

Hamilton-Jacobi theory of quantized biological function

Francisco Monroy *

*Departamento de Química Física, [Universidad Complutense de Madrid](#), 28040 Madrid, Spain
and Translational Biophysics, [Instituto de Investigación Sanitaria Hospital 12 de Octubre \(imas12\)](#),
28041 Madrid, Spain*



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Living systems display organized, context-dependent dynamics through a restricted repertoire of persistent functional states. At mesoscopic scales, where biological motion is classical, dissipative, and noisy, such discreteness cannot be attributed to microscopic quantum effects alone. Here, we formulate a mesoscopic theory of functional closure for thermodynamically open episodes in which regulatory organization confines the dynamics to an approximately invariant informational class. Within a constrained open Hamilton-Jacobi framework, exact dissipative composability requires the complete irreversible realization of a biological act—including bulk, interfacial, regulatory, composite, and unresolved internal contributions—to admit an exhaustive, nonredundant thermodynamic partition. In an approximately isothermal regime, causal accumulation of the total irreversible power defines a unique positive dissipative-action scale, $\hbar_B$. This act-dependent scale sets the resolution of distinguishable functional realizations, while closure defines the domain on which a global additive action invariant can be constructed. If the nontrivial closure spectrum has a least positive action scale, its admissible values form an exact lattice. Quantization in biological dissipative-action units occurs only when this primitive closure scale saturates the independently constructed thermodynamic resolution, so that the spectrum is resolved in integer multiples of $\hbar_B$. Coherent functional modes are the closure-compatible, persistent, and action-resolved realizations of these discrete classes. The theory predicts clustering of completed trajectories in actionlike observables and separates state correlation, hidden dissipation, and information-geometric distinguishability from the exact composability of irreversible physical cost.

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I. INTRODUCTION: FUNCTIONAL-STATE DISCRETENESS IN DISSIPATIVE BIOLOGICAL DYNAMICS

Living systems operate through organized, context-dependent dynamics that often appear as a restricted repertoire of persistent functional states. Cell fates are confined by high-dimensional regulatory landscapes [1], viable transition paths connect such states [2], modular programs organize cellular function [3], and network motifs stabilize and switch regulatory states [4,5]. These observations suggest that biological function is not realized by arbitrary trajectories in state space,

*Contact author: monroy@ucm.es

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but by dynamically admissible histories constrained by regulation, organization, and irreversible cost [6,7]. This discreteness is not the microscopic discreteness of quantum biology. Quantum effects such as excitonic coherence in photosynthetic complexes and proton tunneling in enzymes are well established in specific molecular regimes [8–13]. The problem addressed here is different. At mesoscopic scales, biological motion is effectively classical, noisy, and dissipative. Yet cells, tissues, and active biological materials frequently realize reproducible functional states rather than a continuum of indistinguishable driven histories. The relevant question is therefore how open, dissipative biological dynamics can remain organized into distinguishable functional realizations.

Nonequilibrium physics provides the natural setting for this question. Driven systems sustain organization through irreversible fluxes, from dissipative structures [14,15] to stochastic thermodynamics [16] and entropy production in active stochastic fields [17]. Entropy-production bounds and sum rules further connect irreversible cost to living systems [18,19]. Active-matter theory describes internally driven collective dynamics [20,21], while organizational approaches emphasize autopoiesis, closure of constraints, and informational individuality as conditions for biological identity [6,7,22,23]. Together, these frameworks explain persistence, organization, and dissipation, but not why admissible biological dynamics should be resolved into discrete functional states. Coarse-grained stochastic thermodynamics shows that unresolved degrees of freedom may carry hidden entropy production [24–27]. In parallel, Hamilton-Jacobi structures arise in macroscopic nonequilibrium fluctuation theory [28], while Fisher geometry provides complementary measures of statistical distinguishability and trajectory-level thermodynamic bounds [29–31]. These results motivate separating observable state statistics from the complete irreversible cost of a functional act.

Here, we formulate this problem as one of finite action resolution under functional closure. We consider thermodynamically open episodes in which regulatory organization confines the dynamics to an approximately invariant informational class. Biological action is treated in the Hamilton-Jacobi sense as the action accumulated by organized mesoscopic trajectories, while σ denotes the internal entropy-production rate associated with their irreversible cost. Three structural ingredients are required: informational invariance selects the admissible class; regulated openness allows non-potential transport compatible with that class; and exact dissipative composability requires the complete irreversible realization to admit an exactly summable dissipative partition. The complete variational derivations, thermodynamic construction, treatment of hidden dissipation, closure-lattice theorem, and modal-coherence and breakdown criteria are provided in the Supplemental Material [32]; specific sections are cited below where needed.

Within this constrained open Hamilton-Jacobi framework, the total irreversible power accumulated over the common act horizon defines the act-dependent dissipative-action scale

$$\hbar_B \equiv \mathcal{A}_\Sigma^B[\sigma_\Sigma; \tau_B].$$

Functional closure defines the domain on which a global action invariant can be constructed. If its nontrivial spectrum has a least positive generator J_* , then $\mathcal{J}_\Gamma = nJ_*$; quantization in biological dissipative-action units follows only under thermodynamic saturation, $J_* = \hbar_B$, yielding $\mathcal{J}_\Gamma = n\hbar_B$. Thus, the proposed quantization is not microscopic quantum quantization, but the discretization of organized biological function when closure, regulated open transport, and exact dissipative composability coexist and the primitive closure-action scale matches the independently constructed thermodynamic resolution.

II. FUNCTIONAL CLOSURE AS AN ADMISSIBLE REGIME

We consider living systems in a regime of *functional closure*: a thermodynamically open regime in which the admissible informational class $\mathcal{I}$ is preserved to good approximation over a functional episode. The system exchanges energy, matter, and entropy with its environment while remaining within the same class of organized biological acts (Fig. 1). Functional closure therefore selects the dynamical sector in which different realizations of the same act can be compared.

