

Stationary $1/f^\gamma$ noise in discrete models of the Kardar-Parisi-Zhang class

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In discrete models describing growing rough interfaces in the Kardar-Parisi-Zhang universality class, we examine height fluctuations at a fixed site as a function of time, measured in monolayer units. For small systems, we show that it is possible to reach the stationary state. We compute the two-time autocorrelation and power spectra independently. The correlation function remains nonexponential and vanishes after a correlation time that diverges with system size. As a result, the power spectra display a lower cutoff with constant power. In the nontrivial frequency regime, we observe $1/f^\gamma$ -type scaling with the spectral exponent $5/3$. Finite-size scaling reveals that the temporal correlation function follows dynamic scaling. Our findings, supported by scaling-theoretical arguments, establish that the fluctuations are wide-sense stationary, implying the applicability of the Wiener-Khinchin theorem. The scaling arguments also reveal a scaling relation $\gamma = 1 + 2\alpha/z$, where α and z denote the roughness and dynamical exponents.

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I. INTRODUCTION

Astonishingly diverse nonequilibrium systems exhibit the emergence of long-range correlation. In the dynamical context, one can recognize such behavior as $1/f$ noise [1], a scaling feature of the power spectral density (PSD) of a noisy signal. For a signal $X(t)$ measured up to a finite observation time T , the PSD is $S(f, T) = \langle |\tilde{X}(f, T)|^2 \rangle / T$, where $\tilde{X}(f, T)$ is the truncated Fourier transform of the signal and $\langle \cdot \rangle$ denotes ensemble average. While signals ranging from voltage across electronic devices [2,3] to biological activities [4–8] display the $1/f$ scaling behavior, no general explanation exists. Based on the stationary or nonstationary characteristics, the study of $1/f$ noise falls under two approaches [9].

(1) If the signal represents stationary or long-time dynamics, then $S(f) = \lim_{T \rightarrow \infty} S(f, T)$. The two-time autocorrelation function $C(t_1, t_2) = \langle X(t_1)X(t_2) \rangle$ for a real signal becomes a function of an argument that is the absolute value of the difference between two times, $C(\tau = |t_1 - t_2|)$, because of time translation symmetry. For such wide-sense stationary processes, the Wiener-Khinchin theorem suggests that the PSD is the Fourier (cosine) transform of the correlation function [10]. We emphasize that the underlying temporal correlation function in several systems exhibits non-exponential behavior and dynamical scaling in terms of correlation time due to finite-size effects [11]. Correspondingly, the PSD displays a signature of a lower cutoff frequency (the inverse of the correlation time) below which the power remains constant [12,13].

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(2) In experimental and theoretical studies, the PSD $S(f, T)$ of many processes (such as unrestricted Brownian motion and blinking quantum dots [14]) lacks a lower cutoff frequency that remains elusive for a finite observation time. Such processes are typically nonstationary (can describe a transient toward a stationary state of an infinite-size system). The Wiener-Khinchin theorem becomes invalid in this context [10,15]. The PSD can (or does not) show dependence on the finite observation time. The two-time autocorrelation function does not satisfy the time-translational symmetry and remains a function of two times. Also, the ensemble average and time average are not the same. Recent works have emphasized that the Wiener-Khinchin theorem for nonstationary scale-invariant processes can yield the corresponding PSD if the correlation function satisfies dynamical scaling [15–17].

In particular, we focus on kinetic rough interfaces that remain extensively studied nonequilibrium systems and exhibit interlinked space-time scaling features [18]. Among many relevant interfaces, the Kardar-Parisi-Zhang (KPZ) universality class [19,20] describes an interesting phenomenon with diverse and growing applications found in isotropic spin chains [21] and exciton-polariton condensates [22]. Experimentally, the study of such interfaces has been relevant to thin-film deposition [23,24], liquid crystal films [25,26], growth of bacterial colonies [27,28], and cancer cell growth dynamics [29]. Numerical work relied on examining discrete models such as ballistic deposition (BD) [30,31] and the single-step (SS) model [32]. Analytically, a continuum equation for the KPZ class [33–36] describing the space-time evolution of the height profile $h(x, t)$ is

$$\partial_t h = \nu \partial_x^2 h + \lambda (\partial_x h)^2 + \eta, \quad (1)$$

where ν is the surface tension of the interface and λ is the strength of nonlinear term. The white Gaussian noise term η has zero mean. The nonlinear term in Eq. (1) accounts for growth locally normal to the surface. The square of the width $w^2(t, L) = \langle [h - \langle h \rangle]^2 \rangle$ satisfies dynamic or Family-Vicsek scaling [37] as

$$w^2(t, L) \sim L^{2\alpha} F_{w^2}(t/L^z), \quad (2)$$

where the scaling function varies as $F_{w^2}(u) \sim u^{2\alpha/z}$ for $u \ll 1$ (growth regime) and constant for $u \gg 1$ (saturation regime). z and α are the dynamic and roughness exponents, and the angular brackets $\langle \cdot \rangle$ denote the average over space and configurations. As is known, the width fluctuations exhibit a saturation regime after a transient. The spatial correlation length varies as $\xi \sim L$. It also reveals the existence of a correlation time $\mathcal{T} \sim L^z$.

