

Dissipative charging of tight-binding quantum batteries

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We investigate dissipative charging of a single particle in finite tight-binding lattices coupled to engineered nonequilibrium Markovian reservoirs. The engineered number-conserving bond jumps convert local antisymmetric bond modes into symmetric modes, thereby steering the system toward upper spectral-edge states with finite steady-state ergotropy. In an even periodic chain, this mechanism admits a simple bond-mode interpretation and gives $\mathcal{E}_{ss} = 4t$ in the clean limit, where t denotes the uniform nearest-neighbor hopping amplitude. We further show that the same dissipative charging principle applies to a two-dimensional honeycomb tight-binding lattice. For on-site Anderson disorder, disorder-averaged calculations show that the mixed steady state becomes more spatially concentrated, while the final ergotropy decreases and the charging transient shortens over the examined parameter range. Additional local dephasing produces a similar finite-time trade-off in the displayed rate window. The finite-time metric $P_{0.99}$, defined by $P_{0.99} = 0.99\mathcal{E}_{ss}/\tau_{0.99}$, with $\tau_{0.99}$ the first time at which the ergotropy reaches 99% of its steady-state value, is therefore interpreted as a battery-side charging measure rather than the net thermodynamic power or efficiency of the full reservoir-engineering apparatus.

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I. INTRODUCTION

Quantum batteries, finite-dimensional quantum systems designed to store and deliver useful work, provide a concrete setting to explore fundamental questions at the interface of thermodynamics and quantum physics [1–4]. In this context, the performance of a quantum battery is naturally characterized by its ergotropy, defined as the maximum work extractable from a quantum state via unitary operations with respect to a fixed system Hamiltonian [5–9]. States with zero ergotropy are passive, while nonpassive states encode useful nonequilibrium resources that can be converted into work. A central goal is to identify mechanisms that generate and stabilize highly nonpassive states with large ergotropy.

A large body of work has shown that coherence, correlations, and collective effects can significantly enhance the charging power and extractable work in closed many-body systems [10–23]. In realistic settings, however, quantum batteries operate as open quantum systems subject to decoherence and dissipation. Standard Markovian thermalization or amplitude-damping processes typically drive the system toward Gibbs states or the ground state, which are completely passive and thus possess vanishing ergotropy for a single reservoir [6–9]. This leads to self-discharging and aging

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effects, observed both in theoretical models and in experimental platforms such as superconducting circuits and NV centers [24–27]. A key challenge is therefore to devise charging protocols that remain effective in the presence of unavoidable environmental noise.

Dissipation, however, can also be engineered as a resource. Recent studies have demonstrated that suitably engineered environments can be harnessed for quantum batteries. Non-Markovian reservoirs or structured spectral densities can slow down energy leakage or induce partial revivals [28–39]. Moreover, following the seminal proposal of dissipative charging in Ref. [40], autonomous dissipative charging schemes have been proposed in which engineered reservoirs drive the battery toward nonpassive steady states with finite ergotropy, without explicit time-dependent control on the battery degrees of freedom [10–13, 41–45]. Related driven-dissipative schemes, where coherent driving and engineered dissipation act jointly, have also been shown to enhance power and ergotropy [16, 46]. These works establish reservoir engineering as a powerful tool for mitigating dissipation-induced degradation.

Within this broad landscape, lattice and spin-chain quantum batteries offer a particularly appealing platform. Tight-binding and spin-chain Hamiltonians arise naturally in condensed-matter systems and quantum simulators, and their band structures provide additional degrees of freedom for energy storage. Previous studies have shown that interactions, hopping, topology, or feedback can enhance stored energy or stabilize charged states under dissipation [32, 34, 35, 41, 47–54]. In most of these proposals, however, dissipation primarily plays a stabilizing or protective role, while the actual charging is driven by coherent dynamics or external resources.

From the perspective of ergotropy, this raises a simple but important question: Can a stationary engineered reservoir itself act as the charger of a tight-binding quantum battery by selecting nonpassive high-energy states? For a fixed Hamiltonian, occupation near the upper part of the spectrum can support large ergotropy, whereas conventional thermalization under a single equilibrium reservoir produces passive states with no extractable work [5–9]. Realizing such a state therefore requires a nonthermal dissipative process that breaks detailed balance in a controlled manner and whose energetic cost is understood as part of the external nonequilibrium reservoir that sustains the effective Lindblad dynamics.

In this work, we study this problem in a minimal single-particle tight-binding lattice battery driven by bond dissipation. The engineered jump operators locally convert antisymmetric bond modes into symmetric ones, thereby favoring upper spectral-edge occupation and generating finite steady-state ergotropy. In the clean one-dimensional (1D) chain, this mechanism admits a simple bond-mode interpretation and gives $\mathcal{E}_{ss} = 4t$ for an even periodic lattice. Here, $\mathcal{E}_{ss}$ denotes the steady-state ergotropy and t is the uniform nearest-neighbor hopping amplitude. We then show that the same dissipative charging principle extends to a two-dimensional honeycomb tight-binding lattice. Finally, we analyze on-site Anderson disorder and local dephasing as perturbations of this effective charging protocol. The disorder results are averaged over independent realizations and are characterized by both the slowest nonzero Liouvillian decay rate and the real-space inverse participation ratio of the mixed steady-state density profile. We find that disorder and dephasing can shorten the finite-time charging transient over the examined parameter ranges, although they reduce the final steady-state ergotropy. Accordingly, the finite-time metric $P_{0.99}$, defined as $0.99\mathcal{E}_{ss}/\tau_{0.99}$ with $\tau_{0.99}$ the first time required to reach $0.99\mathcal{E}_{ss}$, should be interpreted as a battery-side charging measure, not as the net thermodynamic power or efficiency of the full reservoir-engineering apparatus.

II. THEORETICAL FRAMEWORK FOR OPEN-SYSTEM QUANTUM BATTERY CHARGING

We set $\hbar = k_B = 1$ throughout. The finite-dimensional battery Hamiltonian H_B has the spectral decomposition

$$H_B = \sum_{n=1}^D E_n |E_n\rangle\langle E_n|. \quad (1)$$

Here, D is the Hilbert-space dimension, and the eigenvalues are ordered increasingly, $E_{n+1} \geq E_n$. For a density matrix ρ , the ergotropy is the maximum work extractable by a cyclic unitary operation [55]:

$$\mathcal{E}(\rho) = \text{Tr}[H_B \rho] - \min_U \text{Tr}[H_B U \rho U^\dagger]. \quad (2)$$

The minimization runs over all unitaries on the system Hilbert space. Here, cyclic means that the control is switched off at the beginning and end of the extraction step, so the system Hamiltonian returns to H_B . The minimum of $\text{Tr}[H_B U \rho U^\dagger]$ defines the associated passive state σ_ρ , which is diagonal in the energy eigenbasis,

$$\sigma_\rho = \sum_n r_n |E_n\rangle\langle E_n|, \quad (3)$$

with populations arranged in nonincreasing order, $r_{n+1} \leq r_n$, as the energy increases. Consequently, $\mathcal{E}(\rho) = \text{Tr}[H_B(\rho - \sigma_\rho)]$. Recent work by Malavazi *et al.* [56] has introduced the concepts of isoergotropic states and ergotropy-preserving operations, providing a systematic framework for understanding how stored energy can be manipulated and protected in quantum batteries. Beyond ergotropy, the fluctuations in extractable work also carry operational significance. Recent work by Sarkar *et al.* [57] has shown that open-system dynamics offer superior control over such fluctuations compared to closed unitary processes, revealing a fundamental trade-off between resource cost and energy extraction reliability.

In a closed-system charging protocol, the battery evolves under a time-dependent Hamiltonian $H(\tau) = H_B + V(\tau)$, where $V(\tau)$ is a control potential applied over a finite interval $[0, \tau']$. The resulting unitary evolution is

$$U(\tau') = \mathcal{T}_+ \exp \left[-i \int_0^{\tau'} H(s) ds \right], \quad (4)$$

which transforms the initial state ρ into $\rho(\tau') = U(\tau')\rho U^\dagger(\tau')$. The work extracted in such a cycle is $W = \text{Tr}[H_B(\rho - \rho(\tau'))]$, and the ergotropy is obtained by optimizing over admissible controls $V(\tau)$, equivalently over unitaries U .

