Enhancement of weak interactions in phase transitions in condensed matter and early Universe

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Parity-violating weak interactions produce extremely small energy differences between left- and right-handed chiral systems. We show that these microscopic effects may be amplified during collective phenomena such as phase transitions. The enhancement factor is proportional to the critical number of atoms, N_c, in the nucleus of the new phase. Measurement of the ratio of produced left- and right-handed chiral structures may provide a way to determine N_c. Experiments in which spin-chiral structures are formed in $ZnCr_2Se_4$ during a phase transition in crossed electric and magnetic fields indicate an effective amplification corresponding to $N_c \sim 10^9$–10^{10}. We emphasize, however, that such large values cannot be inferred from homogeneous capillary nucleation alone, since the same parameters imply a very large nucleation barrier; in real crystals heterogeneous nucleation, defects, strain, and domain-growth dynamics may be essential. We also discuss, more cautiously, whether analogous collective enhancement could operate in first-order cosmological phase transitions beyond the standard model. The standard-model electroweak transition for the observed Higgs mass is a crossover and therefore has no critical bubble nucleus.

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I. INTRODUCTION

Parity-violating (PV) weak interactions are tiny at the energy scales of condensed matter. We demonstrate that phase transitions—specifically, nucleation at first order and large correlation volumes near criticality—provide a generic amplifier that converts microscopic PV energy differences into macroscopic chiral biases. This makes parity violation experimentally accessible in chiral magnets and chiral crystallization without external bias fields, and it yields testable predictions (e.g., handedness statistics recorded by neutron or optical probes).

As known, the standard model gives a baryon-to-photon ratio several orders of magnitude smaller than observations. In this situation, it is important to check that we have not overlooked any enhancement mechanism that can play a role in electroweak baryogenesis. Indeed, systems are very sensitive to weak fields during phase transitions. A very interesting experiment was performed in Ref. [1]. One may interpret its result as a *nine-orders-of-magnitude* enhancement of the effects of small perturbations in phase transitions in condensed matter (see below). If a similar enhancement of *CP* violation existed in the electroweak phase transition (which presumably created the baryon

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asymmetry of the Universe), *CP* violation might produce a baryon-to-photon ratio compatible with observations. However, we no longer assume that the standard-model electroweak transition itself is a first-order nucleation process. For the observed Higgs mass it is a crossover, and the nucleation discussion can apply only to possible first-order transitions in extensions of the standard model.

One may think about using enhancement of weak interaction in phase transition to search for time reversal (T)- and parity (P)-violating interactions that were searched for in the electric dipole moment experiments with neutrons, nuclei, atoms, and molecules for e.g., see Refs. [2–5].

In nature, many organic molecules prefer one chiral form over another. A striking example is that almost all DNA is right-handed, while nearly all amino acids in living organisms are left-handed for e.g., see Ref. [6]. One possible explanation of this phenomenon is the small energy difference between left- and right-handed molecules for e.g., see Refs. [2,7–9]. However, it is generally believed that the parity-violating energy difference, $\Delta E_{\rm PV}$, is too small to affect the homochirality of life.

Energy difference between molecules of different chirality is studied using spectroscopy methods. In principle, there are other possibilities. One may study resonance chemical reactions and enhancement of weak interactions in collisions of cold molecules, where chiral molecules are formed in collision of other molecules. Weak interaction produces energy difference $\delta E_{\rm PV}$ in the cross-section resonance position in right-hand and left-hand molecules, which results in difference in their numbers formed [10].

In a more general case, formation of chiral molecules from nonchiral components in chemical reactions creates nonequal number of right-hand and left-hand molecules with the relative difference that may exceed equilibrium value $\sim \exp(-\delta E_{\rm PV}/T)$. This leads to the optical activity of molecular gas or solution.

A natural place to search for enhancement of such small effects is in collective phenomena such as phase transitions for e.g., see Ref. [1]. Systems are highly sensitive to weak perturbations near criticality. In the following, we develop this idea within the Zeldovich nucleation model of first-order phase transitions presented, e.g., in the book [11].

The nucleation theory may also be applied to formation of crystals. The strongly enhanced effects of weak interactions may appear in the formation of chiral crystals from concentrated solutions of nonchiral components, for example, $NaClO_3$ from solutions containing ions Na^+ and ClO_3^-. Some remarkable experiments with $NaClO_3$ solutions were described in Refs. [12,13]. In principle, the enhancement effect in this problem may be related to the chirality of biological molecules.