We focus on the autocorrelation function of a stationary process $\delta h_L(t)$ that describes fluctuations in the height of a one-dimensional (1D) rough interface of length L about the spatially averaged mean level at time t . Since the driving noise η in Eq. (1) is stationary, it is expected that the process $\delta h_L(t)$ is also stationary. Then, the autocorrelation function $C(\tau, L) = \langle \delta h_L(t) \delta h_L(t + \tau) \rangle$ becomes a function of a single variable τ properly scaled by the system size, and consequently, the Wiener-Khinchin theorem applies.

In discrete models belonging to the KPZ universality class, Takeuchi [9] examined a temporal signal representing the interface's height fluctuations by computing the power spectra $S(f, T)$ for different values of the finite observation time T . He also supported the simulation results with the PSD characterization by experimentally studying growing interfaces in liquid-crystal turbulence. Interestingly, the height displays a signature of the $1/f^\gamma$ noise with a spectral exponent of $5/3$. The power does not explicitly depend on the finite observation time. However, the power spectrum lacks a lower cutoff frequency, implying that the underlying signals are nonstationary or T is much smaller than the underlying correlation time. We emphasize that a power-law tail in the PSD does not immediately imply that the underlying process is nonstationary.

In the discrete KPZ model with large system size $L \rightarrow \infty$, Henkel *et al.* [38] computed the ensemble-averaged two-time autocorrelation of the height noise. The correlation function reveals nonexponential behavior and dynamical scaling, but time-translational symmetry is violated. These findings further support that the dynamics remain transient toward a stationary state. If we assume an infinite-size system, it is not possible to reach a stationary state in the finite observation time,

implying that the dynamics of the nonequilibrium system would be nonstationary. The underlying two-time autocorrelation function shows an aging behavior. The typical aging behavior characterized by the two-time function exhibits the following characteristics: (1) nonexponential relaxation; (2) dynamic scaling; and (3) lack of time-translational invariance [38–40].

In this contribution, we examine the question of whether the $1/f^\gamma$ noise is a transient feature of the dynamics (nonstationary) or a genuine property of the stationary state. We consider discrete models belonging to the KPZ universality class. We reexamine the interface height fluctuations at a fixed site as a function of time in the monolayer unit. For a small system, we show that it is possible to reach the long-time limit or stationary state dynamics within a finite (long) observation time. We compute the two-time autocorrelation and power spectra independently. The correlation vanishes after a correlation time that diverges with system size. Moreover, the correlation function remains nonexponential and satisfies dynamic scaling. Consequently, the power spectra exhibit a lower cutoff displaying constant power. The lower cutoff frequency scales with the system size as $\sim L^{-z}$. This explains that, for larger systems, previous studies did not observe any cutoff and considered $1/f^\gamma$ noise as nonstationary fluctuations. In the nontrivial frequency regime, the power spectrum exhibits $1/f$ -type scaling with the spectral exponent $5/3$. Our findings, supported by scaling-theoretic arguments, establish that the dynamics are wide-sense stationary and the Wiener-Khinchin theorem applies.

The paper organization is as follows. In Sec. II, we begin by recalling four discrete models that describe rough interfaces in the KPZ universality class. We present numerical results for the temporal correlation and PSD of the height noises in the stationary state, along with finite-size scaling (FSS) in Sec. III. We also add a scaling argument here to support our results for the two characterizations in Sec. IV. Finally, the paper concludes with a discussion in Sec. V.

II. KPZ LATTICE MODELS

To illustrate the signature of stationary fluctuations in a finite system, we simulate the following discrete models (cellular automata) belonging to the KPZ universality class. Particularly, we examine the Kim-Kosterlitz (KK) model [41], the BD model [30], the etching model (EM) [42], and the SS model [32]. We consider a substrate as a 1D lattice of size L with periodic boundary conditions. To each site, i.e., $x \in [1, L]$, we associate a height variable $h(x, t)$ representing the number of deposited particles. Initially, the height variable h is set to 0, indicating that the lattice is empty. A random column is selected to deposit a particle, and the particle follows well-defined rules to stick to the surface. The rules are defined below for each model. Since the particle does not always stick to a random site, it creates a nonuniform (rough) growth of the interface. We consider a monolayer time unit; i.e., L deposition attempts constitute one Monte Carlo time step.