Beyond this operational definition, thermodynamic principles impose universal constraints on extractable work. Here, $S(\rho) := -\text{Tr}[\rho \ln \rho]$ denotes the von Neumann entropy. For a system with Hamiltonian H_B in state ρ , the relevant bound follows directly from the positivity of quantum relative entropy. When an entropy-matched Gibbs state exists on the non-negative-temperature branch, let $\bar{\beta} \geq 0$ be chosen such that $S(\rho_{\bar{\beta}}) = S(\rho)$, where $\rho_{\bar{\beta}} = e^{-\bar{\beta} H_B} / \mathcal{Z}_{\bar{\beta}}$ and $\mathcal{Z}_{\bar{\beta}} = \text{Tr}(e^{-\bar{\beta} H_B})$. For finite $0 < \bar{\beta} < \infty$, positivity of the quantum relative entropy $D(\rho \| \sigma) = \text{Tr}[\rho(\ln \rho - \ln \sigma)]$ gives

$$D(\rho \| \rho_{\bar{\beta}}) = \bar{\beta}(\text{Tr}[H_B \rho] - \text{Tr}[H_B \rho_{\bar{\beta}}]) \geq 0, \quad (5)$$

where $S(\rho_{\bar{\beta}}) = S(\rho)$ has been used. The end point cases $\bar{\beta} = 0$ and $\bar{\beta} \rightarrow \infty$ are understood by continuity. This motivates the bound

$$W_{\text{bound}} := \text{Tr}[H_B \rho] - \text{Tr}[H_B \rho_{\bar{\beta}}]. \quad (6)$$

For a single battery in state ρ , the maximum work extractable by unitary operations is the ergotropy $\mathcal{E}(\rho) = \text{Tr}[H_B(\rho - \sigma_\rho)]$. The quantity W_{bound} is the asymptotic many-copy value. For the noninteracting N -copy Hamiltonian $H_B^{(N)} = \sum_{r=1}^N H_B^{(r)}$, global unitaries can asymptotically approach this value per copy,

$$\lim_{N \rightarrow \infty} \frac{1}{N} \mathcal{E}(\rho^{\otimes N}; H_B^{(N)}) = W_{\text{bound}},$$

whereas $\mathcal{E}(\rho)$ is the optimized single-copy value [1].

In open-system charging scenarios, the battery interacts with an environment. One route to engineered dissipation uses repeated interactions with auxiliary systems prepared in thermal states. In this collisional model, the battery interacts sequentially with identical ancillas in state $\omega_\beta(H_A)$, each for a time τ . One collision produces the completely positive and trace-preserving map $\Phi(\rho) = \text{Tr}_A[U(\rho \otimes \omega_\beta(H_A))U^\dagger]$. Here, $U = e^{-i\tau(H_B+H_A+V)}$. If the unitary operator U satisfies $[U, H_0 + H_A] = 0$ for a battery operator H_0 , the map has the fixed point $\pi = e^{-\beta H_0} / \text{Tr}(e^{-\beta H_0})$. When $H_0 \neq H_B$, this steady state can be nonpassive with respect to H_B . For example, $H_0 = -H_B$ gives the population-inverted state $\pi \propto e^{\beta H_B}$. The cost of preparing the ancillas is then part of the charging resource.

We now specialize to a standard Markovian description of dissipative charging. We consider a composite system described by the total Hamiltonian

$$H_{\text{tot}} = H_B + H_R + H_{BR}, \quad (7)$$

where H_R is the reservoir Hamiltonian and H_{BR} denotes the battery-reservoir coupling. We assume weak coupling, Markovianity, and a secular approximation, so the reduced battery dynamics has the time-homogeneous Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) form [58–61]

$$\frac{d\rho(\tau)}{d\tau} = \mathcal{L}[\rho(\tau)] = -i[H_B, \rho(\tau)] + \sum_k \gamma_k \left(L_k \rho(\tau) L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho(\tau)\} \right). \quad (8)$$

The jump operators L_k and rates $\gamma_k > 0$ describe dissipative exchange with the environment. The superoperator $\mathcal{L}$ is the Liouvillian, and the formal solution is $\rho(\tau) = e^{\mathcal{L}\tau} \rho(0)$. A stationary state ρ_{ss} satisfies $\mathcal{L}[\rho_{\text{ss}}] = 0$. When the zero mode is unique and all other Liouvillian eigenvalues have negative real parts, the dynamics relaxes to ρ_{ss} for any initial state. A primitive thermal generator satisfying quantum detailed balance relaxes to the Gibbs state, which is passive with respect to H_B .

Engineered dissipators can stabilize selected Hamiltonian eigenstates, localized density-matrix profiles, and other nonthermal steady states in disordered, quasiperiodic, flat-band, gauge, and scarred settings [62–72]. We apply this idea using a bond-resolved reservoir that selects a nonpassive state near the upper edge of a tight-binding band.

III. CHARGING IN TIGHT-BINDING LATTICE MODELS

All calculations below are carried out in the single-particle sector. For the one-dimensional chain, L denotes the number of lattice sites and hence the battery size; the density matrix ρ is a positive semidefinite $L \times L$ operator satisfying $\text{Tr} \rho = 1$. The second-quantized notation makes the locality of the bond jumps transparent. The battery Hamiltonian is

$$H_B = \sum_{\langle i, j \rangle} (t_{ij} c_i^\dagger c_j + \text{H.c.}). \quad (9)$$

Here, $c_i^\dagger$ (c_i) creates (annihilates) a particle on site i . The coefficient t_{ij} is the nearest-neighbor hopping amplitude on bond $\langle i, j \rangle$. For a clean periodic lattice, translational invariance gives discrete Bloch momenta that sample the corresponding band structure $E(\mathbf{k})$. Populating high-energy states can increase stored energy, but the ergotropy depends on the full density matrix and its passive rearrangement.

We engineer a charging process by coupling the lattice to a structured environment via bond dissipation. Specifically, on each bond $\langle i, j \rangle$ we introduce a jump operator

$$L_{ij} = (c_i^\dagger + \eta^* c_j^\dagger)(c_i - \eta c_j), \quad (10)$$

with $\eta = e^{i\phi}$ and $\phi \in \mathbb{R}$. The jump conserves the total particle number $N = \sum_i c_i^\dagger c_i$, since $[L_{ij}, N] = 0$. Defining the local bond modes

$$b_{ij,+} = \frac{c_i + \eta c_j}{\sqrt{2}}, \quad b_{ij,-} = \frac{c_i - \eta c_j}{\sqrt{2}}, \quad (11)$$

one obtains

$$L_{ij} = 2b_{ij,+}^\dagger b_{ij,-}. \quad (12)$$

Thus, each jump converts the antisymmetric bond component into the symmetric one. For $\eta = 1$, a dark state obeys $b_{ij,-}|\psi\rangle = 0$ on every bond and therefore has equal neighboring amplitudes. In a periodic clean chain, this condition selects the $k = 0$ state. With the hopping convention used here and $t > 0$, that state has energy $E = 2t$ and lies at the upper band edge.

We first analyze a 1D homogeneous chain with L sites, periodic boundary conditions, and uniform hopping $t_{ij} = t$. The Bloch dispersion is $E(k_\ell) = 2t \cos(k_\ell)$, where $k_\ell = 2\pi\ell/L$ and $\ell = 0, 1, \dots, L-1$. The band spans $[-2t, 2t]$, and the states at $k_\ell = 0$ ($k_\ell = \pi$) lie at the band top (bottom). For the even periodic chain considered here, the clean steady state selected by the bond jumps is the pure $k = 0$ state. Its passive counterpart occupies the band bottom, giving $\mathcal{E}_{ss} = E_{\max} - E_{\min} = 4t$; unless stated otherwise, the numerical calculations use $t = \gamma = 1$, where γ is the uniform bond-dissipation rate acting uniformly on every bond, and hence $\mathcal{E}_{ss} = 4$.

