

Vacuum pair production in spatially antisymmetric electric fields

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Vacuum electron-positron pair production is investigated using the Dirac-Heisenberg-Wigner formalism in spatially inhomogeneous electric fields. The external field, given by $E(x, t)\mathbf{e}_x$, exhibits a strong positional dependence and a strict spatial antisymmetry—i.e., $E(-x) = -E(x)$. Our results demonstrate that this antisymmetry fundamentally reshapes the pair-creation dynamics, yielding results that differ qualitatively and quantitatively from those obtained under spatially symmetric or fully asymmetric field configurations. We further analyze multipulse field profiles with systematically varied field separations and show that the separation distance exerts a decisive influence on both the integrated pair yield and the detailed structure of the momentum spectrum. Crucially, precise control over the relative spatial positioning of the fields proves essential for engineering desired features in the pair momentum spectrum. We also find that numerical results are in excellent agreement with the analytical approximation for both small and large separation distances. These findings provide robust theoretical guidance for optimizing vacuum pair production in experimentally accessible spatiotemporal field configurations and reveal dynamical effects arising specifically from spatial field antisymmetry.

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I. INTRODUCTION

Vacuum electron-positron pair production is a fundamental nonperturbative prediction of quantum electrodynamics (QED), providing a rigorous first-principles description of electromagnetic interactions between charged fermions and photons [1–3]. The quantum vacuum constitutes a dynamical medium sustained by transient virtual electron-positron fluctuations, and ultraintense electromagnetic fields can convert pure electromagnetic energy into real, on-shell electron-positron pairs. Although Dirac’s 1928 relativistic wave equation predicted the positron and introduced the “Dirac sea” as the physical vacuum [1], thereby laying the foundation for field-induced pair creation nearly a century ago, direct experimental verification remains elusive. This is primarily due to the required electric field strength approaching the Schwinger critical field ($E_{\text{cr}} \approx 1.3 \times 10^{18}$ V/m) [4], a regime currently inaccessible to state-of-the-art high-intensity lasers. Nevertheless, advances in laser technology have intensified theoretical and experimental interest in this phenomenon, with multiple dedicated experiments planned or underway at major international laser facilities [5,6].

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The first quantitative relativistic treatment of vacuum breakdown in external electromagnetic fields was proposed by Sauter in 1931 [7]. In 1951, Schwinger employed the proper-time method to derive the exact nonperturbative rate of vacuum decay in a constant electric field, thereby establishing the Schwinger mechanism as a fundamental prediction of QED [4]. Collectively, Sauter and Schwinger laid a rigorous theoretical foundation for vacuum electron-positron pair production: Sauter provided the earliest relativistic quantum-mechanical description of field-induced barrier penetration [7], while Schwinger's gauge-invariant QED framework yielded a nonperturbative result that transcends the semiclassical approximation [4, 8, 9]. A distinct perturbative channel (multiphoton pair production) emerges under time-varying fields [10, 11], involving the coherent absorption of n photons satisfying $n\hbar\omega \geq 2m_e c^2$ (SI units) or $n\omega \geq 2m_e$ (natural units, $\hbar = c = 1$); the vacuum pair production rate is exponentially suppressed with increasing n [12–15]. The nonperturbative tunneling and perturbative multiphoton regimes are distinguished by the Keldysh parameter $\gamma = m_e\omega/eE_0$: Tunneling dominates for $\gamma \ll 1$, whereas multiphoton absorption prevails for $\gamma \gg 1$ [16]. Recent dynamically assisted configurations enhance the pair yield at fields far below the Schwinger critical field $E_{\text{cr}} = m_e^2 c^3/(e\hbar)$, offering a promising pathway toward the experimental observation of the Schwinger effect [17–28].

The enhancement of the electron-positron pair production rate and its associated momentum-space spectral signatures has been extensively investigated using a broad range of externally applied electromagnetic field configurations [29–39]. A general theoretical framework is based on the superposition of two or more electric field modes with distinct frequencies, chirps, and phases. To date, theoretical and numerical investigations have predominantly employed spatially homogeneous fields or symmetrically inhomogeneous profiles—motivated primarily by analytical tractability and computational efficiency [40–44]. It is well established that generic spatial inhomogeneities suppress the Schwinger pair yield by imparting net momentum to the Dirac sea, thereby effectively widening the dynamical mass gap [45–48]. In contrast, the effects of temporal asymmetry—including the carrier-envelope phase, frequency chirp, and pulse envelope shape—have been extensively characterized both analytically and numerically [49–52]. Our previous study demonstrated that vacuum pair production efficiency can be significantly enhanced in spatially asymmetric electric fields when combined with coherent temporal oscillations [53, 54]. Concurrently, several independent studies have explored pair-creation dynamics in spatially inhomogeneous fields [41–43, 55].

Meanwhile, vacuum pair production has been extensively studied using a broad range of theoretical methods, such as the worldline instanton (WI) technique [27, 45, 56–74], the real-time Dirac-Heisenberg-Wigner (DHW) formalism [19, 43, 44, 52, 75–84], computational quantum field theory (CQFT) [85–92], the imaginary-time method [93], the quantum Vlasov equation approach [94–99], the Wentzel-Kramers-Brillouin (WKB) approach [100–107], the scattering matrix approach [108–129], and the quantum two-level model [130–141], among others. Among these, the WI, DHW, CQFT, and WKB approaches are capable of treating spacetime-dependent electromagnetic fields. In particular, the DHW formalism is, in principle, applicable to arbitrary spatiotemporal field configurations. Nevertheless, the influence of antisymmetric field configurations on the pair production rate and momentum distribution has been rarely reported.

