| r_{+1} \right| 2s_{1/2}, \frac{1}{2} \right\rangle = \frac{27\,648}{15\,625}. \quad (\text{A39})$$

The reduced E2 matrix elements are

$$\langle 3d_{3/2} \| r^2 C^{(2)} \| 1s_{1/2} \rangle = \frac{81\sqrt{6}}{128}, \quad (\text{A40})$$

$$\langle 3d_{3/2} \| r^2 C^{(2)} \| 2s_{1/2} \rangle = -\frac{10\,616\,832\sqrt{3}}{390\,625}, \quad (\text{A41})$$

$$\langle 3d_{5/2} \| r^2 C^{(2)} \| 1s_{1/2} \rangle = \frac{243}{128}, \quad (\text{A42})$$

$$\langle 3d_{5/2} \| r^2 C^{(2)} \| 2s_{1/2} \rangle = -\frac{15\,925\,248\sqrt{2}}{390\,625}. \quad (\text{A43})$$

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