

Free extension of topological states via double-zero-index media

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Topological states, known for their robustness against disorder, offer promising avenues for disorder-resistant devices. However, their intrinsic spatial confinement at interfaces imposes geometric constraints that limit the scalability of topological functionalities. Here, we propose a strategy to overcome this limitation by using double-zero-index media to expand topological interfaces. Although occupying finite space, these media are optically equivalent to infinitesimal points at their working frequencies, effectively altering the geometry of topological interfaces and breaking conventional bulk-edge correspondence. This strategy enables the spatial expansion of uniform topological states beyond their native interface, offering new possibilities for topological photonic devices. Validated by simulations and microwave experiments, our findings offer a universal framework to break the inherent dimensional limitations of topological states, with implications extending to general wave systems.

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In recent decades, topological insulators have attracted widespread interest in artificial systems due to their unique bulk-edge correspondence [1–8], which has been harnessed to manipulate classical waves in innovative ways and offered exciting opportunities for practical applications. For instance, photonic analogues of topological systems have demonstrated robust unidirectional edge states that are immune to defects and disorder [9–19]. Recent developments in higher-order topological states and disclination states have enabled novel light confinement mechanisms, providing the foundation for next-generation topological lasers distinguished by low thresholds and strong directionality [20–27]. However, the inherent spatial localization of topological states at interfaces results in a highly restricted mode volume. As a result, phenomena such as Purcell enhancement [28] are significantly suppressed. Moreover, their working wavelengths are typically comparable to the lattice constant [29,30]. Thus, despite the achievement of high-quality factors, the spatial extent of

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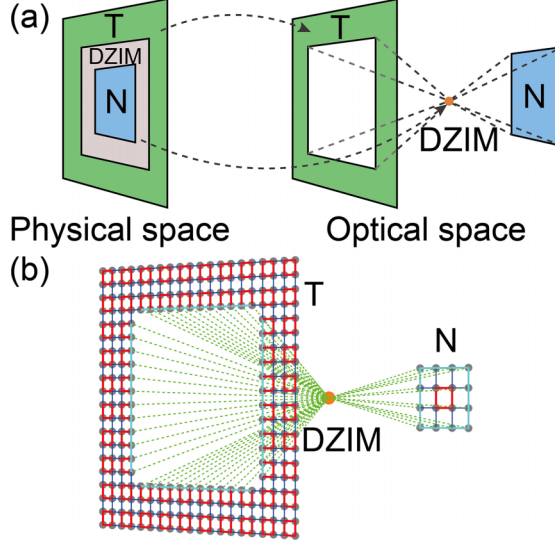


FIG. 1. The reshaping of topological interfaces by DZIM. (a) Left: In physical space, a DZIM interlayer (in gray) is inserted between topological trivial (labeled “T”) and nontrivial (labeled “N”) bulk materials. Right: In optical space, the DZIM is equivalent to a point with the principles of transformation optics, which connects the topological trivial and non-trivial bulk materials. (b) The tight-binding framework of the system in optical space.

topological functionalities is limited. This poses a major challenge for the scalability and integration of topological photonic devices, particularly those based on higher-order topological phases. Despite extensive research, strategies to overcome this fundamental limitation remain scarce.

Double-zero-index media (DZIM), characterized by simultaneously near-zero permittivity and permeability, can be comprehended by isotropically expanding a zero-dimensional (0D) point into a finite region with the principles of transformation optics [31,32]. Although it occupies a finite space, such a medium is optically equivalent to the original point. Thus, a plane wave passing through this space does not accumulate any phase. In a two-dimensional (2D) electromagnetic DZIM, Maxwell’s equations force the electric (or magnetic) field to be uniform due to the vanishing curl, reflecting its equivalence with a point in optical space. Thanks to this zero-index nature, DZIM has attracted significant interest, such as wavefront tailing [33–36], impurity cloaking [37,38], wave tunneling [39–42], photonic doping [43–47], and the “antidoping” effect [31]. Very recently, a novel bulk-spatiotemporal vortex correspondence, unlike the conventional bulk-edge correspondence, was proposed as a bridge between the DZIM and topological photonics [48].

