

Resetting optimized competitive first-passage outcomes in non-Markovian systems

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We investigate the role of stochastic resetting in non-Markovian systems, where memory effects arise due to slow relaxation, rugged energy landscapes, disordered environments, and molecular crowding. Using the celebrated continuous-time random walk framework, we analyze first-passage processes with multiple competing outcomes and examine how resetting can selectively enhance desired outcomes. We characterize the efficiency of resetting through conditional mean first-passage times and demonstrate that its impact is highly sensitive to the underlying waiting-time statistics. Furthermore, we derive an inequality that quantifies how resetting controls fluctuations in conditional first-passage times, revealing regimes where variability is significantly suppressed. Our results provide a systematic understanding of how long-term memory influences competitive first-passage outcomes and establish resetting as a powerful control mechanism beyond the conventional Markovian setting.

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I. INTRODUCTION

In classical Markovian processes, the time evolution of a system depends only on the present state and not on the history of its trajectory. Ordinary Brownian motion is a paradigmatic example of such memoryless dynamics, where the waiting-time distribution (WTD) is typically exponential and successive increments are statistically independent. However, many realistic systems involve interactions with complex, disordered, or crowded environments that generate temporal correlations, trapping events, or slow relaxation, so that the effective dynamics of the observed degree of freedom become non-Markovian [1,2].

A widely used framework for describing one important class of non-Markovian dynamics is the continuous-time random walk (CTRW)—a wait-and-jump model, in which both the waiting time and jump lengths can be governed by general distributions [2,3]. In particular, heavy-tailed WTDs with Gaussian jump distribution with a characteristic length σ , such as stretched-exponential or power-law forms with exponent β , can lead to subdiffusive behavior characterized by the mean-squared displacement (MSD) $\langle x^2(t) \rangle \sim t^\beta$ with $0 < \beta < 1$ [4–6]. Such dynamics are relevant

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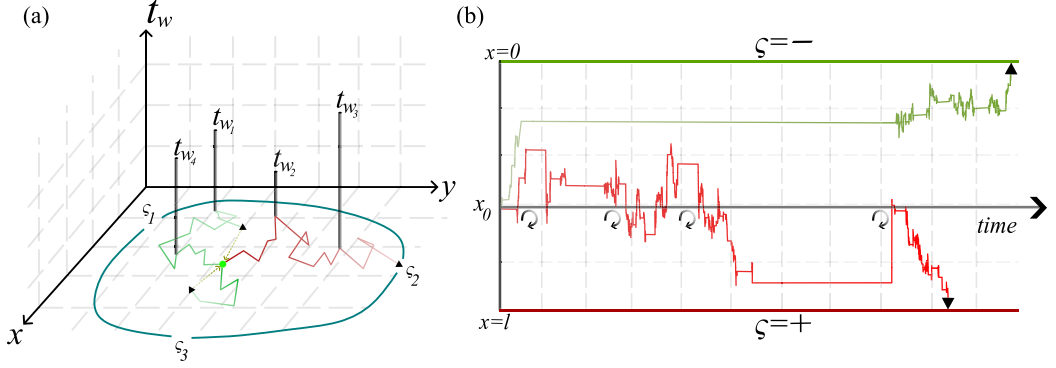


FIG. 1. (a) Schematic illustration of CTRW process inside a bounded domain in the presence of three outcomes $\{\zeta_1, \zeta_2, \zeta_3\}$. The waiting times t_{w_i} represent the intermittent pauses before the jumps. These random pause epochs $\{t_{w_1}, t_{w_2}, t_{w_3}, \dots\}$ follow a WTD distribution $\phi(t_w)$. Panel (b) represents the same CTRW but in the presence of resetting, in a one-dimensional (1D) domain with two outcomes $\zeta \in \{-, +\}$.

to transport in crowded polymer networks, structural glasses with broadly distributed energy barriers, tagged-monomer motion in polymeric systems, and related complex media [1,7–11]. Similar memory effects also arise in biological processes including DNA-binding protein searches and intracellular transport [12,13].

In such non-Markovian processes, where heavy-tailed waiting-time statistics also, in effect, produce broad, fat-tailed first-passage or reaction time distributions [2], particles may remain immobilized for exceptionally long intervals due to disorder or energetic variations in the medium [14–16]. According to the big jump principle, a single, unusually long trapping event can dominate the fluctuations and determine the far tail of the first-passage time (FPT) distribution, leading to large deviations from standard Markovian behavior [17]. *Controlling* these rare, extreme events is therefore essential for improving completion kinetics in realistic systems. Several approaches have been developed to limit such effects, including tempered or truncated power-law waiting times, which introduce natural temporal cutoffs and restore finite statistical moments [2], removal or modification of deep traps to reduce the dominance of extremely long waiting events in glassy or disordered landscapes [9,18,19], external driving or applied fields, which help particles escape traps more readily [20], and intermittent search strategies, where particles alternate between slow, local exploration and faster relocation modes to avoid being dominated by a single unfavorable trap [12,21]. Together, these methods underscore the importance of managing extreme waiting events to optimize first-passage performance in complex, heterogeneous environments.

Much like other control mechanisms, stochastic resetting, where the system is intermittently returned to a prescribed state, has emerged as a successful strategy for expediting first-passage events [22,23] by preventing rare, exceptionally long excursions that dominate completion times [24–61] (see the reviews [33,35,52,62,63] on this topic). Resetting acts to suppress the influence of extreme fluctuations and narrow the distribution of outcomes without requiring detailed microscopic intervention. In this work, we extend this idea to the CTRW models [8–10,64–67]. Since CTRWs allow arbitrary WTDs [see Fig. 1(a)], unlike classical diffusion with exponential waiting times, they become well suited to describe anomalous diffusion in disordered and heterogeneous media where history-dependent dynamics cannot be neglected (see the earlier works on the application of CTRW in first-passage processes [7,9,11,67,68]). By integrating stochastic resetting into this framework, we aim to regulate the long trapping events responsible for broad tails, thereby mitigating variability and optimizing first-passage performance in complex environments.

In particular, our focus is on first-passage processes in which multiple outcomes, denoted by a set $\{\zeta\}$, are possible—some favorable and others undesirable [69]—in contrast to the case where

the completion is indifferent to the nature of the outcome [65]. Multiple first-passage outcomes naturally emerge in soft-matter and disordered systems [70–74], where only a subset corresponds to successful performance [75]. In disordered semiconductors [76], porous geological media [77], and colloids in quenched optical landscapes [78], particles may either reach a productive exit channel or become trapped for long intervals, generating heavy-tailed first-passage statistics. Since many anomalous systems are governed by broad waiting-time statistics [79,80], naturally the competing outcomes can also exhibit large variability dominated by rare, long-lived trapping events. A central question, therefore, is how to systematically bias the flux toward [81,82] the desired outcomes while suppressing the undesired ones.

Recently, we addressed this question by showing that stochastic resetting at a rate r provides a systematic means of controlling competitive first-passage outcomes, as quantified by the conditional CV⁵ criterion (discussed below) [39,69] and demonstrated for Markovian diffusion. A primary objective of the present work is to establish the relevance and robustness of this criterion in non-Markovian systems. Beyond enhancing favorable outcomes through the optimization of mean completion times, we ask whether resetting can also suppress fluctuations in completion times, thereby narrowing the first-passage distribution and improving the reliability of the process. To this end, we derive κ^5 criterion that provides a general condition for fluctuation reduction under stochastic resetting. We demonstrate both criteria using a CTRW confined to a one-dimensional box of length l , with two exits representing competing outcomes. We further examine how the conditional first-passage statistics and the associated optimal behavior depend on the waiting-time distribution—characterized by the exponent β and jump length σ , the scaled initial position $u = x_0/l$, and the resetting rate r .

We now outline the structure of the paper for the convenience of the readers. In Sec. II, we briefly describe the CTRW model in a one-dimensional setting and discuss the qualitative effects of various WTDs. In Sec. III, we set up the renewal structure for the probability currents or fluxes, which, in turn, yields the conditional FPT densities under Poissonian resetting. Section IV investigates how distinct classes of WTDs affect resetting-mediated conditional mean first-passage times (MFPTs). In Sec. V, we analyze a universal criterion that constrains system parameters, determining the conditions under which resetting effectively minimizes fluctuations in conditional FPTs. Finally, we conclude in Sec. VI with a brief discussion and future outlook. Additional results and supporting calculations are discussed in the Appendixes.

II. MODEL SETUP AND DETAILS

We assume the CTRW to be confined in a one-dimensional arena in the presence of two absorbing boundaries at $x = 0$ and $x = l$. The absorbing events through these boundaries are, respectively, denoted with two mutually exclusive outcomes, i.e., $\varsigma = \{-, +\}$ [see Fig. 1(b)]. Initiating from x_0 , the walker undergoes a series of transitions characterized by random waiting times followed by discrete jumps. This *wait-and-jump* sequence effectively models the continuous interaction between the particle and its environment. In particular, we consider the jump lengths and the waiting times to be independent and identically distributed random variables according to the probability density functions (PDFs) $\psi(x)$ and $\phi(t_w)$, respectively. Further, we assume the characteristic jump length σ to be small compared to the dimension of the domain, i.e., $\sigma \ll l$. While the Markovian case is characterized by the exponential distribution $\phi(t_w) = \tau^{-1}e^{-t_w/\tau}$, memory effects emerge as a dominant feature when the WTDs deviate from this exponential form. In what follows, we consider a broad class of $\phi(t_w)$, characterized by their corresponding Laplace transforms, which mark a strong departure from the Markovianity, namely [2],

$$\tilde{\phi}(s) \approx \begin{cases} 1 - a_\beta s^\beta, & \text{class I, } 0 < \beta < 1, \\ 1 - s\langle\mathcal{T}\rangle_\phi + b_\beta s^\beta, & \text{class II, } 1 < \beta < 2, \\ 1 - s\langle\mathcal{T}\rangle_\phi + \frac{s^2}{2}\langle\mathcal{T}^2\rangle_\phi, & \text{class III, } \beta > 2, \end{cases} \quad (1)$$

where $\langle \mathcal{T} \rangle_\phi$ and $\langle \mathcal{T}^2 \rangle_\phi$ represent the mean and the second moment of the WTD, respectively. These statistics of random waiting times are associated with the reaction kinetics and often characterize long pauses between successive jumps. This results in diverging mean and second moment (class I), finite mean and diverging second moment (class II), and finite mean and second moment (class III). We seek to see whether intermittent stochastic resetting events can curtail the long trapping intervals (pertinent to classes I and II), and also the class III processes under general conditions. Finally, we derive a universal criterion that characterizes the impact of resetting on the fluctuations of the random times associated with a specific outcome, extending existing results beyond MFPT optimization.