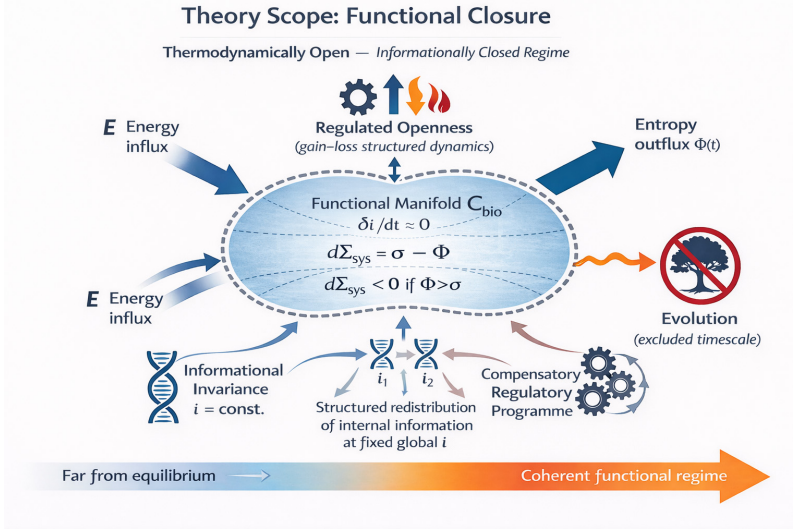


FIG. 1. Functional closure as an open, informationally constrained regime. A living system is represented as an admissible mesoscopic domain $\mathcal{Q}_{\text{bio}}$ that exchanges energy, matter, and entropy with its surroundings while preserving, to good approximation, an informational class $\mathcal{I}$ over the duration of a functional episode. The entropy balance $d\Sigma_{\text{sys}}/dt = \sigma - \Phi$ separates internal entropy production, $\sigma \geq 0$, from outward entropy flux Φ . Functional closure requires approximate informational invariance, $d\mathcal{I}/dt \simeq 0$, as a regime condition rather than an absolute constraint. Local regulatory sectors may reorganize and compensate each other, provided that the global informational class remains effectively preserved. Evolutionary change lies outside the fixed-closure timescale considered here. The variational formulation begins in Method M1 of the Appendix 1.

The corresponding entropy balance is

$$\frac{d\Sigma_{\text{sys}}}{dt} = \sigma_{\Sigma}(t) - \Phi(t), \quad \sigma_{\Sigma}(t) \geq 0, \quad (1)$$

where Σ_{sys} is the mesoscopic state entropy, σ_{Σ} is the total internal entropy-production rate, and Φ is the outward entropy flux. Thus, σ_{Σ} measures irreversible production, whereas $\sigma_{\Sigma} - \Phi$ measures the net entropic load retained by the system. A stationary open state may therefore have $d\Sigma_{\text{sys}}/dt \simeq 0$ while $\sigma_{\Sigma} > 0$.

At the same time, admissible dynamics remains confined to an approximately invariant informational class,

$$\frac{d\mathcal{I}}{dt} \simeq 0. \quad (2)$$

Here, $\mathcal{I}$ denotes the distributed regulatory program through which biological identity is functionally realized, not isolated genetic content alone [23,33,34]. Trajectories outside this sector are not biologically impossible, but they do not realize the same functional act.

For the reduced formal theory, we distinguish the exact reference leaf from its operational neighborhood,

$$\begin{aligned} \mathcal{M}_{\mathcal{I}_0} &= \{(\rho, S) : \mathcal{I}[\rho, \Pi] = \mathcal{I}_0\}, \\ \mathcal{U}_{\varepsilon_I}(\mathcal{M}_{\mathcal{I}_0}) &= \{(\rho, S) : |\mathcal{I}[\rho, \Pi] - \mathcal{I}_0| \leq \varepsilon_I\}. \end{aligned} \quad (3)$$

Exact geometric statements refer to $\mathcal{M}_{\mathcal{I}_0}$, whereas operational fixed-closure trajectories may lie within $\mathcal{U}_{\varepsilon_I}(\mathcal{M}_{\mathcal{I}_0})$.

The global informational descriptor need not be separable across correlated domains. For $Q = (Q_1, \dots, Q_N)$ and optional unresolved variables Y , let $\mathcal{I} = \mathcal{I}(Q, Y)$, so that

$$\frac{d\mathcal{I}}{dt} = \sum_{i=1}^N \chi_i(t) + \chi_Y(t), \quad \chi_i \equiv \nabla_{Q_i} \mathcal{I} \cdot \dot{Q}_i, \quad \chi_Y \equiv \nabla_Y \mathcal{I} \cdot \dot{Y}. \quad (4)$$

These signed regulatory contributions need not cancel instantaneously. Causal completion requires only restoration of the global informational class within the common functional horizon,

$$\Delta \mathcal{I}_B \equiv \mathcal{I}(t_B) - \mathcal{I}(t_0) \simeq 0, \quad \tau_B = t_B - t_0. \quad (5)$$

A retarded kernel representation is given in Method SM2 of the Supplemental Material [32].

Functional closure is therefore neither equilibrium nor ontological immutability, but the mesoscopic regime in which open and dissipative dynamics remains informationally confined long enough for organized action to be defined. Its variational implementation is given in Method M1 of the Appendix 1.

III. BIOLOGICAL ACTION: STRUCTURE AND AXIOMS

Biological action is defined on a manifold of admissible mesoscopic observables,

$$Q = (Q^1, \dots, Q^n) \in \mathcal{Q}_{\text{bio}}(\mathcal{I}), \quad (6)$$

in the Hamilton-Jacobi sense of action-based dynamics on a configuration manifold [35–37]. At mesoscopic nonequilibrium scales, Hamilton-Jacobi structure also arises independently in macroscopic fluctuation theory [28]. The mesoscopic state is specified by a probability density $\rho(Q, t)$ and a local action field $S(Q, t)$, with integrable kinematic momentum

$$P_i(Q, t) = \partial_{Q^i} S(Q, t). \quad (7)$$

Because biological systems are open, admissible transport need not be purely integrable. Regulated exchange may introduce nonpotential components, but functional closure requires them to remain compatible with the preserved informational class. We denote by A_i^{np} the corresponding closure-compatible correction to the integrable momentum.

The admissible biological action is fixed by three structural axioms.

