

Entanglement control in confined two-particle systems: Mass imbalance, trap asymmetry, interaction range, and localized potentials

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We present a systematic study of entanglement control in one-dimensional systems of two distinguishable interacting particles in harmonic confinement. Using density matrix renormalization group calculations, we examine how the von Neumann entropy evolves under variations of mass ratio, trap-frequency asymmetry, interaction range, and localized external fields. Our results reveal a consistent picture connecting the real-space density overlap, the redistribution of quantum fluctuations, and the resulting entanglement. Within the parameter regimes investigated, mass imbalance produces more pronounced changes in entanglement than trap-frequency asymmetry. Furthermore, the competition between interaction range and field-induced polarization determines whether short- or long-range correlations dominate. By connecting the von Neumann entropy with measurable quantities such as the density difference $\Delta n(x)$ and the phase-space products $\langle x_i^2 \rangle \langle p_{x_i}^2 \rangle$, we establish experimentally accessible fingerprints of quantum correlation. Our analysis demonstrates how external and geometrical parameters can tune entanglement in confined two-particle models, providing a basis for exploring related control protocols in suitable experimental settings.

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I. INTRODUCTION

Quantum entanglement is a fundamental concept in modern physics, connecting quantum information, condensed-matter, and atomic systems through nonclassical correlations between subsystems. Foundational reviews have established its theoretical framework and its central role in quantum information and condensed-matter research [1–3]. Recent developments expanded this view: Wang *et al.* employed an entanglement microscopy protocol to reveal multipartite entanglement in spin and fermionic many-body systems [4], while França and Capelle analyzed spatially inhomogeneous fermionic systems and identified universal and nonuniversal components

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of block entanglement [5,6]. Further contributions explored entanglement in few-electron atoms [7], non-Gaussian correlations in ultracold gases [8], and the use of quantum neural networks to detect entanglement transitions [9]. Other studies examined the geometry and dynamics of entanglement in complex many-body quantum systems [10] or its behavior near phase transitions in disordered superfluids and fermionic lattices [11–13]. These works have established entanglement entropy as a quantitative measure of correlation strength and as an indicator of coherence and criticality across quantum systems.

The seminal solution by Busch *et al.* [14] established a foundational framework for understanding two interacting particles under harmonic confinement and motivated a broad literature on trapped few-body correlations. Within this broader context, harmonically confined few-particle systems provide a minimal yet insightful setting for investigating the interplay between interactions, confinement, and quantum entanglement [15]. Sanino *et al.* recently showed that entanglement can act as a topological marker in harmonically confined Fermi-Hubbard chains [16], while Toroš *et al.* investigated relativistic effects on entangled trapped electrons [17]. Earlier studies by Zhang, Vidmar, and Rigol analyzed entanglement signatures of local quantum phase transitions in confined lattice bosons [18], and McKenney *et al.* explored the time evolution of entanglement entropy in driven harmonic traps [19]. Kościk [20] and Sun *et al.* [21] provided exact results for the von Neumann entropy in few-particle harmonic traps, revealing the crossover from delocalized to Wigner-molecule-like states. Deuretzbacher *et al.* [22] extended this approach to dipolar bosons, highlighting the role of long-range correlations. In a broader scope, Mistakidis *et al.* reviewed recent theoretical advances in the static and dynamical properties of low-dimensional few-body Bose gases [23]. Furthermore, Brouzos and Schmelcher explored two-component few-fermion mixtures in one-dimensional traps, comparing numerical and analytical approaches to describe the crossover from weak interactions to fermionization [24]. Becker *et al.* demonstrated how to dynamically control the transport and trapping of an impurity in a double well by alternating the interaction strength with two self-segregated majority species [25].

These studies collectively demonstrate that even within minimal models, such as two interacting particles in a one-dimensional confinement, it is possible to capture the essential competition between kinetic delocalization, external confinement, and interparticle coupling. By tuning parameters such as interaction strength, mass ratio, and trap asymmetry, one can explore how geometric and dynamical asymmetries shape the degree of quantum correlation. Moreover, this system provides a direct route to analyze the transition between localized and delocalized regimes, the redistribution of uncertainty between particles, and the emergence of bipartite entanglement.

From the experimental perspective, ultracold-atom and trapped-ion platforms allow for unprecedented control over these parameters. In particular, recent advances in optical tweezer platforms have enabled the preparation, manipulation, and detection of ultracold atoms at the single-particle level [26]. Tightly focused laser beams generate highly localized optical trapping potentials whose positions, depths, and geometries can be precisely controlled and dynamically manipulated. Such capabilities provide a realistic route for engineering local confinement and controlling the spatial distribution of individual particles. In addition, neutral atoms in optical tweezers can have their interactions tuned via Feshbach resonances [27], while trapped ions [28] enable the realization of long-range Coulomb couplings and localized electric fields.

From the theoretical side, such few-body trapped models can be treated with high-accuracy methods such as the density matrix renormalization group (DMRG) [29,30], matrix-product-state (MPS) algorithms, and exact-diagonalization approaches [31]. These techniques yield highly accurate ground-state wavefunctions, reduced density matrices, and entanglement spectra, revealing how correlations evolve with system parameters. Recent works have shown that entanglement can act as a sensitive indicator of interaction-driven crossovers and structural changes in confined systems [32–34]. In particular, it has been demonstrated that entanglement entropies can detect dimensional crossovers and finite Hilbert space effects by interpreting entropy divergences in terms of the available space in both real and phase space [32,34]. This conceptual framework establishes that entanglement effectively probes the volume of the configuration space accessible to interact-

ing particles, providing a deeper link between quantum correlations and the system's effective dimensionality. Beyond their intrinsic interest, these models can also serve as benchmarks for the construction of functional approaches to quantum correlations, including entanglement-density functionals and extensions to density-functional theory [35–39].

Furthermore, foundational studies have demonstrated that in the strong interacting limit, entanglement entropies exhibit logarithmic divergences when the system reaches the isotropic limit [32,33]. These works established that the available space in phase space for relative correlations is the primary driver of such divergences, providing a conceptual link between entanglement signatures and finite Hilbert space effects [32,34]. Our study builds upon this framework by examining how such correlations are physically redistributed under diverse asymmetries and localized external fields.