II. NUCLEATION MODEL AND ENHANCEMENT FACTOR

In the nucleation picture, fluctuations create small droplets of a new phase. Droplets smaller than a critical size r_{cr} quickly disappear, while larger ones grow to fill the system. The probability of forming a nucleus of radius r is [11]

$$S \sim e^{-W_{\rm min}/kT}, \tag{1}$$

where $W_{\rm min}$ is the minimal work required to form a nucleus, which may be presented as a sum of the negative volume term, reflecting advantage of the new phase, and positive surface term: $W_{\rm min}(r) = -B\,4\pi r^3/3 + \alpha_t\,4\pi r^2$. This function has maximum at the critical size $r_c = 2\alpha_t/B$. This looks like $W_{\rm min}(r)$ produces a potential barrier for formation of the nucleus bigger than the critical size, with $r > r_c$.

Using formulas presented in the book [11], it is easy to express $W_{\rm min}(r_c)$ in terms of the chemical potentials per particle of the old and new phases, μ_1 and μ_2:

$$W_{\rm min} = \tfrac{1}{2} N_c(\mu_1 - \mu_2), \tag{2}$$

where N_c is the number of particles in the critical nucleus.

If the new phase may exist in two nearly degenerate forms (e.g., left- and right-handed chirality), then weak interactions induce a small energy difference ΔE per particle. This results in slightly

different $W^{\pm}_{\rm min}$ for the two structures. The asymmetry in their nucleation rates can be expressed as

$$P \equiv \frac{S_+ - S_-}{S_+ + S_-} = \frac{e^{-W^+_{\rm min}/kT} - e^{-W^-_{\rm min}/kT}}{e^{-W^+_{\rm min}/kT} + e^{-W^-_{\rm min}/kT}} = \tanh\left(-\frac{W^+_{\rm min} - W^-_{\rm min}}{2kT}\right). \tag{3}$$

For $|W^+_{\rm min} - W^-_{\rm min}| \ll kT$, the asymmetry is small and reduces to

$$P \approx -\frac{W^+_{\rm min} - W^-_{\rm min}}{2kT}. \tag{4}$$

When the phase transition proceeds from a nonchiral to a chiral phase,

$$P \sim -\frac{N_c \Delta E}{2kT}. \tag{5}$$

Thus, the small microscopic energy difference ΔE is enhanced by the collective factor N_c, the number of particles in the critical size nucleus, which may be very large.

Here, we see the potential for large enhancement: Normally in statistical physics energies per degree of freedom are compared to kT, but a critical nucleus may contain billions of degrees of freedom, and the *total* difference $W^+_{\rm min} - W^-_{\rm min}$ is compared with kT.

At the same time, N_c is not an independent free parameter. From Eq. (2), $W_{\rm min} = N_c(\mu_1 - \mu_2)/2$, so a very large N_c near the transition also implies a large homogeneous nucleation barrier. Efficient homogeneous nucleation on laboratory timescales usually requires $W_{\rm min}/kT$ not to be too large. Therefore, when a very large effective N_c is inferred experimentally, one must also consider heterogeneous nucleation, defects, finite-size domains, strain, or domain-growth dynamics.

Note that $N_c \propto (\mu_1 - \mu_2)^{-3}$ [11]. The difference in $\mu_1 - \mu_2$ appears in metastable state of overcooled system (we assume that ordered phase appears for $T < T_c$). Therefore, N_C diverges as $\mu_1 \to \mu_2$ for $T \to T_C$. Purely theoretically, if the temperature is reduced very slowly, giving sufficient time for very improbable, exponentially suppressed formation of very large nuclei of ordered phase, the enhancement may be nearly infinite, and the system always comes to the lower-energy chiral state. However, this statement is challenged by the fluctuations and effects of defects.

Another way to formulate this enhancement: The transition temperature is different for transitions to states of different chirality. So, in an ideal situation, with a very slow cooling process we may achieve transition specifically to the lower-energy chiral state that has higher T_C.

In higher-order phase transitions, there are no metastable phases for $T < T_C$, and transition always happens at $\mu_1 = \mu_2$. In the first-order transitions $N_c \propto (\mu_1 - \mu_2)^{-3}$ [11], so one could conclude that the second-order transitions can in principle provide bigger enhancement. However, in a genuine second-order transition or crossover, there is no metastable critical droplet and hence no critical nucleus in the Zeldovich sense. Large correlation volumes near criticality may still enhance the response to small symmetry-breaking fields in condensed matter, but this is a different mechanism and should not be identified with the first-order nucleation factor N_c. In particular, this argument cannot by itself provide electroweak baryogenesis in the standard-model crossover.