Axiom I (Informational invariance). A functional act preserves a global informational class over its causal completion window.

$$\begin{aligned} \mathcal{M}_{\mathcal{I}_0} &= \{(\rho, S) \mid \mathcal{I}[\rho, \Pi] = \mathcal{I}_0\}, \\ (\rho, S)_{\text{op}} &\in \mathcal{U}_{\varepsilon_I}(\mathcal{M}_{\mathcal{I}_0}), \quad |\Delta \mathcal{I}_B| \leq \varepsilon_I. \end{aligned} \quad (8)$$

Local regulatory contributions may have either sign and need not cancel instantaneously; admissibility requires their causal compensation to be completed within τ_B .

Axiom II (Regulated openness). The system is thermodynamically open, but open transport remains compatible with functional closure.

$$\Pi_i = P_i + A_i^{\text{np}} = \partial_{Q^i} S + A_i^{\text{np}}. \quad (9)$$

Openness modifies organized transport without making it arbitrary: The nonpotential contribution carries the admissible part of open flow within the informational sector.

Axiom III (Exact dissipative composability). For the complete physical realization of a biological act, the total irreversible production is the exact sum of all dissipative channels required by that act, each counted once and only once.

$$\sigma_{\Sigma}(t) = \sum_{\alpha \in \mathcal{D}_{\text{full}}} \sigma_{\alpha}^{\text{dis}}(t), \quad \sigma_{\alpha}^{\text{dis}}(t) \geq 0. \quad (10)$$

Each α denotes a nonredundant irreversible thermodynamic block. Strongly coupled fluxes are grouped into composite blocks when separate non-negative productions cannot be assigned. The complete partition may include bulk, interfacial, regulatory, composite, and unresolved internal contributions. If no exhaustive, nonoverlapping partition exists even after such contributions are retained, Axiom III fails and quantization in units of $\hbar_B$ does not follow.

The three axioms constrain complementary aspects of the same biological act: informational invariance selects the admissible class, regulated openness determines its transport, and dissipative composability specifies its irreversible physical realization. Local regulatory contributions may compensate in the signed informational balance, whereas their irreversible implementation accumulates positively: regulation compensates; dissipation accumulates.

Statistical state additivity and dissipative process composability are distinct. For a correlated state $p(Q_1, \dots, Q_N) \neq \prod_i p_i(Q_i)$, the Gibbs-Shannon entropy is generally nonadditive, while irreversible production remains exactly composable when all physical channels are exhaustively accounted for [38–40].

These axioms select the minimal biological generator

$$H_{\text{bio}}(Q, \rho, P, \Pi, t) = H_0(Q, P, t) + H_{\text{drive}}(Q, \Pi, t) + \Phi_{\Sigma}(Q, \rho; t), \quad (11)$$

where H_0 generates conservative transport, H_{drive} encodes regulated open forcing and nonpotential transport, and Φ_{Σ} is an energylike scalar deformation. Its nontransporting role is fixed by

$$\frac{\partial \Phi_{\Sigma}}{\partial \Pi_i} = 0. \quad (12)$$

The biological act is represented by the local action density

$$\mathcal{L}_{\text{bio}}(Q, t) = -\rho(Q, t)[\partial_t S(Q, t) + H_{\text{bio}}(Q, t)] \quad (13)$$

and by the mesoscopic action functional over $[t_0, t_B]$,

$$\mathcal{A}_{\text{tot}}[\rho, S] = \int_{t_0}^{t_B} dt \int_{\mathcal{Q}_{\text{bio}}} dQ \mathcal{L}_{\text{bio}}(Q, t). \quad (14)$$

The minus sign follows from the Hamilton-Jacobi convention $\partial_t S + H = 0$; its consistency with the irreversible sector is given in Method SM0 of the Supplemental Material [32].

Informational invariance is imposed by constrained stationarity,

$$\delta \left[\mathcal{A}_{\text{tot}} - \int_{t_0}^{t_B} \lambda(t) (\mathcal{I}[\rho, \Pi] - \mathcal{I}_0) dt \right] = 0. \quad (15)$$

This yields the open Hamilton-Jacobi field equations derived in Method M1 of the Appendix 1 and Method SM1 of the Supplemental Material [32].

The irreversible scalar sector is kept dimensionally distinct from its thermodynamic rate representation. At each running time s , define the accumulated irreversible energy carried by this sector through the ensemble normalization

$$\Delta E_{\Sigma}(s) \equiv \int_{\mathcal{Q}_{\text{bio}}} \rho(Q, s) \Phi_{\Sigma}(Q, s) dQ. \quad (16)$$

Thus, Φ_Σ is an energylike scalar deformation of the local Hamilton-Jacobi generator, while $\Delta E_\Sigma(s)$ is its corresponding act-level accumulated irreversible energy. Its causal construction from irreversible power and entropy production is developed below.

IV. OPEN HAMILTON-JACOBI STRUCTURE

The constrained variational structure yields an open Hamilton-Jacobi system for admissible biological dynamics. Stationarity with respect to S gives

$$\partial_t \rho + \nabla_Q \cdot J = 0, \quad J^i = \rho \left(\frac{\partial H_0}{\partial P_i} + \frac{\partial H_{\text{drive}}}{\partial \Pi_i} \right) + \lambda(t) \frac{\delta \mathcal{I}}{\delta \Pi_i}, \quad (17)$$

where $P_i = \partial_{Q^i} S$ and $\Pi_i = P_i + A_i^{\text{np}}$. The last term is the regulatory current induced by the momentum dependence of the global informational constraint; it preserves probability under the same no-flux boundary conditions as the mechanical current.

Stationarity with respect to ρ gives

$$\partial_t S + H_0 + H_{\text{drive}} + \Phi_\Sigma = -\rho \frac{\delta \Phi_\Sigma}{\delta \rho} - \lambda(t) \frac{\delta \mathcal{I}}{\delta \rho}. \quad (18)$$

The scalar Φ_Σ deforms the action balance but, by Eq. (12), generates no independent current. Equations (17) and (18) therefore separate organized transport, regulatory constraint response, and irreversible scalar cost. The full variational derivation is given in Method SM1 of the Supplemental Material [32].