In this work, we propose a strategy to overcome the intrinsic spatial localization of topological states by separating topological interfaces and filling the gap with DZIM. We consider a 2D Su-Schrieffer-Heeger (SSH) model implemented in a dielectric photonic crystal (PC). By tuning a geometric parameter in each unit cell, the PC undergoes a topological transition between configurations with different quantized bulk polarizations, resulting in the emergence of one-dimensional (1D) edge states and 0D corner states. As illustrated in Fig. 1(a), when we insert the DZIM between topologically trivial and nontrivial bulk materials, they are effectively linked by a point-like connection in optical space. The DZIM effectively reshapes the geometry of the topological interface, thus modifying the manifestation of bulk-edge correspondence. To make this reshaping clearer, we introduce a tight-binding illustration based on the 2D SSH model, as shown in Fig. 1(b), in which all boundary sites coupled through the DZIM are effectively coupled onto a single site with equalized coupling strengths [32]. The detailed formulation and physical implications of this model are discussed later. Nevertheless, distinct bulk polarizations across the interface remain essential

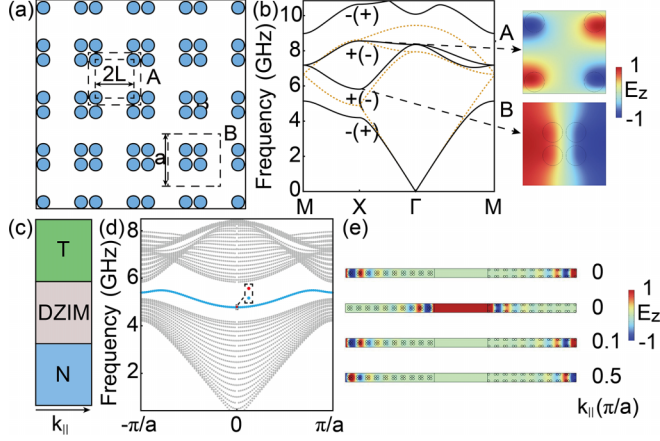


FIG. 2. A topological edge state extended by the DZIM layer. (a) A PC with different unit cell selections, labeled A and B. Each unit cell is composed of four rods, which are spaced at a distance of $2L$ from each other. (b) Black solid lines: The calculated band structures for $L = 7.2(2.8)$ mm. The yellow dotted lines: The calculated band structures for $L = 5$ mm. The symbols “ $\pm$ ” represent the parity of the states under π rotation, obtained from the eigenfields. The cases of $L = 7.2$ mm and $L = 2.8$ mm share the same band structure but have different bulk polarizations. (c) Scheme for a DZIM layer sandwiched by the topological trivial PC (labeled “T”) and the nontrivial PC (labeled “N”). The upper/lower and left/right boundaries of the three-layered structure are two pairs of periodic boundary conditions. (d) The projected band structure along the direction of $k_{\parallel}$ in (c) exhibits bulk states (gray), 1D topological edge states (blue) at the upper/lower interface, and the spatially extended edge state (red) that spans across the DZIM layer. (e) The electric field distributions for the edge states when $k_{\parallel} = 0$, $k_{\parallel} = 0.1\pi/a$, and $k_{\parallel} = 0.5\pi/a$, respectively.

for the emergence of topological states, ensuring that their topological origin is preserved. This approach is validated through numerical simulations and microwave experiments. Our findings demonstrate that DZIM enables the spatial extension of topological states beyond their interfaces while retaining their topological characteristics. This strategy may provide a foundation for photonic platforms that benefit from enhanced spatial scalability and mode delocalization.