In the KK model, at each randomly selected column, a Kim-Kosterlitz slope constraint $h(x, t) - h(x \pm 1, t) \leq 1$ is checked. The height of the selected column increases by one unit only if the constraint is satisfied; otherwise, we similarly pick a new column. The KK model was presented as a simple system to understand the essential features of the KPZ universality class [41].

The BD was initially introduced to model colloidal aggregation. In this model, the particle falls vertically to the randomly selected column; however, it adheres upon first contact. The simulation can be easily performed by following a simple rule: If at time t the height is $h(x, t)$ to a randomly selected column, then $h(x, t + 1) = \max\{h(x \pm 1, t), h(x, t) + 1\}$ [30]. The simple rule yields a nonlinear growth in the vertical direction. Interestingly, the model falls into the KPZ universality class.

The third model we consider is the etching model. It was introduced as an atomistic model by Mello *et al.* to study the etching of a solid [42]. We consider the deposition version of the EM. We increase the height of a randomly selected column $h(x, t)$ by one unit, i.e., $h(x, t + 1) = h(x, t) + 1$, and the nearest-neighbor columns modify as $h(x \pm 1, t + 1) = h(x, t)$.

The last model that we studied is the SS model. In the SS model, we start with the groove initial condition $h(x, 0) = [1 + (-1)^x]/2$. If the randomly selected column is a local minimum (or

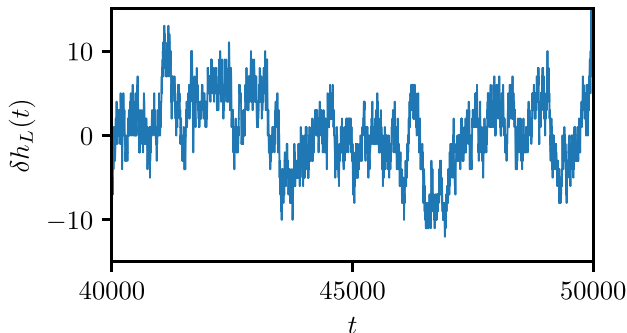


FIG. 1. A typical profile of the height noise signal $\delta h_L(t)$ in the KK model with the system size $L = 2^8$. The signal $\delta h_L(t)$ represents the difference between the height at a site and the spatially averaged mean height at time t . The time unit is in monolayers; i.e., one Monte Carlo time step is equivalent to L deposition attempts.

maximum), then we add (or remove) two particles with probability p (or $1 - p$). It is interesting to note that the height difference between neighboring sites always remains 1. We set $p = 1$, so the SS model belongs to the KPZ universality class [32].

III. RESULTS

Let $h(x, t)$ be the height of the fluctuating rough interface at position x after time t , which we measured in monolayer units. We consider L particles deposited per unit time for a 1D substrate of size L . As the mean height $\bar{h}(t) = \sum_{x=1}^L h(x, t)/L$ (spatially averaged) grows with time typically linearly, we consider a well-behaved temporal signal as height fluctuations,

$$\delta h_L(t) = h(t) - \bar{h}(t). \quad (3)$$

To uncover the underlying temporal correlations of the signal described in Eq. (3), we first compute the two-time autocorrelation function

$$C(\tau, L) = \langle \delta h_L(t) \delta h_L(t + \tau) \rangle, \quad (4)$$

where the angular brackets $\langle \cdot \rangle$ in Eq. (4) denote average over independent realizations of the interfaces. Since the periodic boundary conditions ensure that the system remains homogeneous in space, the correlation function of the temporal fluctuations $\delta h_L(t)$ remains the same for all sites.

Numerically, starting from a flat initial condition $h(x, t = 0) = 0$ and discarding transients up to 10^3 for a process with system size $L = 2^{10}$, we record the interface height $h(x, t)$ at a fixed position x with varying time t . We compute the height noise $\delta h_L(t)$ by removing the mean height (cf. Fig. 1). We set the finite observation time T much larger than the underlying correlation time of the process, which scales as $\mathcal{T} \sim L^z$, where z is the dynamic exponent. It implies that the dynamics reached a stationary state. To determine the two-time autocorrelation function numerically, we follow the time average and the ensemble average, both as

$$C(\tau, L) = \left\langle (T - \tau)^{-1} \sum_{t=1}^{T-\tau} \delta h_L(t) \delta h_L(t + \tau) \right\rangle. \quad (5)$$