III. MAGNETIC CRYSTALS WITH SPIN-SCREW STRUCTURES

In magnetic crystals, the spin-screw structures play the role of molecular chirality. The energy difference between left- and right-handed spin-screw states arises from spin-orbit coupling and external fields for e.g., see the book [2].

For the spinel $ZnCr_2Se_4$, the phase transition is classified as a weak first-order phase transition (i.e., close to the second-order phase transition). Near its Néel temperature $T_N = 21$ K, experiment in crossed electric field $E = 2.5$ kV/cm and magnetic field $H = 12$ kOe measured an asymmetry $P \simeq -0.9$ [1]. The effect is produced by interactions with magnetic field $\hat{H}_H$, electric field $\hat{H}_E$, and

spin-orbit interaction $\hat{H}_{\rm SO}$ [1,2]. A rough estimate yields

$$\Delta E \sim \frac{\langle n|\hat{H}_{\rm SO}|m\rangle\langle m|\hat{H}_E|k\rangle\langle k|\hat{H}_H|n\rangle}{\left(E_n^{(0)}-E_m^{(0)}\right)\left(E_n^{(0)}-E_k^{(0)}\right)} \tag{6}$$

$$\sim \eta_{EH}\frac{Z^2\alpha^2(e^2/a_B)\times ea_BE\times\mu_BH}{(e^2/a_B)^2} \sim \eta_{EH}\times 10^{-12}\,\text{eV} \tag{7}$$

$$\approx \eta_{EH}\times 10^{-8}\,\text{K}. \tag{8}$$

Geometric suppression factor $\eta_{EH} < 1$ may reduce this estimate. Using Eq. (5), we obtain

$$N_c \gtrsim 10^9. \tag{9}$$

Since the observed asymmetry is large, the exact expression (3) should be used rather than the linear approximation. If one nevertheless parametrizes the observation by an effective critical number, then

$$N_c^{\rm eff} = \frac{2kT_N}{|\Delta E_{EH}|}\tanh^{-1}|P| \simeq 5\times 10^9\left(\frac{10^{-12}\,\text{eV}}{|\Delta E_{EH}|}\right). \tag{10}$$

Thus, the data correspond phenomenologically to $N_c^{\rm eff}\sim 10^9$–10^{10} for $|\Delta E_{EH}|\sim 10^{-12}$ eV. This should not be interpreted as a proof that homogeneous capillary nucleation occurs with this value of N_c. As shown in the appendix, homogeneous nucleation with $N_c\sim 10^9$ would have an enormous barrier. The experiment therefore motivates the enhancement mechanism, but a quantitative microscopic explanation of the observed domain selection requires a model of heterogeneous nucleation, defects, strain, or nonequilibrium domain growth.

Note that heterogeneous nucleation typically rescales W_c and N_c by the same shape factor, so it does not obviously lift the tension between a large N_c and efficient nucleation.

IV. EFFECT OF PARITY-VIOLATING WEAK INTERACTION

The estimate of the collective enhancement factor N_c from the Zeldovich nucleation theory [11] is not unique, since several material parameters are not known accurately. In particular, the result depends strongly on the interfacial energy and on the actual cooling protocol, which determines the undercooling temperature. As discussed in the appendix, a capillary estimate gives $N_c\sim 10^9$ only for very small undercooling. However, the corresponding homogeneous nucleation barrier is then extremely large. Thus, this value should be interpreted as an effective collective scale suggested by the observed chirality selection, not as a self-contained homogeneous nucleation calculation.

We may estimate the ratio of the effect produced by the parity-violating weak interaction (in the absence of external fields) to the observed effect in the crossed electric and magnetic fields. PV electron-nuclear interaction can be written as a sum of the nuclear-spin-independent and nuclear-spin-dependent (NSD) parts:

$$H_{\rm PV} = \frac{G_F}{\sqrt{2}}\left(-\frac{Q_W}{2}\gamma_5+\frac{\kappa}{I}\alpha\cdot\mathbf{I}\right)\rho(r), \tag{11}$$

where G_F is the Fermi weak interaction constant, Q_W is the nuclear weak charge, α and γ_5 are the Dirac matrices, I is the nuclear spin, and $\rho(r)$ is the nuclear density normalized to 1. The effective nuclear weak charge, including radiative corrections, is equal to [14]

$$Q_W = -0.988\,97N + 0.070\,605Z. \tag{12}$$

Here, N is the number of neutrons and Z is the number of protons. The dimensionless constant $\kappa\sim 1$ is dominated by the interaction of electrons with the nuclear anapole moment and determines the strength of the NSD PV interaction [15,16].