In this work, we present a systematic DMRG study of two distinguishable interacting particles confined in one-dimensional harmonic potentials. We investigate how interaction strength, mass imbalance, trap-frequency asymmetry, interaction range, and a particle-selective localized potential jointly determine the von Neumann entanglement entropy. The central contribution of our analysis is the identification of how these distinct asymmetries compete to enhance or suppress quantum correlations. In particular, we show that mass imbalance has a stronger influence on the entanglement than trap-frequency asymmetry and that the localized potential provides an external control parameter capable of polarizing the particle densities and reducing their spatial overlap. Moreover, in specific parameter regimes, the field-induced polarization reverses the relative entanglement efficiency of short- and long-range interactions, producing a switching between qualitatively distinct correlation regimes. By connecting the entropy to the density difference and to single-particle phase-space quantities, our results provide parameter maps for identifying comparatively high- and low-entanglement configurations through the combined control of the interaction strength, trap-frequency ratio, and localized-potential amplitude.

The paper is organized as follows. In Sec. II, we introduce the model Hamiltonian and formalism; in Sec. III, we present the DMRG methodology and results; and in Sec. IV, we summarize the main conclusions and perspectives.

II. FORMALISM AND BACKGROUND

Consider a system of two quantum particles with masses m_1 and m_2 confined in one-dimensional harmonic traps and interacting via a potential V_{int} . The full Hamiltonian reads

$$\hat{H} = -\frac{\hbar^2}{2m_1} \frac{\partial^2}{\partial \tilde{x}_1^2} - \frac{\hbar^2}{2m_2} \frac{\partial^2}{\partial \tilde{x}_2^2} + \frac{1}{2}m_1\omega_1^2\tilde{x}_1^2 + \frac{1}{2}m_2\omega_2^2\tilde{x}_2^2 - \tilde{E} e^{-(\tilde{x}_2-\tilde{x}_1)^2/\tilde{s}^2} + \tilde{V}_{\text{int}}(\tilde{x}_2 - \tilde{x}_1), \quad (1)$$

where $\tilde{x}_1$ and $\tilde{x}_2$ are the particle positions, with ω_1 and ω_2 labeling the trapping frequencies. The first two terms correspond to the kinetic energies, the next two to the harmonic confinements, and the fifth term indicates a Gaussian-like potential acting only on particle 2, whose experimental realization can be made through spatially focused optical or electrostatic potentials [40] and with $\tilde{E}$ related to the magnitude of the corresponding electric field. From an experimental perspective, such a localized potential is precisely realized in platforms such as optical tweezers for neutral atoms or focused electrostatic potentials in trapped-ion and hybrid atom-ion systems. The last term denotes the particle-particle interaction potential, which may be long or short ranged depending on the physical system. We can state that this general Hamiltonian can be applied to a wide range of physical settings, from trapped ions with Coulomb interactions to neutral atoms in quasi-one-dimensional optical traps [28,40,41].

The Hamiltonian of Eq. (1) can be conveniently rewritten in terms of dimensionless variables, specifically in the form

$$\hat{\mathcal{H}} = \frac{\hat{H}}{\hbar\omega_1} = -\frac{1}{2} \frac{\partial^2}{\partial x_1^2} - \frac{1}{2b} \frac{\partial^2}{\partial x_2^2} + \frac{1}{2}x_1^2 + \frac{1}{2}b\gamma^2 x_2^2 - E e^{-(x_2-x_1)^2/s^2} + V_{\text{int}}(x_2 - x_1), \quad (2)$$

where $b = m_2/m_1$, $\gamma = \omega_2/\omega_1$, $x_1 = [(m_1 \omega_1)/\hbar]^{1/2} \tilde{x}_1$, $x_2 = [(m_1 \omega_1)/\hbar]^{1/2} \tilde{x}_2$, $s = [(m_1 \omega_1)/\hbar]^{1/2} \tilde{s}$, $x_c = [(m_1 \omega_1)/\hbar]^{1/2} \tilde{x}_c$, and $E = \tilde{E}/(\hbar \omega_1)$. Therefore, $\mathcal{H}$, b , γ , x_1 , x_2 , s , x_c , and E are dimensionless parameters. This representation provides a useful scale-invariant framework commonly used in theoretical studies involving trapped-particle systems [14,15,33,42–44]. Throughout this paper, we shall consider $x_c = 0.5$ and $s = 0.1$.

In order to simulate the particle-particle interaction, two classes of interaction potentials are considered: For long-range interactions, we use the soft-Coulomb potential

$$V_{\text{int}}^{\text{Long}}(x_2 - x_1) = \frac{g}{2\sqrt{(x_2 - x_1)^2 + 0.1^2}}, \quad (3)$$

which is commonly employed to describe ion-ion couplings in reduced dimensions [33,44–48]. For short-range interactions, typical of neutral atoms confined in quasi-one-dimensional traps, we employ a smooth soft-core potential:

$$V_{\text{int}}^{\text{Short}}(x_2 - x_1) = \frac{g}{0.2} \left[1 + \left(\frac{x_2 - x_1}{0.1} \right)^6 \right]^{-1}, \quad (4)$$

which effectively mimics a contact interaction and reproduces, for example, the physics of the van der Waals forces [49]. Both potentials have the same value at particle coincidence, $V_{\text{int}}(0) = 5g$.

The dimensionless coupling g describes repulsive ($g > 0$) or attractive ($g < 0$) model interactions. For neutral-atom systems, magnetic Feshbach resonances tune low-energy scattering properties and effective interaction strengths [27]. For a given pair of ions, the Coulomb coefficient is fixed and is not Feshbach tunable. For the soft-Coulomb model, its strength relative to confinement can be varied by adjusting the trap frequency: With the normalization of Eq. (3), $g = 2C/(\hbar\omega_1 a_1)$, where $C = q_1 q_2 / (4\pi\epsilon_0)$ and $a_1 = \sqrt{\hbar/(m_1\omega_1)}$. Thus, $g \propto \omega_1^{-1/2}$ for fixed charges and masses. Changing the trap frequency can also modify other dimensionless parameters and therefore need not reproduce a variation of g alone. The attractive branch $g < 0$ is motivated by Coulomb attraction between oppositely charged particles.