III. GENERAL FRAMEWORK FOR RESETTING-MEDIATED COMPETITIVE FIRST-PASSAGE PROCESS

To set up the general formalism, let us consider a stochastic process with a set of multiple outcomes, denoted with a set $\{\zeta\}$ in the presence of a constant resetting with rate r . We start by defining a probability current $J_r^\zeta(u, t)$ assigned with each outcome or target ζ up to a time t . Since the system is reset at a rate r , we can write a renewal equation for the individual probability current following the first renewal formalism [43,83], which reads

$$J_r^\zeta(u, t) = e^{-rt} J_0^\zeta(u, t) + \int_0^t d\tau_f r e^{-r\tau_f} Q_0(u, \tau_f) J_r^\zeta(u, t - \tau_f), \quad (2)$$

where $J_0^\zeta(u, t)$ is the probability current and $Q_0(u, \tau_f)$ is the probability that the process survives up to time τ_f , both in the absence of resetting. Additionally, $f_{T_0}(u, t) = -\partial Q_0(u, t)/\partial t$ is the unconditional FPT density of the underlying process. The renewal equation (2) follows from decomposing the process into two mutually exclusive scenarios: (1) The first-passage event occurs before any resetting event, corresponding to the first term on the right-hand side (RHS), and (2) the process undergoes one or more resetting events before eventually reaching the target, represented by the second term on the right-hand side. It is worth emphasizing that such renewal framework requires the resetting of all relevant dynamical variables of the system, including position, velocity, orientation, or any other internal degrees of freedom that carry memory, thereby ensuring a complete renewal of the process after each resetting event. To proceed further, we perform a Laplace transformation on both sides of Eq. (2) to arrive at [43,69]

$$\tilde{J}_r^\zeta(u, s) = \frac{\tilde{J}_0^\zeta(u, r + s)}{1 - r\tilde{Q}_0(u, r + s)}. \quad (3)$$

To ensure completeness, we introduce a comprehensive renewal structure by integrating a generalized resetting framework, going beyond simple Poissonian resetting, in Appendix A. For the current, we have denoted the Laplace transform of a function $f(t)$ as $\tilde{f}(s) = \int_0^\infty dt e^{-st} f(t)$. Furthermore, for the conditional and unconditional first-passage time densities, we use the following notation: $\tilde{T}_0^\zeta(u, s) = \int_0^\infty dt e^{-st} f_{T_0}^\zeta(u, t)$ and $\tilde{T}_0(u, s) = \int_0^\infty dt e^{-st} f_{T_0}(u, t)$, respectively.

Recalling the relation between the conditional FPT and the corresponding current, one can write [69,84]

$$f_{T_{r(0)}^\zeta}(u, t) = \frac{J_{r(0)}^\zeta(u, t)}{\epsilon_{r(0)}^\zeta(u)}, \quad (4)$$

where $\epsilon_{r(0)}^\zeta(u) = \int_0^\infty J_{r(0)}^\zeta(u, t) dt$ is the exit or splitting probability that starting from u , the process exits through a specific boundary ζ in the presence (absence) of resetting. The above relations essentially allow us to connect the resetting-mediated conditional FPT density with the underlying FPT densities as follows:

$$\tilde{T}_r^\zeta(u, s) = \frac{\tilde{T}_0^\zeta(u, r + s)}{\tilde{T}_0^\zeta(u, r)} \frac{(s + r)\tilde{T}_0(u, r)}{s + r\tilde{T}_0(u, r + s)}, \quad (5)$$

using which we can obtain the conditional MFPT under resetting

$$\langle T_r^\varsigma(u) \rangle = - \frac{\partial \tilde{T}_r^\varsigma(u, s)}{\partial s} \Big|_{s \rightarrow 0} = \langle T_r(u) \rangle + \frac{\partial}{\partial r} \ln \left[\frac{\tilde{T}_0(u, r)}{\tilde{T}_0^\varsigma(u, r)} \right], \quad (6)$$

where $\langle T_r(u) \rangle$ represents the unconditional MFPT in the presence of resetting, which was derived in earlier works [28,29,33]:

$$\langle T_r(u) \rangle = \frac{1 - \tilde{T}_0(u, r)}{r \tilde{T}_0(u, r)}. \quad (7)$$

Similarly, using Eq. (5), one can find the second moment of the resetting-induced conditional FPT density, i.e., $\langle (T_r^\varsigma(u))^2 \rangle = \frac{\partial^2 \tilde{T}_r^\varsigma(u, s)}{\partial s^2} \Big|_{s \rightarrow 0}$, which reads

$$\langle (T_r^\varsigma(u))^2 \rangle = \langle T_r(u)^2 \rangle \left[\tilde{T}_0(u, r) + \frac{\langle T_r^\varsigma(u) \rangle}{\langle T_r(u) \rangle} (1 - \tilde{T}_0(u, r)) \right] - \left[\frac{\frac{\partial^2 \tilde{T}_0(u, r)}{\partial r^2}}{\tilde{T}_0(u, r)} - \frac{\frac{\partial^2 \tilde{T}_0^\varsigma(u, r)}{\partial r^2}}{\tilde{T}_0^\varsigma(u, r)} \right], \quad (8)$$

where $\langle T_r(u)^2 \rangle = \frac{2(1 - \tilde{T}_0(u, r) + r \partial_r \tilde{T}_0(u, r))}{r^2 \tilde{T}_0^2(u, r)}$ denotes the second moment of the unconditional FPT density in the presence of resetting. These expressions, given by Eqs. (3)–(8), require that we first evaluate both the conditional and unconditional FPT quantities for the underlying process. One key observable of our interest, namely, the variance, is defined as

$$(\Delta T_r^\varsigma)^2 = \langle (T_r^\varsigma(u))^2 \rangle - \langle T_r^\varsigma(u) \rangle^2. \quad (9)$$

IV. EXPLICIT RESULTS FOR CTRW IN 1D

In this section, we present explicit results for CTRW using the generic formalism presented above. As mentioned earlier, the renewal structure assumes that each resetting event completely renews the dynamics from scratch. This includes not only the resetting of the spatial coordinate but also the renewal of the waiting-time clock $\phi(t_w)$. Consequently, after each reset, the process starts afresh from the prescribed initial state with a newly initialized waiting-time clock.

To quantify the effect of resetting on the first-passage-time statistics, we first need to determine the underlying FPT density in the absence of resetting, which serves as the basic building block of the renewal description. For our model system, i.e., the 1D CTRW with two absorbing boundaries, i.e., $\varsigma = \{-, +\}$, the underlying conditional FPT densities read (see Appendix B)

$$\tilde{T}_0^-(u, s) = \frac{1}{1 - u} \frac{\sinh[(1 - u)l\alpha(s)]}{\sinh[l\alpha(s)]}, \quad (10)$$

$$\tilde{T}_0^+(u, s) = \frac{1}{u} \frac{\sinh[ul\alpha(s)]}{\sinh[l\alpha(s)]}, \quad (11)$$

where $u = x_0/l$ and the corresponding splitting probabilities read $\epsilon_0^- = 1 - u$ and $\epsilon_0^+ = u$ [see Fig. 4(a)], respectively, where $\alpha(s) = \sqrt{2}/\sigma \sqrt{(1/\tilde{\phi}(s)) - 1}$. We note that the underlying splitting probabilities are found to be independent of the microscopic details, as illustrated in Fig. 4(a). Irrespective of the time span at a particular location between two successive jumps, the number of trajectories through the left or right exit solely depends on the initial location. Conversely, resetting breaks this universality, as can be seen in Fig. 4(b). Using these, we obtain the most general expressions for the conditional MFPT for any WTD:

$$\langle T_r^-(u) \rangle = \langle T_r(u) \rangle + \alpha'[r] \times \frac{\mathcal{N}(1 - u)}{\mathcal{D}(1 - u)}, \quad (12)$$

$$\langle T_r^+(u) \rangle = \langle T_r(u) \rangle + \alpha'[r] \times \frac{\mathcal{N}(u)}{\mathcal{D}(u)}, \quad (13)$$

where $\mathcal{N}(u) = (1-u)l \cosh[(1-u)l\alpha(r)] - ul \coth[ul\alpha(r)] \sinh[(1-u)l\alpha(r)]$ and $\mathcal{D}(u) = \sinh[(1-u)l\alpha(r)] + \sinh[ul\alpha(r)]$. We also need the resetting-mediated unconditional MFPT that can be written as

$$\langle T_r(u) \rangle = \frac{\mathcal{D}(0) - (\mathcal{D}(u) + \mathcal{D}(1-u))}{r(\mathcal{D}(u) + \mathcal{D}(1-u))}, \quad (14)$$

whose exact expression can be found in Appendix C [see Eq. (C2)]. In what follows, we will now discuss the different three classes of WTDs and their intricate role toward the MFPT.

A. Diverging first and second moments of the WTD: class I

Let us first consider the WTD to be the Mittag-Leffler (ML) distribution

$$\phi(t_w) = \frac{t_w^{\beta-1}}{\tau^\beta} \mathcal{E}_{\beta,\beta} \left(-\frac{t_w^\beta}{\tau^\beta} \right), \quad (15)$$

with $0 < \beta < 1$, and diverging first and second moments; also $\mathcal{E}_{\mu,\nu}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\mu z + \nu)}$ is the two parameter Mittag-Leffler function. In the limit $r \rightarrow 0$, the conditional MFPTs read, following Ref. [65],

$$\langle T_r^- \rangle \simeq \frac{u(1-u)}{2r} l^2 \alpha(r)^2 + \mathcal{A}(u, r), \quad (16)$$

$$\langle T_r^+ \rangle \simeq \frac{u(1-u)}{2r} l^2 \alpha(r)^2 + \mathcal{A}(1-u, r), \quad (17)$$

where $\mathcal{A}(u, r) = \frac{l^2}{3} u(2u-1) \alpha(r) \partial_r \alpha(r)$. In particular, the WTD in Eq. (15) can be expressed as $\tilde{\phi}(r) = 1/[1 + (r\tau)^\beta]$, which results in $\alpha(r) = \frac{\sqrt{2}}{\sigma} (r\tau)^{\beta/2}$. Notably, both conditional MFPTs diverge as $1/r^{1-\beta}$ as $r \rightarrow 0$, directly reflecting the heavy-tailed waiting-time statistics. In this context, the efficiency of resetting can be determined by examining the slope of the conditional MFPTs at $r \rightarrow 0$, yielding

$$\partial_r \langle T_r^- \rangle|_{r \rightarrow 0} \sim -\frac{(1-\beta)\tau^\beta}{3r^{2-\beta}} \frac{\mathcal{F}(u)l^2}{\sigma^2} < 0, \quad (18)$$

$$\partial_r \langle T_r^+ \rangle|_{r \rightarrow 0} \sim -\frac{(1-\beta)\tau^\beta}{3r^{2-\beta}} \frac{\mathcal{F}(1-u)l^2}{\sigma^2} < 0, \quad (19)$$

with $\mathcal{F}(u) = u(3 - \beta - u(3 - 2\beta)) > 0$. Since the slopes are negative, an infinitesimal introduction of resetting necessarily decreases the corresponding first-passage time. Thus, in this regime, resetting provides an immediate speedup, indicating that it is beneficial for the process. This behavior is consistent with the trends shown in Figs. 2(a) and 2(b), where the conditional first-passage times decrease upon increasing the resetting rate from zero.