The energy difference $\Delta E_{\rm PV}$ between opposite enantiomers originates from the PV interaction (11) between electrons and nuclei. The weak interaction mixes $s_{1/2}$ and $p_{1/2}$ orbitals, creating a spin-helical structure within atoms [2,9]. The spin-orbit interaction then distinguishes between coordinate and spin structures aligned in opposite directions. The estimate gives [17,18]

$$\Delta E_{\rm PV} \sim 10^{-19} Z^5 \eta_{\rm PV}\ ({\rm eV}), \tag{13}$$

where Z is the nuclear charge of the heaviest atom in the molecule and $\eta_{\rm PV}$ is a molecular asymmetry factor. The steep Z^5 dependence results from weak ($\propto Z^3$) and spin-orbit ($\propto Z^2$) scaling, but η is typically small. Comparison with accurate calculations [19–31] indicates $\eta_{\rm PV} \sim 10^{-5}$. In molecules with two heavy atoms, $\eta_{\rm PV}$ can be few orders of magnitude larger than that in molecules with one heavy atom [17,18]. In molecules containing heavy atoms, $\Delta E_{\rm PV}$ may exceed 10^{-11} eV for e.g., see Refs. [19–31].

Now we can estimate the ratio of the energy difference produced by the parity-violating interaction (13) to that produced by crossed electric and magnetic fields (6):

$$\frac{\Delta E_{\rm PV}}{\Delta E_{EH}} \sim 10^{-4} Z^3 \frac{\eta_{\rm PV}}{\eta_{EH}}, \tag{14}$$

For $ZnCr_2Se_4$, the largest nuclear charge is $Z = 34$, and therefore

$$\frac{\Delta E_{\rm PV}}{\Delta E_{EH}} \sim \frac{\eta_{\rm PV}}{\eta_{EH}}. \tag{15}$$

Thus, if the structural factors for the PV and crossed-field mechanisms are not too different, the PV interaction in systems with a heavy atom may produce a bias comparable to, or even larger than, the crossed-field effect observed in Ref. [1]. This estimate is only qualitative: In a magnetic crystal, the relevant "chirality" is the spin-screw chirality rather than ordinary molecular chirality, and the factors $\eta_{\rm PV}$ and η_{EH} require a microscopic calculation. Nevertheless, Eq. (15) suggests that parity-violating weak interactions in heavy magnetic crystals could be observable through the same phase-transition enhancement mechanism.

V. POSSIBLE IMPLICATIONS FOR BARYOGENESIS

Several studies have concluded that the standard model predicts a baryon-to-photon ratio several orders of magnitude smaller than observed in electroweak-baryogenesis estimates for e.g., see Refs. [32–36]. It is therefore useful to ask whether collective effects during phase transitions could amplify *CP*-violating biases.

The standard-model electroweak transition for the observed Higgs mass is a crossover, not a first-order phase transition. Therefore, there is no critical bubble nucleus in the standard-model electroweak transition. The nucleation argument discussed in this paper can apply to cosmology only in models beyond the standard model in which the electroweak or another cosmological transition is first order.

In ordinary electroweak baryogenesis, bubble walls separate the high-temperature symmetric phase from the low-temperature broken phase. *CP*-violating interactions modify the transport of particles and antiparticles in front of a moving wall, while sphaleron processes in the symmetric phase convert the resulting chiral asymmetry into baryon number for e.g., see Refs. [32–36].

For a first-order transition, the temperature at which bubbles actually nucleate is not exactly the critical temperature. In cosmological applications, it is determined by the condition that the nucleation rate becomes comparable with the Hubble expansion rate. The transition then completes by bubble growth, collision, and percolation, rather than by a single bubble filling the Universe. The amount of supercooling, the bubble-wall velocity, the surface tension, and the latent heat are model dependent.

The condensed-matter mechanism discussed above therefore should be viewed only as an analogy and motivation for further study. It does not constitute a quantitative theory of

baryogenesis. A quantitative cosmological calculation would require a specific first-order model beyond the standard model, together with the finite-temperature effective potential, bubble action, wall profile, hydrodynamics, and *CP*-violating transport. Nevertheless, the analogy suggests a question worth investigating: Whether *CP*-violating effects that modify bubble-wall transport, pressure balance, or the critical action could be amplified by collective properties of a first-order transition.