By means of the total wavefunction $\psi(x_1, x_2)$, the one-body densities read

$$n_1(x_1) = \int_{-\ell}^{\ell} |\psi(x_1, x_2)|^2 dx_2 \quad (5)$$

and

$$n_2(x_2) = \int_{-\ell}^{\ell} |\psi(x_1, x_2)|^2 dx_1, \quad (6)$$

with ℓ labeling the system size. The reduced density matrix of particle 1 is

$$\rho_1(x, x') = \int_{-\ell}^{\ell} \psi(x, x_2) \psi^*(x', x_2) dx_2. \quad (7)$$

Its eigenvalues λ_k yield the von Neumann entanglement entropy

$$S = - \sum_k \lambda_k \ln \lambda_k, \quad (8)$$

which quantifies the spatial correlation between the two particles [1,2]. Equivalently, the Schmidt decomposition

$$\psi(x_1, x_2) = \sum_k s_k u_k(x_1) v_k(x_2), \quad (9)$$

with $\sum_k s_k^2 = 1$, leads to $\lambda_k = s_k^2$.

In summary, the Hamiltonian of Eq. (2) describes two interacting particles characterized by four dimensionless model parameters: (1) the interaction strength g ; (2) the mass ratio $b = m_2/m_1$; (3) the trap-frequency ratio $\gamma = \omega_2/\omega_1$; and (4) the external-field amplitude E . By varying these parameters, one can continuously move between symmetric and asymmetric confinement, localized and delocalized regimes, or weakly and strongly correlated states. This model provides a framework for studying entanglement in confined two-particle systems with soft-Coulomb or soft-core interactions, motivated by charged-particle and neutral-atom settings, respectively. Here, neutral-atom systems refer to two trapped atoms, without assuming molecular dimer formation. Extension to hybrid atom-ion systems requires an appropriate interaction with a long-range tail proportional to r^{-4} [50], and is left for future work.

Throughout this work, the parameters b , g , and γ are independently varied using uniform increments $\Delta b = 0.5$, $\Delta g = 0.5$, and $\Delta \gamma = 0.05$, enabling a controlled and systematic investigation of mass imbalance, interaction strength, and trap asymmetry effects on quantum entanglement.

Particle distinguishability and exchange statistics. We consider two distinguishable constituents, which may correspond to different species, isotopes, or distinguishable internal-state components. The coordinates x_1 and x_2 label these constituents, and the entropy in Eq. (8) quantifies their bipartite entanglement. The localized potential acts selectively on constituent 2; this selectivity is consistent with the assumed distinguishability and does not itself change the particles' exchange statistics.

When $m_1 = m_2$, $\omega_1 = \omega_2$, and $E = 0$, the Hamiltonian is invariant under coordinate exchange, $x_1 \leftrightarrow x_2$. In this limit, our numerical procedure explicitly projects onto the symmetric spatial sector. This projection is consistent with the nondegenerate ground state of the regularized Hamiltonian, which is exchange symmetric and coincides with the spatial ground state of two identical bosons subject to the same Hamiltonian. When the masses or trapping frequencies differ, or when $E \neq 0$, this projection is disabled, and no exchange-symmetry constraint is imposed on the spatial wavefunction. The antisymmetric spatial sector required for two identical spin-polarized fermions is not investigated here.

III. RESULTS AND DISCUSSION

All numerical results presented in this work were obtained by using a DMRG algorithm, specifically developed for continuous one-dimensional systems. The code solves the stationary Schrödinger equation with the Hamiltonian of Eq. (2). Spatial discretization was performed on a uniform grid $x \in [-\ell, \ell]$, with $\ell = 12$ and a spacing of $\Delta x = 0.1$ ensuring the wavefunction to vanish at the domain edges. Instead of performing a full diagonalization of the two-body Hamiltonian matrix, the algorithm applies the Hamiltonian operator directly to the state vector $\psi(x_1, x_2)$ through local matrix-vector multiplications. A total of ten DMRG cycles were performed, and during each cycle, the bipartite state was represented as an MPS with a maximum bond dimension of $\chi_{\max} = 241$. Singular value decompositions (SVDs) were used to obtain the Schmidt representation, retaining up to 241 singular values. An energy-change stopping criterion of 10^{-8} was employed.

The parameters x_c and s determine, respectively, the position and spatial width of the localized perturbation. In the present calculations, they are fixed at $x_c = 0.5$ and $s = 0.1$, while the external control is implemented through the potential amplitude E . The finite value of s defines a smooth localized potential that can be reliably represented on the spatial grid used in the DMRG calculations. Experimentally, the selective application of this potential to particle 2 may be implemented through a species-selective or internal-state-dependent optical potential, so that the two distinguishable constituents experience different local trapping landscapes. A systematic investigation of the dependence on x_c and s is beyond the scope of the present work.

At every DMRG iteration, the two-body wavefunction $\psi(x_1, x_2)$ is reshaped into a matrix form and decomposed as $\psi(x_1, x_2) = \sum_i s_i u_i(x_1) v_i(x_2)$, where s_i are the singular values obtained from the SVD and $\{u_i, v_i\}$ are orthonormal single-particle basis functions for particles 1 and 2, respectively. This expansion is formally equivalent to the Schmidt decomposition of a bipartite pure state, providing direct access to the entanglement spectrum. The squared singular values $p_i = s_i^2 / \sum_j s_j^2$

correspond to the eigenvalues of the reduced density matrix and are used to compute the von Neumann entanglement entropy, $S = -\sum_i p_i \ln p_i$. A single dominant singular value ($s_1 \approx 1$) characterizes a separable, unentangled state, whereas the presence of several comparable singular values signals strong bipartite quantum correlations.

In Fig. 1, we display the von Neumann entanglement entropy S as a function of the interaction parameter g and the mass ratio $b = m_2/m_1$, for both short- and long-range interactions, under two conditions: without external field ($E = 0$) and with a weak localized electric field ($E = 1.0 \times 10^{-2}$). In all cases, the trap frequencies are chosen as $\gamma = 1/\sqrt{b}$, ensuring that both particles experience the same harmonic curvature; thus, the analysis isolates the effect of mass imbalance. The results describe two distinguishable particles. At the fully symmetric point $b = \gamma = 1$ and $E = 0$, the calculated ground state can additionally be identified with the spatial ground state of two identical bosons.

For $E = 0$, the entanglement patterns exhibit distinct behaviors depending on the interaction range and the sign of g . For attractive couplings ($g < 0$), S increases markedly as the attraction strengthens, reflecting the enhanced spatial overlap between the two particles. This effect is particularly pronounced for large b , where the heavier particle remains more localized while the lighter one delocalizes around it, yielding strong spatial correlations and consequently higher S values (orange-red regions). For repulsive couplings ($g > 0$), the entanglement also rises from low to intermediate g , but eventually tends to saturate as the particles become spatially separated. The increase with b persists because the lighter particle explores a wider spatial region, partially compensating for the reduced overlap.