Motivated by this open issue, we perform a systematic investigation of vacuum electron-positron pair production induced by spatially inhomogeneous, strictly antisymmetric electric fields within the $(1+1)$ -dimensional DHW formalism. We consider an external field configuration of the form $E(x)\mathbf{e}_x$, which exhibits both strong spatial variation and exact spatial antisymmetry about the origin—i.e., $E(-x) = -E(x)$. We systematically vary the electric field spatial separation to quantify its impact on the total pair yield and the fine structure of the longitudinal momentum spectrum. Our results reveal that strict spatial antisymmetry gives rise to dynamical features in vacuum pair production, leading to pronounced quantitative and structural deviations relative to those obtained under symmetric or fully asymmetric field profiles. Crucially, precise control over the field spatial separation leads to clear differences in the momentum spectrum, including longitudinal momentum peak splitting, suppression nodes, and controlled asymmetry in the momentum distribution. These

findings establish strict spatial antisymmetry as a decisive control parameter in strong-field QED and provide concrete, experimentally actionable theoretical guidance for enhancing pair yield and improving spectral controllability in future investigations.

The rest of this paper is organized as follows. Section II introduces the theoretical framework: Section II A provides a concise exposition of the DHW formalism and its underlying kinetic equations, and Sec. II B presents our field model. Section III reports a systematic analysis accompanied by physical interpretations of the observed pair production dynamics and momentum spectral features. Finally, Sec. IV summarizes the key findings of this work and outlines perspectives for future research. In the Appendix, we assess the accuracy of the DHW formalism results by comparison with those derived from analytical approximations based on the locally constant field approximation. Throughout this study, natural units ($\hbar = c = 1$) are adopted, and all dimensional quantities are normalized to the electron rest mass m .

II. METHOD AND EXTERNAL FIELD MODEL

A. DHW formalism

The DHW formalism provides a rigorous, gauge-covariant theoretical framework for investigating vacuum electron-positron pair production in time-dependent and spatially inhomogeneous electromagnetic fields. Since comprehensive derivations and technical details of the DHW formalism are available in Refs. [54,55,142–145], this work focuses on summarizing the key governing equations and the associated physical observables.

Beginning with the Dirac spinors Ψ and $\bar{\Psi}$, we construct a gauge-covariant density operator in the Heisenberg picture:

$$\hat{C}_{\alpha\beta}(r, s) = \mathcal{U}(A; r, s) [\bar{\Psi}_{\beta}(r - s/2), \Psi_{\alpha}(r + s/2)], \quad (1)$$

where r and s denote the center of mass and relative coordinates, respectively, and $\mathcal{U}(A; r, s)$ represents the Wilson line factor ensuring manifest gauge invariance [54].

The covariant Wigner operator in phase space is defined as the Fourier transform of the density operator with respect to the relative coordinate s :

$$\hat{W}_{\alpha\beta}(r, p) = \frac{1}{(2\pi)^4} \int d^4s e^{ip \cdot s} \hat{C}_{\alpha\beta}(r, s). \quad (2)$$

Its vacuum expectation value yields the covariant Wigner function $\mathbb{W}(r, p)$.

Under the mean-field approximation and employing standard Dirac algebra, $\mathbb{W}$ admits a unique decomposition into 16 independent Lorentz-covariant components [54,146]:

$$\mathbb{W} = \frac{1}{4} (1 \mathbb{S} + i\gamma_5 \mathbb{P} + \gamma^{\mu} \mathbb{V}_{\mu} + \gamma^{\mu} \gamma_5 \mathbb{A}_{\mu} + \sigma^{\mu\nu} \mathbb{T}_{\mu\nu}). \quad (3)$$

Here, $\mathbb{S}$, $\mathbb{P}$, $\mathbb{V}_{\mu}$, $\mathbb{A}_{\mu}$, and $\mathbb{T}_{\mu\nu}$ correspond to the scalar, pseudoscalar, vector, axial-vector, and antisymmetric tensor components, respectively. Physically, $\mathbb{S}$ encodes the mass density, $\mathbb{P}$ the chiral condensate density, $\mathbb{V}_{\mu}$ the fermionic current density, $\mathbb{A}_{\mu}$ the spin polarization density, and $\mathbb{T}_{\mu\nu}$ the electric dipole-moment density [75].

To describe vacuum pair creation as a causal initial-value problem, we adopt the equal-time formalism [147–149]. The equal-time Wigner function is obtained by integrating over the energy component p_0 :

$$\mathbb{w}(\mathbf{x}, \mathbf{p}, t) = \int \frac{dp_0}{2\pi} \mathbb{W}(r, p). \quad (4)$$

In this representation, $\mathbf{x}$ and $\mathbf{p}$ correspond to the position and kinetic momentum variables. For vacuum-initiated processes, only the equal-time Wigner function captures physically observable dynamics; all subsequent quantities refer to its components.

We specialize the formalism to $(1+1)$ -dimensional spacetime, with a single spatial coordinate x and an external electric field $E(x, t)$. In this setting, Lorentz covariance reduces the full set of

Wigner components to four nontrivial ones: the scalar $\mathbb{s}$, temporal vector $\mathbb{v}_0$, spatial vector $\mathbb{v}_x$, and pseudoscalar $\mathbb{p}$. Their coupled equations of motion read [29,40,54,75]

$$D_t \mathbb{s} - 2p_x \mathbb{p} = 0, \quad (5)$$

$$D_t \mathbb{v}_0 + \partial_x \mathbb{v}_x = 0, \quad (6)$$

$$D_t \mathbb{v}_x + \partial_x \mathbb{v}_0 = -2m \mathbb{p}, \quad (7)$$

$$D_t \mathbb{p} + 2p_x \mathbb{s} = 2m \mathbb{v}_x. \quad (8)$$

The pseudodifferential time derivative D_t is explicitly given by

$$D_t = \partial_t + e \int_{-1/2}^{1/2} d\xi E_x(x + i\xi \partial_{p_x}, t) \partial_{p_x}. \quad (9)$$

In the $(1 + 1)$ -dimensional configuration, transverse momentum degrees of freedom are absent by construction; hence, no approximation is required to neglect them [29,38,44,53,55].