B. Finite first moment and diverging second moment of the WTD: class II

In this case, we consider the WTD to be Pareto distribution, which reads

$$\phi(t_w) = \frac{\beta}{\tau(1 + t_w/\tau)^{\beta+1}}, \quad 1 < \beta < 2. \quad (20)$$

In this case, the first moment remains finite. In the Laplace domain, it can be expressed as $\tilde{\phi}(r) = \beta \mathcal{U}(1, 1 - \beta; r\tau)$. Subsequently, in $r \rightarrow 0$, $\alpha(r)$ reads

$$\alpha(r) \sim \sqrt{r} A_\beta [1 + B_\beta r^{\beta-1}], \quad (21)$$

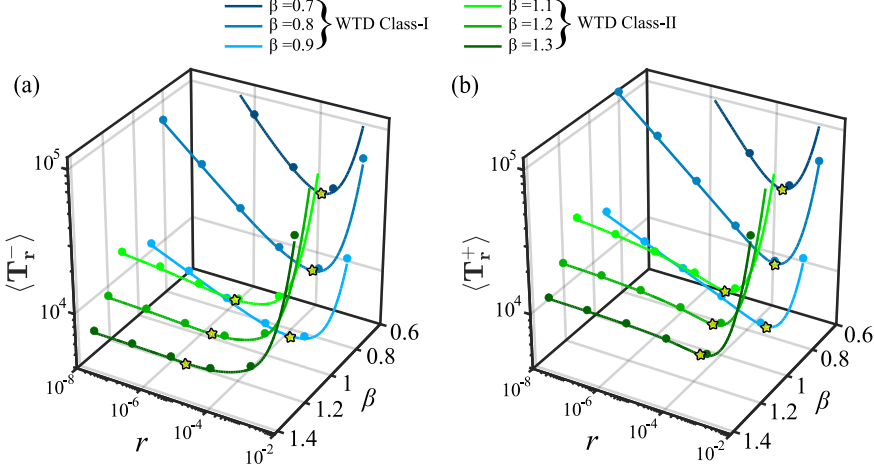


FIG. 2. Variation of the conditional MFPTs with resetting rate. Each panel represents the behavior of the conditional MFPTs for different classes of WTDs with resetting rate, indicated with different colors. Note that $0 < \beta < 1$ and $1 < \beta < 2$ indicate class I and class II, respectively. Other parameters are fixed to $\sigma = 0.01$, $\tau = 1$, and domain length $l = 1$. For each figure, we use the initial condition $u = 0.3$. Panel (a) illustrates the behavior of the conditional MFPT through the left boundary. Consequently, following the inequalities in Eqs. (18) and (23), here we find resetting to be useful in optimizing the MFPTs. The corresponding optimal resetting rates are indicated with a star marker (in yellow) for each β . Similarly, in panel (b), we find the resetting protocol to be beneficial in optimizing the conditional MFPT through the right boundary, which is reported in Eqs. (19) and (24). We have taken Mittag-Leffler PDF with exponent $0 < \beta < 1$ as class I WTD, and the Pareto distribution with exponent $1 < \beta < 2$ as class II WTD. Here, the markers indicate the Monte Carlo simulation results with the corresponding class of WTD and they are in good agreement with the theoretical results.

where $A_\beta = \frac{\sqrt{2\tau}}{\sigma\sqrt{(\beta-1)}}$ and $B_\beta = \frac{\pi(\beta-1)\tau^{\beta-1}}{2\sin(\pi\beta)\Gamma(1+\beta)}$. Incorporating Eq. (21) in Eqs. (10) and (11), conditional MFPTs for the underlying dynamics yield

$$\langle T_{r \rightarrow 0}^-(u) \rangle = \frac{(2-u)u}{3(\sigma/l)^2} \langle \mathcal{T} \rangle_\phi, \quad \langle T_{r \rightarrow 0}^+(u) \rangle = \frac{1-u^2}{3(\sigma/l)^2} \langle \mathcal{T} \rangle_\phi, \quad (22)$$

with $\langle \mathcal{T} \rangle_\phi = \tau/(\beta-1)$ denoting the first moment of the WTD. Similar to the previous analysis, one can estimate the efficiency of resetting by examining the slope of conditional MFPTs at $r \rightarrow 0$. Replacing $\alpha(r)$ with Eq. (21) in Eqs. (16) and (17), one can find the slopes of the conditional MFPTs [see Appendix D, Eqs. (D3) and (D4)]:

$$\partial_r \langle T_r^- \rangle|_{r \rightarrow 0} \sim \frac{(\beta-1)\tau^\beta f(u)l^2}{r^{2-\beta}} \frac{\pi}{6\sigma^2 \sin(\pi\beta)\Gamma(1+\beta)} < 0, \quad (23)$$

$$\partial_r \langle T_r^+ \rangle|_{r \rightarrow 0} \sim \frac{(\beta-1)\tau^\beta f(1-u)l^2}{r^{2-\beta}} \frac{\pi}{6\sigma^2 \sin(\pi\beta)\Gamma(1+\beta)} < 0, \quad (24)$$

with $f(u) = u(2-u) > 0$ and $\sin(\pi\beta) < 0$ for $1 < \beta < 2$. Here too, the negative slopes imply that introducing resetting immediately reduces the corresponding first-passage times, indicating that resetting is beneficial in this regime. This is consistent with Figs. 2(a) and 2(b), where the conditional first-passage times decrease with increasing resetting rate from zero. Thus, we find that for $0 < \beta < 2$, introduction of resetting is always useful.

C. Finite first and second moments of the WTD: class III

In contrast to the cases discussed above, the effect of resetting needs to be carefully monitored when the WTD possesses finite first and second moments, which can be obtained for Pareto with $\beta > 2$ in Eq. (20). It turns out that resetting is not always guaranteed to be useful. To reflect on this, in Refs. [39,69], we had derived a criterion that turns out to be a sufficient condition for resetting to be beneficial for any conditional exit. For completeness, we rederive this criterion in the following. In particular, we consider an underlying first-passage process, introduce an infinitesimal resetting rate $\delta r \rightarrow 0$, and see its effect. By performing linear expansion of the conditional MFPTs, we find

$$\langle T_{\delta r}^\varsigma(u) \rangle = \langle T_0^\varsigma(u) \rangle - \delta r \langle T_0^\varsigma(u) \rangle^2 [(\text{CV}_0^\varsigma(u))^2 - (\Lambda_0^\varsigma(u))^2], \quad (25)$$

where CV_0^ς denotes the statistical dispersion of the underlying conditional FPT density for the respective outcome ς and $\Lambda_0^\varsigma = [\frac{\langle T_0^\varsigma \rangle^2}{2\langle T_0^\varsigma \rangle^2} (1 + \text{CV}_0^2)]^{1/2}$ is the threshold, where CV_0 denotes the statistical dispersion of the underlying FPT irrespective of boundaries. Imposing $\langle T_{\delta r}^\varsigma(u) \rangle < \langle T_0^\varsigma(u) \rangle$ leads to

$$\text{CV}_0^\varsigma(u) > \Lambda_0^\varsigma(u). \quad (26)$$

Computing the quantities on the left-hand side (LHS) and RHS of Eq. (26) for Pareto with $\beta > 2$ leads to the following inequalities:

$$\mathcal{H}^-(u, \beta) > 0, \quad (27)$$

$$\mathcal{H}^+(1-u, \beta) > 0, \quad (28)$$

with $\mathcal{H}(u, \beta) = (1 - 24u + 46u^2 - 19u^3)(\beta - 2) + 30(1 + u)(\sigma/l)^2$; one can find detailed derivation of Eqs. (27) and (28) in Appendix E. As described in Fig. 3(a), the inequality in Eq. (27) marks the phase domains of efficient versus inefficient resetting. For example, at $u = 0.8$ a global minimum in $\langle T_r^- \rangle$ emerges at finite r , whereas at $u = 0.3$ resetting fails to accelerate the process. A similar behavior can be observed for the escape through the right boundary governed by Eq. (28)—this is illustrated in Appendix G.

V. A SUFFICIENT CRITERION FOR REGULATING FPT FLUCTUATIONS

Much like the mean first-passage time, resetting can also influence the spread of the conditional FPT distribution. A natural question is the following: When does introducing resetting make the completion times more predictable by suppressing their fluctuations, rather than just reducing the mean? To answer this, we introduce a small resetting rate δr and examine the initial response of the conditional FPT variance as resetting is switched on, i.e., its slope in the limit $\delta r \rightarrow 0$. We perform a linear expansion of $(\Delta T_r^\varsigma)^2$, given by Eq. (9), to obtain

$$(\Delta T_{\delta r \rightarrow 0}^\varsigma)^2 = (\Delta T_0^\varsigma)^2 - (\mu_2^\varsigma)^{3/2} \tilde{\gamma}_1 \kappa^\varsigma \delta r + \mathcal{O}((\delta r)^2), \quad (29)$$

where $\kappa^\varsigma = \gamma_1^\varsigma / \tilde{\gamma}_1 - \frac{1}{3}(\mu_2/\mu_2^\varsigma)^{3/2}$, where μ_2^ς and μ_3^ς are the second and third central moments of the underlying FPT density $T_0^\varsigma(s)$, respectively, and γ_1^ς and $\tilde{\gamma}_1$ are the central skewness and standardized third raw moment of $\tilde{T}_0^\varsigma(s)$ and $\tilde{T}_0(s)$, respectively.

If resetting is to suppress fluctuations in the process, one must require that

$$(\Delta T_{\delta r \rightarrow 0}^\varsigma)^2 < (\Delta T_0^\varsigma)^2, \quad (30)$$

which, upon expanding to linear order, leads to the condition [see Eq. (29)]

$$\kappa^\varsigma > 0. \quad (31)$$

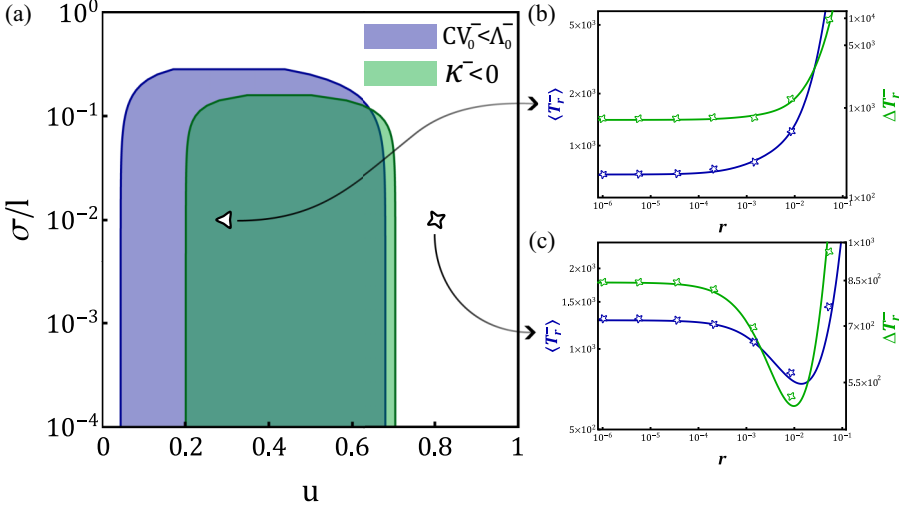


FIG. 3. Illustration of the system-parameter domains obtained from the criteria defined in Eqs. (27) and (31) for realizing the outcome “ $\zeta = -$ ”, with a Pareto WTD of exponent $\beta = 3.5$. Panel (a) shows the regions where $CV_0^- < \Lambda_0^-$ (shaded in violet) and $\kappa^- < 0$ (shaded in green). Panel (b) demonstrates that resetting has a negligible effect on both the conditional MFPT and the exit-time fluctuations through the left boundary for the initial condition $u = 0.3$ and domain length $l = 1$. In contrast, for the initial condition $u = 0.8$, resetting significantly modifies the dynamics and becomes effective in optimizing these metrics, as illustrated in panel (c). Simulation results, indicated by markers in panels (b) and (c), show an excellent agreement with the theoretical results.