Figure 2(a) shows the correlation function $C(\tau, L)$ for different system sizes $L = 2^6, 2^7, \dots, 2^{10}$. As shown in Fig. 2(b), the zero-lag correlation, or variance, varies linearly with the system size $C(0, L) \sim L$. Finally, we plot the scaling function obtained by FSS as the variation of the normalized correlation function $C(\tau, L)/C(0, L)$ with the reduced lag time with respect to the correlation time

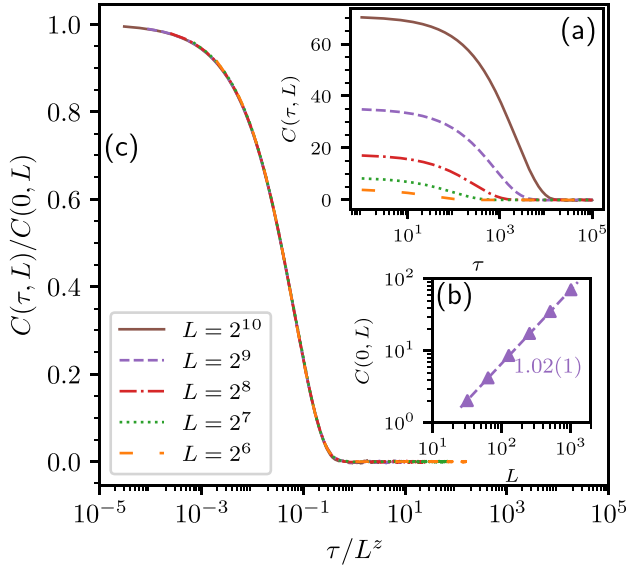


FIG. 2. (a) The two-time autocorrelation function $C(\tau, L)$ of the signal $\delta h_L(t)$ [cf. Eqs. (5) and (7)]. The lag time τ ranges from 1 to 10^5 , and the signal length T is 2^{22} . We used 120 independent realizations of the process for the ensemble average. (b) As the system size increases, the zero-lag correlation $C(0, L)$ grows linearly. (c) The data collapse is obtained by plotting $C(\tau, L)/C(0, L)$ with τ/L^z , where z is the dynamic exponent.

τ/L^z . The data collapse of the correlation function suggests that we can write a scaling ansatz,

$$C(\tau, L) = L F_C\left(\frac{\tau}{L^z}\right), \quad (6)$$

where F_C is a scaling function.

The correlation function consists of two contributions, one from zero lag time $C(0, L)$ and another from nonzero lag time $C(|\tau| > 0, L)$. As shown in Fig. 3(a), we further plotted the shifted correlation function $C(0, L) - C(\tau, L)$ versus τ and found a power-law growth for the short-time regime and a saturation-type behavior beyond the correlation time, for a fixed system size. Also, the saturation value depends on the system size. We also plot the normalized shifted correlation function $1 - C(\tau, L)/C(0, L)$ with the scaled lag time τ/L^z and find a reasonable data collapse [cf. Fig. 3(b)]. The corresponding scaling function behaves as $\nu^{1/z}$ for $\nu \ll 1$ and 1 for $\nu \gg 1$. Then, we can also write Eq. (6) as

$$C(\tau, L) \sim \begin{cases} L[1 - |\frac{\tau}{L^z}|^{1/z}], & \text{for } \tau \ll L^z, \\ \rightarrow 0, & \text{for } \tau \gg L^z. \end{cases} \quad (7)$$

Equation (7) suggests that the scaling function in Eq. (6) behaves as

$$F_C(\nu) \sim 1 - \nu^{1/z}, \quad \text{for } \nu \ll 1. \quad (8)$$

In Eq. (8), the scaling function decays exponentially in the regime $\nu \gg 1$ and tends to 0 when $\nu \rightarrow \infty$.

Alternatively, we can easily understand the scaling behavior of the correlation function. Notice that $2[C(0, L) - C(\tau, L)]$ is the same as the mean-squared displacement (MSD) of the interface height fluctuations,

$$2[C(0, L) - C(\tau, L)] = \langle [\delta h_L(t) - \delta h_L(t + \tau)]^2 \rangle.$$