Among the four dynamical components, only $\mathbb{s}$ and $\mathbb{v}_x$ acquire nonzero vacuum expectation values [142]:

$$\mathbb{s}_{\text{vac}} = -\frac{2m}{\Omega}, \quad \mathbb{v}_{x,\text{vac}} = -\frac{2p_x}{\Omega}, \quad (10)$$

where $\Omega = \sqrt{p_x^2 + m^2}$ denotes the on-shell single-particle energy. Physical observables are extracted from the renormalized (vacuum-subtracted) Wigner components:

$$\widetilde{\mathbb{W}} = \mathbb{W} - \mathbb{W}_{\text{vac}}. \quad (11)$$

The phase-space particle number density is formally defined as [29]

$$n(x, p_x, t) = \frac{m \mathbb{s}(x, p_x, t) + p_x \mathbb{v}_x(x, p_x, t)}{\Omega(p_x)}. \quad (12)$$

The momentum distribution and the total particle yield are obtained via integration as follows:

$$n(p_x, t) = \int \frac{dx}{2\pi} n(x, p_x, t), \quad (13)$$

$$N(t) = \int dp_x n(p_x, t). \quad (14)$$

To facilitate scaling analysis and cross-parameter comparisons, particularly with respect to the characteristic field length scale λ , we introduce reduced quantities $\bar{n}(p_x, t) = n(p_x, t)/\lambda$ and $\bar{N}(t) = N(t)/\lambda$.

B. External field model

The efficiency of electron-positron pair creation is governed jointly by the temporal and spatial characteristics of the applied external electromagnetic field [36,44,51,52,142]. In our earlier works [53,54], we systematically investigate how distinct spatial configurations—specifically, two spatially separated, position-dependent electric fields—affect both the total pair production yield and the associated momentum distribution.

In this study, we analyze electron-positron pair production in $1 + 1$ dimensions by considering a spatially antisymmetric electric field with a variable spatial distance, as illustrated in Fig. 1(c). The field configuration of the antisymmetric electric field is described as

$$E(x, t) = \epsilon E_{\text{cr}} [E_1(x) - E_2(x)] g(t) = \epsilon E_{\text{cr}} \left[\text{sech}^2\left(\frac{x}{\lambda} - d\right) - \text{sech}^2\left(\frac{x}{\lambda} + d\right) \right] \text{sech}^2\left(\frac{t}{\tau}\right), \quad (15)$$

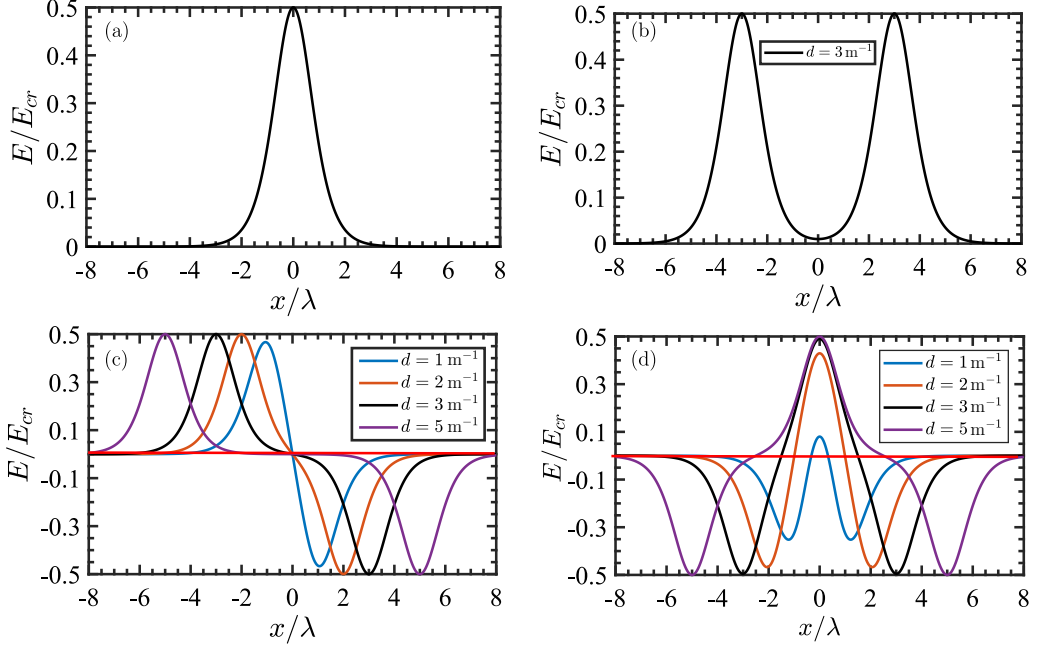


FIG. 1. Schematic diagrams of the spatial profiles of the electric fields (normalized to the critical field E_{cr}): (a) symmetric single-field configuration, (b) two-field symmetric configuration, (c) two-field antisymmetric configuration, and (d) three-field configuration. For the two- and three-field configurations, the fields are separated by a variable spatial distance d . For all field configurations, the dimensionless amplitude is fixed at $\epsilon = 0.5$. The spatial distance d and the spatial field extent λ are treated as variable parameters, while the temporal scale is fixed at $\tau = 10 \text{ m}^{-1}$ in all numerical computations.

where $g(t) = \text{sech}^2(t/\tau)$ denotes the temporal envelope of the electric field, and λ and τ correspond to the characteristic spatial and temporal scales, respectively. Here, E_{cr} signifies the Schwinger critical field strength. We fix the dimensionless field amplitude at $\epsilon = E_0/E_{\text{cr}} = 0.5$, while systematically varying the spatial separation d of the two fields. All DHW calculations employ a peak field amplitude $E_0 = 0.5 E_{\text{cr}}$ and a Sauter-type temporal profile $g(t)$, with a fixed temporal scale $\tau = 10 \text{ m}^{-1}$. The electric field is polarized along the x axis and exhibits spatiotemporal dependence, i.e., $\mathbf{E}(x, t) = E(x, t) \mathbf{e}_x$.