To quantify the charging performance, we solve $\mathcal{L}[\rho_{ss}] = 0$ and compute the ergotropy from the passive-state rearrangement. We diagonalize $\rho_{ss} = \sum_\alpha p_\alpha |r_\alpha\rangle\langle r_\alpha|$ with $p_1 \geq p_2 \geq \dots$. We order the Hamiltonian eigenstates as $H_B = \sum_\alpha E_\alpha |E_\alpha\rangle\langle E_\alpha|$ with $E_1 \leq E_2 \leq \dots$. The associated passive state is

$$\sigma_{ss} = \sum_\alpha p_\alpha |E_\alpha\rangle\langle E_\alpha|. \quad (13)$$

This gives $\mathcal{E}_{ss} = \text{Tr}[H_B(\rho_{ss} - \sigma_{ss})]$. Formally, the optimizing passifying unitary is

$$U_{\text{opt}} = \sum_\alpha |E_\alpha\rangle\langle r_\alpha|, \quad (14)$$

up to rotations within degenerate subspaces. All ergotropy values below use this optimization. Equation (14) specifies the target unitary mathematically; its platform-dependent decomposition into control pulses is separate from the stored-ergotropy analysis considered here.

When $\eta = 1$, the dissipator converts out-of-phase bond components ($k \approx \pi$) into in-phase components ($k \approx 0$). This process prepares a steady state concentrated near the band top, as illustrated in Figs. 1(a) and 1(b). Starting from the passive state associated with ρ_{ss} , the ergotropy

$$\mathcal{E}(\tau) = \text{Tr}[H_B \rho(\tau)] - \min_U \text{Tr}[H_B U \rho(\tau) U^\dagger] \quad (15)$$

grows from zero and approaches its steady-state value, as shown in Fig. 1(c).

The finite-time charging metric is

$$P_{0.99} = \frac{0.99\mathcal{E}_{ss}}{\tau_{0.99}}. \quad (16)$$

Here, $\tau_{0.99}$ is the first time at which $\mathcal{E}(\tau_{0.99}) = 0.99\mathcal{E}_{ss}$. This metric combines the stored ergotropy and the relaxation time. The independent diagnostic of long-time relaxation is the Liouvillian spectral gap

$$\Delta_{\mathcal{L}} := -\max_{\lambda_\alpha \neq 0} \text{Re } \lambda_\alpha. \quad (17)$$

Here, $\{\lambda_\alpha\}$ are the Liouvillian eigenvalues, and the stationary zero mode is excluded. When that mode is unique and no other eigenvalue lies on the imaginary axis, $\Delta_{\mathcal{L}} > 0$ sets the slowest asymptotic exponential decay rate.

We next add on-site Anderson disorder, $H_{\text{dis}} = \sum_i \epsilon_i c_i^\dagger c_i$, with independent $\epsilon_i \in [-W/2, W/2]$. For every nonzero W in Fig. 2, we average over $N_{\text{dis}} = 100$ realizations and report the standard error of the mean (SEM). In many cases, the SEM is smaller than the marker size or line width and is therefore not separately visible. The clean point is deterministic. Over the range studied, disorder

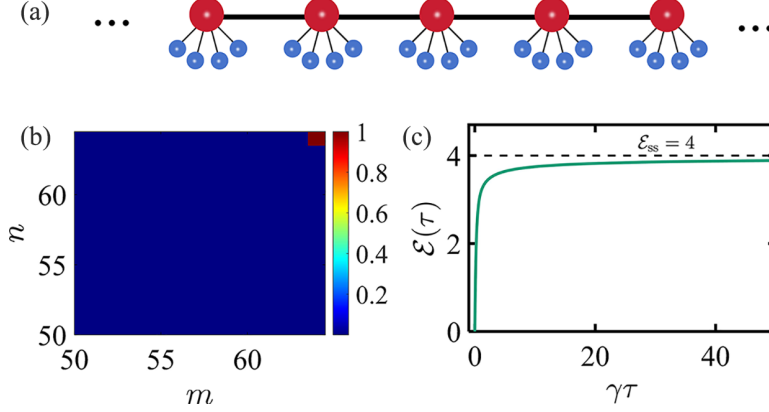


FIG. 1. Clean tight-binding quantum battery. (a) Schematic of a dissipative tight-binding chain. Red spheres denote lattice sites, and the blue structures schematically denote the engineered reservoir degrees of freedom whose phase-matched coupling realizes the bond jump. (b) Absolute value of the steady-state density matrix in the energy basis, $|\langle E_n | \rho_{ss} | E_m \rangle|$, for a periodic one-dimensional chain. Both axes are ordered energy-eigenstate indices n, m ; the displayed block contains the 15 highest-energy states ($n, m = 50, \dots, 64$). (c) Ergotropy $\mathcal{E}(\tau)$ for the passive state associated with ρ_{ss} . The dashed line marks $\mathcal{E}_{ss} = 4$. Unless stated otherwise, numerical results are obtained for $L = 64$ with periodic boundary conditions.

produces an overall reduction in $\mathcal{E}_{ss}$ and shortens the approach to the steady state, yielding an overall increase in $P_{0.99}$.

Appendix A shows that finite disorder raises $\Delta_{\mathcal{L}}$ above the clean value. After the initial increase, the disorder-averaged gap approximately plateaus with small nonmonotonic fluctuations. The corresponding finite-size trends are presented in Appendix B.

To characterize the spatial structure of the mixed steady state, we use the real-space inverse participation ratio (IPR) I_p^{ss} of the diagonal steady-state density profile, following density-matrix-based diagnostics used in previous studies of localization in open quantum systems [62–64]. This quantity is not the IPR of a single Hamiltonian eigenstate; it is the IPR of the physical spatial occupation distribution selected by the Liouvillian steady state. Its definition and a direct comparison with the eigenstate-averaged Hamiltonian IPR benchmark are given in Appendix C. The same Appendix also

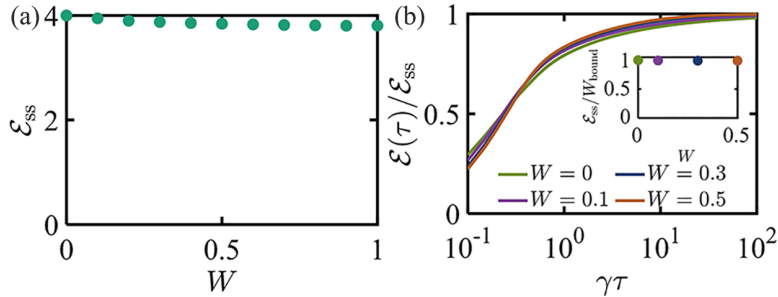


FIG. 2. Effect of on-site Anderson disorder in a periodic one-dimensional chain with $L = 64$. (a) Disorder-averaged steady-state ergotropy $\mathcal{E}_{ss}$ as a function of W . (b) Disorder-averaged charging curves $\mathcal{E}(\tau)/\mathcal{E}_{ss}$ for $W = 0, 0.1, 0.3$, and 0.5 . Each realization is normalized by its own steady-state ergotropy before averaging. The inset shows the disorder-averaged ratio $\mathcal{E}_{ss}/W_{\text{bound}}$. Every nonzero- W point or curve uses $N_{\text{dis}} = 100$ independent realizations; error bars or bands denote SEM and are often smaller than the line width or markers, so they are not separately visible. The clean result is calculated once.

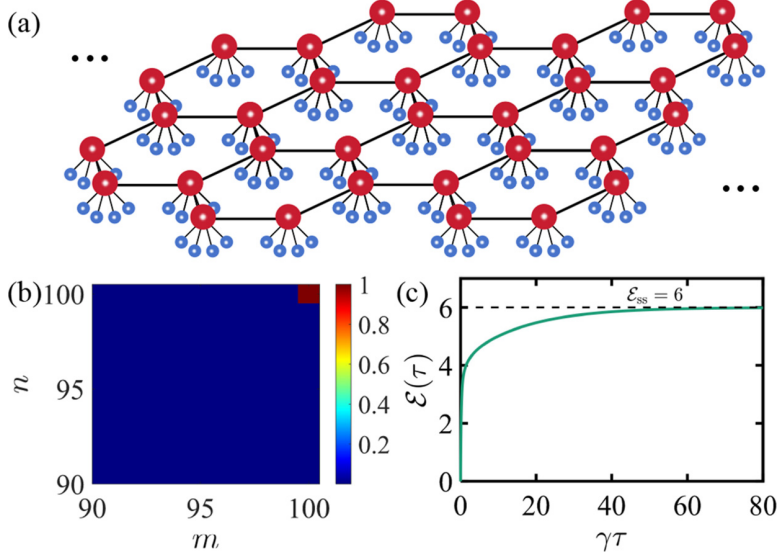


FIG. 3. Honeycomb tight-binding lattice as a quantum battery. (a) Schematic of the dissipative honeycomb lattice. Red spheres denote lattice sites, and the blue structures schematically denote the engineered reservoir degrees of freedom whose phase-matched coupling realizes the bond jump. (b) Absolute values of the steady-state density-matrix elements in the energy eigenbasis, $|\langle E_n | \rho_{ss} | E_m \rangle|$, for $\eta = 1$. The displayed 11×11 block contains the 11 highest-energy states, $n, m = 90, \dots, 100$. (c) Ergotropy $\mathcal{E}(\tau)$ for the passive state associated with ρ_{ss} . All data in this figure are computed for a clean lattice with $W = 0$, $N_{\text{site}} = 100$, and periodic boundary conditions.

reports the normalized steady-state energy and the highest-energy-quarter occupation, showing that the steady state remains biased toward the upper part of the spectrum over the on-site-disorder range considered.