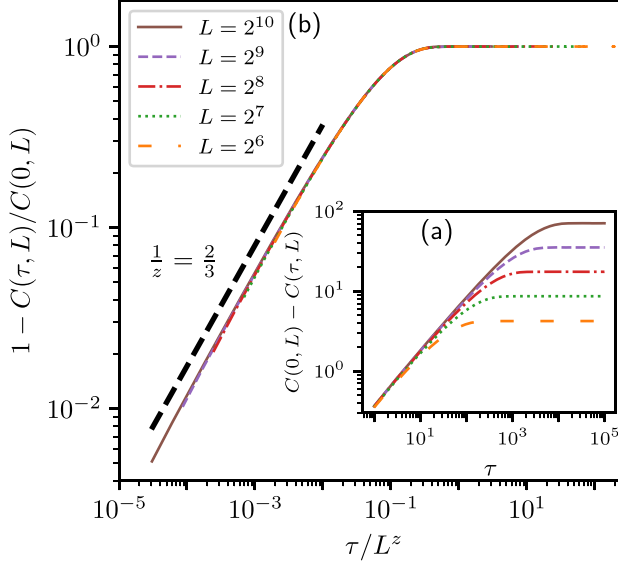


FIG. 3. (a) The shifted correlation function $C(0, L) - C(\tau, L)$, with the lag time τ , grows in a power-law manner in the short-time regime. Beyond the correlation time, the function saturates, with a value that depends on the system size. (b) A data collapse obtained by normalizing the shifted correlation function $1 - C(\tau, L)/C(0, L)$ and plotting it against the scaled lag time τ/L^z . The corresponding scaling function scales as $\sim \nu^{1/z}$ for $\nu \ll 1$ and tends to 1 for $\nu \gg 1$.

Since the MSD scales for the KPZ process as $\sim L^{2\alpha}(\tau/L^z)^{2\alpha/z}$ [cf. Eq. (2)], with $\alpha = 1/2$ and $z = 3/2$, it becomes immediately clear that the scaling function in Eq. (6) behaves as $F_C(\nu) \sim 1 - \nu^{1/z}$ for $\nu \ll 1$. Thus, the spectral exponent is $1/z + 1 = 5/3$.

Since the two-time autocorrelation function depends on the difference between the two times, the underlying signal is wide-sense stationary. We can apply the Wiener-Khinchin theorem to obtain the PSD by taking the Fourier transform of the correlation function,

$$S(f, L) \sim \int C(\tau, L) \exp(-i2\pi f \tau) d\tau. \quad (9)$$

Plugging Eq. (6) into Eq. (9), we can write

$$S(f, L) \sim L^{z+1} \int F_C\left(\frac{\tau}{L^z}\right) \exp\left(-i2\pi f L^z \frac{\tau}{L^z}\right) d\left(\frac{\tau}{L^z}\right).$$

Corresponding to the reduced lag time τ/L^z , we identify a reduced frequency fL^z . Then, the PSD follows a scaling ansatz

$$S(f, L) \sim L^{z+1} F_S(fL^z), \quad (10)$$

where the scaling function $F_S(fL^z)$ in Eq. (10) is the Fourier transform of the scaling function $F_C(\cdot)$ [cf. Eq. (8)]. After simplifying, we find

$$F_S(u) \sim \begin{cases} \text{constant}, & \text{for } u \ll 1, \\ u^{-(1+1/z)}, & \text{for } u \gg 1. \end{cases} \quad (11)$$

It is easy to note [cf. Eqs. (10) and (11)] that

$$S(f, L) \sim \begin{cases} L^{z+1}, & \text{for } f \ll L^{-z}, \\ f^{-(1+1/z)}, & \text{for } f \gg L^{-z}. \end{cases} \quad (12)$$

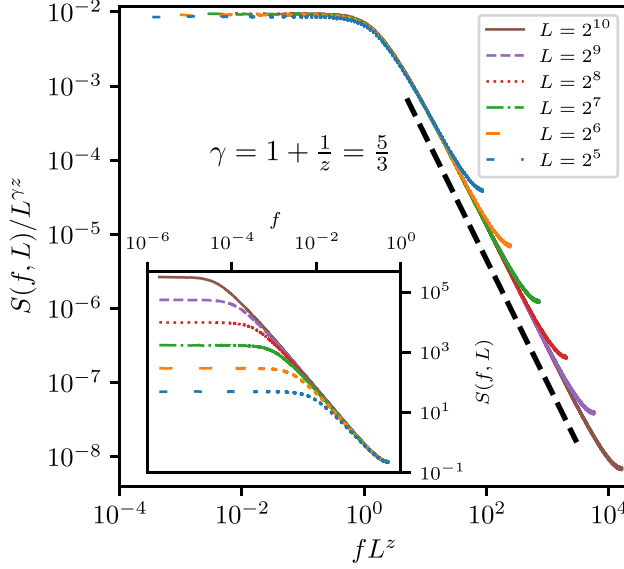


FIG. 4. Inset: The power spectrum $S(f, L)$ of the height noise $\delta h_L(t)$ in the KK model. The system size varies from 2^5 to 2^{10} . The initial 10^3 measurements are discarded as transient, and the signal length is $T = 2^{20}$. To reduce noise in the PSD for each curve, we perform an ensemble average over 10^4 independent realizations. Main panel: The data collapse of the power spectra, as shown in the inset [cf. Eqs. (10)–(12)]. The dashed (black) line guides the slope of $5/3$.