We compare the results with the simple symmetric single-field case [see Fig. 1(a)] and the two-field symmetric case [see Fig. 1(b)], as well as with the antisymmetric case [see Fig. 1(c)]. The single electric field case in Fig. 1(a) is obtained from Eq. (15) simply by setting $E_2(x) = 0$ and $d = 0$. The symmetric field case in Fig. 1(b) is obtained from Eq. (15) simply by replacing $-E_2(x)$ with $E_2(x)$ [53]. At the same time, we also consider a three-field combination [Fig. 1(d)] in order to extend our study to more general cases. The field configuration of the three-field combination is described as

$$\begin{aligned} E(x, t) &= \epsilon E_{\text{cr}} [\mp E_1(x) \pm E_2(x) \mp E_3(x)] g(t) \\ &= \epsilon E_{\text{cr}} \left[\mp \text{sech}^2\left(\frac{x}{\lambda} - d\right) \pm \text{sech}^2\left(\frac{x}{\lambda}\right) \mp \text{sech}^2\left(\frac{x}{\lambda} + d\right) \right] \text{sech}^2\left(\frac{t}{\tau}\right), \end{aligned} \quad (16)$$

where the symbols $\mp$ and $\pm$ denote the orientation of the electric field—specifically, alignment parallel to or antiparallel with the x axis.

Figure 1 illustrates that the superposition of the two or three fields gives rise to a spatially antisymmetric or effective field, which substantially alters both the pair production dynamics and the

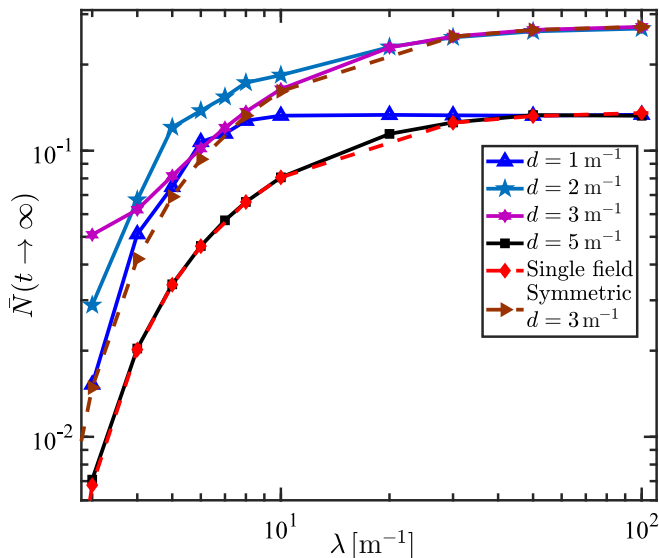


FIG. 2. Reduced total particle number $\bar{N}$ as a function of the electric field spatial scale λ for different field separations d . The brown (red) curve corresponds to the symmetric two-field (single-field) configuration; other curves correspond to the antisymmetric field configuration with different d . All other parameters are identical to those in Fig. 1.

associated phase-space distribution of the produced particles. Within this idealized framework, we systematically investigate how a spatially antisymmetric electric field, characterized by a prescribed time-dependent envelope, modulates the pair production rate, the final momentum spectrum, and the total reduced particle yield in phase space.

III. RESULTS AND ANALYSIS

In this section, we present the numerical results obtained using the DHW formalism introduced in Sec. II A, under the external field configurations specified in Sec. II B. A comprehensive analysis of these results is subsequently provided with respect to the key field parameters.

We begin by comparing the reduced total particle number $\bar{N}$ across distinct field configurations, as illustrated in Fig. 2. The results reveal that, for small spatial separations d and small field extents λ , the reduced particle number $\bar{N}$ attains higher values under the spatially antisymmetric field configuration. However, as λ increases, $\bar{N}$ saturates at two distinct values for the different field configurations. Specifically, for $d = 1 \text{ m}^{-1}$, $\bar{N}$ increases monotonically with increasing λ and saturates at approximately $\lambda = 10 \text{ m}^{-1}$. In contrast, for $d = 2 \text{ m}^{-1}$ and $d = 3 \text{ m}^{-1}$, $\bar{N}$ initially increases gradually with λ , converges to the same asymptotic value by $\lambda = 20 \text{ m}^{-1}$, and subsequently saturates beyond $\lambda = 30 \text{ m}^{-1}$. For the largest separation considered, $d = 5 \text{ m}^{-1}$, $\bar{N}$ likewise increases with λ and ultimately saturates at the same value observed for $d = 1 \text{ m}^{-1}$ beyond $\lambda = 30 \text{ m}^{-1}$. However, the saturated value is lower than that corresponding to $d = 2 \text{ m}^{-1}$ and $d = 3 \text{ m}^{-1}$.

It is observed from Fig. 2 that the reduced particle number $\bar{N}$ converges to two distinct asymptotic values for different field configurations, especially for $\lambda \gtrsim 30 \text{ m}^{-1}$. This is because, for $d = 2 \text{ m}^{-1}$ and $d = 3 \text{ m}^{-1}$, the two electric fields are separated but the distance is not very large, and the total effective field strength may be the same for both symmetric and antisymmetric configurations. However, for $d = 1 \text{ m}^{-1}$, the effective field of the antisymmetric two-field configuration is reduced to some extent [Fig. 1(c)], so that the pair production efficiency becomes the same as in the single-field case. For $d = 5 \text{ m}^{-1}$, the two fields are completely separated and the distance between

them is so large that this may reduce the pair production efficiency. Therefore, the final value of $\bar{N}$ converges to the same value as in the single-field case.