We emphasize that this density-profile diagnostic is not used to claim that localization universally enhances battery performance. Rather, it characterizes the spatial concentration of the actual mixed steady state. The increase of $P_{0.99}$ in Fig. 2 should instead be understood as a finite-time effect: On-site disorder lowers the final steady-state ergotropy but shortens the charging transient. We therefore restrict this conclusion to the on-site-disorder ensemble and parameter range examined here. This interpretation is consistent with the increase of the slowest nonzero Liouvillian decay rate shown in Appendix A.

The inset of Fig. 2(b) compares the optimized single-copy ergotropy with the entropy-matched bound in Eq. (6). The ratio remains high for the parameters shown.

Next, we consider a two-dimensional honeycomb tight-binding lattice, whose nearest-neighbor Hamiltonian has the standard graphene band structure. The dissipative lattice geometry is shown schematically in Fig. 3(a). Applying Eq. (10) with $\eta = 1$ on every nearest-neighbor bond produces a steady state weighted toward the top of the upper band [Fig. 3(b)]. The resulting state has finite ergotropy. The bond jump operator is applied to every nearest-neighbor bond of the honeycomb lattice.

Starting from the passive state associated with ρ_{ss} , the ergotropy increases toward its steady-state value, as shown in Fig. 3(c).

The honeycomb calculation uses the same on-site Anderson distribution as the chain. Figure 4(a) shows that the disorder-averaged steady state retains weight in the upper spectral sector at $W = 0.6$. In Fig. 4(b), disorder decreases the final ergotropy but shortens the transient sufficiently to increase $P_{0.99}$ over the displayed range. The honeycomb lattice exhibits acceleration over a finite disorder interval, as does the chain.

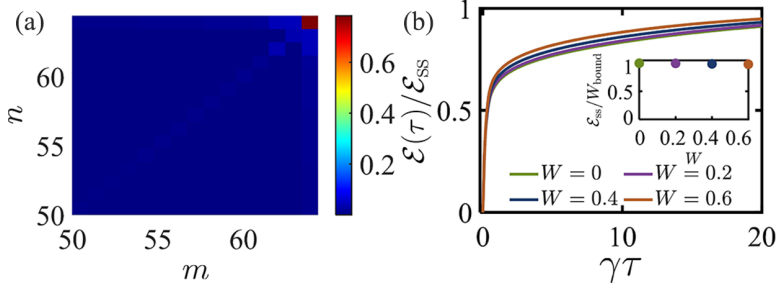


FIG. 4. On-site Anderson disorder in a periodic honeycomb tight-binding lattice with 64 sites. (a) Disorder-averaged $|\langle E_n | \rho_{ss} | E_m \rangle|$ at $W = 0.6$ for the 15 highest-energy states ($n, m = 50, \dots, 64$). The energy basis is ordered separately for each realization before averaging. (b) Disorder-averaged charging curves $\mathcal{E}(\tau)/\mathcal{E}_{ss}$ for $W = 0, 0.2, 0.4$, and 0.6 ; each realization is normalized by its own $\mathcal{E}_{ss}$. The inset shows the disorder-averaged $\mathcal{E}_{ss}/W_{\text{bound}}$. Every nonzero- W dataset uses $N_{\text{dis}} = 100$ independent realizations. Error bars or bands denote SEM and are often smaller than the line width or markers, so they are not separately visible; the clean result is calculated once.

IV. RESERVOIR-ENGINEERING ROUTE AND LOCAL DEPHASING

A microscopic reservoir-engineering construction produces the effective jump in Eq. (10). For bond $\langle i, j \rangle$, introduce the phase-resolved states

$$|b_{ij,\pm}\rangle = \frac{|i\rangle \pm \eta^* |j\rangle}{\sqrt{2}}, \quad J_{ij} = |b_{ij,+}\rangle \langle b_{ij,-}| = \frac{1}{2} L_{ij}. \quad (18)$$

Raman or microwave phases can then select only $|b_{ij,-}\rangle$ for excitation to an auxiliary state $|a_{ij}\rangle$,

$$H_{SA}^{(ij)} = \Delta |a_{ij}\rangle \langle a_{ij}| + \Omega_{ij} |a_{ij}\rangle \langle b_{ij,-}| + \text{H.c.} \quad (19)$$

The return step is generated microscopically by coupling $|a_{ij}\rangle$ to the two bond sites through the same reservoir modes. With $S_{ij,\mu} = |\mu\rangle \langle a_{ij}|$ and reservoir operators $d_{ij,q}$, use the reservoir Hamiltonian

$$H_R^{(ij)} = \sum_q \omega_q d_{ij,q}^\dagger d_{ij,q}. \quad (20)$$

The rotating-wave coupling to the auxiliary state is

$$H_{aR}^{(ij)} = \sum_{q,\mu=i,j} g_{\mu q}^{(ij)} d_{ij,q}^\dagger S_{ij,\mu} + \text{H.c.} \quad (21)$$

Tracing out a vacuum Markov reservoir gives

$$\mathcal{L}_R^{(ij)}[\rho] = \sum_{\mu,v=i,j} \Gamma_{\mu v}^{(ij)} \left(S_{ij,\mu} \rho S_{ij,v}^\dagger - \frac{1}{2} \{ S_{ij,v}^\dagger S_{ij,\mu}, \rho \} \right), \quad (22)$$

with coefficients

$$\Gamma_{\mu v}^{(ij)} = 2\pi \sum_q g_{\mu q}^{(ij)} g_{v q}^{(ij)*} \delta(\omega_q - \omega_a). \quad (23)$$

When the two resonant return paths are indistinguishable and satisfy $g_{jq}^{(ij)} = \eta^* g_{iq}^{(ij)}$, the coefficient matrix and its only nonzero jump channel are

$$\mathbf{\Gamma}^{(ij)} = \frac{\Gamma_{ij}}{2} \begin{pmatrix} 1 & \eta \\ \eta^* & 1 \end{pmatrix}, \quad (24)$$

$$R_{ij} = \sqrt{\Gamma_{ij}} |b_{ij,+}\rangle \langle a_{ij}|. \quad (25)$$

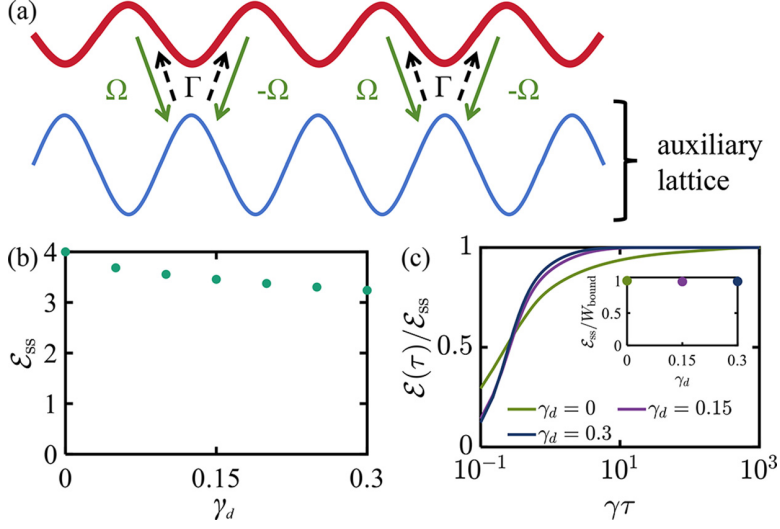


FIG. 5. Reservoir-engineering schematic and local dephasing. (a) Phase-selective coupling with amplitude Ω and collective reset with rate Γ . (b) Steady-state ergotropy as a function of the dephasing rate γ_d . (c) Normalized charging dynamics $\mathcal{E}(\tau)/\mathcal{E}_{ss}$ from the corresponding passive initial states. The $P_{0.99}$ values were evaluated by propagating each trajectory until the $0.99\mathcal{E}_{ss}$ threshold was reached. The corresponding values are $P_{0.99}(\gamma_d = 0) = 0.0170$, $P_{0.99}(\gamma_d = 0.15) = 0.570$, and $P_{0.99}(\gamma_d = 0.3) = 0.868$. The inset compares the optimized ergotropy with W_{bound} . The calculations use a periodic chain with $L = 64$, $W = 0$, and $t = \gamma = 1$.