When a weak localized electric field is applied ($E = 1.0 \times 10^{-2}$), the entanglement maps change significantly, reflecting the interplay between field-induced polarization and interparticle correlation. For attractive couplings ($g < 0$), S remains large for both interaction ranges (red regions). For repulsive couplings ($g > 0$), the electric field enhances spatial separation and amplifies the contrast between the two interaction types. In the short-range case, S decreases moderately with increasing b , since the field and repulsion act together to localize each particle in distinct regions, suppressing quantum coherence. In the long-range case, however, the external field reinforces the natural tendency of the Coulomb-like repulsion to separate the particles, producing a broad low- S region (blue area) that extends toward large b values. In this regime, the system becomes strongly polarized, and spatial separation dominates over delocalization-induced quantum correlations.

The two last panels of Fig. 1 show the difference between the entanglement entropies obtained for long- and short-range interactions, defined as $\Delta S = S_{\text{Long}} - S_{\text{Short}}$. The dashed lines mark the contour $\Delta S = 0$, separating the regions where the long-range interaction produces higher ($\Delta S > 0$) or lower ($\Delta S < 0$) entanglement compared to the short-range case. For $E = 0$, the region to the left of the dashed curve corresponds to $\Delta S < 0$, indicating that for sufficiently attractive couplings ($g \lesssim -2$), the short-range potential yields slightly higher S . This occurs because, in the strongly attractive regime, the short-range coupling confines both particles to a narrow spatial region where their wavefunctions overlap more efficiently, leading to higher entanglement despite the interaction's local nature. However, for most of the parameter space—particularly for repulsive interactions ($g > 0$) and large b —the long-range interaction dominates, producing $\Delta S > 0$ (yellow-red region). This behavior reinforces that the Coulomb-like tail sustains correlations even when the particles are spatially separated. When the external field is applied ($E = 1.0 \times 10^{-2}$), the ΔS landscape becomes more complex and strongly asymmetric. The regions to the left of the first dashed curve and to the right of the second dashed curve—predominantly blue—correspond to $\Delta S < 0$, revealing that under polarization, the short-range interaction can generate stronger entanglement than the long-range one. This remarkable inversion arises because the external field enhances spatial asymmetry: The long-range potential, being more sensitive to such polarization, accentuates the spatial separation between the particles and thus reduces their mutual overlap. In other words, it ceases to promote delocalized states and instead reinforces the localization induced by the field. In contrast, the short-range potential confines both particles near the field minimum, preserving partial overlap and hence higher S . Between the two dashed boundaries, $\Delta S > 0$, meaning that in this intermediate