A detailed derivation of this criterion is provided in Appendix F. Importantly, Eq. (31), analogous to the conditional CV^ζ criterion, enables us to identify parameter regimes where intermittent resetting becomes effective in controlling fluctuations.

For the WTDs with $0 < \beta < 3$, we find that stochastic resetting is always effective in optimizing fluctuations in this domain, indicating $\partial_r(\Delta T_r^\zeta)^2|_{r \rightarrow 0} < 0$ (see Appendix H for the complete analysis). However, to obtain the efficiency of resetting beyond $\beta = 3$, we use Eq. (31)—This yields partitioning of the phase space according to the initial condition u , as illustrated in Fig. 3.

For example, choosing $u = 0.3$ [triangular marker inside the shaded region of Fig. 3(a)], where neither criterion (27) nor criterion (31) (with $\zeta = -$) is satisfied, resetting becomes ineffective in controlling fluctuations associated with exits through the left boundary. In this regime, both the conditional MFPT and the corresponding fluctuations increase monotonically with the resetting rate, indicating that intermittent restarts hinder rather than improve the process, as indicated in Fig. 3(b). In contrast, for $u = 0.8$ (outside the shaded region), where both criteria are satisfied, resetting efficiently reduces fluctuations while simultaneously optimizing the conditional MFPT, as shown in Fig. 3(c).

Thus, similar to the conditional CV_0^ζ criterion, the κ^ζ criterion also exhibits a universal character and provides a systematic framework to regulate fluctuations based on the chosen performance measure. More importantly, it serves as a diagnostic tool for identifying parameter domains where resetting suppresses variability and enhances efficiency, thereby offering a practical route to control competitive first-passage processes, particularly in systems with memory and multiple outcomes.

VI. CONCLUSION

In summary, we have explored how stochastic resetting influences rare-event kinetics in non-Markovian systems characterized by memory, disorder, and anomalous transport. Within the CTRW

framework, we have shown that resetting can serve as an effective control mechanism for systems with multiple competing first-passage outcomes, selectively enhancing the likelihood of desired events while suppressing the impact of long trapping events. Our analysis of conditional MFPTs reveals that the benefits of resetting are strongly dependent on the underlying WTDs, highlighting a nontrivial interplay between memory effects and external control.

Another central result of our work is the identification of an inequality governing fluctuations in conditional FPTs, demonstrating that resetting can significantly reduce variability in regimes dominated by heavy-tailed dynamics. This provides a quantitative framework for understanding how resetting mitigates extreme events and stabilizes system performance. More broadly, our findings extend the applicability of resetting-based optimization strategies beyond Markovian settings, offering new insights into transport, reaction kinetics, and search processes in complex environments.

We finally comment on the possible experimental realization of our results in non-Markovian systems. CTRWs have long provided a successful framework for describing transport in disordered and heterogeneous media, beginning with the seminal work of Scher and Montroll on anomalous transit-time dispersion of photoinjected carriers in amorphous semiconductors [76], where the underlying time-of-flight geometry is itself a first-passage problem in a finite interval. Related CTRW-like dynamics have subsequently been observed in a variety of systems, including solute transport through geological formations [85], intermittent fluorescence of single nanocrystals [86], membrane-protein motion [87], glassy trap models [4], protein folding [88], intracellular transport [89–91], and even in blinking quantum dots [92]. Independently, stochastic resetting has become an experimentally controllable protocol, with resetting of colloidal particles demonstrated using optical tweezers [34,55,56,60,61] and resetting protocols implemented in light-controlled robotic systems [59]. Recent experiments have further begun to explore resetting in many-particle systems [58,93]. These developments suggest a natural experimental route for testing our predictions: A tracer particle exhibiting slow dynamics could be subjected to controlled resetting, with competing absorbing targets introduced to realize multiple first-passage outcomes. The conditional first-passage times, splitting probabilities, and their fluctuations could then be measured as functions of the resetting rate [34,55,56] and the underlying waiting-time statistics. We believe that such experiments would provide a direct test of our predicted criteria for optimizing desired outcomes and suppressing fluctuations in non-Markovian first-passage processes.

The framework developed here opens several avenues for future research, including extensions to interacting systems [47], spatial resetting protocols [25,33,46], non-Markovian resetting [48–51], and newly introduced threshold resetting [38,54]. Given the ubiquity of non-Markovian dynamics in natural and engineered systems, we anticipate that these results will have broad implications for controlling stochastic processes in physics, biology, and beyond.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: DETAILED DERIVATION OF EQS. (3)–(8)

Let us consider a first-passage process, starting initially from u , and being further subjected to resetting with a time-dependent rate $r(t)$, which compels the process to restart from the initial configuration. The survival probability $Q_r(u, t)$ for the resetting-mediated dynamics follows a renewal structure given by [83]

$$Q_r(u, t) = e^{-R(t)} Q_0(u, t) + \int_0^t d\tau_f r(\tau_f) e^{-R(\tau_f)} Q_0(u, \tau_f) Q_r(u, t - \tau_f), \quad (\text{A1})$$

where recall that $Q_0(u, t)$ is the survival probability of the resetting-free dynamics. Since resetting rate $r(t)$ can be generically time dependent, the quantity $e^{-R(t)}$ measures the probability of no

resetting event until time t . Subsequently, the probability of reinitiating a trajectory between τ and $\tau_f + d\tau_f$ is $r(\tau_f)e^{-R(\tau_f)}$ for the first time. To quantify the overall behavior of survival probability in the presence of restarts, one has to integrate over time. Since the above relation belongs to the class of Wiener-Hopf integrals, this can be solved in the Laplace domain. By considering the following set of functions:

$$\mathcal{A}_{Q_0}(u, t) = e^{-R(t)}Q_0(u, t), \quad \text{and} \quad \mathcal{K}_{Q_0}(u, t) = r(t)e^{-R(t)}Q_0(u, t),$$

with $R(t) = \int_0^t r(z) dz$, one can rewrite the renewal equation as follows:

$$Q_r(u, t) = \mathcal{A}_{Q_0}(u, t) + (\mathcal{K}_{Q_0} * Q_r)(u, t), \quad (\text{A2})$$

where $(f * g)(t) = \int_0^t f(\tau_f)g(t - \tau_f)d\tau_f$ is simply the convolution between two functions. Performing Laplace transformation on both sides of the above equation, we arrive at

$$\tilde{Q}_r(u, s) = \frac{\tilde{\mathcal{A}}_{Q_0}(u, s)}{1 - \tilde{\mathcal{K}}_{Q_0}(u, s)} = \frac{\int_0^\infty e^{-st} e^{-R(t)} Q_0(u, t) dt}{1 - \int_0^\infty r(t) e^{-st} e^{-R(t)} Q_0(u, t) dt}. \quad (\text{A3})$$

This equation can be simplified further when the rate is time independent, i.e., $r(t) = r$, so that [24]

$$\tilde{Q}_r(u, s) = \frac{\tilde{Q}_0(u, r + s)}{1 - r\tilde{Q}_0(u, r + s)}. \quad (\text{A4})$$

Using the relation between the FPT density and the survival probability, namely, $f_{T_r}(u, t) = -\partial_t Q_r(u, t)$, the above relation can be further simplified to

$$\tilde{T}_r(u, s) = \frac{(r + s)\tilde{T}_0(u, r + s)}{s + r\tilde{T}_0(u, r + s)}, \quad (\text{A5})$$

where $\tilde{T}_{r(0)}(u, s)$ denotes the FPT density in the Laplace space with and without (indicated by “0”) resetting. The first and second moments can easily be extracted with the following forms:

$$\langle T_r(u) \rangle = -\partial_s \tilde{T}_r(u, s)|_{s \rightarrow 0} = \frac{1 - \tilde{T}_0(u, r)}{r\tilde{T}_0(u, r)}, \quad (\text{A6})$$

$$\langle T_r^2(u) \rangle = \partial_s^2 \tilde{T}_r(u, s)|_{s \rightarrow 0} = \frac{2}{r^2} \frac{1 - \tilde{T}_0(u, r) + r\partial_r \tilde{T}_0(u, r)}{\tilde{T}_0^2(u, r)}. \quad (\text{A7})$$

So far, we have developed the framework for the unconditional first-passage time under resetting, where the completion time is averaged over all possible trajectories irrespective of the eventual outcome. We now extend this discussion to competitive first-passage processes, where multiple exits or outcomes coexist, and revisit the formalism developed in Ref. [69] to analyze the impact of resetting on conditional first-passage statistics.

Extending the formalism for conditional first-passage times

Unlike the unconditional first-passage time, which integrates over all outcomes and therefore cannot distinguish between successful and unsuccessful events, the corresponding survival probability $Q_r(u, t)$ only captures the probability that the process has not terminated up to time t , without retaining information about the specific exit through which completion eventually occurs. Consequently, it cannot resolve the statistics associated with individual outcomes. To characterize the dynamics conditioned on a particular exit, one must instead monitor the probability flux through each absorbing boundary separately. Such currents encode the rate at which trajectories terminate through a given outcome and therefore provide direct access to the corresponding conditional first-passage times. Following Ref. [39], the renewal equation for the probability current associated

with each exit can be written as

$$J_r^\zeta(u, t) = e^{-R(t)} J_0^\zeta(u, t) + \int_0^t d\tau_f r(\tau_f) e^{-R(\tau_f)} J_r^\zeta(u, t - \tau_f) Q_0(u, \tau_f), \quad (\text{A8})$$

which can be further written by considering the following time-dependent functions:

$$\mathcal{A}^\zeta(u, t) = e^{-R(t)} J_0^\zeta(u, t) \quad \text{and} \quad \mathcal{K}_{Q_0}(u, t) = r(t) e^{-R(t)} Q_0(u, t).$$

Incorporating these, Eq. (A8) can be rewritten as

$$J_r^\zeta(u, t) = \mathcal{A}^\zeta(u, t) + (\mathcal{K}_{Q_0} * J_r^\zeta)(u, t). \quad (\text{A9})$$

Performing the Laplace transformation, we get

$$\tilde{J}_r^\zeta(u, s) = \frac{\tilde{\mathcal{A}}^\zeta(u, s)}{1 - \tilde{\mathcal{K}}_{Q_0}(u, s)} = \frac{\int_0^\infty e^{-st} e^{-R(t)} J_0^\zeta(u, t) dt}{1 - \int_0^\infty r(t) e^{-st} e^{-R(t)} Q_0(u, t) dt}. \quad (\text{A10})$$

For the time-independent resetting rate $r(t) = r$, the above equation reads

$$\tilde{J}_r^\zeta(u, s) = \frac{\tilde{J}_0^\zeta(u, r + s)}{1 - r \tilde{Q}_0(u, r + s)}. \quad (\text{A11})$$