V. DISSIPATIVE-ACTION SCALE

Thermodynamic consistency requires distinguishing entropy production, irreversible power, accumulated irreversible energy, and action. In the approximately isothermal regime at physical reference temperature T_0 ,

$$P_\Sigma^{\text{dis}}(t) = T_0 \sigma_\Sigma(t) = \sum_{\alpha \in \mathcal{D}_{\text{full}}} P_\alpha^{\text{dis}}(t) \geq 0. \quad (19)$$

A generic far-from-equilibrium effective temperature is not required; its operational meaning is not unique [41–44]. The first causal integral gives the irreversible energy accumulated up to a running time s ,

$$\Delta E_\Sigma(s) = \int_{t_0}^s P_\Sigma^{\text{dis}}(t) dt, \quad (20)$$

which is the ensemble energy represented by the scalar Hamiltonian deformation through Eq. (16). Integrating this accumulated cost over the remaining act defines the causal dissipative action

$$\mathcal{A}_\Sigma^B = \int_{t_0}^{t_B} \Delta E_\Sigma(s) ds \quad (21)$$

$$= \int_{t_0}^{t_B} (t_B - t) P_\Sigma^{\text{dis}}(t) dt. \quad (22)$$

The kernel $G_B(t_B, t) = t_B - t$ is the causal depth of an irreversible contribution inside the act: Earlier expenditure remains accumulated for a larger fraction of τ_B . Dimensionally, $[P](t_B - t)dt = E t$, so $\mathcal{A}_\Sigma^B$ is an action.

Exact dissipative composability is inherited by the action because all channels share the same causal horizon:

$$\mathcal{A}_\Sigma^B = \sum_{\alpha \in \mathcal{D}_{\text{full}}} \int_{t_0}^{t_B} (t_B - t) P_\alpha^{\text{dis}}(t) dt \equiv \sum_{\alpha} \mathcal{A}_{\Sigma, \alpha}^B. \quad (23)$$

Correlations do not alter this identity; an irreversible interaction is retained as an interfacial or composite channel rather than discarded as a nonadditive remainder. Coarse graining may, however, make part of the complete physical cost unobservable [24–27].

The positive, act-dependent thermodynamic action scale is therefore

$$\boxed{\hbar_B \equiv \mathcal{A}_\Sigma^B[\sigma_\Sigma; \tau_B]} . \quad (24)$$

It is a single physical scale defined by the total entropy production of the complete act, not a family of separate observed and hidden quanta. An incomplete experiment may only reconstruct or bound this same $\hbar_B$.

As a constitutive operational postulate, we identify $\hbar_B$ with the minimum organized-action separation resolvable between admissible realizations of the same act:

$$\Delta\mathcal{A}_{\text{org}} \gtrsim \hbar_B . \quad (25)$$

This comparison is restricted to a fixed act sector, with $\mathcal{I}_0$, τ_B , and $\hbar_B$ held fixed. Below this scale, organized-action differences are not resolved as distinct admissible realizations. Finite resolution alone does not prove an exact lattice; the independent closure-spectrum condition is introduced next.

VI. MINIMAL WORKED BIOLOGICAL REALIZATIONS

Two minimal mesoscopic examples illustrate causal compensation and the independent construction of thermodynamic and closure-action scales (Fig. 2).

A. Causally delayed compensatory regulation

Consider two regulatory domains, D_1 and D_2 , participating in the same functional act. Their signed informational contributions are

$$\chi_1(t) = af_1(t - t_1), \quad \chi_2(t) = -af_2(t - t_2), \quad t_2 > t_1, \quad (26)$$

with both pulse supports contained in $[t_0, t_B]$ and

$$\int_{t_0}^{t_B} f_1(t) dt = \int_{t_0}^{t_B} f_2(t) dt = 1.$$

The compensation may be delayed but is completed within the act,

$$\Delta\mathcal{I}_B = \int_{t_0}^{t_B} [\chi_1(t) + \chi_2(t)] dt = 0. \quad (27)$$

Let the corresponding irreversible powers be

$$P_1^{\text{dis}}(t) = E_1 f_1(t - t_1), \quad P_2^{\text{dis}}(t) = E_2 f_2(t - t_2), \quad E_1, E_2 > 0. \quad (28)$$

Exact dissipative composability then gives

$$\mathcal{A}_\Sigma^B = \int_{t_0}^{t_B} (t_B - t) [P_1^{\text{dis}}(t) + P_2^{\text{dis}}(t)] dt. \quad (29)$$

In the impulsive limit,

$$\mathcal{A}_\Sigma^B = E_1(t_B - t_1) + E_2(t_B - t_2) > 0. \quad (30)$$

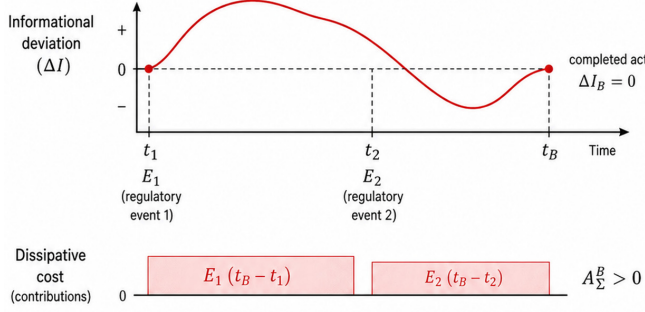
Thus, $\Delta\mathcal{I}_B = 0$ while $\mathcal{A}_\Sigma^B > 0$: Informational compensation does not cancel its irreversible physical cost.

If interfacial or unresolved processes contribute,

$$P_\Sigma^{\text{dis}} = P_1^{\text{dis}} + P_2^{\text{dis}} + P_{12}^\partial + P_Y^{\text{hid}}, \quad (31)$$

with each physical contribution counted once.

(a) Causally delayed compensatory regulation



(b) Two-variable regulated cycle

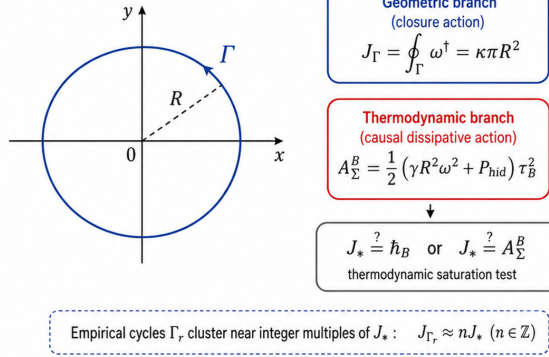


FIG. 2. Minimal biological realizations of causal compensation and thermodynamic saturation. (a) A deviation generated at t_1 is compensated at t_2 , so that $\Delta \mathcal{I}_B = 0$ at t_B , while the causally weighted irreversible cost remains positive. (b) A closure-compatible cycle defines independently a geometric closure action J_Γ and the thermodynamic dissipative action $A_\Sigma^B \equiv \hbar_B$. Quantization in biological action units requires the saturation condition $J_* = \hbar_B$. Repeated cycles permit independent reconstruction of both quantities and a trajectory-level test of this equality.