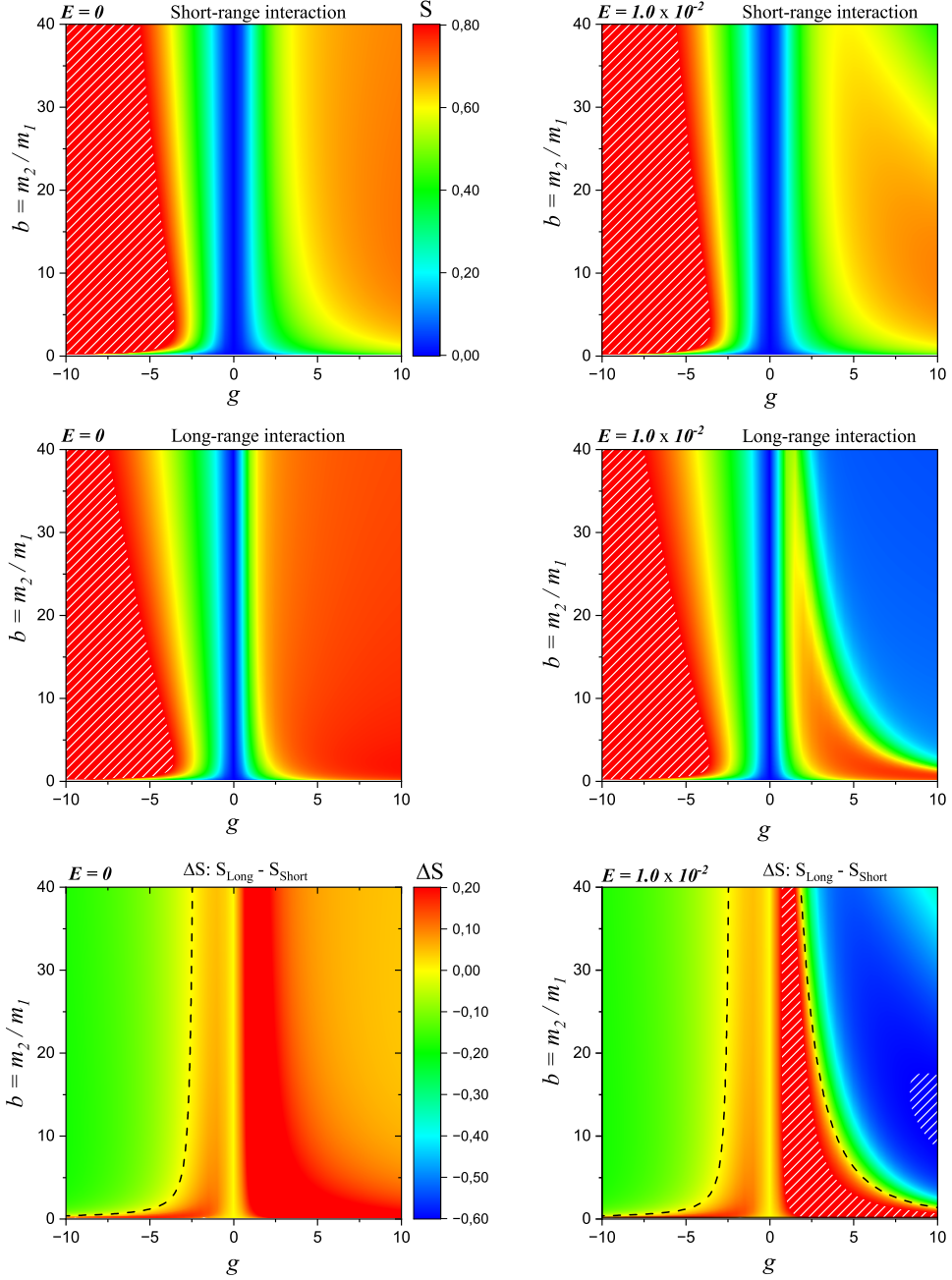


FIG. 1. Mass-imbalance effects: von Neumann entropy, as given by Eq. (8), as a function of ratio between the masses, $b = m_2/m_1$, and the interaction parameter g , for short- and long-range interactions as given by Eqs. (3) and (4). Two different electric fields with effective parameters $E = 0$ and $E = 1.0 \times 10^{-2}$ are considered. In addition, we make $\gamma = \omega_2/\omega_1 = 1/\sqrt{b}$; that is, both particles are confined under the same harmonic curvature. The results describe two distinguishable particles. At the fully symmetric point $b = \gamma = 1$ and $E = 0$, the calculated ground state can additionally be identified with the spatial ground state of two identical bosons. The crosshatched areas highlight regions where the plotted values exceed the maximum or minimum limits of the color scale for visual emphasis and contrast enhancement.

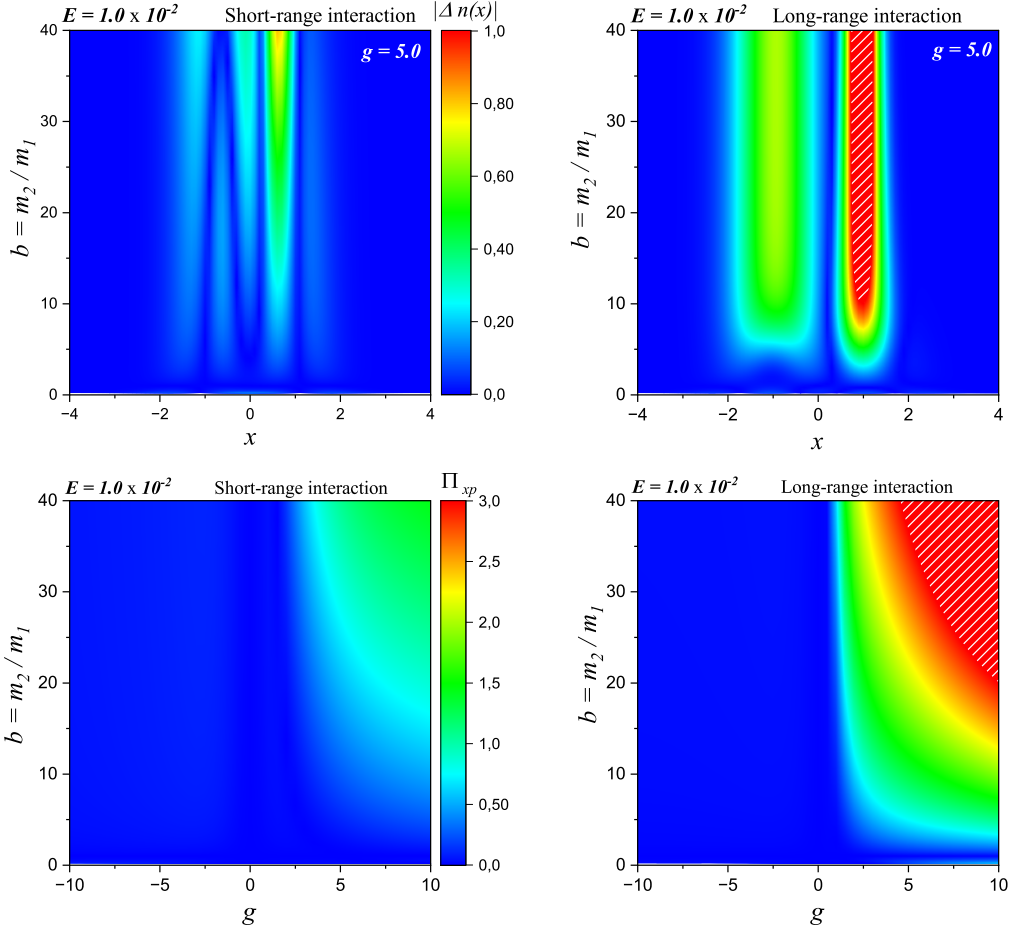


FIG. 2. For the situations considered in Fig. 1, the two upper panels display the difference between the densities $|\Delta n(x)| = |n_1(x) - n_2(x)|$ for $E = 1.0 \times 10^{-2}$ and $g = 5.0$, as a function of the position and $b = m_2/m_1$, with $\gamma = 1/\sqrt{b}$. In the two lower panels, we display the difference Π_{xp} , as defined in Eq. (10), for each type of interaction. Crosshatched areas are defined as in Fig. 1.

region, long-range correlations still prevail. These ΔS maps demonstrate how the interplay between interaction range, mass imbalance, and external polarization defines distinct domains in which either local or delocalized correlations dominate the quantum entanglement.

The dependence on mass imbalance is therefore regime dependent. At $E = 0$, increasing b enhances the entropy in portions of the repulsive regime for both interactions, whereas at $E = 10^{-2}$ the long-range interaction exhibits pronounced suppression at larger b over a broad repulsive region. Thus, departure from mass equality does not universally suppress entanglement.

The results presented in Fig. 2 complement the analysis of Fig. 1 by showing, first, the spatial density difference $|\Delta n(x)| = |n_1(x) - n_2(x)|$ and, second, the difference between the phase-space products of each particle, given by

$$\Pi_{xp} = |\langle x_1^2 \rangle \langle p_{x_1}^2 \rangle - \langle x_2^2 \rangle \langle p_{x_2}^2 \rangle|. \quad (10)$$

The quantities $\langle x_i^2 \rangle \langle p_{x_i}^2 \rangle$ used here are products of raw single-particle second moments rather than centered uncertainty products. They are employed as structural indicators of the spatial and momentum distributions. In the presence of the localized potential, the particle densities may be

displaced from the trap center, so that $\langle x_i \rangle$ is not necessarily zero and $\langle x_i^2 \rangle$ does not generally coincide with the position variance $(\Delta x_i)^2$.