The splitting probabilities through the specific outcome can be defined as

$$\epsilon_r^\zeta(u) = \int_0^\infty dt J^\zeta(u, t) = \tilde{J}_r^\zeta(u, s \rightarrow 0) = \frac{\tilde{J}_0^\zeta(u, r)}{1 - r \tilde{Q}_0(u, r)} = \frac{\tilde{J}_0^\zeta(u, r)}{\tilde{T}_0(u, r)}. \quad (\text{A12})$$

Substituting $\tilde{J}_0^\zeta(u, r) = \epsilon_0^\zeta(u) \tilde{T}_0^\zeta(u, r)$ for the resetting-free process, the above relation can be recast as

$$\epsilon_r^\zeta(u) = \epsilon_0^\zeta(u) \frac{\tilde{T}_0^\zeta(u, r)}{\tilde{T}_0(u, r)}, \quad (\text{A13})$$

which essentially connects the splitting probability of the resetting process with that of the resetting-free process. Finally, using Eq. (A13) in Eq. (A11), the resetting-mediated conditional FPT densities yield

$$\tilde{T}_r^\zeta(u, s) = \frac{\tilde{T}_0^\zeta(r + s)}{\tilde{T}_0^\zeta(r)} \frac{(r + s) \tilde{T}_0(r)}{s + r \tilde{T}_0(r + s)}, \quad (\text{A14})$$

which was announced in Eq. (5). Accordingly, the first moment of the MFPT reads

$$\langle T_r^\zeta(u) \rangle = \langle T_r(u) \rangle + \frac{\partial}{\partial r} \ln \left[\frac{\tilde{T}_0(u, r)}{\tilde{T}_0^\zeta(u, r)} \right]. \quad (\text{A15})$$

Similarly, the second moment of the conditional FPT in the presence of resetting can be found in terms of the underlying FPT densities:

$$\langle (T_r^\zeta(u))^2 \rangle = \langle T_r(u)^2 \rangle \left[\tilde{T}_0(u, r) + \frac{\langle T_r^\zeta(u) \rangle}{\langle T_r(u) \rangle} (1 - \tilde{T}_0(u, r)) \right] - \left[\frac{\partial^2 \tilde{T}_0(u, r)}{\partial r^2} - \frac{\partial^2 \tilde{T}_0^\zeta(u, r)}{\partial r^2} \right], \quad (\text{A16})$$

where $\langle T_r^2(u) \rangle = \frac{2}{r^2} \frac{1 - \tilde{T}_0(u, r) + r \frac{\partial \tilde{T}_0(u, r)}{\partial r}}{\tilde{T}_0^2(u, r)}$ is the second moment of the resetting-mediated unconditional FPT density [28]. This completes the derivation of Eqs. (3)–(8).

APPENDIX B: DERIVATION OF SPLITTING PROBABILITIES AND THE FPT DENSITIES FOR THE CTRW IN THE ABSENCE OF RESETTING

We consider a CTRW confined within a finite one-dimensional domain of length l , bounded by two absorbing barriers located at $x = 0$ and $x = l$. The absorption event in these two barriers can be considered as two mutually exclusive outcomes of a first-passage process, indicated with $\varsigma = \{-, +\}$, i.e., exit pathways through the left and right boundaries, respectively. We followed Ref. [65] to derive some of our results below. While Ref. [65] discussed only the unconditional outcome, our aim here is to derive expressions for the conditional observables.

Starting from the initial position x_0 , the walker remains stationary for a random time before making its first jump. After each displacement, it waits for another random time interval before performing the next jump, and this process continues until the walker reaches one of the absorbing boundaries. The jump lengths and waiting times are drawn independently from the respective PDFs $\Psi(x)$ and $\phi(t_w)$. In particular, we are interested in studying the trade-off between the natural and resetting-mediated absorption, followed by the statistical properties of the WTDs. To study the effect of intermittent restarts on the first-passage process, first, we need to find the conditional FPT densities without resetting, and furthermore, using those, one can estimate the FPT densities with resetting.

The propagator $p_0(x, t|x_0)$ that describes the position of the particle at time t given that initially it was located at x_0 is given by the famous Montroll-Weiss equation [64]. In the Fourier-Laplace space, this reads

$$\check{p}_0(k, s|x_0) = \frac{e^{ikx_0}[1 - \tilde{\phi}(s)]}{s[1 - \tilde{\phi}(s)\hat{\Psi}(k)]}, \quad (\text{B1})$$

where $[\widetilde{\cdot}] = \mathcal{L}[\cdot]$ and $[\hat{\cdot}] = \mathcal{F}[\cdot]$ denote Laplace and Fourier transformation, respectively. Rearranging the above equation, one can write

$$s\check{p}_0(k, s|x_0) - e^{ikx_0} = \tilde{K}(s)[\hat{\Psi}(k) - 1]\check{p}_0(k, s|x_0); \quad (\text{B2})$$

in Laplace space, the memory kernel is related to the WTD in the following manner:

$$\tilde{K}(s) = \frac{s\tilde{\phi}(s)}{1 - \tilde{\phi}(s)}. \quad (\text{B3})$$

Consider the characteristic jump length σ to be small compared to the length of the domain length l . This allows the jump statistics to be in the Gaussian limit, i.e., $\hat{\Psi}(k) \simeq 1 - \sigma^2 k^2/2$. Finally, inverting Eq. (B2) in Fourier-Laplace space, we obtain the master equation for nonstandard diffusion [65]:

$$\frac{\partial p_0(x, t|x_0)}{\partial t} = \frac{\sigma^2}{2} \int_0^t K(t-t') \frac{\partial^2 p_0(x, t'|x_0)}{\partial x^2} dt', \quad (\text{B4})$$

with $K(t) = \mathcal{L}^{-1}[\tilde{K}(s)]$; notably $\mathcal{L}^{-1}[\cdot]$ indicates inverse Laplace transformation. Incorporating the initial condition $p_0(x, t=0|x_0) = \delta(x-x_0)$, in the Laplace domain Eq. (B4) reads

$$\frac{d^2 \tilde{p}_0(x, s)}{dx^2} - \frac{2s}{\sigma^2 \tilde{K}(s)} \tilde{p}_0(x, s) = -\frac{2\delta(x-x_0)}{\sigma^2 \tilde{K}(s)}. \quad (\text{B5})$$

Utilizing the boundary conditions $p_0(x=0, t|x_0) = p_0(x=l, t|x_0) = 0$, one can find the following solution of the probability density function, which reads

$$\tilde{p}_0(x, s) = \frac{1}{\sigma} \sqrt{\frac{2}{s\tilde{K}(s)}} \frac{1}{\sinh[\alpha(s)l]} \times \begin{cases} \sinh[\alpha(s)(l-x_0)] \sinh[\alpha(s)x], & 0 \leq x \leq x_0, \\ \sinh[\alpha(s)(l-x)] \sinh[\alpha(s)x_0], & x_0 \leq x \leq l, \end{cases} \quad (\text{B6})$$

where $\alpha(s) = \frac{1}{\sigma} \sqrt{\frac{2s}{\tilde{K}(s)}} = \frac{\sqrt{2}}{\sigma} \sqrt{\frac{1}{\tilde{\phi}(s)} - 1}$. From Eq. (B4), probability density currents through the right and left boundaries can be written in the following way:

$$J_0^-(x_0, t) = \frac{\sigma^2}{2} \int_0^\infty K(t-t') \frac{\partial p_0(x, t')}{\partial x} \Big|_{x \rightarrow 0}, \quad (\text{B7})$$

$$J_0^+(x_0, t) = -\frac{\sigma^2}{2} \int_0^\infty K(t-t') \frac{\partial p_0(x, t')}{\partial x} \Big|_{x \rightarrow l}. \quad (\text{B8})$$

Eventually, in the Laplace domain, the above quantities can be written in the following way:

$$\tilde{J}_0^\pm(x_0, s) = \pm \tilde{K}(s) \frac{\sigma^2}{2} \frac{\partial \tilde{p}_0(x, s)}{\partial x} \Big|_{x \rightarrow 0(l)}; \quad (\text{B9})$$

using Eq. (B6), the probability currents in the Laplace domain read

$$\tilde{J}_0^-(u, s) = +\tilde{K}(s) \frac{\sigma^2}{2} \frac{\partial \tilde{p}_0(x, s)}{\partial x} \Big|_{x \rightarrow 0} = \frac{\sinh[(1-u)l\alpha(s)]}{\sinh[l\alpha(s)]}, \quad (\text{B10})$$

$$\tilde{J}_0^+(u, s) = -\tilde{K}(s) \frac{\sigma^2}{2} \frac{\partial \tilde{p}_0(x, s)}{\partial x} \Big|_{x \rightarrow l} = \frac{\sinh[ul\alpha(s)]}{\sinh[l\alpha(s)]}. \quad (\text{B11})$$

Here, we assigned a new variable u , which is the scaled initial condition with length of the domain, i.e., $u = x_0/l$. In the rest of the article, we will follow this notation. In terms of the probability currents, the splitting probabilities can be written in the following way:

$$\epsilon_0^-(u) = \tilde{J}_0^-(u, s \rightarrow 0) = \frac{\sinh[(1-u)l\alpha(s)]}{\sinh[l\alpha(s)]} \Big|_{s \rightarrow 0}, \quad (\text{B12})$$

$$\epsilon_0^+(u) = \tilde{J}_0^+(u, s \rightarrow 0) = \frac{\sinh[ul\alpha(s)]}{\sinh[l\alpha(s)]} \Big|_{s \rightarrow 0}. \quad (\text{B13})$$

For instance, consider $\tilde{\phi}(s \rightarrow 0) \approx 1 - a_\beta s^\beta$ (representing class I WTD), where all nonzero moments are diverging. Consequently, $\alpha(s) \approx \frac{\sqrt{2}}{\sigma} \sqrt{a_\beta s^{\beta/2}} + \mathcal{O}(s^\beta)$ as $s \rightarrow 0$. From Eq. (B12), one can expand $\frac{\sinh[(1-u)l\alpha(s)]}{\sinh[l\alpha(s)]} \approx (1-u) + \mathcal{O}(s^\beta)$. Eventually, we find

$$\epsilon_0^-(u) = 1 - u, \quad \epsilon_0^+(u) = u.$$

Remarkably, this result holds irrespective of the underlying WTDs, revealing a universal structure for the splitting probabilities with multiple outcomes. It should be noted that the introduction of resetting breaks this universality and the behavior becomes system dependent. See Fig. 4 for details.