As shown in Fig. 2(a), we consider a 2D PC consisting of a square lattice of dielectric rods with relative permittivity $\varepsilon = 7.7$, radius $r = 2.4$ mm, and lattice constant $a = 20$ mm. A geometric parameter L defines the displacement of each rod from the center of the unit cell in both the horizontal and vertical directions. The dashed boxes in Fig. 2(a) are two unit cells: configuration A with $L = 7.2$ mm and configuration B with $L = 2.8$ mm, with reversed choices of center and corner, respectively. Along the high symmetry lines of the irreducible Brillouin zone, the band structure for transverse electric (E_z) polarized waves is calculated by COMSOL MULTIPHYSICS and plotted in Fig. 2(b). Although the eigenfrequencies are invariant under different unit cell choices, the corresponding eigenfields exhibit distinct spatial symmetries, as illustrated by the insets. The symbols “+” and “−” indicate the eigenvalues “+1” and “−1” under π rotation for the $i_{\text{th}} (i = 1, 2, 3, 4)$ eigenstates at the X point.

The PC can be described by a 2D SSH model, which undergoes a topological phase transition when L crosses $a/4$ [49,50]. The topological phase of the PC can be characterized by a 2D bulk polarization P in terms of the parities of eigenstates at the high-symmetry points

$$P_n = \frac{1}{2} \left(\sum_i q_n^i \bmod 2 \right), \quad (-1)^{q_n^i} = \frac{\eta(X_n^i)}{\eta(\Gamma^i)}, \quad (1)$$

where the summation is taken over all the occupied bands, η denotes the parity associated with π rotation, and n stands for x or y . Under the first band gap, we obtain quantized bulk polarizations

$P = (1/2, 1/2)$ and $P = (0, 0)$ for unit cell selections A and B, which indicate nontrivial and trivial topologies, respectively. At $L = 5$ mm, the two unit cell configurations merge, and the band gap closes, marking the topological phase transition. This transition point enables the double-zero-index property in wave manipulation, which will be demonstrated later.

When two PCs with distinct bulk polarizations are bonded together, a hierarchy of topological states emerges within the band gap [49,51]. To spatially extend these states, we introduce a DZIM layer at the interface. As illustrated in Fig. 2(c), a DZIM slab is inserted between two PC bulks with trivial (green) and nontrivial (blue) topologies. The structure is bounded by periodic boundary conditions along both the vertical and horizontal directions. There exist two different types of interfaces between PC bulks with distinct polarizations: one is the interface at the top and bottom of the slab; the other is the interface in the middle, extended by the DZIM layer. Along the direction of $k_{\parallel}$ in Fig. 2(c), the projected band structure of this three-layered structure is shown in Fig. 2(d). Due to the curl-free nature of a 2D electromagnetic DZIM, its insertion effectively transforms the interface into a point-like connection in optical space, thereby modifying the bulk-edge correspondence. The corresponding wave equation inside the DZIM can be written as

$$\nabla \cdot \left(\frac{1}{\mu} \nabla E_z \right) = -\varepsilon \frac{\omega^2}{c^2} E_z, \quad (2)$$

where ω , c , and μ are the angular frequency, light speed in a vacuum, and the relative permeability, respectively. When both ε and μ are zero, the gradient of E_z must be zero, indicating a uniform E_z in the DZIM. Before the insertion of the DZIM layer, each of the two interfaces between topologically distinct bulks supports edge states at the same frequency, consistent with the bulk-edge correspondence. The insertion of DZIM does not change topological invariants of the bulk PCs or imply any violation of causality; rather, the DZIM's curl-free constraint enables only the in-phase edge state to extend with equal phase and nearly uniform amplitude across the DZIM, while out-of-phase states are suppressed by mismatch. At $k_{\parallel} = 0$, the two interfaces become optically equivalent, the in-phase topological state (red dot) extends into the DZIM layer and maintains nearly the same frequency as the original edge state (blue dots); the bulk states are degenerate. However, when $k_{\parallel} \neq 0$, the out-of-phase topological edge states are suppressed; the bulk states are no longer degenerate. This behavior is confirmed by the electric-field distributions of the topological edge states shown in Fig. 2(e), where only the state at $k_{\parallel} = 0$ is extended through the DZIM layer.