Considering $E = 1.0 \times 10^{-2}$ and $g = 5.0$, the two upper panels of Fig. 2 show that the short-range potential (upper left) yields nearly symmetric densities with small oscillations around the trap center, while the long-range potential (upper right) produces clear polarization, with the two particles displaced to opposite sides as $b = m_2/m_1$ increases. This separation, caused by the combined action of the external field and the nonlocal tail, reduces spatial overlap and thus the entanglement, consistent with the trends in Fig. 1. The two lower panels of Fig. 2 show how mass imbalance redistributes the single-particle second-moment products between the two constituents. The results follow the qualitative picture obtained from the density maps and from the entanglement behavior in Fig. 1: Regions where $\langle x_1^2 \rangle \langle p_{x_1}^2 \rangle$ and $\langle x_2^2 \rangle \langle p_{x_2}^2 \rangle$ differ significantly generally correspond to weaker entanglement, whereas more balanced phase-space distributions are associated with larger S . A large value of Π_{xp} indicates that the two particles have substantially different spatial and momentum spreads, with one constituent being more spatially extended or kinetically broadened than the other. Such an imbalance is associated with a polarized configuration and reduced bipartite coherence. Conversely, comparable second-moment products indicate a more balanced distribution of the single-particle phase-space spreads and are correlated with larger entanglement.

In Fig. 3, we display the von Neumann entropy as a function of the interaction strength g and the trap-frequency ratio $\gamma = \omega_2/\omega_1$, with $b = 1$ for short- and long-range interactions. In the absence of external fields ($E = 0$), in both cases, S reaches its maximum near the symmetric confinement ($\gamma \simeq 1$) and for moderate repulsive couplings ($g \approx 5$ – 10), where the harmonic curvatures are comparable and the particles develop balanced spatial correlations. As γ deviates from unity, the trap asymmetry breaks this balance: One particle becomes increasingly localized while the other delocalizes, leading to a reduction of S . For attractive interactions ($g < 0$), both short- and long-range potentials exhibit a rapid increment of S , associated with stronger pair localization under attractive interactions. In these regions, the crosshatched (shaded) areas indicate where the entropy values exceed the predefined limits of the color-bar scale. This visual coding is employed to enhance contrast in the transition regions. Notably, a comparison with Fig. 1 reveals that the overall entanglement is smaller in this geometric-asymmetry case, suggesting that varying γ (trap curvature asymmetry) has a weaker effect on quantum correlations than the mass-imbalance mechanism discussed earlier.

When an external electric field is applied ($E = 1.0$), the entanglement landscape shown in Fig. 3 undergoes a noticeable redistribution. For both short- and long-range interactions, for $g > 0$ the entropy decreases compared to the $E = 0$ case, reflecting the field-induced polarization that partially suppresses quantum correlations by shifting the single-particle densities toward opposite sides of the trap. In the short-range case, S remains nearly symmetric around $\gamma \simeq 1$ and exhibits a moderate enhancement only for repulsive interactions, where the external field competes with the confinement asymmetry to balance the spatial localization of each particle. For the long-range potential, a similar trend is observed, but the Coulomb-like tail amplifies the field's effect, creating a narrow region of enhanced entropy near $g \approx 2$ and $\gamma \simeq 1$. Notably, substantial variations in S (in comparison with the case of $E = 0$) now require a field strength two orders of magnitude larger than that used in the mass-imbalance case of Fig. 1, indicating that geometric asymmetry ($\gamma \neq 1$) alone couples more weakly to the field-induced polarization mechanism.

The two last panels of Fig. 3 display the difference $\Delta S = S_{\text{Long}} - S_{\text{Short}}$ for the same parameter sets, highlighting the relative impact of the interaction range on the entanglement. For $E = 0$ (left panel), the dashed line marks the condition $\Delta S = 0$. In the region to its left ($g \lesssim -5$), $\Delta S < 0$, meaning that short-range interactions yield slightly larger entropies than their long-range counterparts, the same as seen in Fig. 1. For repulsive interactions ($g > 0$), $\Delta S > 0$ dominates, as the long-range potential maintains correlations even at larger separations. When the electric field is increased to $E = 1.0$ (right panel), two regions with $\Delta S < 0$ appear—one at strong attraction ($g \lesssim -5$) and another at strong repulsion ($g \gtrsim 5$). The latter region, dominated by blue tones, is