B. Two-variable regulated cycle and trajectory-level reconstruction

Let two mesoscopic regulatory coordinates execute the closed cycle:

$$x(t) = R \cos \omega t, \quad y(t) = R \sin \omega t, \quad \tau_B = \frac{2\pi}{\omega}. \quad (32)$$

For isotropic effective friction γ and unresolved expenditure $P_{\text{hid}} \geq 0$,

$$P_\Sigma^{\text{dis}} = \gamma(\dot{x}^2 + \dot{y}^2) + P_{\text{hid}} = \gamma R^2 \omega^2 + P_{\text{hid}}, \quad (33)$$

so that

$$\hbar_B = A_\Sigma^B = \frac{1}{2}(\gamma R^2 \omega^2 + P_{\text{hid}})\tau_B^2. \quad (34)$$

Choose the minimal closure-compatible nonpotential 1-form

$$\mathcal{A}^{\text{np}} = \frac{\kappa}{2}(-y dx + x dy). \quad (35)$$

For this closed cycle, the exact dS contribution integrates to zero and the regulatory descriptor is constant along the completed orbit, so the closure action reduces to

$$\mathcal{J}_\Gamma = \oint_\Gamma \mathcal{A}^{\text{np}} = \kappa \pi R^2. \quad (36)$$

If the once-wound orbit is the primitive positive closure class,

$$J_* = \mathcal{J}_\Gamma = \kappa \pi R^2, \quad \mathcal{J}_n = nJ_*. \quad (37)$$

Thermodynamic saturation is therefore tested independently through

$$\kappa \pi R^2 \stackrel{?}{=} \frac{1}{2}(\gamma R^2 \omega^2 + P_{\text{hid}}) \tau_B^2. \quad (38)$$

Only under this equality does $\mathcal{J}_n = n\hbar_B$. Saturation is a matching condition, not a minimum-dissipation principle.

For repeated trajectories $Q^{(r)}(t)$, the ensemble irreversible power

$$\widehat{P}_\Sigma^{\text{dis}}(t) = \frac{1}{N} \sum_{r=1}^N P_\Sigma^{(r),\text{dis}}(t) \quad (39)$$

gives

$$\widehat{h}_B = \int_{t_0}^{t_B} (t_B - t) \widehat{P}_\Sigma^{\text{dis}}(t) dt. \quad (40)$$

Each completed trajectory independently provides

$$\mathcal{J}_\Gamma^{(r)} = \oint_{\Gamma_r} \omega_{\text{adm}}. \quad (41)$$

Clustering of $\{\mathcal{J}_\Gamma^{(r)}\}$ tests for a primitive spacing $\widehat{J}_*$, with the decisive comparison

$$\widehat{J}_* \stackrel{?}{=} \widehat{h}_B. \quad (42)$$

VII. QUANTIZATION OF ADMISSIBLE ACTS

Finite action resolution is necessary but not sufficient for quantization. The scale $\hbar_B$ becomes relevant to an exact lattice only after identifying a global invariant of the completed functionally closed act. By Axiom I, closure means causal completion within the preserved informational class, not temporal periodicity.

For an admissibly completed contour Γ , define the compensated action 1-form and closure action

$$\omega_{\text{adm}} = \Pi_i dQ^i + \sum_\alpha \eta_\alpha dI_\alpha, \quad \mathcal{J}_\Gamma = \oint_\Gamma \omega_{\text{adm}}, \quad (43)$$

where the second term represents pathwise regulatory compensation without requiring an additive decomposition of the global informational functional.

Under closure-compatible deformations,

$$\iota^*(d\omega_{\text{adm}}) = 0 \quad (44)$$

makes $\mathcal{J}_\Gamma$ invariant on closure classes, while concatenation and oriented completion make its image $\Lambda_\chi \subset \mathbb{R}$ an additive subgroup.

If the nontrivial image has a least positive element,

$$J_* = \min(\Lambda_\chi \cap \mathbb{R}_{>0}), \quad (45)$$

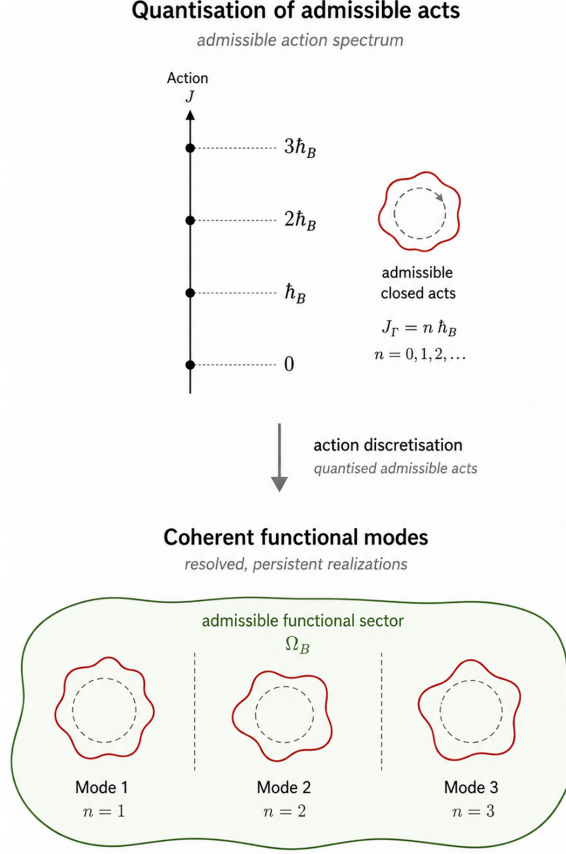


FIG. 3. From quantized closure action to discrete functional modes. Under thermodynamic saturation, admissibly completed acts occupy the spectrum $J_\Gamma = n\hbar_B$. Quantization defines discrete closure-action classes but does not by itself guarantee dynamical coherence. A class becomes a coherent functional mode only when its realization remains closure compatible, persistent and action resolved. The algebraic construction uses $n \in \mathbb{Z}$; physically realized nontrivial modes belong to the positive sector after fixing forward biological orientation.