Furthermore, we apply the concept of spatial extension to 0D corner states by introducing a DZIM layer between two topologically distinct PC bulks. As illustrated in the inset of Fig. 3(a), the structure consists of three concentric regions: a topologically nontrivial PC square of 6×6 unit cells, surrounded by a DZIM shell of six-unit-cell thickness, which is further enclosed by a topologically trivial PC shell with six layers of unit cells. The calculated spectrum of this composite structure is shown in Fig. 3(a). Without the DZIM layer, four nearly degenerate 0D corner states and 12 1D edge states emerge at the closed interface between the topologically trivial and nontrivial PC [32], in accordance with bulk-edge correspondence. Interestingly, the insertion of the DZIM layer enforces a uniform electric field distribution along the closed interface. As a result, the bulks with distinct polarizations become effectively connected through a point-like connection in optical space. The original corner states are coupled through the DZIM and hybridize with the original edge states, leaving only an in-phase topological state at the point-like connection. We should emphasize that the uniform field constraint is far more restrictive than symmetry constraints, which would otherwise permit other corner state combinations with phase or amplitude variations. Importantly, the hybridization demonstrates that the insertion of DZIM layer reshapes the manifestation of bulk-edge correspondence by modifying the topological states. Consequently, the frequency of topological states deviates from that of the original corner states, falling at 5.3 GHz between the original corner states and the original edge states. Figure 3(b) shows the spectrum when the innermost square region is replaced with a topologically trivial PC. In this case, the corner state within the band gap disappears, confirming that the emergence of the corner state is primarily

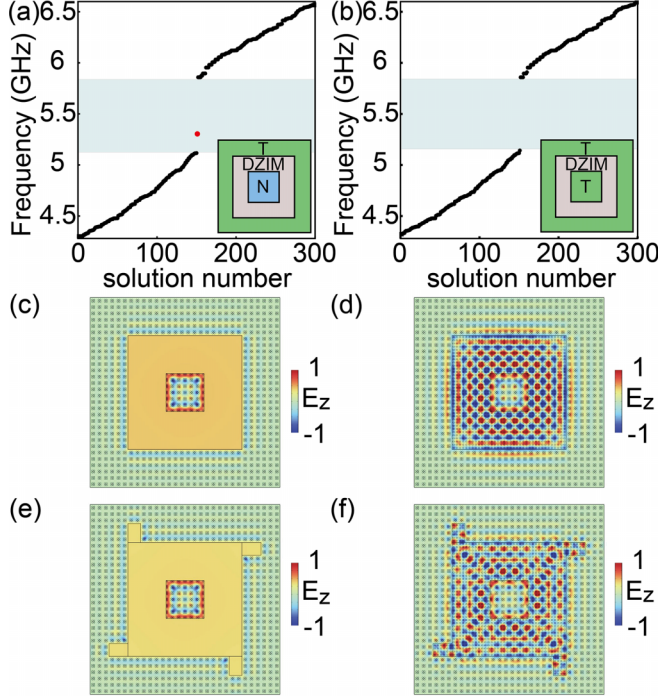


FIG. 3. A topological corner state extended by DZIM layer. (a) The eigenfrequency spectrum when the DZIM is embedded in a square interface between the topologically trivial and nontrivial bulks. (b) The eigenfrequency spectrum when the DZIM is embedded between two topologically trivial bulks. (c), (d) The electric field distribution E_z of the spatially extended corner state when the filling is set as (c) ideal DZIM and (d) a PC with effective double-zero-index property, respectively. (e), (f) The electric field distribution E_z of the spatial extension of the corner state when the shape of (e) the ideal DZIM and (f) the PC are changed.

determined by the topological difference between the inner and outer PC layers. Figure 3(c) shows the electric field distribution E_z of the spatially extended topological corner state within the entire three-layer concentric structure. In the DZIM layer, the electric field exhibits both uniform amplitude and uniform phase, owing to the inherent uniform-field condition of the DZIM. This condition enables the in-phase topological corner state to be extended over the entire boundary of the nontrivial PC.