We now proceed to derive the expressions for the conditional FPT densities in the Laplace domain using the expressions for the conditional currents and the splitting probabilities:

$$\tilde{T}_0^-(u, s) = \frac{\tilde{J}_0^-(u, s)}{\epsilon_0^-(u)} = \frac{1}{1-u} \frac{\sinh[(1-u)l\alpha(s)]}{\sinh[l\alpha(s)]}, \quad (\text{B14})$$

$$\tilde{T}_0^+(u, s) = \frac{\tilde{J}_0^+(u, s)}{\epsilon_0^+(u)} = \frac{1}{u} \frac{\sinh[ul\alpha(s)]}{\sinh[l\alpha(s)]}. \quad (\text{B15})$$

In passing, we mention that the unconditional FPT distribution can be written in terms of the weighted sum of the conditional first-passage distributions; thus, we recover the following expression reported in Ref. [65]:

$$\tilde{T}_0(u, s) = \epsilon_0^-(u) \tilde{T}_0^-(u, s) + \epsilon_0^+(u) \tilde{T}_0^+(u, s) = \frac{\sinh[(1-u)l\alpha(s)] + \sinh[ul\alpha(s)]}{\sinh[l\alpha(s)]}. \quad (\text{B16})$$

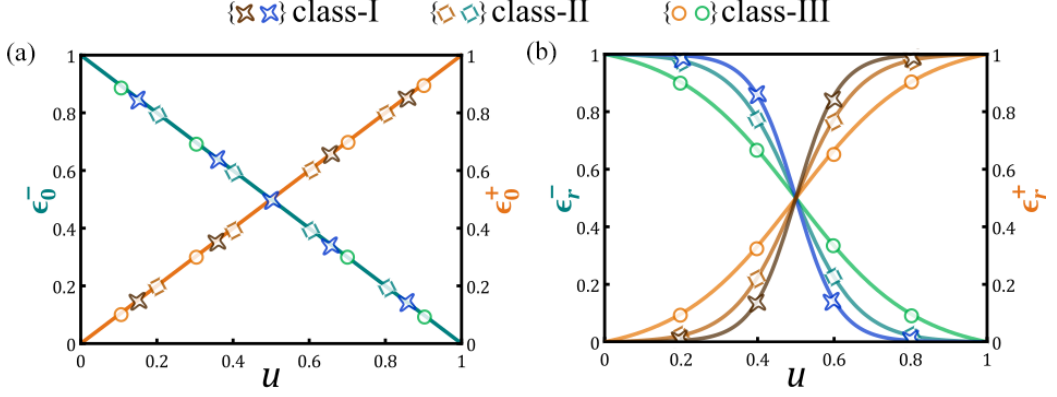


FIG. 4. Considering three distinct WTDs, i.e., class I, class II, and class III, we illustrate the behavior of the splitting probabilities without and with resetting. Panel (a) represents the escape probabilities, i.e., $\epsilon_0^- = 1 - u$ and $\epsilon_0^+ = u$, which are independent of the nature of the underlying WTDs. However, resetting can reshape these probabilities as shown in panel (b), also breaking the universality. Markers indicate simulation results in both the panels.

Effects of the WTD on the MSD

The nature of the WTD also decides the behavior of the particle in long time. For unbounded CTRW on a one-dimensional line, let us again consider the propagator from Eq. (B1). Setting $x_0 = 0$, the propagator reads

$$\check{p}_0(k, s|x_0) = \frac{[1 - \tilde{\phi}(s)]}{s[1 - \tilde{\phi}(s)\hat{\Psi}(k)]} \quad (\text{B17})$$

in the Fourier-Laplace space. We now consider the jump distribution $\Psi(\Delta x)$ to be Gaussian with mean *zero* and variance σ^2 . Under this assumption, the propagator takes the following form:

$$\langle x^2(s) \rangle = \frac{\tilde{\phi}(s)}{s[1 - \tilde{\phi}(s)]} \sigma^2 \quad (\text{B18})$$

in the Laplace space. Now, by considering different waiting-time distributions, one gets the following behavior of the MSD: subdiffusive behavior for $0 < \beta < 1$ and linear diffusive behavior for $\beta > 1$; the details are provided in Table I.

TABLE I. Behavior of the MSD with time for different WTDs.

$\phi(t_w)$	$\langle x^2(t) \rangle$	$\langle x^2(t) \rangle$ (large t behavior)
Class I: $0 < \beta < 1$	$\frac{\sigma^2}{\tau^\beta} \frac{t^\beta}{\Gamma(1+\beta)}$	$\sim t^\beta$: Subdiffusive
Class II: $1 < \beta < 2$	$\frac{\sigma^2}{\langle T \rangle_\phi} t - \frac{\sigma^2 \Gamma(1-\beta) \tau^\beta}{\langle T \rangle_\phi^2 \Gamma(3-\beta)} t^{2-\beta}$	$\sim t$: Diffusive
Class III: $2 < \beta < 3$	$\frac{\sigma^2}{\langle T \rangle_\phi} t + \sigma^2 \left(\frac{\langle T^2 \rangle_\phi}{2 \langle T \rangle_\phi^2} - 1 \right) - \frac{\sigma^2 \Gamma(1-\beta) \tau^\beta}{\langle T \rangle_\phi^2 \Gamma(3-\beta)} t^{2-\beta}$	$\sim t$: Diffusive

APPENDIX C: RESETTING-INDUCED FPT DENSITIES AND RESPECTIVE MFPTs

In this Appendix, we provide the functional form of the (un)conditional FPT densities with their first moments in the presence of resetting. Using Eq. (B16) in Eq. (A6), the unconditional FPT reads

$$\tilde{T}_r(u, s) = \frac{r + s}{r + \frac{s \sinh[l\alpha(r+s)]}{\sinh[(1-u)l\alpha(r+s)] + \sinh[ul\alpha(r+s)]}}. \quad (C1)$$

Furthermore, recalling Eq. (A6), the resetting-mediated MFPT irrespective of outcome reads

$$\langle T_r(u) \rangle = \frac{1}{r} \left[\frac{\sinh[l\alpha(r)]}{\sinh[(1-u)l\alpha(r)] + \sinh[ul\alpha(r)]} - 1 \right]. \quad (C2)$$

Similarly, one can find the second moment in terms of the underlying unconditional FPT density, which reads

$$\langle (T_r(u))^2 \rangle = \frac{2}{r^2} \frac{1 - \tilde{T}_0(u, r) + r \frac{\partial \tilde{T}_0(u, r)}{\partial r}}{\tilde{T}_0^2(u, r)}. \quad (C3)$$

Similarly, utilizing Eqs. (B14)–(B16) in Eq. (A14), we find

$$\begin{aligned} \tilde{T}_r^-(u, s) &= \frac{\sinh[(1-u)l\alpha(r+s)] \sinh[l\alpha(r)]}{\sinh[(1-u)l\alpha(r)] \sinh[l\alpha(r+s)]} \times \frac{\sinh[l\alpha(r+s)]}{\sinh[l\alpha(r)]} \\ &\times \frac{(s+r)[\sinh[ul\alpha(r)] + \sinh[(1-u)l\alpha(r)]]}{s \sinh[l\alpha(r+s)] + r[\sinh[ul\alpha(r+s)] + \sinh[(1-u)l\alpha(r+s)]]}, \end{aligned} \quad (C4)$$

$$\begin{aligned} \tilde{T}_r^+(u, s) &= \frac{\sinh[ul\alpha(r+s)] \sinh[l\alpha(r)]}{\sinh[ul\alpha(r)] \sinh[l\alpha(r+s)]} \times \frac{\sinh[l\alpha(r+s)]}{\sinh[l\alpha(r)]} \\ &\times \frac{(s+r)[\sinh[ul\alpha(r)] + \sinh[(1-u)l\alpha(r)]]}{s \sinh[l\alpha(r+s)] + r[\sinh[ul\alpha(r+s)] + \sinh[(1-u)l\alpha(r+s)]]}. \end{aligned} \quad (C5)$$

Readily, from Eq. (A15), we get the conditional MFPTs

$$\langle T_r^-(u) \rangle = \langle T_r(u) \rangle + \alpha'[r] \times \frac{\mathcal{N}(1-u)}{\mathcal{D}(1-u)}, \quad (C6)$$

$$\langle T_r^+(u) \rangle = \langle T_r(u) \rangle + \alpha'[r] \times \frac{\mathcal{N}(u)}{\mathcal{D}(u)}, \quad (C7)$$

where

$$\begin{aligned} \mathcal{N}(u) &= (1-u)l \cosh[(1-u)l\alpha(r)] - ul \coth[ul\alpha(r)] \sinh[(1-u)l\alpha(r)], \\ \mathcal{D}(u) &= \sinh[(1-u)l\alpha(r)] + \sinh[ul\alpha(r)]. \end{aligned}$$

APPENDIX D: SUPPORTING DERIVATION OF EQS. (23) AND (24)

In this Appendix, we describe the behavior of the MFPT related to a particular outcome for WTD with a finite first moment but a diverging second moment. Recalling Eq. (12) in the small resetting rate limit gives

$$\langle T_r^- \rangle \simeq \underbrace{\frac{u(1-u)}{2r} l^2 \alpha(r)^2}_{F_1} + \underbrace{\frac{u(2u-1)}{3} l^2 \alpha(r) \partial_r \alpha(r)}_{F_2}. \quad (D1)$$

Moreover, Eq. (12) reads $\alpha(r) \sim \sqrt{r} A_\beta [1 + B_\beta r^{\beta-1}]$, with necessary constants, described in the main text. From this relation, we can write $\alpha(r)^2 \sim r A_\beta^2 [1 + 2B_\beta r^{\beta-1}]$ and

$$\partial_r \alpha(r) \sim \frac{1}{2\sqrt{r}} A_\beta [1 + B_\beta r^{\beta-1}] + \sqrt{r} A_\beta [(\beta-1) B_\beta r^{\beta-2}].$$

The first term on the RHS, i.e., F_1 of Eq. (D1), can be further simplified to

$$F_1 \sim \frac{u(1-u)}{2} l^2 A_\beta^2 [1 + 2B_\beta r^{\beta-1}].$$

The second term can be written as

$$\begin{aligned} F_2 &\sim \frac{u(2u-1)}{3} l^2 \sqrt{r} A_\beta^2 [1 + B_\beta r^{\beta-1}] \left[\frac{1}{2\sqrt{r}} [1 + B_\beta r^{\beta-1}] + \sqrt{r} B_\beta (\beta-1) r^{\beta-2} \right] \\ &\sim \frac{u(2u-1)}{6} l^2 A_\beta^2 [1 + 2B_\beta r^{\beta-1}]. \end{aligned}$$

Finally, Eq. (D1) can be written as

$$\langle T_r^- \rangle \sim \frac{u(2-u)}{6} l^2 A_\beta^2 B_\beta r^{\beta-1}. \quad (\text{D2})$$

Finally, recalling $A_\beta = \frac{\sqrt{2\tau}}{\sigma\sqrt{(\beta-1)}}$ and $B_\beta = \frac{\pi(\beta-1)\tau^{\beta-1}}{2\sin(\pi\beta)\Gamma(1+\beta)}$, we get

$$\partial_r \langle T_r^- \rangle \sim \frac{(\beta-1)\tau^\beta}{r^{2-\beta}} \frac{u(2-u)l^2}{6\sigma^2} \frac{\pi}{\sin(\pi\beta)\Gamma(1+\beta)}. \quad (\text{D3})$$

Similarly, one can examine the slope of the conditional MFPT at $r \rightarrow 0$, which reads

$$\partial_r \langle T_r^+ \rangle \sim \frac{(\beta-1)\tau^\beta}{r^{2-\beta}} \frac{(1-u^2)l^2}{6\sigma^2} \frac{\pi}{\sin(\pi\beta)\Gamma(1+\beta)}. \quad (\text{D4})$$

This completes the derivation of Eqs. (23) and (24).