then

$$\Lambda_\chi = J_*\mathbb{Z}, \quad \mathcal{J}_\Gamma = nJ_*, \quad n \in \mathbb{Z}. \quad (46)$$

This algebraic result is independent of the thermodynamic definition of $\hbar_B$. Quantization in biological dissipative-action units requires the additional saturation condition

$$\hbar_B \in \Lambda_\chi, \quad \Lambda_\chi \cap (0, \hbar_B) = \emptyset \implies J_* = \hbar_B, \quad (47)$$

and therefore

$$\boxed{\mathcal{J}_\Gamma = n\hbar_B}. \quad (48)$$

The theorem and saturation corollary are proved in Method SM3 of the Supplemental Material [32]. The algebraic construction uses $n \in \mathbb{Z}$; after fixing forward biological orientation, physically realized nontrivial acts occupy $n \in \mathbb{Z}_+$, whereas $n < 0$ denotes only the formal orientation inverse.

Quantization therefore supplies discrete closure-action classes, whereas coherence determines which of them persist as identifiable biological modes (Fig. 3).

A one-field representation can then be introduced as a derived local encoding, in the standard polar form used to combine density and action phase into a single complex field [45–48],

$$\psi(Q, t) = \sqrt{\rho(Q, t)} \exp\left(\frac{i}{\hbar_B} S(Q, t)\right). \quad (49)$$

This representation neither generates the lattice nor defines $\hbar_B$.

Correlated states carry a distinct information-geometric structure. Their total correlation,

$$\mathcal{C}[p] = D_{\text{KL}}\left(p \parallel \prod_i p_i\right) \geq 0, \quad (50)$$

gives $S[p] = \sum_i S[p_i] - k_B \mathcal{C}[p]$, while locally

$$D_{\text{KL}}[p_{\theta+d\theta} \parallel p_{\theta}] = \frac{1}{2} g_{ab}^F d\theta^a d\theta^b + O(d\theta^3). \quad (51)$$

Fisher geometry therefore characterizes statistical distinguishability, including cross-domain correlations, and stochastic-Fisher formulations provide trajectory-level thermodynamic bounds [29–31]. It neither defines entropy production nor proves Axiom III: Its role here is inferential, complementary to the physical dissipative resolution $\hbar_B$. Any quantitative Fisher-derived modification of $\hbar_B$ would require additional system-specific assumptions.

VIII. COHERENT FUNCTIONAL MODES

Before admissible completion and assignment to an integer closure class, let $\Gamma_a[t_1, t] \subset \mathcal{U}_{\varepsilon_l}(\mathcal{M}_{\mathcal{T}_0})$ denote a generic closure-compatible candidate with $0 < \tau_a \leq \tau_B$. Its accumulated admissible action is

$$\mathcal{A}_{\text{adm},a}(t) = \int_{\Gamma_a[t_1, t]} \omega_{\text{adm}}, \quad (52)$$

with resolvability index

$$\mathcal{C}_a(t) = \frac{|\mathcal{A}_{\text{adm},a}(t)|}{\hbar_B}. \quad (53)$$

Modal distinguishability requires $\mathcal{C}_a(t_1 + \tau_a) \gtrsim 1$.

Persistence is controlled by the net retained entropic load,

$$\Sigma_{\text{load},a}(t) = \int_{t_1}^t [\sigma_{\Sigma,a}(t') - \Phi_a(t')] dt', \quad (54)$$

where Φ_a is the outward entropy flux. This quantity measures net entropy accumulation after externalization and may therefore be positive, zero, negative, or nonmonotonic. Depending on the system, it may represent transient heat, metabolic by-products, chemical or oxidative load, or other retained consequences of irreversible activity.

Let Σ_a^* be an independently calibrated closure-tolerance threshold. Persistence requires

$$\sup_{t \in [t_1, t_1 + \tau_a]} \Sigma_{\text{load},a}(t) \leq \Sigma_a^*. \quad (55)$$

Only after admissible completion does the integer closure index become available. If the least-positive-element condition holds,

$$\mathcal{A}_{\text{adm},a}(t_1 + \tau_a) = \mathcal{J}([\Gamma_a]) = nJ_*, \quad (56)$$

and under thermodynamic saturation, $J_* = \hbar_B$,

$$\mathcal{J}([\Gamma_n]) = n\hbar_B, \quad \mathcal{C}_n = |n|. \quad (57)$$

Quantization supplies discrete closure classes; coherence selects which persist as identifiable biological modes. The compact criteria are given in Method M4 of the Appendix 4 and Method SM4 of the Supplemental Material [32].

IX. CLASSICAL LIMIT: MODAL BREAKDOWN

The formally reversible limit,

$$\sigma_\Sigma \rightarrow 0 \implies \mathcal{A}_\Sigma^B \rightarrow 0, \quad \hbar_B \rightarrow 0,$$

removes the finite dissipative scale required to resolve neighboring acts.

For a generic candidate a , sufficient routes to modal breakdown are

$$\mathcal{C}_a(t_1 + \tau_a) \ll 1, \quad \sup_{t \in [t_1, t_1 + \tau_a]} \Sigma_{\text{load}, a}(t) > \Sigma_a^*, \quad \Gamma_a[t_1, t_1 + \tau_a] \not\subset \mathcal{U}_{\varepsilon_l}(\mathcal{M}_{\mathcal{I}_0}), \quad \tau_a > \tau_B. \quad (58)$$

Breakdown also occurs if the common dissipative partition ceases to be admissible.

Modal breakdown does not by itself imply a continuum. Continuity additionally requires a sufficiently dense family of admissible realizations to remain unresolved on the common action scale. Method SM5 of the Supplemental Material [32] gives the corresponding formal criteria.