In Fig. 2, we also compare the results for the symmetric two-field configuration (brown curve) and the antisymmetric two-field configuration (magenta curve), both with the identical spatial separation $d = 3 \text{ m}^{-1}$. The two curves exhibit nearly identical behavior and converge to the same saturated value of $\bar{N}$. We further compare these results with those obtained for the single electric field configuration (red curve). In the latter case, pair production is suppressed due to the reduced field strength—specifically, half that of each individual field in the two-field setups. Notably, the $\bar{N}$ distribution under the single-field configuration closely resembles that observed in the two-field case with $d = 5 \text{ m}^{-1}$, suggesting that the dominant factor limiting pair production is not the field symmetry but rather the effective spatial separation. Indeed, increasing d suppresses pair production. Among the configurations examined, $d = 3 \text{ m}^{-1}$ yields the highest $\bar{N}$, indicating that it is the most favorable separation for efficient pair production.

The symmetric and antisymmetric two-field configurations yield nearly identical results, as demonstrated in Fig. 2. However, one can also consider the electric field with strict antisymmetry in space. This spatial antisymmetry exerts a profound influence on the nonperturbative QED process of electron-positron pair creation from the vacuum. Accordingly, it is essential to systematically investigate the impact of spatially antisymmetric electric fields endowed with simple Sauter-type temporal profiles on the pair production dynamics. The observed trends indicate increasingly complex, nonlinear dynamics at small spatial scales, whereas the large- λ , moderate- d region displays robust stability. These results highlight the necessity of deeper theoretical and numerical investigations into the physical mechanisms underlying the stable, high- $\bar{N}$ regimes for different spatially dependent fields.

It is well established that particle momentum represents a measurable physical observable. To advance our understanding of the pair production mechanism in a spatially antisymmetric electric field configuration, we systematically analyze the momentum spectra of spontaneously generated electron-positron pairs as functions of the electric field separation distance d and the characteristic spatial extent λ of the field. Figure 3 displays these momentum spectra for the antisymmetric field profile defined in Eq. (15), with panels (a)–(d) corresponding to fixed separation distances $d = 1, 2, 3, 5 \text{ m}^{-1}$, respectively. Within each panel, five representative values of λ are considered: 3, 5, 10, 50, 100 m^{-1} .

We begin by examining the smallest separation distance, $d = 1 \text{ m}^{-1}$ [Fig. 3(a)]. For the largest spatial extents ($\lambda = 50$ and 100 m^{-1}), the momentum distributions are nearly indistinguishable and exhibit approximate symmetry. In contrast, at intermediate scales ($\lambda = 5$ and 10 m^{-1}), the spectra become markedly asymmetric and broaden significantly, indicating increased complexity in the momentum structure as the field inhomogeneity begins to resolve individual production events. Most notably, for the smallest scale ($\lambda = 3 \text{ m}^{-1}$), a sharp peak emerges, with an amplitude substantially exceeding those observed at larger λ but with narrower width. This correlates quantitatively with the elevated normalized total particle yield $\bar{N}$ reported in Fig. 2 for identical parameters.

Upon increasing the separation distance to $d = 2 \text{ m}^{-1}$ [Fig. 3(b)], the peak amplitude for $\lambda = 3 \text{ m}^{-1}$ remains comparable to the $d = 1 \text{ m}^{-1}$ case and the width of the momentum spectrum broadens to some extent, while modest increases are observed for other λ values. Concurrently, spectral broadening becomes more pronounced for $\lambda = 3, 5, 10 \text{ m}^{-1}$, suggesting enhanced momentum dispersion under reduced field coherence. As λ increases, the $\lambda = 3 \text{ m}^{-1}$ distribution flattens progressively and yields at larger λ grow relatively. This evolution is consistent with the physical picture that increasing d establishes more stable localized field gradients responsible for resonant enhancement, thereby distributing the production probability more uniformly across the momentum spectrum.

A qualitative transition occurs at $d = 3 \text{ m}^{-1}$ [Fig. 3(c)], where symmetric momentum distributions reemerge across all λ . This restoration of symmetry marks a definitive departure from the asymmetric profiles observed at smaller d , indicating that the field inhomogeneity has weakened sufficiently to allow balanced pair production at positive and negative momenta. The peak for