APPENDIX E: EXACT EXPRESSIONS OF $\mathcal{H}^\mp(u, \beta)$ FROM EQS. (27) AND (28)

Starting from the moment-generating function $\tilde{\phi}(s)$, for a given WTD $\phi(t_w)$ can be expanded in the following way:

$$\tilde{\phi}(s)|_{s \rightarrow 0} \approx 1 - s\langle \mathcal{T} \rangle_\phi + \frac{1}{2}s^2\langle \mathcal{T}^2 \rangle_\phi + \mathcal{O}(s^3), \quad (\text{E1})$$

where $\langle \mathcal{T} \rangle_\phi$ and $\langle \mathcal{T}^2 \rangle_\phi$ are the first and second moments of the WTD. Recalling the definition of $\text{CV}_\phi = \sqrt{\frac{\langle \mathcal{T}^2 \rangle_\phi - \langle \mathcal{T} \rangle_\phi^2}{\langle \mathcal{T} \rangle_\phi^2}}$, the above relation can be rewritten as

$$\tilde{\phi}(s)|_{s \rightarrow 0} \approx 1 - s\langle \mathcal{T} \rangle_\phi + \frac{1}{2}s^2\langle \mathcal{T} \rangle_\phi^2 (1 + \text{CV}_\phi^2) + \mathcal{O}(s^3). \quad (\text{E2})$$

Furthermore, using the above result, $\alpha(s)$ reads

$$\alpha^2(s) \simeq \frac{2\langle \mathcal{T} \rangle_\phi}{\sigma^2} s + \frac{\langle \mathcal{T} \rangle_\phi^2 (1 - \text{CV}_\phi^2)}{\sigma^2} s^2 + \mathcal{O}(s^3). \quad (\text{E3})$$

Notably, one can find the first and second moments of exit times through left boundary for the underlying process. From Eq. (B14), one can derive

$$\langle T_0^-(u) \rangle = \frac{(2-u)u}{3(\sigma/l)^2} \langle \mathcal{T} \rangle_\phi, \quad \langle (T_0^-(u))^2 \rangle = \frac{u(2-u)}{45(\sigma/l)^4} \langle \mathcal{T} \rangle_\phi^2 [4 + 3u(2-u) + 15(\text{CV}_\phi^2 - 1)(\sigma/l)^2]. \quad (\text{E4})$$

Similarly, from Eq. (B15), first and second moments of exit times through the right boundary read

$$\langle T_0^+(u) \rangle = \frac{1-u^2}{3(\sigma/l)^2} \langle \mathcal{T} \rangle_\phi, \quad \langle (T_0^+(u))^2 \rangle = \frac{1-u^2}{45(\sigma/l)^4} \langle \mathcal{T} \rangle_\phi^2 [(7-3u^2) + 15(\text{CV}_\phi^2 - 1)(\sigma/l)^2]. \quad (\text{E5})$$

Also from Eq. (B16), first and second moments of the unconditional FPT read

$$\langle T_0(u) \rangle = \frac{u(1-u)}{(\sigma/l)^2} \langle \mathcal{T} \rangle_\phi, \quad \langle (T_0(u))^2 \rangle = \frac{u(1-u)}{3(\sigma/l)^4} \langle \mathcal{T} \rangle_\phi^2 [(1+u(1-u)) + 3(\text{CV}_\phi^2 - 1)(\sigma/l)^2]. \quad (\text{E6})$$

Consider the Pareto WTD

$$\phi(t_w) = \frac{\beta}{\tau(1 + \frac{t_w}{\tau})^{\beta+1}}, \quad (\text{E7})$$

which has finite moments up to an integer order n if $\beta > n$. With $\beta > 2$, the first and second moments read

$$\langle \mathcal{T} \rangle_\phi = \frac{\tau}{\beta - 1}, \quad \langle \mathcal{T}^2 \rangle_\phi = \frac{2\tau^2}{(\beta - 1)(\beta - 2)}, \quad (\text{E8})$$

and the coefficient of variation reads

$$\text{CV}_\phi = \sqrt{\frac{\beta}{\beta - 2}}. \quad (\text{E9})$$

From the definitions $\text{CV}_0^\mp(u) = \sqrt{\frac{\langle (T_0^\mp(u))^2 \rangle - \langle T_0^\mp(u) \rangle^2}{\langle T_0^\mp(u) \rangle^2}}$ and $\Lambda_0^\mp(u) = \sqrt{\frac{\langle T_0(u) \rangle^2}{2\langle T_0^\mp(u) \rangle^2} (1 + \text{CV}_0(u)^2)}$ and using Eqs. (E4)–(E6), one gets the following:

$$\text{CV}_0^-(u) = \left[\frac{4 - 4u + 2u^2 + 15(\sigma/l)^2(\text{CV}_\phi^2 - 1)}{5u(2 - u)} \right]^{1/2}, \quad (\text{E10})$$

$$\text{CV}_0^+(u) = \left[\frac{2 + 2u^2 + 15(\sigma/l)^2(\text{CV}_\phi^2 - 1)}{5(1 - u^2)} \right]^{1/2}. \quad (\text{E11})$$

along with

$$\Lambda_0^-(u) = \sqrt{\frac{3}{2}} \left[\frac{(1 - u)(1 + u - u^2 + 3(\sigma/l)^2(\text{CV}_\phi^2 - 1))}{u(2 - u)^2} \right]^{1/2}, \quad (\text{E12})$$

$$\Lambda_0^+(u) = \sqrt{\frac{3}{2}} \left[\frac{u(1 + u - u^2 + 3(\sigma/l)^2(\text{CV}_\phi^2 - 1))}{(1 - u)(1 + u)^2} \right]^{1/2}. \quad (\text{E13})$$

Using Eq. (E9) in Eqs. (E10) and (E12), one can find the domain of initial conditions for given σ in an interval of length l in terms of the following inequality:

$$(1 - 24u + 46u^2 - 19u^3)(\beta - 2) + 30(1 + u)(\sigma/l)^2 > 0, \quad (\text{E14})$$

for the trajectories through the left exit, where $(1 - 24u + 46u^2 - 19u^3)(\beta - 2) + 30(1 + u)(\sigma/l)^2$ is identified as $\mathcal{H}^-(u, \beta)$ in Eq. (27). Similarly from Eqs. (E11) and (E13), we get

$$(4 - 11u - 11u^2 + 19u^3)(\beta - 2) + 30(2 - u)(\sigma/l)^2 > 0, \quad (\text{E15})$$

for the right exit pathways, with $\mathcal{H}^+(1 - u, \beta) = (4 - 11u - 11u^2 + 19u^3)(\beta - 2) + 30(2 - u)(\sigma/l)^2$ [see Eq. (28)].

APPENDIX F: DERIVATION OF THE CRITERION IN EQ. (31)

In this Appendix, we provide the detailed derivation of Eq. (31). Starting with the variance (centralized second moment) of the conditional FPT density, one can write

$$(\Delta T_{\delta r \rightarrow 0}^\zeta)^2 = (\Delta T_0^\zeta)^2 + \delta r \frac{1}{3} (\langle (T_0)^3 \rangle - 6\langle (T_0^\zeta) \rangle^3 + 9\langle T_0^\zeta \rangle \langle (T_0^\zeta)^2 \rangle - 3\langle (T_0^\zeta)^3 \rangle) + \mathcal{O}((\delta r)^2). \quad (\text{F1})$$

Further, recall the definitions of third central moment, i.e., $\mu_3^\varsigma = \langle (T_0^\varsigma - \langle T_0^\varsigma \rangle)^3 \rangle$, and the second central moment $\mu_2^\varsigma = (\Delta T_0^\varsigma)^2 = \langle (T_0^\varsigma - \langle T_0^\varsigma \rangle)^2 \rangle$ of the underlying FPT density $T_0^\varsigma(s)$. Also, denoting $\tilde{\gamma}_1 = \langle T_0^3 \rangle / (\Delta T_0)^3$ as standardized third raw moment and $\gamma_1 = \mu_3 / (\Delta T_0)^3$ as the central skewness of $\tilde{T}_0(s)$, one can write

$$(\Delta T_{\delta r}^\varsigma)^2 = (\Delta T_0^\varsigma)^2 - \delta r (\mu_2^\varsigma)^{3/2} \underbrace{\tilde{\gamma}_1 \left[\frac{\gamma_1^\varsigma}{\tilde{\gamma}_1} - \frac{1}{3} \left(\frac{\Delta T_0}{\Delta T_0^\varsigma} \right)^3 \right]}_{\kappa^\varsigma} + \mathcal{O}((\delta r)^2). \quad (\text{F2})$$

It is easy to see from above that the inequality

$$\kappa^\varsigma > 0 \quad (\text{F3})$$

ensures $\Delta T_{\delta r}^\varsigma < \Delta T_0^\varsigma$; i.e., resetting becomes useful in reducing the fluctuation in competitive FPTs, as described by Eq. (31). Similarly, one can analyze the behavior of the fluctuations for the unconditional process:

$$\langle (T_{\delta r})^2 \rangle - \langle T_{\delta r} \rangle^2 \equiv (\Delta T_{\delta r})^2 = (\Delta T_0)^2 + \delta r \Delta T_0^3 \left[\frac{1}{3} \tilde{\gamma}_1 - \gamma_1 \right] + \mathcal{O}(\delta r^2). \quad (\text{F4})$$

For resetting to suppress the fluctuation, one should have $\Delta T_{\delta r} < \Delta T_0$, which leads to $\gamma_1 > \frac{1}{3} \tilde{\gamma}_1$. This inequality is counterpart for the fluctuations akin to the $\text{CV}_0 > 1$ criterion for the unconditional MFPTs [29].

APPENDIX G: SIMULTANEOUS REGULATION OF MFPT AND FLUCTUATION IN RIGHT EXIT VIA STOCHASTIC RESETTING

In this Appendix, we systematically delineate the parameter regimes in which intermittent restarts become pronounced in regulating the conditional MFPT and fluctuations at the right exit. The inequality in Eq. (28) puts a strong bound on the system parameters $u, \beta, \sigma/l$, in which resetting becomes useful in optimizing the MFPT through the right exit. Conversely, outside this optimal window—specifically in the regime characterized by $\mathcal{H}^+ < 0$ —resetting has a detrimental effect, prolonging the expected exit time, as shown in Fig. 5. Furthermore, we demonstrate that a structurally analogous boundary regime, governed by Eq. (31) evaluated at $\varsigma = +$, closely mirrors the behavior of the MFPT and is also visually corroborated in Fig. 5.