Numerically, we directly compute the PSD of the temporal signal $\delta h_L(t)$ of length T (the finite observation time) under the condition that $T \gg L^z$ as

$$S(f, L) = \lim_{T \rightarrow \infty} \frac{1}{T} \langle |\tilde{\delta h}_L(f)|^2 \rangle,$$

where the Fourier transform of the signal is

$$\tilde{\delta h}_L\left(f = \frac{k}{T}\right) = \sum_{t=0}^{T-1} \delta h_L(t) \exp\left(-i2\pi \frac{k}{T} t\right),$$

with $k = 0, 1, 2, \dots, T - 1$. We do not normalize the PSD because the power in the low-frequency component shows an explicit system-size dependence. We consider a frequency window $f \in [1/T, 1/2]$, with the lowest-frequency bins $\Delta f = 1/T$. Numerically, we implemented the standard fast Fourier transform (FFT) algorithm to compute the Fourier transform.

Figure 4 (inset) presents the power spectra for the signal $\delta h_L(t)$ for different system sizes. We note that for a fixed system size, the power spectrum remains constant below a lower cutoff frequency (inverse of the correlation time $\mathcal{T}$) and shows a power-law decaying behavior $\sim 1/f^\gamma$ in the nontrivial frequency regime $L^{-z} \ll f \ll 1/2$. We also note that below the lower cutoff frequency, the power in the low-frequency component grows with the system size in a power-law manner as $S(f \rightarrow 0, L) \sim L^{\gamma z}$ and the total power of the process scales as $P(L) \sim L^{(\gamma-1)z}$ (cf. Fig. 5). Numerically, we find that $\gamma z = 2.53(1)$ and $(\gamma - 1)z = 1.02(1)$. It is easy to note that $z = 1.51(2)$. As shown in Fig. 4 (main panel), applying FSS, we can obtain data collapse for the power spectra with the aid of the two exponents γz and z by plotting scaled power $S(f, L)/L^{\gamma z}$ with the reduced frequency $f L^z$. The clean data collapse obtained provides a precise estimate of the critical exponents.

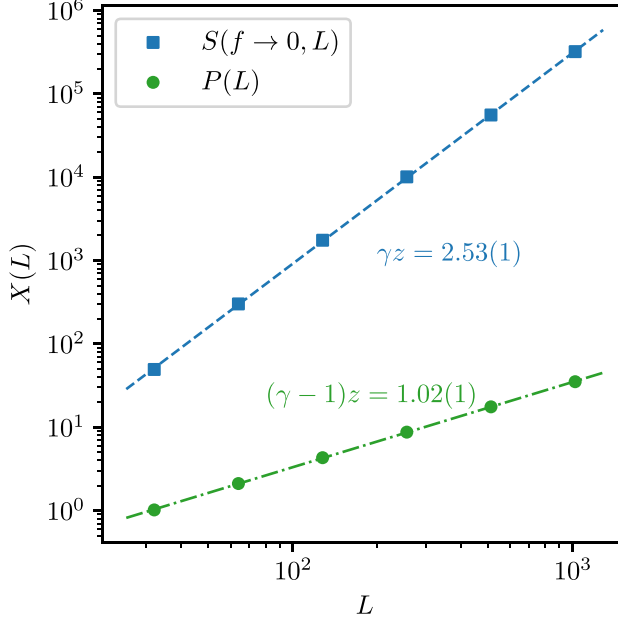


FIG. 5. The system-size scaling of the power in the low-frequency regime below the cutoff frequency, $\sim L^{-z}$. The lower curve shows the scaling of total power with system size. We numerically find the dynamic exponent $z = 1.51(2)$, which agrees well with the theoretical value of $3/2$.

Finally, to examine the extent of the scaling features observed for the KK model of the KPZ universality class, we also studied several discrete models (such as the EM, BD, and SS models) belonging to the same class. As shown in Fig. 6, the data collapse of the power spectra confirms that the scaling features are not model specific in the KPZ class.