The nucleation framework and its N_c-based enhancement is not directly applicable to a crossover. Note, however, that even in a crossover, dropletlike regions ("bubbles") of the emergent phase appear within the symmetric phase for e.g., see Ref. [36] and references therein). Therefore, a possibility of a collective enhancement in the crossover case may also deserve investigation.

VI. CONCLUSION

We have discussed a simple mechanism by which very small energy differences may be amplified during phase transitions. In the Zeldovich nucleation picture, the probability of forming a critical nucleus depends exponentially on the minimum work $W_{\min}$. Therefore, if two possible structures differ in energy by a small amount ΔE per particle, the asymmetry in their formation probabilities contains the collective factor N_c, the number of particles in the critical nucleus:

$$P \sim -\frac{N_c\,\Delta E}{2kT}.$$

This formula links a microscopic splitting ΔE to a collective factor N_c—the number of particles in the critical nucleus of ordered phase. Enhancement factor N_c grows rapidly as the temperature of overcooled material approaches critical transition temperature, $T \to T_c$, and, in principle, could become extremely large. However, N_c is tied to the nucleation barrier and cannot be chosen independently. Large N_c values are possible near a weakly first-order transition, but homogeneous nucleation with such large nuclei may be exponentially suppressed. Therefore, the large effective enhancement suggested by $ZnCr_2Se_4$ should be regarded as phenomenological unless heterogeneous nucleation, defects, strain, or domain-growth dynamics are included.

This mechanism suggests experimental searches for parity-violating effects based not only on high-resolution spectroscopy, but also on measurements of chiral domain statistics or optical activity generated during crystallization. A similar mechanism may enhance the difference in the formation of chiral crystals from concentrated solutions of nonchiral components. Measurement of the ratio of produced left- and right-handed chiral structures, combined with calculations of the corresponding microscopic energy differences, may provide a way to determine an effective collective enhancement factor.

The possible cosmological implications have also been mentioned. The standard-model electroweak transition is a crossover and has no critical bubble nucleus. Hence, the first-order nucleation mechanism considered here can be relevant to cosmology only in appropriate extensions of the standard model. A quantitative baryogenesis calculation would require a specific finite-temperature model, including the bubble action, wall profile, hydrodynamics, sphaleron processes, and *CP*-violating transport. In a finite condensed-matter sample, a supercritical nucleus grows until it collides with other domains or reaches the sample boundary; in a cosmological first-order transition, the transition proceeds by nucleation, growth, collision, and percolation of many bubbles.

Thus, the main result of the present work is a general amplification mechanism and its order-of-magnitude consequences, rather than a complete microscopic calculation for a particular material or a quantitative theory of baryogenesis. The estimates presented here motivate detailed numerical calculations and new measurements in systems where small parity-violating energy differences may be converted into observable macroscopic asymmetries.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the author.

APPENDIX: ESTIMATE OF THE CRITICAL NUCLEUS SIZE IN $ZnCr_2Se_4$

In this Appendix, we give an order-of-magnitude estimate of the critical size of the nucleus of the helical phase in $ZnCr_2Se_4$. The estimate is intended only to show that a very large critical number of spins, N_c, is plausible near a weak first-order phase transition.

The free energy of a spherical nucleus of radius r is

$$W(r) = 4\pi r^2 \alpha_t - \frac{4\pi}{3} r^3 B, \tag{A1}$$

where α_t is the interfacial energy between the paramagnetic and helical phases, and B is the bulk free-energy gain per unit volume. The critical radius is determined by $dW/dr = 0$, giving

$$r_c = \frac{2\alpha_t}{B}. \tag{A2}$$

Close to the transition temperature, the bulk driving force may be written as

$$B \simeq \Delta s_v \, \delta T, \tag{A3}$$

where Δs_v is the entropy change per unit volume and

$$\delta T = T_N - T$$

is the undercooling below the transition temperature. Therefore,

$$r_c = \frac{2\alpha_t}{\Delta s_v \, \delta T}. \tag{A4}$$

The corresponding number of Cr spins in the critical nucleus is

$$N_c = \frac{4\pi}{3} n_{\mathrm{Cr}} r_c^3 = \frac{4\pi}{3} n_{\mathrm{Cr}} \left(\frac{2\alpha_t}{\Delta s_v \, \delta T} \right)^3, \tag{A5}$$

where n_{Cr} is the density of Cr spins.