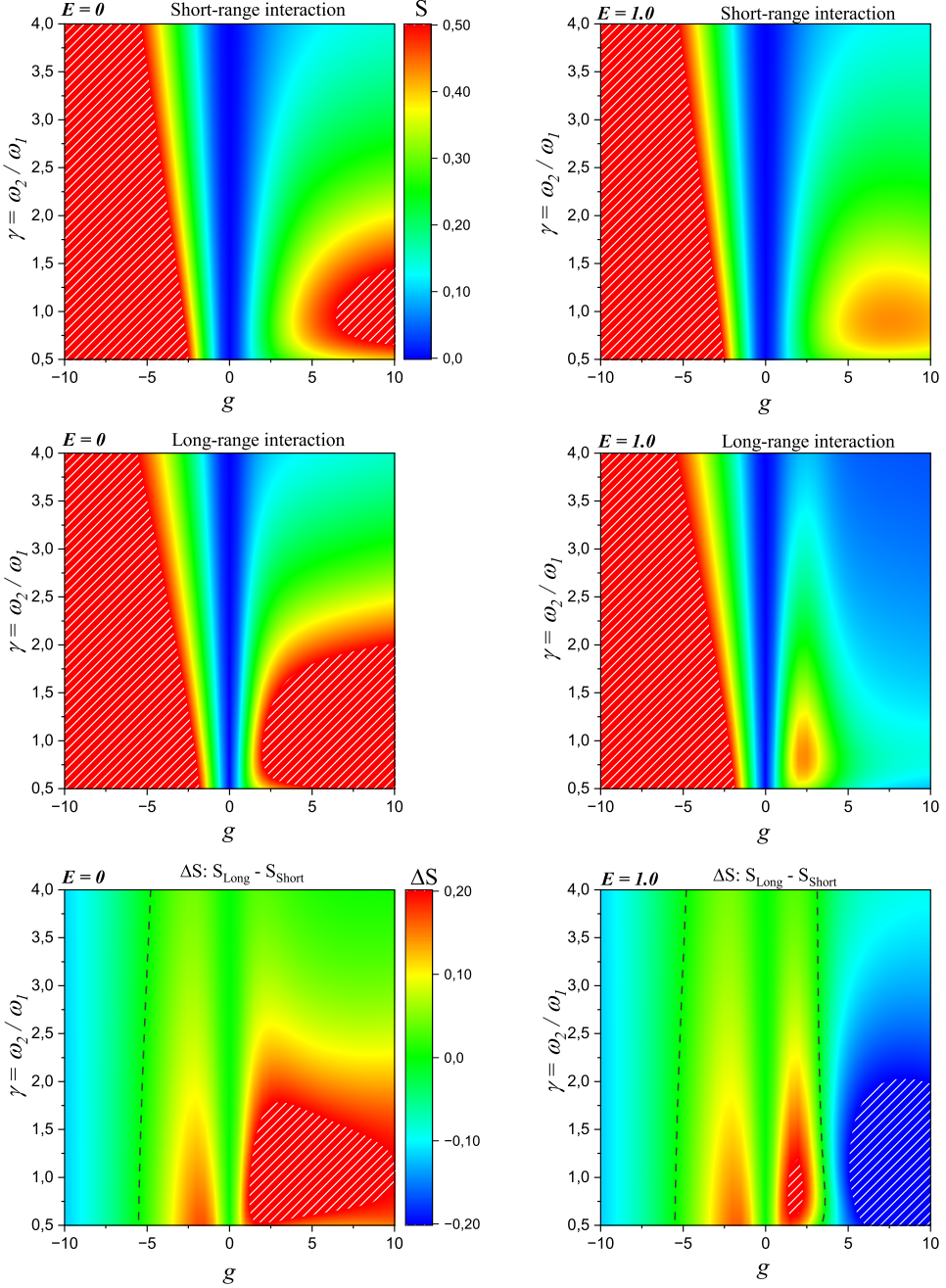


FIG. 3. Trap asymmetry effects: von Neumann entropy, as given by Eq. (8), as a function of ratio between the frequencies, $\gamma = \omega_2/\omega_1$, and the interaction parameter g , for short- and long-range interactions as given by Eqs. (3) and (4). Two different electric fields with effective parameters $E = 0$ and $E = 1.0$ are considered. In order to measure the effect of the trap asymmetry alone, we consider $b = 1$. The results describe two distinguishable particles. At the fully symmetric point $\gamma = 1$ and $E = 0$, with $b = 1$, the calculated ground state can additionally be identified with the spatial ground state of two identical bosons. Crosshatched areas are defined as in Fig. 1.

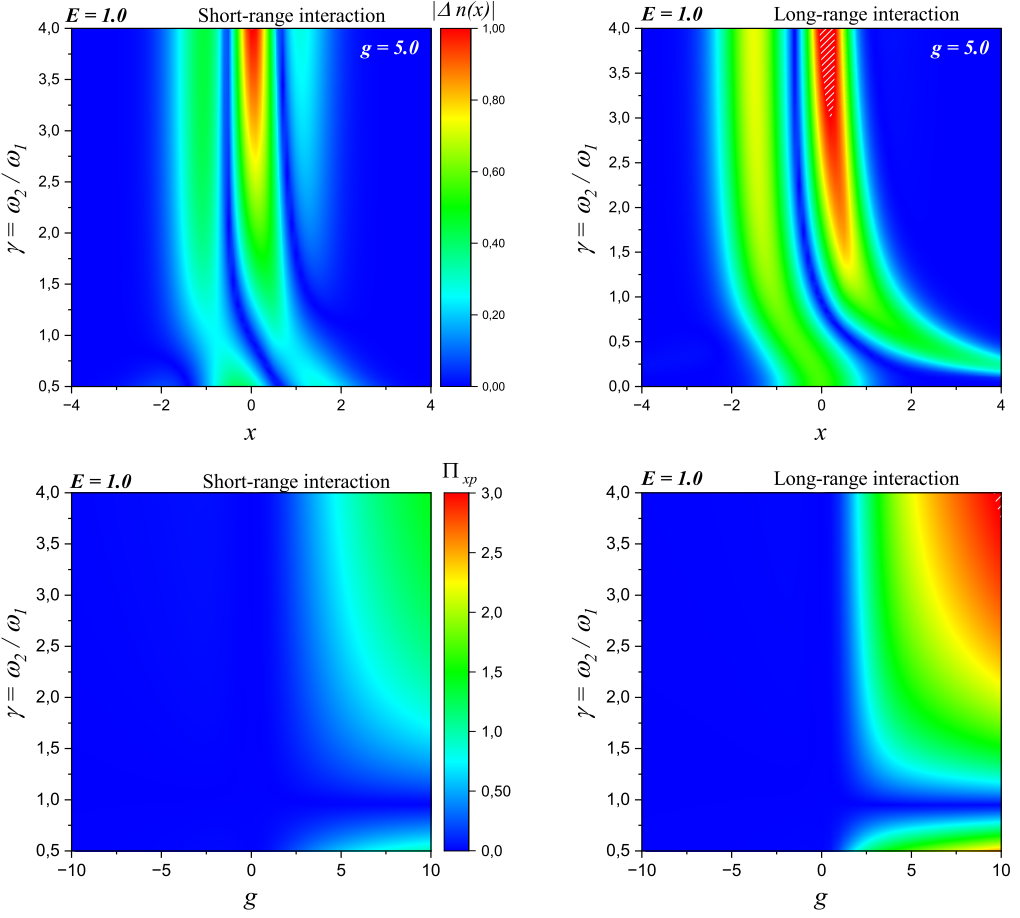


FIG. 4. For the situations considered in Fig. 3, the two upper panels display the difference between the densities $|\Delta n(x)| = |n_1(x) - n_2(x)|$ for $E = 1.0$ and $g = 5.0$, as a function of the position and $\gamma = \omega_2/\omega_1$, with $b = 1$. In the two lower panels, we display the difference Π_{xp} , as defined in Eq. (10), for each type of interaction. Crosshatched areas are defined as in Fig. 1.

particularly striking: Despite the nonlocal character of the Coulomb-like potential, the short-range interaction produces larger S . This counterintuitive behavior, also visible in Fig. 1, arises from the interplay between the strong field polarization and geometric confinement, which suppresses the spatial overlap more effectively for the long-range case, thereby reducing its entanglement efficiency. Between the two dashed boundaries, $\Delta S > 0$, meaning that in this intermediate region, long-range correlations still prevail. Overall, the ΔS maps emphasize that while long-range forces generally favor stronger correlations, their advantage can be reversed under external fields or extreme confinement asymmetries.