X. QUANTIZED FUNCTIONAL ACTION

Under functional closure, the three axioms play distinct roles: informational invariance defines the admissible class, regulated openness determines closure-compatible transport, and exact dissipative composability makes the irreversible realization exactly summable over the completed act.

Axiom III is required for quantization specifically in thermodynamic units of $\hbar_B$. If an exhaustive, nonredundant dissipative partition cannot be constructed, the geometric closure spectrum may still possess an independent generator J_* , but its identification with $\hbar_B$ is no longer justified. Hence, $\mathcal{J}_\Gamma = n\hbar_B$ does not follow.

The discretized objects are therefore closure-compatible biological acts, not microscopic quantum states or arbitrary trajectories. Biological quantization occurs when an exact closure-action lattice exists and its primitive positive generator saturates the independently constructed dissipative-action scale.

XI. DISCUSSION AND OUTLOOK

Biological functional identity is maintained here not by thermodynamic isolation, but by irreversible physical realization under global informational closure. Regulatory contributions may have either sign and compensate across domains and time, whereas their irreversible costs accumulate: regulation compensates; dissipation accumulates.

This distinction separates correlated state structure from dissipative process composability. State entropy may be nonadditive and Fisher geometry may contain cross-domain correlations without violating Axiom III. Exact dissipative composability instead requires an exhaustive, nonredundant partition of the irreversible physical processes participating in the same act horizon. Its failure is therefore a physical failure of the axiom, not an accounting remainder.

The same distinction clarifies hidden degrees of freedom. Writing the complete realization as $X = (Q, Y)$, elimination of unresolved variables Y may generate memory, delayed response, coloured noise, effective forces, and hidden entropy production in the reduced Q dynamics [24–27]. Functional closure constrains which hidden dynamics is compatible with causal completion, while thermodynamic inference estimates its irreversible cost. There remains a single physical $\hbar_B = \mathcal{A}_\Sigma^B[\sigma_\Sigma; \tau_B]$; coarse graining affects its reconstruction, not its definition.

The causal kernel $G_B(t_B, t) = t_B - t$ gives the dissipative-action information not contained in total dissipated energy alone. Two acts may dissipate the same energy yet have different $\mathcal{A}_\Sigma^B$ if that

dissipation is distributed differently in time. The action scale therefore encodes both the magnitude and the temporal organization of irreversible expenditure.

Quantization results from matching two independently constructed structures:

$$P_{\Sigma}^{\text{dis}} \longrightarrow \mathcal{A}_{\Sigma}^B \equiv \hbar_B, \quad \Gamma \longrightarrow \mathcal{J}_{\Gamma} \longrightarrow J_*$$

Neither finite dissipation nor the existence of the lattice fixes the other. Biological quantization in the form $\mathcal{J}_{\Gamma} = n\hbar_B$ occurs only under thermodynamic saturation, $J_* = \hbar_B$. Saturation is therefore a matching condition between closure geometry and irreversible resolution, not a minimum-dissipation principle.

Information geometry operates at a complementary inferential level. Fisher and stochastic Fisher information quantify statistical distinguishability, trajectory-level bounds, and inference limits [29–31], but do not define $\hbar_B$. They may instead constrain how reliably neighboring closure classes can be inferred from finite data.

The worked examples provide an operational test of the theory: Causal compensation can restore informational closure while retaining positive irreversible cost, whereas repeated regulated cycles permit independent reconstruction of $\hbar_B$ and of the primitive closure spacing J_* .

Three experimental consequences follow. First, completed trajectory ensembles should cluster in actionlike observables when a primitive closure spectrum exists. Second, changes in irreversible throughput or in the temporal placement of dissipation modify $\hbar_B$; the closure spacing follows only while saturation persists. Third, failure of causal completion, excessive retained entropic load, or loss of the common dissipative partition destroys certified modal status. Such breakdown does not by itself imply a continuum.

Candidate systems include oscillatory signaling and metabolic programs, chromatin and nuclear dynamics, organelle motion, actomyosin remodeling, cell-cycle progression, phenotypic switching, developmental transitions, and active biomaterials with tunable internal driving. The present theory applies to fixed-closure acts; adaptive and evolutionary regimes require a slower description in which the informational class itself evolves.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX

These methods provide the minimal construction supporting the results in the main text. The three structural axioms define the biological framework; the constitutive resolution postulate and the explicit hypotheses required for the algebraic lattice and thermodynamic saturation are stated separately where used. Full variational details, theorem-level quantization, and modal breakdown criteria are given in the methods of the Supplemental Material [32].

1. Constrained open Hamilton-Jacobi formulation (Method M1)

Throughout, $S(Q, t)$ obeys the Hamilton-Jacobi convention

$$\partial_t S + H = 0. \tag{A1}$$

The exact reference sector and its operational tolerance tube are

$$\mathcal{M}_{\mathcal{I}_0} = \{(\rho, S) : \mathcal{I}[\rho, \Pi] = \mathcal{I}_0\}, \quad \mathcal{U}_{\varepsilon_I}(\mathcal{M}_{\mathcal{I}_0}) = \{(\rho, S) : |\mathcal{I}[\rho, \Pi] - \mathcal{I}_0| \leq \varepsilon_I\}. \quad (\text{A2})$$

Exact geometric statements refer to $\mathcal{M}_{\mathcal{I}_0}$, while operational realizations may occupy $\mathcal{U}_{\varepsilon_I}(\mathcal{M}_{\mathcal{I}_0})$. The biological generator is

$$H_{\text{bio}} = H_0(Q, P, t) + H_{\text{drive}}(Q, \Pi, t) + \Phi_{\Sigma}(Q, \rho; t), \quad \frac{\partial \Phi_{\Sigma}}{\partial \Pi_i} = 0, \quad (\text{A3})$$

and the constrained action is

$$\tilde{\mathcal{A}} = - \int_{t_0}^{t_B} dt \int_{\mathcal{Q}_{\text{bio}}} dQ \rho (\partial_t S + H_{\text{bio}}) - \int_{t_0}^{t_B} \lambda(t) [\mathcal{I}[\rho, \Pi] - \mathcal{I}_0] dt. \quad (\text{A4})$$

With $P_i = \partial_{Q^i} S$ and $\Pi_i = P_i + A_i^{\text{np}}$, variation gives

$$\partial_t \rho + \partial_{Q^i} \left[\rho \left(\frac{\partial H_0}{\partial P_i} + \frac{\partial H_{\text{drive}}}{\partial \Pi_i} \right) + \lambda(t) \frac{\delta \mathcal{I}}{\delta \Pi_i} \right] = 0 \quad (\text{A5})$$

and

$$\partial_t S + H_0 + H_{\text{drive}} + \Phi_{\Sigma} = -\rho \frac{\delta \Phi_{\Sigma}}{\delta \rho} - \lambda(t) \frac{\delta \mathcal{I}}{\delta \rho}. \quad (\text{A6})$$

The complete derivation is given in Method SM1 of the Supplemental Material [32].