The spinel unit cell contains eight formula units and therefore 16 Cr ions. Taking the lattice constant

$$a \simeq 10.5 \text{ Å},$$

one obtains

$$n_{\mathrm{Cr}} = \frac{16}{a^3} \simeq 1.38 \times 10^{28} \text{ m}^{-3}. \tag{A6}$$

The molar volume is

$$V_m = \frac{N_A a^3}{8} \simeq 8.7 \times 10^{-5} \text{ m}^3 \text{ mol}^{-1}. \tag{A7}$$

Using the entropy change at the transition,

$$\Delta S \simeq 1.8\text{–}3.1\ \mathrm{J\,mol^{-1}\,K^{-1}}, \tag{A8}$$

gives

$$\Delta s_v = \frac{\Delta S}{V_m} \simeq (2.1\text{–}3.6)\times 10^4\ \mathrm{J\,m^{-3}\,K^{-1}}. \tag{A9}$$

The least certain parameter is the interfacial energy α_t. Since no direct measurement of the interface energy between the paramagnetic and helical phases appears to be available, we estimate its scale from the micromagnetic expression

$$\alpha_t \sim 4\sqrt{A_{\mathrm{ex}} K_u}, \tag{A10}$$

where A_{ex} is the exchange stiffness and K_u is the anisotropy energy density. Taking

$$A_{\mathrm{ex}} \sim (1\text{–}3)\times 10^{-12}\ \mathrm{J\,m^{-1}}, \quad K_u \sim (3\text{–}7)\times 10^3\ \mathrm{J\,m^{-3}}, \tag{A11}$$

one obtains the order-of-magnitude range

$$\alpha_t \sim (2\text{–}6)\times 10^{-4}\ \mathrm{J\,m^{-2}}. \tag{A12}$$

Substitution of Eqs. (A6), (A9), and (A12) into Eq. (A5) gives

$$N_c \simeq (0.08\text{–}11)\times 10^6 \left(\frac{1\ \mathrm{K}}{\delta T}\right)^3. \tag{A13}$$

A representative central value is

$$N_c \simeq 1.4\times 10^6 \left(\frac{1\ \mathrm{K}}{\delta T}\right)^3. \tag{A14}$$

Thus, the critical nucleus size is extremely sensitive to the undercooling:

$$N_c \propto \delta T^{-3}.$$

For example, if the transition occurs with an undercooling of order

$$\delta T \sim 0.1\ \mathrm{K},$$

Eq. (A14) gives

$$N_c \sim 10^9.$$

The corresponding critical radius is

$$r_c = \left(\frac{3N_c}{4\pi n_{\mathrm{Cr}}}\right)^{1/3} \sim 3\times 10^{-7}\ \mathrm{m}, \tag{A15}$$

i.e., a few tenths of a micron.

We now add an important qualification. The same capillary parameters also determine the homogeneous nucleation barrier,

$$W_c = \frac{16\pi\alpha_t^3}{3B^2} = \frac{16\pi\alpha_t^3}{3(\Delta s_v \delta T)^2}. \tag{A16}$$

For the central values used above and $\delta T \simeq 0.1$ K, one finds

$$\frac{W_c}{kT} \sim 5\times 10^5, \tag{A17}$$

so homogeneous thermal nucleation would be exponentially suppressed. Conversely, requiring an efficient homogeneous nucleation rate, roughly $W_c/kT \sim 40$–100 on laboratory timescales, would then give only

$$N_c \sim 10^3\text{–}10^4, \tag{A18}$$

and the asymmetry produced by a microscopic energy difference $\Delta E \sim 10^{-12}\,\mathrm{eV}$ at $T \simeq 21\,\mathrm{K}$ would be only

$$|P| \sim \frac{N_c \Delta E}{2kT} \lesssim 10^{-6} \text{ to } 10^{-5}, \tag{A19}$$

far below the observed $|P| \simeq 0.9$. Therefore, the simple homogeneous capillary estimate cannot by itself explain the $ZnCr_2Se_4$ experiment. The large value $N_c \sim 10^9$ should instead be regarded as an effective collective scale inferred phenomenologically from the observed chirality selection. A quantitative explanation requires additional physics, such as heterogeneous nucleation on defects or surfaces, strain, domain-wall motion, sample history, or other nonequilibrium mechanisms.

This estimate is necessarily approximate because the interfacial energy α_t and the actual undercooling in the experiment are not known accurately. Its main purpose is to display the tension between large critical nuclei and efficient homogeneous nucleation, and to identify the additional material physics that must be included in a realistic calculation.

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