The two upper panels of Fig. 4 show the spatial density difference $|\Delta n(x)| = |n_1(x) - n_2(x)|$ for $E = 1.0$ and $g = 5.0$, under short- and long-range interactions. In both cases, varying the trap-frequency ratio $\gamma = \omega_2/\omega_1$ changes the magnitude and spatial structure of the density difference. For the short-range potential (left panel), the density imbalance remains moderate and smoothly distributed, indicating that the local interaction confines both particles near the center while still allowing partial spatial overlap. In contrast, the long-range case (right panel) displays a more pronounced polarization, with a stronger separation of the two density profiles as γ increases. These

features are directly connected to the entropic behavior observed in Fig. 3, where strong repulsion and long-range coupling under external fields lead to a suppression of S .

Consistent with the isotropic-limit analysis in Ref. [32], the entropy generally takes higher values near $\gamma = 1$ than at larger confinement asymmetries in the repulsive regime, where the available phase space for correlations is less restricted by the confinement asymmetry. However, within the parameter regimes investigated, varying the mass ratio produces more pronounced changes in the entanglement entropy than varying the trap-frequency ratio.

The two lower panels of Fig. 4 present the phase-space difference Π_{xp} , as defined in Eq. (10), for each interaction type, illustrating how the external field and trap asymmetry affect the single-particle second-moment products. The short-range results display smoother variations and more comparable values for the two particles. In contrast, the long-range case exhibits a pronounced increase of Π_{xp} for repulsive couplings, indicating a larger imbalance between their spatial and momentum spreads. Thus, Fig. 4 reinforces the qualitative picture obtained from Fig. 2: Comparable second-moment products are associated with more balanced spatial and momentum distributions and generally larger entanglement, whereas a marked imbalance corresponds to weaker S and more polarized configurations. These associations refer to the interacting regimes compared here and do not constitute a general criterion for entanglement. In particular, comparable second-moment products do not imply a nonzero entropy, as illustrated by the separable ground state at $g = 0$.

The phase-space interpretation of entanglement divergences, as discussed in Ref. [34], provides a broader framework for interpreting the results presented in Figs. 1–4. While foundational studies use these divergences to characterize the “available space” in isotropic limits [32,33], we employ the second-moment-product imbalance Π_{xp} and the density difference $|\Delta n(x)|$ as structural indicators of the changes induced by mass imbalance, trap-frequency asymmetry, and the localized potential.

The present single-particle description is complementary to the formulation in relative and center-of-mass coordinates employed in Refs. [32–34]. Defining the relative coordinate as $r = x_1 - x_2$ and the center-of-mass coordinate as $R = (m_1 x_1 + m_2 x_2)/(m_1 + m_2)$ provides the natural framework for analyzing interaction-induced correlations and the entropy behavior discussed in those works. In the present model, however, mass and confinement asymmetries, together with the particle-selective localized potential, generally couple the relative and center-of-mass motions. A complete quantitative analysis in these coordinates would therefore require additional calculations. The single-particle second-moment products used here should therefore be understood as complementary structural indicators.

The parameter maps in Figs. 1–4 suggest a simple quasistatic control scenario for switching between configurations with different degrees of bipartite entanglement. For a chosen pair of distinguishable particles, the mass ratio b is fixed by the selected species or isotopes. The trap-frequency ratio γ and the localized-potential amplitude E provide external control parameters in suitable implementations. The experimental tunability of the interaction parameter g depends on the platform, as discussed following Eq. (4). The system may first be prepared close to a balanced-confinement configuration, $\gamma \simeq 1$, with the interaction strength chosen inside a region of comparatively large entropy. The localized potential acting on particle 2 can then be increased to displace its density and reduce the density overlap, thereby driving the system toward a lower-entanglement, polarized configuration. Reversing the ramp restores the larger-overlap regime. The density difference $|\Delta n(x)|$ and the single-particle second moments provide experimentally accessible indicators of this switching process. In addition, the ΔS maps show that the localized field can change which interaction range produces the larger entropy, providing a second control mechanism based on the competition between field-induced polarization and interaction range.

IV. SUMMARY AND CONCLUSIONS

In this work, we have analyzed the quantum correlations of two interacting particles confined in one-dimensional harmonic traps, focusing on the combined effects of interaction range, trap asymmetry, and localized external fields. Using accurate DMRG calculations, we obtained

systematic trends for the von Neumann entanglement entropy and the single-particle densities under both long- and short-range particle-particle interactions. The results reveal a consistent physical picture linking spatial overlap, trap geometry, and the degree of entanglement in confined quantum systems. Most importantly, the localized particle-selective potential provides a switching parameter that connects high-overlap, comparatively entangled configurations to polarized configurations with reduced entanglement and can reverse the relative entanglement efficiency of short- and long-range interactions.

Figures 1–4 establish a unified physical picture of how mass imbalance, trap geometry, and external fields collectively govern the spatial structure and quantum correlations of two interacting particles in one-dimensional harmonic traps. For two distinguishable particles, the results show that entanglement depends on mass and confinement asymmetries in a regime-dependent manner. These asymmetries can either enhance or suppress the entropy, depending on the interaction strength, interaction range, and external-field amplitude. Within the parameter regimes investigated, mass imbalance has a more pronounced influence on the entanglement entropy than trap-frequency asymmetry, emphasizing the distinct roles of these two control parameters. In particular, when an asymmetry or polarization reduces the spatial overlap between the particles, the entanglement is generally suppressed. Long-range interactions often sustain stronger correlations over a broad region of parameter space, but their entanglement advantage can be reversed when external fields or geometric asymmetries strongly polarize the densities.

The density difference $|\Delta n(x)|$ and the second-moment-product imbalance Π_{xp} (Figs. 2 and 4) provide complementary structural information about the regimes identified in the entropy maps (Figs. 1 and 3). In the interacting regimes discussed, pronounced imbalances often accompany reduced entanglement, although these observables alone do not determine the entropy. These findings emphasize that geometric and field-induced asymmetries can be exploited to manipulate entanglement in confined two-particle models, motivating the exploration of related control protocols in suitable neutral-atom or charged-particle settings.

Future extensions of this work may include time-dependent external fields to study entanglement dynamics and control protocols, the incorporation of spin degrees of freedom, and the generalization to three-body or multicomponent systems where collective effects may emerge. It would also be interesting to benchmark DMRG results against time-dependent density functional theory approaches and to design effective functionals of the entanglement entropy, bridging many-body methods and quantum information frameworks. The results presented here contribute to a unified understanding of how interaction range, geometry, and external control collectively determine the structure of quantum correlations in low-dimensional quantum systems.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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