2. Thermodynamic construction and exact dissipative composability (Method M2)

For an approximately isothermal act at reference temperature T_0 , the physical irreversible power is

$$P_{\Sigma}^{\text{dis}}(t) = T_0 \sigma_{\Sigma}(t) = \sum_{\alpha \in \mathcal{D}_{\text{full}}} P_{\alpha}^{\text{dis}}(t) \geq 0. \quad (\text{A7})$$

The complete set is a nonredundant partition into irreversible thermodynamic blocks. Strongly coupled elementary fluxes are grouped when necessary so that each retained block has non-negative production. The set may include bulk, interfacial, regulatory, composite, and hidden contributions, and its existence is required by Axiom III rather than guaranteed by definition. The causal construction is

$$\Delta E_{\Sigma}(s) = \int_{t_0}^s P_{\Sigma}^{\text{dis}}(t) dt, \quad \mathcal{A}_{\Sigma}^B = \int_{t_0}^{t_B} (t_B - t) P_{\Sigma}^{\text{dis}}(t) dt. \quad (\text{A8})$$

Linearity immediately gives

$$\mathcal{A}_{\Sigma}^B = \sum_{\alpha \in \mathcal{D}_{\text{full}}} \mathcal{A}_{\Sigma, \alpha}^B. \quad (\text{A9})$$

The biological dissipative-action scale is the unique total

$$\hbar_B \equiv \mathcal{A}_{\Sigma}^B[\sigma_{\Sigma}; \tau_B], \quad (\text{A10})$$

and its operational resolution postulate is

$$\Delta \mathcal{A}_{\text{org}} \gtrsim \hbar_B. \quad (\text{A11})$$

State correlations, hidden variables, and the complete proof of Eq. (A9) are treated in Method SM2 of the Supplemental Material [32].

3. Closure spectrum and thermodynamic saturation (Method M3)

The compensated 1-form and completed-act invariant are

$$\omega_{\text{adm}} = \Pi_i dQ^i + \sum_{\alpha} \eta_{\alpha} dI_{\alpha}, \quad \mathcal{J}_{\Gamma} = \oint_{\Gamma} \omega_{\text{adm}}. \quad (\text{A12})$$

Under the reduced closure condition $\iota^*(d\omega_{\text{adm}}) = 0$, contour action is invariant on admissible closure classes and its image Λ_{χ} is an additive subgroup. If this image has a least positive element J_* , then

$$\Lambda_{\chi} = J_* \mathbb{Z}, \quad \mathcal{J}_{\Gamma} = nJ_*, \quad n \in \mathbb{Z}. \quad (\text{A13})$$

Only if the independent thermodynamic scale saturates the spectrum, $J_* = \hbar_B$, does

$$\mathcal{J}_{\Gamma} = n\hbar_B \quad (\text{A14})$$

follow. The negative branch belongs to the formal oriented algebraic completion; after fixing forward biological orientation, physically realized nontrivial acts occupy $n \in \mathbb{Z}^+$. Method SM3 of the Supplemental Material [32] proves the lattice theorem and states the saturation condition.

4. Modal coherence and modal breakdown (Method M4)

Before algebraic completion, let $\Gamma_a[t_1, t] \subset \mathcal{U}_{\varepsilon_l}(\mathcal{M}_{\mathcal{I}_0})$ denote a generic closure-compatible candidate with $0 < \tau_a \leq \tau_B$. Its accumulated admissible action and resolvability index are

$$\mathcal{A}_{\text{adm},a}(t) = \int_{\Gamma_a[t_1, t]} \omega_{\text{adm}}, \quad C_a(t) = \frac{|\mathcal{A}_{\text{adm},a}(t)|}{\hbar_B}. \quad (\text{A15})$$

Dynamical persistence is limited by the net retained entropic load,

$$\Sigma_{\text{load},a}(t) = \int_{t_1}^t [\sigma_{\Sigma,a}(t') - \Phi_a(t')] dt', \quad (\text{A16})$$

where Φ_a is the outward entropy flux, not the scalar action deformation Φ_{Σ} . This load may be positive, zero, negative, or nonmonotonic.

Let Σ_a^* be an independently calibrated closure-tolerance threshold. Modal coherence requires

$$\Gamma_a[t_1, t_1 + \tau_a] \subset \mathcal{U}_{\varepsilon_l}(\mathcal{M}_{\mathcal{I}_0}), \quad C_a(t_1 + \tau_a) \gtrsim 1, \quad \sup_{t \in [t_1, t_1 + \tau_a]} \Sigma_{\text{load},a}(t) \leq \Sigma_a^*, \quad 0 < \tau_a \leq \tau_B. \quad (\text{A17})$$

Only after admissible completion does an integer closure label become available. If the least-positive-element condition holds, $\mathcal{J}([\Gamma_a]) = nJ_*$; under saturation, $\mathcal{J}([\Gamma_n]) = n\hbar_B$.

Modal breakdown occurs whenever a required condition in Eq. (A17) fails. Sufficient routes are

$$C_a(t_1 + \tau_a) \ll 1, \quad \sup_t \Sigma_{\text{load},a}(t) > \Sigma_a^*, \quad \Gamma_a \not\subset \mathcal{U}_{\varepsilon_l}(\mathcal{M}_{\mathcal{I}_0}), \quad \tau_a > \tau_B \quad (\text{A18})$$

or loss of the admissible dissipative partition. Such breakdown removes modal certification; a continuum additionally requires a dense unresolved family of realizations. Formal definitions and sufficient criteria are given in Methods SM4 and SM5 of the Supplemental Material [32].

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