To verify the stationarity of the height fluctuations in Eq. (1), we numerically integrate the KPZ equation using the Euler-Maruyama discretization scheme [43]. In 1D, we use the following discretization for the Laplacian and the square of the gradient:

$$\begin{aligned}\partial_x^2 h(x_i, t) &= (h_{i+1} + h_{i-1} - 2h_i)/(\Delta x)^2, \\ (\partial_x h(x_i, t))^2 &= (h_{i+1} - h_{i-1})^2/(2\Delta x)^2,\end{aligned}$$

where Δx is the lattice spacing. Let h_i^t be the height at site i at time t . Then, Eq. (1) can be written as

$$h_i^{t+1} = h_i^t + \frac{\nu \Delta t}{(\Delta x)^2} (h_{i+1}^t + h_{i-1}^t - 2h_i^t) + \frac{\lambda \Delta t}{4(\Delta x)^2} (h_{i+1}^t - h_{i-1}^t)^2 + \sqrt{2\Gamma \Delta t} \eta_i^t,$$

where Δt is the time increment and η is Gaussian noise with zero mean and unit variance, generated by the Box-Muller method. The nonlinear term can create intrinsic instabilities in the discretized equation for large λ . For this, one can apply the Lam and Shin method [44] or replace the term $(\nabla h)^2$ with a function $f((\nabla h)^2)$ [45,46]. We numerically choose the parameters as $\nu = 1$, $\lambda = 1$, $\Gamma = 1$, $\Delta x = 1$, and $\Delta t = 0.1$. For the discretized KPZ process, we compute the power spectra of height fluctuations and data collapse, as shown in Fig. 7. FSS reveals the same scaling exponents within statistical error. We again observe a lower cutoff frequency below which the power becomes constant. The presence of a lower cutoff implies that the underlying process is stationary.

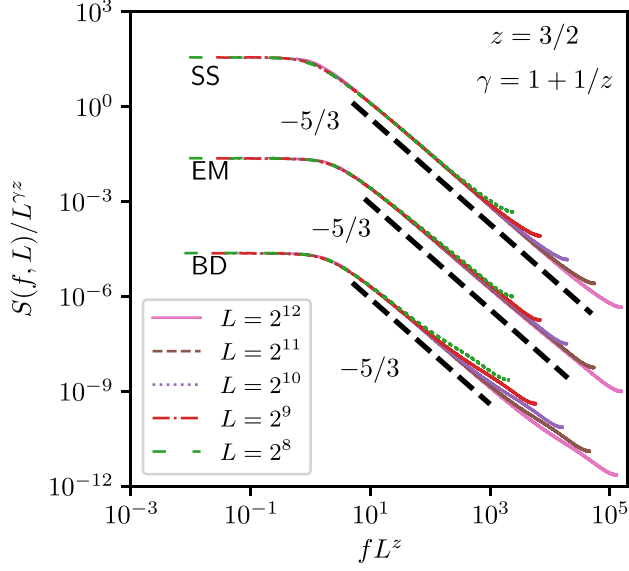


FIG. 6. The data collapse of the power spectrum $S(f, L)$ for the signal $\delta h_L(t)$ in different models. The BD and EM models belong to the KPZ universality class, and the results are consistent with the presented framework.

IV. CORRELATION FUNCTION SCALING ANSATZ

In this section, we present a simple scaling argument explaining the spectral exponent in terms of the growth exponents of the width fluctuations. We assume that the temporal correlation function $C(\tau, \mathcal{T})$ of the stationary noisy signal $\delta h_L(t)$ that exhibits temporally scale-invariant features is a generalized homogeneous function of its arguments [47], the lag time τ and the correlation time $\mathcal{T}$.

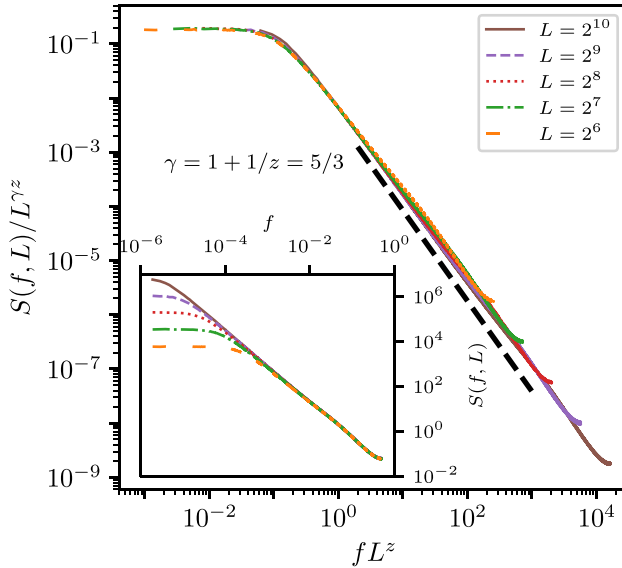


FIG. 7. Same as Fig. 4, but for the discretized KPZ process.