Using an effective parameter method based on field averaging at the boundary of eigenstates [31,41], we find that the critical configuration between the topologically trivial and nontrivial PC, i.e., $L = 5$ mm, shows nearly vanishing effective permittivity and permeability around the frequency of the X point in band structure [32]. In view of this, we select the PC with $L = 5$ mm as the filling. Figure 3(d) is the electric field distribution E_z when the ideal DZIM in Fig. 3(c) is replaced by the PC with $L = 5$ mm. The essential characteristics of the DZIM and the spatially extended corner state are well preserved, making it possible to experimentally realize this phenomenon.

Furthermore, the spatial extension of the topological corner state remains robust even when the geometry of the DZIM shell changes. As shown in Fig. 3(e), the four corners of the DZIM shell are modified by adding rectangular protrusions with an area equivalent to 2×3 unit cells. The in-phase topological corner state still extends into the deformed DZIM region, and its eigenfrequency remains nearly unchanged. Figure 3(f) presents the electric field distribution E_z when the ideal DZIM in Fig. 3(e) is replaced by a PC with $L = 5$ mm. The spatial extension of the corner state remains, indicating that our strategy transcends conventional dimensional constraints of topological

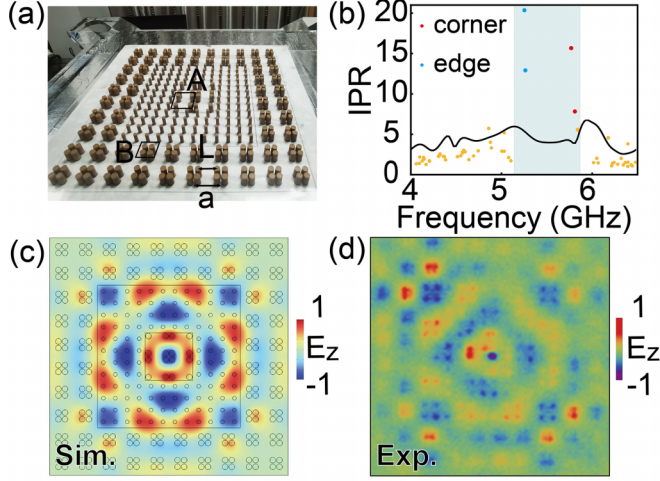


FIG. 4. Experimental realization of spatially extended topological corner states. (a) The PC composed of resin, with two layers of unit cells in each of the topological trivial layer, nontrivial layer, and the effective DZIM layer. (b) The IPR with and without the effective DZIM layer. (c), (d) The electric field distribution E_z of the spatially extended corner state from the (c) simulation and (d) experiment, respectively.

interfaces, enabling flexible geometric control of topological states, which is rarely discussed in previous studies.

To experimentally verify this intriguing spatial extension of topological states, we fabricate a three-layer concentric PC with dielectric rods made of low-loss polyphenyl ether resin with permittivity $\varepsilon = 7.7 + 0.018i$. Due to the size of our scanning platform, each PC layer consists of only two unit cells. Figure 4(a) is a photo of the PC assembled on the surface of a metallic plate. Compared to the case in Fig. 3, the eigenfrequency of the in-phase topological state shifts as the number of layers in the bulk PC changes. Thus, the working frequency of the effective DZIM must be tuned accordingly. We therefore adjust the rod radius in the effective DZIM layer to 1.6 mm to ensure that the effective zero-index frequency (frequency of the X point) matches that of the topological state. The radius of each rod in the remaining region is set to 2.4 mm. In Fig. 4(b), we present the inverse participation ratio (IPR) of the electric fields with and without the DZIM. The IPR is defined as