Thus, the emitted reservoir quantum carries no which-site record, and the particle returns coherently to $|b_{ij,+}\rangle$.

In the weak-coupling regime $|\Omega_{ij}| \ll \sqrt{\Delta^2 + \Gamma_{ij}^2}/4$, eliminating the auxiliary state gives

$$L_{ij}^{\text{eff}} = \frac{\sqrt{\Gamma_{ij}}\Omega_{ij}}{\Delta - i\Gamma_{ij}/2} J_{ij}, \quad (26)$$

$$\gamma_{ij} = \frac{\Gamma_{ij}|\Omega_{ij}|^2}{4(\Delta^2 + \Gamma_{ij}^2/4)}. \quad (27)$$

This rate uses the convention $\gamma_{ij}\mathcal{D}[L_{ij}]$. If the return paths are distinguishable, the off-diagonal coefficients in Eq. (23) disappear and the result is an incoherent pair of local refill channels; ordinary decay to the vacuum instead produces loss of $|b_{ij,-}\rangle$. Bond-selective reservoir modes or frequencies suppress cross-bond correlations. Phase-selective auxiliary-state layouts have been proposed for dissipative Rydberg lattices [72–75], while a cavity, waveguide, Raman-assisted channel, or circuit-QED reservoir can provide the collective return [62,76–87]. Appendix D gives the full Lindblad and Kraus derivation, including the accompanying ac-Stark shift. Figure 5(a) summarizes the two steps.

In experimental settings, quantum systems are exposed to environmental noise, among which local dephasing is a common decoherence mechanism. We therefore examine the effect of additional local dephasing on the proposed dissipative charging scheme. To this end, we include local dephasing channels described by Lindblad operators $O_j = c_j^\dagger c_j$ with dephasing rate γ_d . The resulting dynamics is governed by the master equation

$$\dot{\rho} = -i[H_B, \rho] + \gamma \sum_{\langle i,j \rangle} \mathcal{D}[L_{ij}](\rho) + \gamma_d \sum_j \mathcal{D}[O_j](\rho), \quad (28)$$

where $\mathcal{D}[O](\rho) = O\rho O^\dagger - \frac{1}{2}\{O^\dagger O, \rho\}$.

Figures 5(b) and 5(c) show the one-dimensional chain with both bond dissipation and local dephasing. Over the rates displayed, increasing γ_d reduces $\mathcal{E}_{ss}$ but shortens $\tau_{0.99}$ sufficiently to increase $P_{0.99}$. Appendix E extends the normalized steady-state scan to $\gamma_d/\gamma = 0.5$ and shows the continuing decrease of the normalized steady-state ergotropy.

The engineered reservoir is the charger: preparing the auxiliary state, maintaining the Raman or microwave couplings, and operating the reset bath require external resources. The finite-time metric $P_{0.99}$ is therefore a battery-side charging metric rather than the net thermodynamic output power or full efficiency of the reservoir-engineering apparatus. A thermodynamic-efficiency analysis would combine the stored ergotropy with these external inputs. Appendixes B and F provide finite-size and phase-error checks.

V. CONCLUSION AND OUTLOOK

We have studied dissipative charging in finite tight-binding quantum batteries driven by stationary engineered reservoirs. The central ingredient is a number-conserving bond jump that converts local antisymmetric bond modes into symmetric ones. This dissipative selection mechanism steers the battery toward upper spectral-edge states and produces nonpassive steady states with finite ergotropy. In the clean one-dimensional periodic chain, the mechanism admits a simple bond-mode interpretation and gives the steady-state ergotropy $\mathcal{E}_{ss} = 4t$ for an even lattice, where t denotes the uniform nearest-neighbor hopping amplitude. We also demonstrated that the same principle applies to a two-dimensional honeycomb tight-binding lattice, showing that the mechanism is not tied to a specific one-dimensional geometry.

We further analyzed on-site Anderson disorder and local dephasing as perturbations of this effective charging protocol. For on-site disorder, averages over $N_{\text{dis}} = 100$ realizations show that the mixed steady state becomes more spatially concentrated in its real-space density profile, as characterized by the density-profile inverse participation ratio I_ρ^{ss} . At the same time, the final steady-state ergotropy decreases, while the charging transient shortens over the parameter range examined. The resulting increase of the finite-time metric $P_{0.99}$ is therefore not a consequence of larger stored ergotropy, but of faster relaxation toward the steady state, consistent with the slowest nonzero Liouvillian decay rate remaining above its clean value. Additional local dephasing shows a similar finite-time trade-off in the displayed rate window: It reduces the stored ergotropy but shortens the charging transient.

The proposed charging mechanism should be interpreted as an effective reservoir-engineered protocol. The nonequilibrium reservoir acts as the charger, and the reported $P_{0.99}$ is a battery-side finite-time charging measure rather than the net thermodynamic power or efficiency of the full system-auxiliary-reservoir apparatus. A complete efficiency analysis would require accounting for the energetic and control costs of preparing the auxiliary states, maintaining the coherent couplings, and operating the reset bath. Future directions include extending the construction to interacting many-particle sectors, developing experimentally constrained passification protocols, and quantifying the thermodynamic cost of the reservoir-engineered charging cycle.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

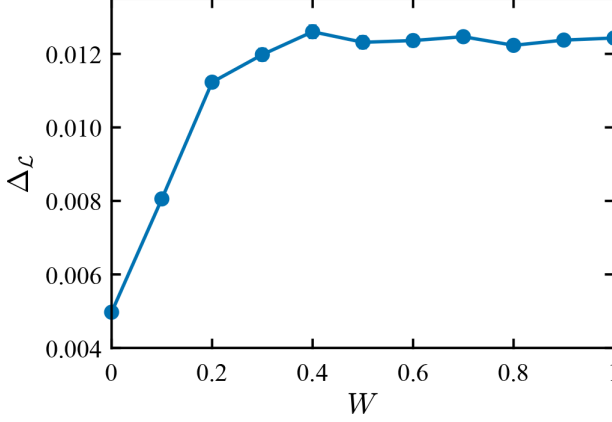


FIG. 6. Disorder dependence of the asymptotic Liouvillian relaxation diagnostic $\Delta_{\mathcal{L}}$ in the one-dimensional periodic chain. The system size is $L = 64$, and $N_{\text{dis}} = 100$ disorder realizations are used for each nonzero disorder strength W . Error bars denote the SEM.

APPENDIX A: DISORDER-AVERAGED LIOUVILLIAN RELAXATION SCALE

The following Appendixes collect the numerical checks and diagnostics used to support the disorder, finite-size, phase-imperfection, and dephasing results in the main text. All calculations are performed in the single-particle sector, where ρ is a positive semidefinite $L \times L$ operator satisfying $\text{Tr} \rho = 1$. The notation follows the main text: H_B is the battery Hamiltonian, ρ_{ss} is the nonequilibrium steady state, γ is the engineered bond-dissipation rate, γ_d is the local dephasing rate, and W is the on-site disorder strength. The finite-time charging metric $P_{0,99}$ is defined in Eq. (16). Unless stated otherwise, error bars show the SEM. In many panels, the SEM is smaller than the marker size or line width and is therefore not separately visible. For clean reference points and deterministic dephasing-rate scans, an ensemble SEM is not applicable and is therefore not shown.