APPENDIX H: EFFECT OF RESETTING IN THE FLUCTUATIONS FOR DIFFERENT WTDs

In this Appendix, we systematically discuss the behavior of the resetting-mediated fluctuation of the conditional FPTs with different classes of WTDs, i.e., $\phi(t_w)$. We start with recalling the definition of $\alpha(r) = \sqrt{2}/\sigma \sqrt{(1/\tilde{\phi}(r)) - 1}$, where $\tilde{\phi}(r)$ represents the Laplace-transformed WTD. With a given $\alpha(r)$, the centralized second moment of the conditional FPT density can be written as

$$(\Delta T_{r \rightarrow 0}^-(u))^2 \simeq \mathcal{G}(u, r), \quad (\text{H1})$$

$$(\Delta T_{r \rightarrow 0}^+(u))^2 \simeq \mathcal{G}(1 - u, r), \quad (\text{H2})$$

where the analytical form of $\mathcal{G}$ reads

$$\begin{aligned} \mathcal{G}(u, r) = & \frac{1}{3} l^2 (1 - 2u) u (\partial_r \alpha(r))^2 + \frac{u l^2}{3r} [r(1 - 2u) \partial_r^2 \alpha(r) - 6(1 - u) \partial_r \alpha(r)] \alpha(r) \\ & + \frac{u(1 - u)}{r^2} l^2 \alpha(r)^2 - \frac{l^4}{30} u(1 - 2u)(7 - u(19 - 14u)) (\partial_r \alpha(r))^2 \alpha(r)^2. \end{aligned}$$

To analyze the efficiency of resetting in controlling fluctuation, one needs to probe the slope of the centralized second moment [refer Eqs. (H1) and (H2)] in the limit $r \rightarrow 0$. Importantly, this analysis

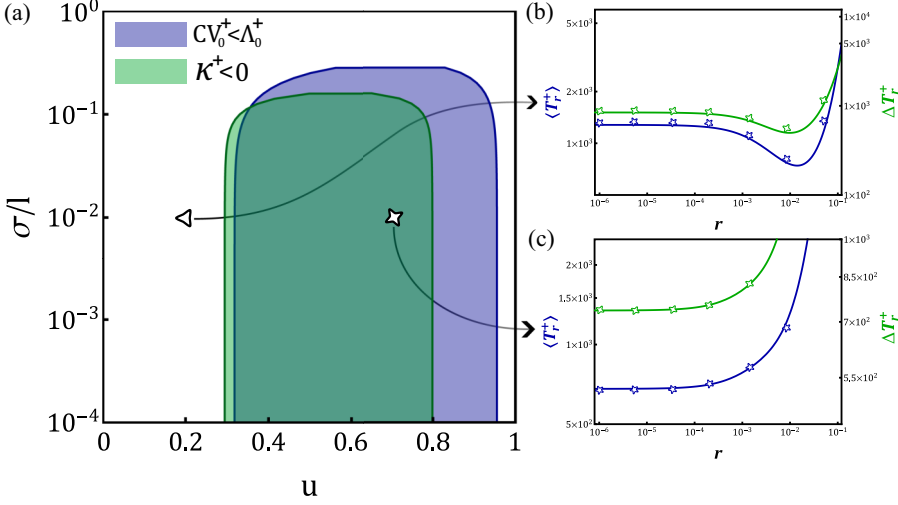


FIG. 5. Illustration of the system-parameter domains associated with the realization of the outcome “ $\varsigma = +$ ”, based on the criteria defined in Eqs. (28) and (31), for a Pareto WTD with exponent $\beta = 3.5$. Panel (a) presents the regions of parameter space where the condition $CV_0^+ < \Lambda_0^+$ is satisfied (shaded in violet), along with the regime $\kappa^+ < 0$ (shaded in green), which together signal parameter ranges where resetting-induced optimization is anticipated. Panel (b) shows that, for the initial condition $u = 0.2$ and a fixed domain length $l = 1$, stochastic resetting exerts a pronounced influence on both the conditional MFPT and the corresponding exit-time fluctuations through the right boundary. In contrast, for the initial condition $u = 0.7$, as illustrated in panel (c), the impact of resetting on the transport properties is significantly reduced, rendering resetting comparatively less effective in this regime. Markers indicate simulation results in panels (b) and (c).

enables the following relation:

$$\begin{aligned} \partial_r \mathcal{G}(u, r)|_{r \rightarrow 0} = & \frac{ul^2}{15r^3} \left[15r^2 \partial_r \alpha(r) (r(1-2u) \partial_r^2 \alpha(r) - 2(1-u) \partial_r \alpha(r)) \right. \\ & + \alpha(r)^2 (l^2 r^3 (1-2u)(u(19-14u)-7) \partial_r \alpha(r) \partial_r^2 \alpha(r) - 30(1-u)) \\ & + r \alpha(r) (60(1-u) \partial_r \alpha(r) + l^2 r^2 (1-2u)(u(19-14u)-7) (\partial_r \alpha(r))^3) \\ & \left. + 5r (r(1-2u) \partial_r^3 \alpha(r) - 6(1-u) \partial_r^2 \alpha(r)) \right]. \end{aligned} \quad (\text{H3})$$

Class I WTD. Let us consider ML WTD with $0 < \beta < 1$. In the Laplace domain, the WTD can be written as $\tilde{\phi}(r) = 1/(1 + (r\tau)^\beta)$; in what follows, $\alpha(r) = \frac{\sqrt{2}}{\sigma} (r\tau)^{\beta/2}$. Utilizing this in Eq. (H3), we get the following:

$$\partial_r (\Delta T_r^-(u))^2|_{r \rightarrow 0} \simeq -\frac{l^2 \tau^\beta (2-\beta)(1-\beta)}{3r^{3-\beta} \sigma^4} \mathcal{H}(u, \beta), \quad (\text{H4})$$

$$\partial_r (\Delta T_r^+(u))^2|_{r \rightarrow 0} \simeq -\frac{l^2 \tau^\beta (2-\beta)(1-\beta)}{3r^{3-\beta} \sigma^4} \mathcal{H}(1-u, \beta), \quad (\text{H5})$$

where $\mathcal{H}(u, \beta) = u(6 - 2u(3 - \beta) - \beta)$, with $u \in [0, 1]$ and $0 < \beta < 1$, indicating $\mathcal{H}(u, \beta) > 0$. Taking this into account, from Eqs. (H4) and (H5), we find

$$\partial_r (\Delta T_r^-(u))^2|_{r \rightarrow 0} < 0, \quad \partial_r (\Delta T_r^+(u))^2|_{r \rightarrow 0} < 0, \quad (\text{H6})$$

which we validate against numerical simulations in Fig. 6(a).

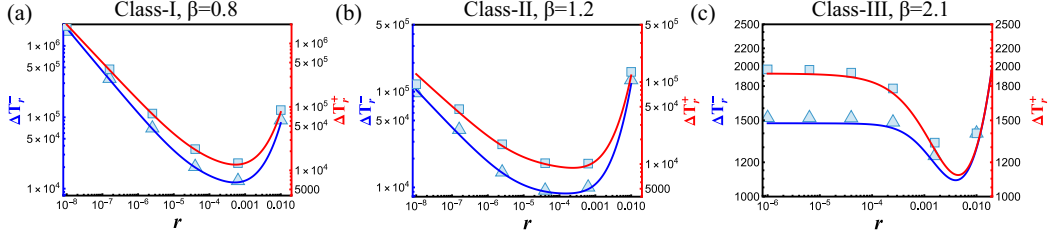


FIG. 6. Behavior of the fluctuations as a function of resetting rate r . In panel (a), we plot the fluctuations of conditional FPTs by considering class I WTD with initial location $u = 0.3$. The negative slope shows the pronounced effect of resetting as predicted by Eq. (H6). Solid lines represent the analytical results, which can be derived by using Eqs. (6) and (8), whereas the markers indicate the Monte Carlo simulation. Similarly, by considering class II and class III WTDs, we validate the theoretical predictions with the Monte Carlo simulations as indicated in panels (b) and (c), respectively.

Class II WTD. In particular, we consider Pareto WTD, i.e., $\phi(t_w) = \frac{\beta}{\tau(1+t_w/\tau)^{\beta+1}}$, with $1 < \beta < 2$. In the Laplace domain, this can be expressed as

$$\tilde{\phi}(r) = \beta \mathcal{U}(1, 1 - \beta; r\tau); \quad (\text{H7})$$

moreover, in the limit $r \rightarrow 0$, we find

$$\alpha(r) \simeq \frac{\sqrt{2}}{\sigma} \frac{\sqrt{r\tau}}{\sqrt{\beta-1}} \left[1 + \frac{\pi(\beta-1)}{2 \sin(\pi\beta)\Gamma(1+\beta)} (r\tau)^{\beta-1} \right].$$

Utilizing this, from Eq. (H3), one can find the leading terms to be

$$\partial_r (\Delta T_r^-(u))^2|_{r \rightarrow 0} \simeq \frac{\pi l^2 \tau^\beta (\beta-1)(2-\beta)}{3r^{3-\beta} \sigma^2 \Gamma(1+\beta) \sin(\pi\beta)} \mathcal{H}(u, \beta), \quad (\text{H8})$$

$$\partial_r (\Delta T_r^+(u))^2|_{r \rightarrow 0} \simeq \frac{\pi l^2 \tau^\beta (\beta-1)(2-\beta)}{3r^{3-\beta} \sigma^2 \Gamma(1+\beta) \sin(\pi\beta)} \mathcal{H}(1-u, \beta). \quad (\text{H9})$$

Remembering the domain of β , i.e., $1 < \beta < 2$ and $u \in (0, 1)$, one can find $\mathcal{H}(u, \beta)$ to be positive and $\sin(\pi\beta) < 0$. In what follows,

$$\partial_r (\Delta T_r^-(u))^2|_{r \rightarrow 0} < 0, \quad \partial_r (\Delta T_r^+(u))^2|_{r \rightarrow 0} < 0. \quad (\text{H10})$$

In Fig. 6(b), we observe that the slope of fluctuations in conditional FPTs at $r \rightarrow 0$ is negative, as indicated by Eq. (H10).

Class III WTD. Let us consider the Pareto distribution as WTD with $2 < \beta < 3$, where the first and the second moment of the underlying FPT become finite as described in Eqs. (E4) and (E5). Recalling Eq. (E3), in the limit $r \rightarrow 0$ one can write

$$\alpha(r) \simeq \frac{\sqrt{2}}{\sigma} \frac{\sqrt{r\tau}}{\sqrt{\beta-1}} \left[1 - \frac{1}{2(\beta-2)} r\tau + \frac{\pi(\beta-1)}{2 \sin(\pi\beta)\Gamma(1+\beta)} (r\tau)^{\beta-1} \right].$$

Using the above relation, from Eq. (H3), one gets

$$\partial_r (\Delta T_r^-(u))^2|_{r \rightarrow 0} \simeq -\frac{\pi l^2 \tau^\beta (\beta-1)(\beta-2)}{3r^{3-\beta} \sigma^2 \Gamma(1+\beta) \sin(\pi\beta)} \mathcal{H}(u, \beta), \quad (\text{H11})$$

$$\partial_r (\Delta T_r^+(u))^2|_{r \rightarrow 0} \simeq -\frac{\pi l^2 \tau^\beta (\beta-1)(\beta-2)}{3r^{3-\beta} \sigma^2 \Gamma(1+\beta) \sin(\pi\beta)} \mathcal{H}(1-u, \beta). \quad (\text{H12})$$

In particular, $0 < \sin(\pi\beta) < 1$ with $2 < \beta < 3$, which essentially indicates that the slope of the variance is negative in the limit $r \rightarrow 0$, as described in Fig. 6(c).

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