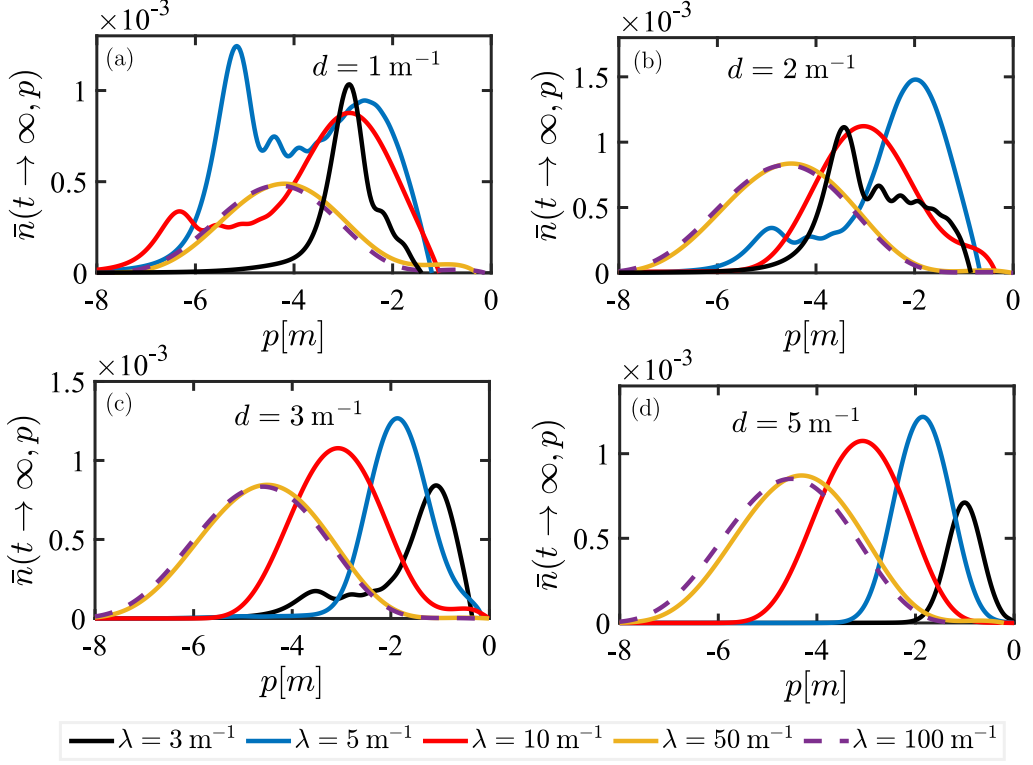


FIG. 3. Momentum distributions for five representative spatial scales λ under a spatially antisymmetric field with various spatial distances d : (a) $d = 1 \text{ m}^{-1}$, (b) $d = 2 \text{ m}^{-1}$, (c) $d = 3 \text{ m}^{-1}$, (d) $d = 5 \text{ m}^{-1}$. Other parameters are the same as those in Fig. 1.

$\lambda = 3 \text{ m}^{-1}$ continues to decrease, whereas those for $\lambda = 5$ and 10 m^{-1} remain largely unchanged relative to $d = 2 \text{ m}^{-1}$, consistent with the suppression of pair creation under rapidly varying fields. The insensitivity of these distributions to increasing d suggests that the characteristic length scale of the field variation has already fallen below the threshold required for efficient particle production, effectively decoupling the yield from the spatial extent of the pulse. For $\lambda = 10 \text{ m}^{-1}$, the peak amplitude stabilizes for $d \geq 3 \text{ m}^{-1}$, establishing a lower bound for the onset of this decoupling. This saturation behavior intensifies for $\lambda = 50$ and 100 m^{-1} , whose spectra converge across the entire momentum range, signaling the onset of a saturation regime in which further increases in λ produce negligible modifications to the momentum distribution. The convergence of these high- λ curves implies that the momentum distribution has become insensitive to the detailed spatial structure of the field, responding instead to an effective average field strength. This asymptotic universality provides strong evidence that, in the limit of large λ , the multiscale field configuration reduces to a homogeneous-field approximation, with pair production governed by the spatially averaged potential rather than local field gradients.

At the largest separation, $d = 5 \text{ m}^{-1}$ [Fig. 3(d)], the spectra adopt a fully symmetric shape, and the $\lambda = 3 \text{ m}^{-1}$ peak continues to decline. Notably, the $d = 3 \text{ m}^{-1}$ configuration yields marginally higher number densities at intermediate momenta than the $d = 5 \text{ m}^{-1}$ case—demonstrating that excessive separation does not universally enhance pair production and that an optimal separation distance exists near $d \approx 3 \text{ m}^{-1}$.

Across all configurations, three robust trends emerge. First, the peak value of the normalized momentum-resolved number density $\bar{n}(p)$ decreases monotonically with increasing d for

$\lambda = 3 \text{ m}^{-1}$, and the peak position shifts progressively toward $p = 0$, as either λ decreases or d increases, confirming the dominance of low-momentum pair production under such conditions. Conversely, for large λ , the peak migrates to increasingly negative p , implying greater energy absorption from the field and acceleration of particles over extended spatial intervals. Second, a self-bunching effect, characterized by momentum-space shrinking, is consistently observed; its strength increases with d for smaller λ , in agreement with prior theoretical studies [29]. Third, spectral saturation is evident for $\lambda \geq 50 \text{ m}^{-1}$, beyond which variations in λ no longer alter the momentum distribution.

These features arise from the quasihomogeneous nature of the field at large λ . In this regime, the local density approximation (LDA) applies, as validated in Refs. [29,36,40]. Under the LDA, the spatially inhomogeneous field is approximated as a superposition of locally uniform fields. Consequently, the asymptotic momentum distribution is obtained via spatial averaging:

$$\tilde{n}(p, t \rightarrow \infty) = \sum_x \frac{n(\epsilon(x) | p, t \rightarrow \infty)}{\lambda}, \quad (17)$$

where $n(\epsilon(x) | p, t \rightarrow \infty)$ denotes the momentum spectrum for a homogeneous field of strength $\epsilon(x)$. This approximation is rigorously valid when the pair-formation length $\ell = 2m/|eE_{\text{cr}}|$ is significantly smaller than the characteristic length scale λ , a condition satisfied for $\lambda \gtrsim 50 \text{ m}^{-1}$. As shown in Fig. 3, the spectral profiles corresponding to $\lambda = 50 \text{ m}^{-1}$ and $\lambda = 100 \text{ m}^{-1}$ are nearly indistinguishable, thereby confirming the onset of quasihomogeneous behavior.

It is important to note that the momentum spectrum of vacuum-produced electron-positron pairs exhibits high sensitivity to both the field variation scale λ and the separation distance d . Efficient pair production requires the simultaneous satisfaction of a minimum spatial scale of field variation and a sufficiently large separation distance. The present analysis is confined to field separation distances in the range $1 \text{ m}^{-1} \leq d \leq 5 \text{ m}^{-1}$. For $d \geq 3 \text{ m}^{-1}$, the pair production efficiency is significantly suppressed, as shown in Fig. 3. A comparable suppression occurs for $d \leq 1 \text{ m}^{-1}$, arising from the antisymmetric spatial configuration of the two electric fields, which reduces the net field strength coupling to the quantum vacuum (see Fig. 1). Consequently, the selected interval constitutes an optimal operational window in which both suppression mechanisms are simultaneously minimized, thereby enabling a systematic characterization of the transition from resonant enhancement to saturation. These findings advance the understanding of spatial constraints governing nonperturbative vacuum decay and establish the momentum-resolved particle yield as a quantitative diagnostic tool for reconstructing the underlying electromagnetic field geometry.