For this scan, the Liouvillian acts on the single-particle density matrix ρ , which is an $L \times L$ matrix. After vectorizing ρ , the Liouvillian is an $L^2 \times L^2$ matrix. For $L = 64$, this gives a 4096×4096 superoperator. We diagonalize this full matrix in the single-particle density-matrix sector of the one-dimensional periodic chain. The on-site disorder is

$$H_{\text{dis}} = \sum_i \epsilon_i c_i^\dagger c_i, \quad \epsilon_i \in [-W/2, W/2], \quad (\text{A1})$$

where ϵ_i are independent on different sites. For each disorder realization, the total single-particle Hamiltonian is $H(W) = H_B + H_{\text{dis}}$. Each nonzero W is averaged over $N_{\text{dis}} = 100$ disorder realizations. The Liouvillian eigenmodes obey

$$\mathcal{L}[\rho_\alpha] = \lambda_\alpha \rho_\alpha, \quad (\text{A2})$$

with the steady state corresponding to $\lambda_0 = 0$. Figure 6 plots the slowest nonsteady decay rate,

$$\Delta_{\mathcal{L}} := -\max_{\lambda_\alpha \neq 0} \text{Re } \lambda_\alpha, \quad (\text{A3})$$

where the stationary zero mode is excluded. Equivalently, if λ_1 denotes the nonzero mode with the largest real part, then $\Delta_{\mathcal{L}} = -\text{Re } \lambda_1$. In this dataset, finite disorder increases $\Delta_{\mathcal{L}}$ relative to the clean value. We use $\Delta_{\mathcal{L}}$ as an asymptotic relaxation diagnostic. It does not by itself determine the finite-time threshold $\tau_{0,99}$, which also depends on the overlap of the initial state with the relevant Liouvillian modes. Nevertheless, the increase of $\Delta_{\mathcal{L}}$ above the clean value is consistent with the observed shortening of the charging transient.

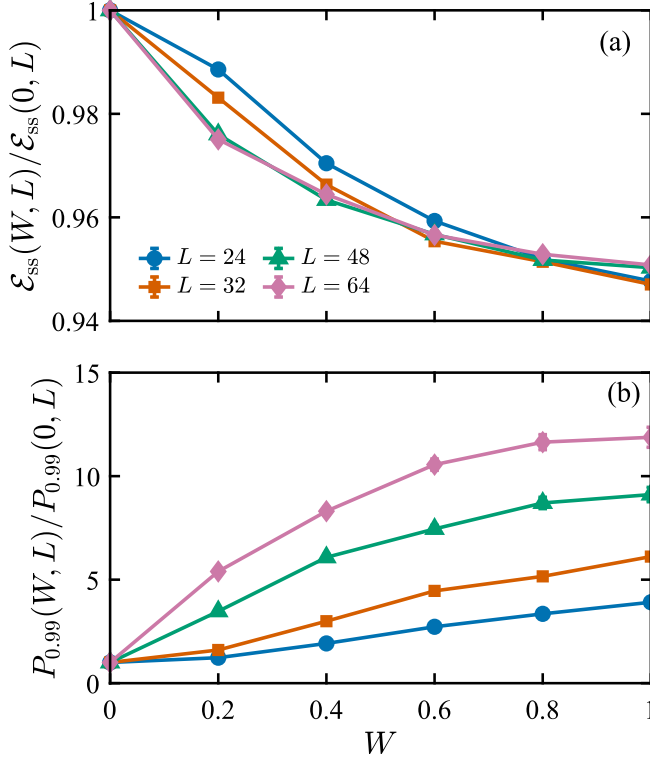


FIG. 7. Finite-size check for the disorder-averaged one-dimensional chain. (a) Normalized steady-state ergotropy $\mathcal{E}_{ss}(W, L)/\mathcal{E}_{ss}(0, L)$. (b) Normalized finite-time charging metric $P_{0.99}(W, L)/P_{0.99}(0, L)$. Calculations use $L = 24, 32, 48$, and 64 , with $N_{\text{dis}} = 100$ realizations for each nonzero W . Error bars denote SEM.

APPENDIX B: FINITE-SIZE CHECK

We repeat the one-dimensional calculation for $L = 24, 32, 48$, and 64 to check the finite-size dependence. For each L , the clean $W = 0$ case is computed once; each nonzero W uses $N_{\text{dis}} = 100$ independent on-site disorder realizations. The plotted quantities are normalized by the clean result at the same L :

$$\frac{\mathcal{E}_{ss}(W, L)}{\mathcal{E}_{ss}(0, L)}, \quad \frac{P_{0.99}(W, L)}{P_{0.99}(0, L)}. \quad (\text{B1})$$

The corresponding finite-size results are shown in Fig. 7. For all four sizes, the normalized steady-state ergotropy decreases with disorder, while $P_{0.99}$ rises over the same range. In this parameter window, disorder lowers the final stored ergotropy and shortens the finite-time charging transient. The consistent trends across $L = 24, 32, 48$, and 64 indicate that the disorder-induced reduction of the final ergotropy and the shortening of the finite-time charging transient are not artifacts of a single system size.

APPENDIX C: STEADY-STATE DENSITY-PROFILE IPR AND HIGH-ENERGY OCCUPATION

In the same $L = 64$ disorder scan, we examine how concentrated the mixed steady state is in the local site basis. For one disorder realization, let ρ_{ss} be the single-particle steady-state density matrix normalized by $\text{Tr}\rho_{ss} = 1$. We define the local steady-state occupation as

$$n_i^{ss} = \langle i | \rho_{ss} | i \rangle = (\rho_{ss})_{ii}, \quad (\text{C1})$$

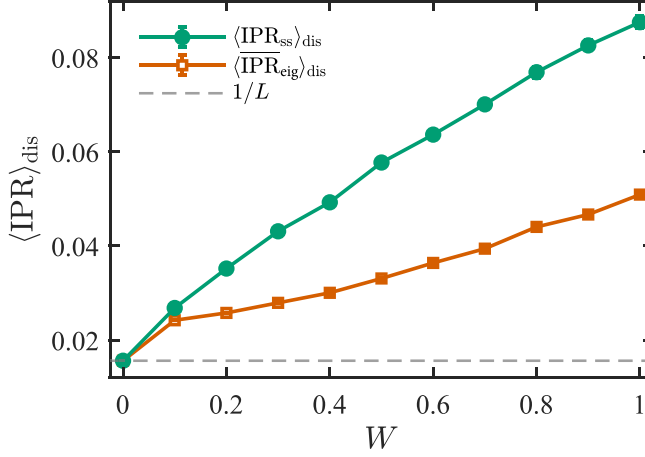


FIG. 8. Real-space inverse participation ratios in the disordered one-dimensional chain with $L = 64$. The green circles show the IPR I_{ρ}^{ss} of the steady-state density profile, averaged over disorder realizations. The orange squares show the average IPR $\bar{I}_{\text{eig}}$ of all eigenstates of the corresponding disordered Hamiltonian without dissipation, also averaged over disorder realizations. The dashed line marks $1/L = 0.015625$, the uniform-density reference. Both IPR measures increase with W , while $\langle I_{\rho}^{\text{ss}} \rangle_{\text{dis}}$ remains larger than $\langle \bar{I}_{\text{eig}} \rangle_{\text{dis}}$, indicating enhanced real-space concentration of the dissipative steady-state density profile. Error bars denote the SEM over $N_{\text{dis}} = 100$ realizations for each nonzero W ; the clean point is calculated once.

where $|i\rangle$ is the orbital localized on site i . To benchmark the spatial structure of the dissipative steady state, we compare two real-space IPRs. The first one is the IPR of the steady-state density profile,

$$I_{\rho}^{\text{ss}} = \sum_i (n_i^{\text{ss}})^2, \quad n_i^{\text{ss}} = \langle i | \rho_{\text{ss}} | i \rangle. \quad (\text{C2})$$

The second one is the average IPR of all single-particle eigenstates of the corresponding disordered Hamiltonian without dissipation,

$$\bar{I}_{\text{eig}} = \frac{1}{L} \sum_{\alpha=1}^L \sum_i |\langle i | \chi_{\alpha} \rangle|^4, \quad (\text{C3})$$

where $|\chi_{\alpha}\rangle$ denotes an eigenstate of the disordered tight-binding Hamiltonian. Both quantities are then averaged over disorder realizations. The dashed line $1/L$ marks the uniform-density reference. The results are shown in Fig. 8.