$$\text{IPR} = \frac{\int_{\text{PC}} |E(r)|^4 dr}{\left(\int_{\text{PC}} |E(r)|^2 dr \right)^2} \cdot A, \quad (3)$$

where $|E(r)|$ represents the amplitude distribution of the electric field and A is the area of PC. The discrete colored points represent the IPR calculated from the eigenfields of the PC when the DZIM layer is replaced by trivial bulk, while the solid curve corresponds to the IPR obtained from the electric field distributions excited by an out-of-plane line source at the center of the PC [32]. Without the DZIM interlayer, the IPRs of the topological states are about 16, indicating strong localization of electric fields. However, when the PC serving as the DZIM layer is inserted at the interface between the topologically trivial and nontrivial bulks, the IPR of the electric field distribution drops significantly to around 4. This substantial reduction in IPR indicates a significant decrease in field localization, consistent with the spatial extension of the topological state. Figures 4(c) and 4(d) are the simulated and experimentally measured distributions of the electric field E_z at 5.70 GHz, respectively. Indeed, the limited number of layers weakens the double-zero-index property, which leads the phase of electric field at the outer boundary less uniform than the case of larger structures in Fig. 3. Nevertheless, it is evident that the essential effect of the DZIM is preserved: the field

distribution along the boundary of the nontrivial PC remains nearly uniform, which illustrates a deformation of the topological state. Moreover, the ring-like extension of topological state is preserved, which is in good agreement with the topological state obtained from a tight-binding framework [32]. When the DZIM layer is removed, the edge and corner of the nontrivial PC will be in opposite phase, the ring-like extension of the topological state disappears, which is quite different from the DZIM-interlayer case [32]. We should also emphasize that the material losses have nearly no influence on the real part of the spectrum, the IPR values, the field map, and the spatial extension of topological state. The effect of material loss on the quality factor of the topological state is also small [32].

In conclusion, we propose and experimentally demonstrate a novel strategy to overcome the intrinsic spatial confinement of topological states by embedding DZIM into topological interfaces. Owing to the curl-free nature of DZIM, this approach enables the free spatial extension of in-phase topological edge states and corner states, while selectively suppressing their out-of-phase counterparts, thereby introducing a new degree of freedom for manipulating topological states. The effective redefinition of the bulk-edge correspondence introduced by the DZIM layer provides new opportunities for engineering the radiation properties [46,52–54] of topological states. In our experiment, material loss plays only a minor role compared with radiation leakage, and the uniform-field nature of DZIM remains well preserved, as reported in low-loss DZIM designs [55,56]. Given the universality of the mechanism, the strategy of introducing DZIM into topological systems is highly versatile and applicable to various classes of topological insulators, such as Chern insulators and topological insulators with chiral edge states [32]. We should emphasize that the point-like connection is inherently incompatible with some topological boundary states whose existence does not rely on global bulk topology. For example, the spatial extension mechanism does not extend to valley-Hall systems, as their valley indices always appear in opposite pairs and collapse to a net-zero value under a point-like optical connection, preventing the formation of topological edge states [32]. Overall, our findings show that the interaction between DZIM and topological phases is governed not by impedance-based mode selection, but by a fundamental modification of bulk-edge correspondence. This expands the design space for compact, integrable, and reconfigurable topological photonic devices, and may inspire analogous developments in acoustic and elastic systems.

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Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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- [32] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/w5cy-d811> for further information on the equivalence of a point and a square of DZIM; the original 0D corner states and 1D edge states between the topologically trivial and nontrivial PC; the derivation of effective parameter method based on field averaging, the experimental setup, the tight-binding framework of the topological state extended by DZIM; the comparison between air and DZIM as interlayer between PC with different bulk polarizations; the impact of material losses in the experiment; the spatially extended one-way propagation in Chern insulators; and topological insulators with chiral edge state, and the suppression of valley Hall effect by the DZIM layer.
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