The three-field configuration produces distinctive structures in the momentum spectrum that are absent in all two-field antisymmetric configurations; these features originate from interference-driven spatial inhomogeneity. In Fig. 4, we present the momentum spectrum for a fixed spatial distance $d = 3 \text{ m}^{-1}$, corresponding to the field profile with maximal spatial structure [Fig. 1(d)], for five different spatial extents λ . The results reveal three important features.

At $d = 3 \text{ m}^{-1}$ with a spatial extent of $\lambda = 10 \text{ m}^{-1}$, the momentum distribution exhibits two well-resolved peaks centered at $p \approx \pm 3 \text{ m}$. This stands in sharp contrast to the single central peak universally observed in antisymmetric two-field configurations (see Fig. 3). The splitting arises directly from the field distribution shown in Fig. 1(d), where the three-field configuration produces transient field enhancements; these sub-cycle-scale regions impart symmetric momentum transfers to nascent electron-positron pairs via the semiclassical acceleration mechanism (for the physical interpretation, see the following paragraph). For longer field extents ($\lambda \geq 50 \text{ m}^{-1}$), the peaks merge into a broadened distribution with the same profile, which is quantitatively consistent with the LDA for quasihomogeneous fields (as discussed in the previous section). For small spatial extents $\lambda = 3, 5 \text{ m}^{-1}$ with the same fixed separation $d = 3 \text{ m}^{-1}$, the momentum spectrum displays a narrowed shape with reduced peak amplitudes. This occurs because destructive interference minimizes the effective field amplitude, thereby suppressing the quantum

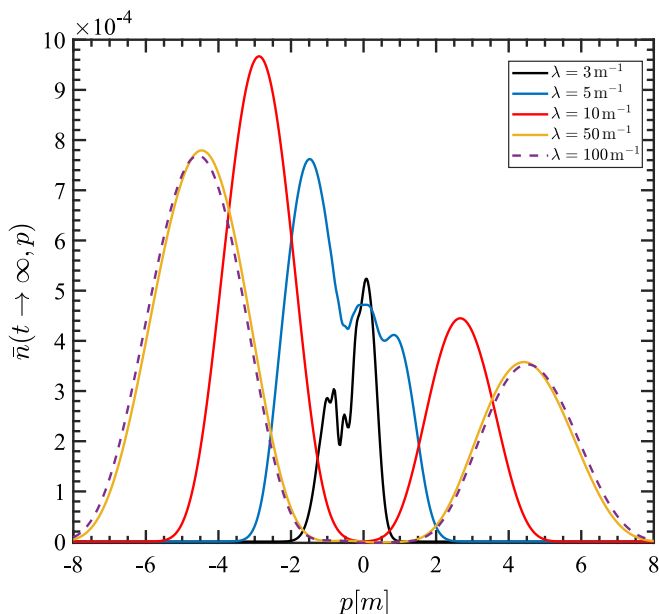


FIG. 4. Momentum distributions for a three-field configuration with fixed field separation distance $d = 3 \text{ m}^{-1}$, evaluated at five representative spatial extents λ . All other parameters are identical to those in Fig. 1.

tunneling probability for pair creation. Such nodal structure has no analog in two-field configurations, whose pairwise interference cannot generate multiple, spectrally isolated structures in the momentum distribution. The normalized total pair yield achieves its global maximum at $d = 3 \text{ m}^{-1}$ (Fig. 4, red curve), in quantitative agreement with the field enhancement documented in Fig. 1(a).

It is worth noting that the heights of the peaks in the negative and positive p regions differ, as shown in Fig. 4. As illustrated in Fig. 1(d), the direction of the first field is opposite to the x axis, and the second field is directed along the x axis ($-E_1$ and E_2). Consequently, the electrons experience field forces directed both along and opposite to the x axis, leading to more electrons accumulating between the two fields (centered at approximately $p = 3 \text{ m}$ for $\lambda = 3 \text{ m}^{-1}$). However, the direction of the third field is opposite to the x axis ($-E_3$); thus, in this region, the electrons only experience a field force directed along the positive x direction. This results in peak splitting, with the height of the left peak being twice that of the right peak, as shown in Fig. 4. These spectral features confirm that the three-field geometry introduces interference pathways, beyond any perturbative extension of two-field configurations, that imprint topologically distinct, interference-encoded signatures onto the quantum vacuum decay spectrum. In Fig. 5, we investigate the dependence of the total reduced pair yield $\bar{N}$ on the field parameters λ (spatial extent) and d (separation distance) within the three-field configuration. Figure 5 depicts the evolution of $\bar{N}$ as a function of λ for distinct values of d . Our results demonstrate that the pair-creation efficiency is maximized in the parameter regime $d \gtrsim 2 \text{ m}^{-1}$ and $\lambda \gtrsim 20 \text{ m}^{-1}$, identifying the optimal separation distance and spatial extent for pair production in this setup. Notably, for $\lambda \gtrsim 30 \text{ m}^{-1}$, $\bar{N}$ exhibits saturation behavior, indicating diminishing returns in pair yield with further increases in the spatial extent of the field. In this saturated regime, the field variation rate is sufficiently slow compared to the characteristic Compton length scale, which validates the quantitative applicability of the LDA approximation introduced in Eq. (17) for analyzing the pair production dynamics. This agreement between the numerical results and the LDA framework confirms the robustness of our theoretical analysis in the high- λ regime.