Both $\langle I_{\rho}^{\text{ss}} \rangle_{\text{dis}}$ and $\langle \bar{I}_{\text{eig}} \rangle_{\text{dis}}$ increase with disorder, consistent with the enhanced real-space localization tendency of the one-dimensional on-site-disordered chain. More importantly, $\langle I_{\rho}^{\text{ss}} \rangle_{\text{dis}}$ is systematically larger than $\langle \bar{I}_{\text{eig}} \rangle_{\text{dis}}$ for nonzero W . This comparison shows that the dissipatively selected mixed steady state is more strongly concentrated in real space than the eigenstate-averaged Hamiltonian benchmark. Thus, the engineered bond dissipation does not merely inherit the average localization tendency of the disordered Hamiltonian; it selects a steady-state density profile with enhanced spatial concentration over the scanned on-site Anderson-disorder interval.

To connect the increased spatial concentration with the occupied energy range, we monitor the normalized steady-state energy

$$\varepsilon_{\text{ss}} = \frac{\text{Tr}[H(W)\rho_{\text{ss}}] - E_{\text{min}}}{E_{\text{max}} - E_{\text{min}}}, \quad (\text{C4})$$

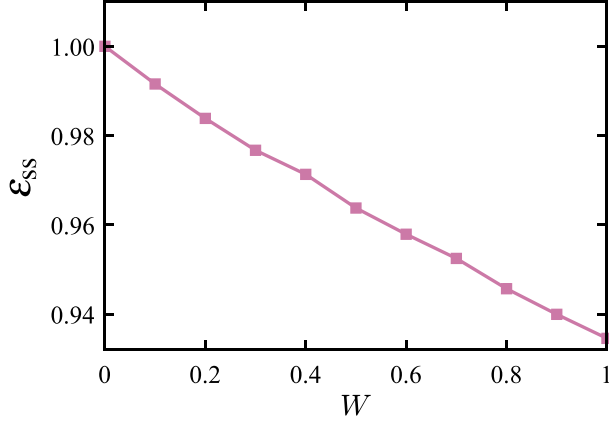


FIG. 9. Normalized steady-state energy ε_{ss} in the disordered one-dimensional chain with $L = 64$. Values near one indicate occupation close to the upper spectral edge. These values stay high over the disorder range in which Fig. 8 shows enhanced spatial concentration relative to the uniform reference. Error bars denote SEM.

where $E_{\min}$ and $E_{\max}$ are the lowest and highest single-particle energies of the same $H(W)$. Here, $\varepsilon_{ss} = 0$ corresponds to the bottom of that spectrum and $\varepsilon_{ss} = 1$ to the top. The disorder dependence of ε_{ss} is shown in Fig. 9.

We also compute the total steady-state occupation weight in the highest-energy quarter of the spectrum. We order the eigenstates of each disorder realization as $H(W)|E_\alpha\rangle = E_\alpha|E_\alpha\rangle$, with $E_1 \leq E_2 \leq \dots \leq E_L$, and define $p_\alpha^{ss} = \langle E_\alpha | \rho_{ss} | E_\alpha \rangle$. The highest-energy-quarter weight is

$$Q_{\text{top}} = \sum_{\alpha=\lfloor 3L/4 \rfloor + 1}^L p_\alpha^{ss}. \quad (\text{C5})$$

The resulting disorder-averaged highest-energy-quarter weight is shown in Fig. 10.

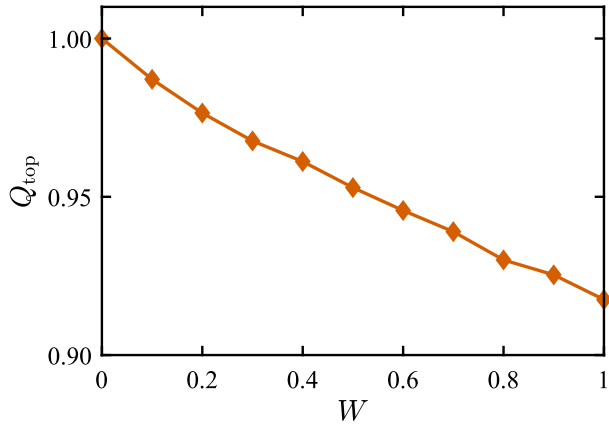


FIG. 10. Total steady-state occupation weight Q_{top} in the highest-energy quarter of the single-particle spectrum for the $L = 64$ disorder scan. The weight remains large over the full range of W , consistent with the high normalized energy shown in Fig. 9. Error bars denote SEM.

APPENDIX D: MICROSCOPIC RESERVOIR-ENGINEERING DERIVATION OF THE EFFECTIVE BOND JUMP

We derive Eq. (10) in the single-particle sector from a system-auxiliary-reservoir model. The construction follows the phase-selective auxiliary-state architecture used in dissipative lattice proposals [72,76,88,89]. Its two ingredients are excitation of the antisymmetric bond state and a common reservoir that returns the auxiliary excitation to the symmetric bond state without retaining which-site information.

For bond $\langle i, j \rangle$, define

$$|b_{ij,\pm}\rangle = \frac{|i\rangle \pm \eta^* |j\rangle}{\sqrt{2}}, \quad \eta = e^{i\phi}. \quad (\text{D1})$$

The target bond transition is

$$J_{ij} = |b_{ij,+}\rangle \langle b_{ij,-}| = b_{ij,+}^\dagger b_{ij,-} = \frac{1}{2} L_{ij}. \quad (\text{D2})$$

We introduce an auxiliary state $|a_{ij}\rangle$ and, in a rotating frame, use the phase-selective coupling

$$H_{\text{SA}}^{(ij)} = \Delta |a_{ij}\rangle \langle a_{ij}| + \Omega_{ij} |a_{ij}\rangle \langle b_{ij,-}| + \text{H.c.} \quad (\text{D3})$$

Only the antisymmetric bond state is excited. We next derive its collective return channel. Let $d_{ij,q}$ annihilate mode q of a reservoir assigned to bond $\langle i, j \rangle$ and define $S_{ij,\mu} = |\mu\rangle \langle a_{ij}|$ for $\mu \in \{i, j\}$. The reservoir Hamiltonian and its rotating-wave coupling to the auxiliary state are

$$H_{\text{R}}^{(ij)} = \sum_q \omega_q d_{ij,q}^\dagger d_{ij,q}, \quad (\text{D4})$$

$$H_{\text{aR}}^{(ij)} = \sum_{q,\mu=i,j} g_{\mu q}^{(ij)} d_{ij,q}^\dagger S_{ij,\mu} + \text{H.c.} \quad (\text{D5})$$

Here, q labels the reservoir modes assigned to bond $\langle i, j \rangle$, whereas $\mu \in \{i, j\}$ labels the two possible return sites on that bond; explicitly, $\sum_{q,\mu=i,j} \equiv \sum_q \sum_{\mu \in \{i,j\}}$. The operator $d_{ij,q}^\dagger S_{ij,\mu}$ emits a reservoir quantum while transferring the particle from the auxiliary state to site μ . For a reservoir initially in its vacuum state, the Born-Markov and secular approximations give

$$\mathcal{L}_{\text{R}}^{(ij)}[\rho] = \sum_{\mu,\nu=i,j} \Gamma_{\mu\nu}^{(ij)} \left(S_{ij,\mu} \rho S_{ij,\nu}^\dagger - \frac{1}{2} \{ S_{ij,\nu}^\dagger S_{ij,\mu}, \rho \} \right), \quad (\text{D6})$$

where the positive Kossakowski matrix is

$$\Gamma_{\mu\nu}^{(ij)} = 2\pi \sum_q g_{\mu q}^{(ij)} g_{\nu q}^{(ij)*} \delta(\omega_q - \omega_a). \quad (\text{D7})$$

The required collective channel is obtained when the two resonant return amplitudes have equal magnitude and the phase relation $g_{jq}^{(ij)} = \eta^* g_{iq}^{(ij)}$. Equivalently,

$$\mathbf{\Gamma}^{(ij)} = \frac{\Gamma_{ij}}{2} \begin{pmatrix} 1 & \eta \\ \eta^* & 1 \end{pmatrix}. \quad (\text{D8})$$

This rank-one matrix has the single jump operator

$$R_{ij} = \sqrt{\frac{\Gamma_{ij}}{2}} (S_{ij,i} + \eta^* S_{ij,j}) = \sqrt{\Gamma_{ij}} |b_{ij,+}\rangle \langle a_{ij}|. \quad (\text{D9})$$