Then, we can write

$$C(\tau, \mathcal{T}) \sim b^{-1} C(b^{y_\tau} \tau, b^{y_\mathcal{T}} \mathcal{T}), \quad (13)$$

where b is a scaling factor. Setting $b^{y_\mathcal{T}} \mathcal{T} = 1$ or $b \sim \mathcal{T}^{-1/y_\mathcal{T}}$ in Eq. (13), we get

$$C(\tau, \mathcal{T}) \sim \mathcal{T}^{1/y_\mathcal{T}} F_C(\tau/\mathcal{T}^{y_\tau/y_\mathcal{T}}). \quad (14)$$

The argument of the scaling function should be dimensionless, suggesting $y_\tau = y_\mathcal{T}$. We express the correlation function in its conventional form as

$$C(\tau, \mathcal{T}) \sim \mathcal{T}^{\gamma-1} F_C(\tau/\mathcal{T}) \sim \tau^{\gamma-1} G_C(\tau/\mathcal{T}), \quad (15)$$

where the scaling functions are $G_C(v) \sim F_C(v)/v^{\gamma-1}$ and γ is the spectral exponent. Comparing Eqs. (14) and (15), we note that the correlation exponent is $\gamma - 1 = 1/y_\mathcal{T}$.

In critical systems, the correlation time diverges with the linear extent of the system, $\mathcal{T} \sim L^z$, with a dynamic exponent z . Writing Eq. (15) in terms of L , we get a scaling ansatz for the correlation function convenient for FSS as

$$C(\tau, L) \sim L^{z(\gamma-1)} F_C(\tau/L^z), \quad (16)$$

where the scaling function becomes zero for $v \gg 1$ and constant for $v = 0$. From Eq. (8), the expected behavior in the nontrivial regime is

$$F_C(v) \sim 1 - v^{(\gamma-1)}, \quad \text{for } v \ll 1.$$

The zero-lag correlation, or the variance, varies as

$$C(0, L) \sim L^{z(\gamma-1)} \sim w^2(L) \sim L^{2\alpha},$$

suggesting $2\alpha/z = \gamma - 1$. As the zero-lag correlation function varies as $\sim L^{z(\gamma-1)}$, one can numerically estimate the critical exponents by examining system size scaling of the variance. Comparing Eqs. (6) and (16), we note that $z(\gamma - 1) = 2\alpha = 1$. Particularly, the spectral exponent is

$$\gamma = 1 + 2\alpha/z, \quad (17)$$

suggesting $\gamma = 5/3$ for the KPZ class as $\alpha = 1/2$ and $z = 3/2$. We also emphasize that for the temporal noise in interface height, the nontrivial value of the spectral exponent is related to the growth exponent of the width fluctuations [cf. Eq. (17)].

V. CONCLUSION

In the KPZ universality class, we studied four discrete systems (Kim-Kosterlitz, ballistic deposition, single-step, and etching models) of kinetic rough interfaces in one spatial dimension, including a discretized version of Eq. (1). In particular, we examine temporal correlations of height fluctuations. Notice that previous studies examined the PSD of the height noise and concluded that the underlying processes are nonstationary. Similarly, the ensemble-averaged two-time correlation functions reveal a signature of aging behavior for large systems. However, we evolve the dynamics for a long time to attain a stationary state, ensuring that the finite observation time remains much larger than the underlying correlation time, which scales as $\mathcal{T} \sim L^z$. We compute the two-time autocorrelation function, which varies nonexponentially $L - \tau^{1/z}$ for $\tau \ll L^z$ and decays to zero thereafter. We can obtain a data collapse for the correlation functions for different system sizes using FSS. We also compute the PSD that displays a power-law scaling $1/f^\gamma$ with the spectral exponent value $\gamma = 1 + 1/z = 5/3$ [cf. Eq. (17)] in the nontrivial frequency regime $L^{-z} \ll f \ll 1/2$. The power shows a lower cutoff frequency, below which it remains constant. Our scaling arguments can relate the two characterizations.

Our main conclusion is that for a small system, it is possible to reach a stationary state, and the Wiener-Khinchin theorem remains valid. We provided numerical evidence that the $1/f^\gamma$ noise is a generic characteristic of the stationary state of the dynamics. The decreasing behavior of the

lower cutoff frequency with the system size, $\sim L^{-z}$, explains the absence of the saturation regime in previous studies. The scaling relation directly relates the temporal correlation to the width-scaling exponents. Experimentally, our approach is useful as single-site fluctuations can reveal the width characteristics. To examine the extent of stationary fluctuations, it would be interesting to investigate the role of dimensionality and anomalous rough interfaces. Our work clarifies the experimental limitation: The lower cutoff of the power spectrum is typically inaccessible due to the diverging feature of the correlation time.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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