Thus, the common reservoir returns the particle coherently to the symmetric bond state. In infinitesimal Kraus form, the reduced system-auxiliary map is

$$K_0 = I - iH_{\text{SA}}^{(ij)} dt - \frac{1}{2} R_{ij}^\dagger R_{ij} dt, \quad K_1 = \sqrt{dt} R_{ij}. \quad (\text{D10})$$

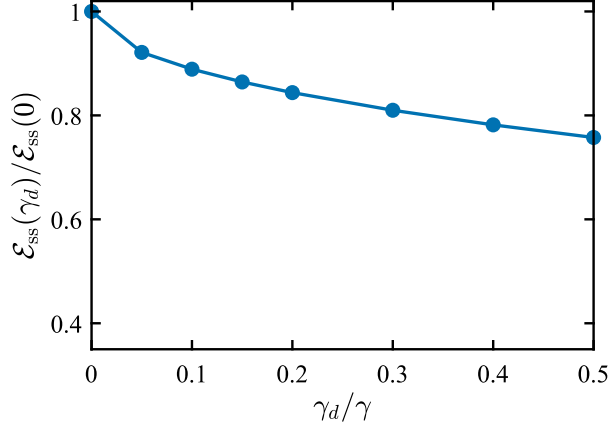


FIG. 11. Clean-chain reference scan for local dephasing. Shown is $\mathcal{E}_{ss}(\gamma_d)/\mathcal{E}_{ss}(0)$ as a function of γ_d/γ for $0 \leq \gamma_d/\gamma \leq 0.5$, with $L = 64$, $W = 0$, and $\gamma = 1$.

This expression holds up to terms of order dt^2 . Reservoir modes or frequency channels that are orthogonal for different bonds make $\Gamma_{\mu\nu}^{(ij),(kl)} \propto \delta_{(ij),(kl)}$, which removes cross-bond dissipative terms.

The non-Hermitian Hamiltonian in the auxiliary subspace is

$$H_{\text{NH}} = (\Delta - i\Gamma_{ij}/2) |a_{ij}\rangle \langle a_{ij}|. \quad (\text{D11})$$

For $|\Omega_{ij}| \ll \sqrt{\Delta^2 + \Gamma_{ij}^2}/4$ and a Markovian reset bath, adiabatic elimination gives

$$L_{ij}^{\text{eff}} = R_{ij} H_{\text{NH}}^{-1} \Omega_{ij} |a_{ij}\rangle \langle b_{ij,-}| = \frac{\sqrt{\Gamma_{ij}} \Omega_{ij}}{\Delta - i\Gamma_{ij}/2} J_{ij}, \quad (\text{D12})$$

and the accompanying ac-Stark shift

$$H_{\text{LS}}^{(ij)} = -\frac{|\Omega_{ij}|^2 \Delta}{\Delta^2 + \Gamma_{ij}^2/4} |b_{ij,-}\rangle \langle b_{ij,-}|. \quad (\text{D13})$$

The shift may be retained in the effective Hamiltonian or compensated by detuning. Since the dimensionless operator in Eq. (10) is $L_{ij} = 2J_{ij}$, the convention $\gamma_{ij}\mathcal{D}[L_{ij}]$ corresponds to

$$\gamma_{ij} = \frac{\Gamma_{ij} |\Omega_{ij}|^2}{4(\Delta^2 + \Gamma_{ij}^2/4)}. \quad (\text{D14})$$

With the collective return derived above, the effective process is the desired number-conserving transition. If the two return paths are distinguishable, the off-diagonal entries of Eq. (D7) vanish, and the reservoir produces two local refill jumps rather than the coherent state $|b_{ij,+}\rangle$. Replacing the return channel with ordinary loss $\sqrt{\Gamma_{ij}} |\text{vac}\rangle \langle a_{ij}|$ instead yields an effective channel proportional to $|\text{vac}\rangle \langle b_{ij,-}|$. The drive and reservoir energetics enter separately in a resource-cost analysis.

APPENDIX E: REFERENCE SCAN FOR CLEAN LOCAL DEPHASING

The main text studies engineered bond dissipation in the presence of local dephasing channels

$$O_j = c_j^\dagger c_j, \quad (\text{E1})$$

with rate γ_d . The clean-chain reference scan shown in Fig. 11 uses $L = 64$, $W = 0$, and $\gamma = 1$, and covers the interval $0 \leq \gamma_d/\gamma \leq 0.5$. It is included to document the reduction of stored ergotropy by

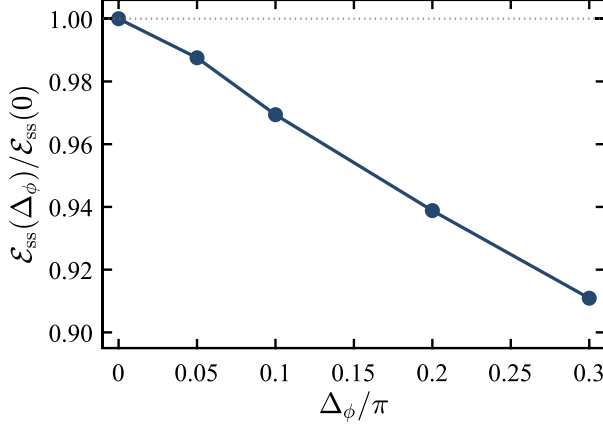


FIG. 12. Effect of phase imperfection in the engineered bond dissipator. Shown is $\mathcal{E}_{ss}(\Delta_\phi)/\mathcal{E}_{ss}(0)$ for the one-dimensional periodic chain with $L = 64$. For each nonzero Δ_ϕ , $N_{\text{samp}} = 100$ phase-error realizations are used. Error bars denote SEM, which is smaller than the marker size or line width for the displayed data. The calculation uses the conjugated jump operator $(c_i^\dagger + \eta_{ij}^* c_j^\dagger)(c_i - \eta_{ij} c_j)$.

local dephasing alone. Over this interval, Fig. 11 shows that increasing γ_d/γ lowers the normalized steady-state ergotropy. This extended reference scan documents the reduction of stored ergotropy beyond the three dephasing rates shown in Fig. 5.

APPENDIX F: EFFECT OF PHASE IMPERFECTION IN THE ENGINEERED JUMP OPERATOR

The ideal bond jump operator is

$$L_{ij} = (c_i^\dagger + \eta^* c_j^\dagger)(c_i - \eta c_j), \quad \eta = \exp(i\phi). \quad (\text{F1})$$

For the phase-error data below, the ideal phase is replaced at the single-particle matrix level by a bond-dependent phase

$$\eta_{ij} = \exp[i(\phi + \delta\phi_{ij})], \quad \delta\phi_{ij} \in [-\Delta_\phi/2, \Delta_\phi/2], \quad (\text{F2})$$

where Δ_ϕ is the width of the random phase-error window. The jump matrix L_{ij} has the nonzero elements

$$(L_{ij})_{ii} = 1, \quad (L_{ij})_{ij} = -\eta_{ij}, \quad (L_{ij})_{ji} = \eta_{ij}^*, \quad (L_{ij})_{jj} = -1. \quad (\text{F3})$$

All other matrix elements are zero. These entries are the single-particle matrix representation of the conjugated jump above. The ideal value is $\phi = 0$. The calculation uses a periodic chain with $L = 64$, $\gamma = 1$, and no local dephasing. For every nonzero Δ_ϕ , we average over $N_{\text{samp}} = 100$ independent phase-error samples.

The ratio in Fig. 12 stays close to unity for small Δ_ϕ . For example, its phase-error-ensemble-averaged values are approximately 0.988, 0.969, 0.939, and 0.911 for $\Delta_\phi/\pi = 0.05, 0.1, 0.2$, and 0.3 , respectively. Larger phase errors reduce the steady-state ergotropy because the random bond phases spoil the bond-by-bond dark-state condition $a_i = \eta_{ij} a_j$ imposed by the annihilation channel $c_i - \eta_{ij} c_j$, where a_i is the single-particle wave-function amplitude on site i .

We note that this test quantifies robustness within the effective Markovian model used throughout this work. All lattice calculations reported in the main text and in the Appendixes use periodic boundary conditions, and the phase-imperfection data correspond to the bond-phase-error ensemble specified above. The robustness conclusion should therefore be understood within this tested parameter window and error model.

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