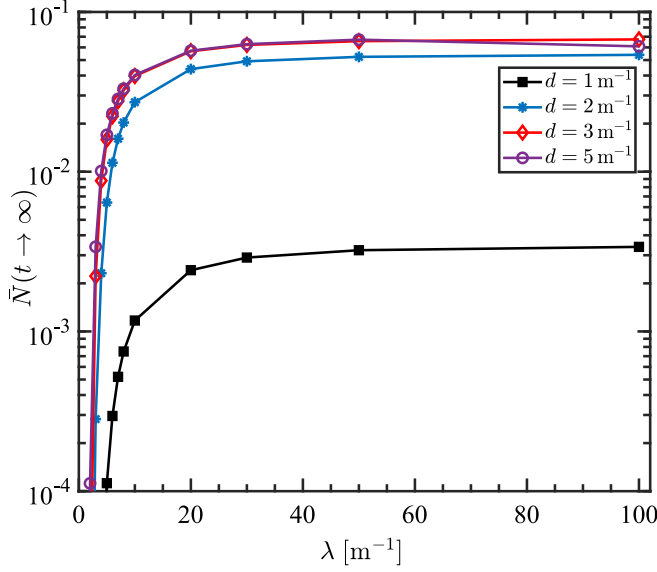


FIG. 5. Total reduced pair yield $\bar{N}$ as a function of the field spatial extent λ and separation distance d for the three-field configuration. The other parameters are the same as in Fig. 1.

IV. CONCLUSION

In this work, we systematically investigate vacuum electron-positron pair production induced by spatially antisymmetric electric fields within the $(1 + 1)$ -dimensional DHW formalism. Our study encompasses both two-field and three-field configurations, with systematic parametric scans over the field separation distance d and the characteristic field scale λ . This enables us to elucidate the regulatory role of spatial antisymmetry in nonperturbative pair production dynamics and to establish concrete theoretical guidelines for future studies.

Strict spatial antisymmetry alters the vacuum pair production mechanism, yielding qualitative and quantitative distinctions from spatially symmetric or fully antisymmetric field configurations. Such antisymmetry generates effective field inhomogeneity and distinctive momentum-space structures absent in alternative setups. Crucially, the field separation d serves as a key control parameter: Maximal total pair yield is achieved at $d = 3 \text{ m}^{-1}$, where constructive interference enhances local field strength while preserving sufficient spatial overlap between field components, thereby optimizing the quantum tunneling probability.

The momentum spectrum of produced pairs exhibits pronounced sensitivity to both λ and d , and three-field configurations afford distinctive spectral engineering capability. Specifically, at $d = 3 \text{ m}^{-1}$, the three-field structure splits the momentum distribution into two well-resolved peaks, a feature absent in two-field configurations. For small spatial scales ($\lambda = 3$ and 5 m^{-1}), sharp suppression nodes emerge, arising from destructive interference at specific spatial locations that reduce the effective field amplitude and suppress tunneling. These spectrally resolved features constitute a robust basis for tailoring pair momentum distributions in upcoming studies.

This study establishes a rigorous theoretical foundation for understanding how spatial asymmetry constraints govern nonperturbative quantum electrodynamical processes in vacuum. It further delivers actionable design principles for optimizing pair production efficiency and spectral properties. Future work will extend the analysis to $(3 + 1)$ -dimensional spacetime to incorporate transverse momentum effects and magnetic field contributions, factors expected to modify pair dynamics under realistic experimental conditions. Additionally, inclusion of time-dependent chirp and

carrier-envelope phase will allow exploration of synergistic spatiotemporal control, advancing toward high-fidelity manipulation of this fundamental quantum phenomenon.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: ANALYTICAL APPROXIMATION

To assess the accuracy of the DHW calculations in the preceding section, we derive an analytical approximation for the particle momentum distribution and compare it with DHW results, additionally investigating the validity of both the DHW formalism and the analytical approximation. For spatially homogeneous systems, quantum kinetic theory (QKT) reduces to the DHW formalism; specifically, for a time-dependent homogeneous electric field and vanishing magnetic field, the DHW formalism recovers the Vlasov equation of QKT for Schwinger pair production. Owing to spatial homogeneity and the absence of a magnetic field, DHW functions are x independent (all spatial derivatives vanish), enabling analytical simplification via two steps: (1) compute the vacuum Wigner function for initial conditions and (2) reduce Eqs. (5)–(8) (partial differential equations) to ordinary differential equations equivalent to the Vlasov equation [55,144], yielding

$$n(x, p_x, p_\perp, t \rightarrow +\infty) = \frac{2 \sinh\left(\frac{\pi\tau}{2}[2e\tau\epsilon(x) + \tilde{Q}(x) - Q(x)]\right) \sinh\left(\frac{\pi\tau}{2}[2e\tau\epsilon(x) - \tilde{Q}(x) + Q(x)]\right)}{\sinh(\pi\tau\tilde{Q}(x)) \sinh(\pi\tau Q(x))}, \quad (\text{A1})$$

where

$$\tilde{Q}(x) = \sqrt{m^2 + (p_x + 2e\tau\epsilon(x))^2}, \quad (\text{A2})$$

$$Q(x) = \sqrt{m^2 + (p_x - 2e\tau\epsilon(x))^2}. \quad (\text{A3})$$

Because the model is $(1+1)$ dimensional, the transverse momentum $p_\perp$ is also neglected in the analytical approximation.

In this investigation, we employ an analytically solvable Sauter-type electric field, as expressed in Eqs. (15) and (16). Therefore, the particle momentum distribution can be obtained analytically using Eq. (A1) by replacing the field strength $E(t, x) = E_0 f(x)g(t)$ in the analytical one-particle distribution solution with the effective field strength $E(t) = \epsilon(x)g(t)$ in a space- and time-dependent field, provided that the spatial scale of the field is large.

As discussed above, the analytical approximation is valid only in the large- λ regime; therefore, we adopt $\lambda = 50 \text{ m}^{-1}$ in our analytical calculations, with results presented in Fig. 6. For the antisymmetric two-field configuration defined in Eq. (15), we select two representative values of the separation parameter, $d = 1 \text{ m}^{-1}$ and $d = 3 \text{ m}^{-1}$, and compare the corresponding analytical predictions against the full DHW numerical results, as shown in Fig. 6(a). For the three-field configuration specified in Eq. (16), we perform analogous comparisons at the same d values, with results displayed in Fig. 6(b). In all cases, the particle momentum distributions obtained from the DHW formalism exhibit excellent quantitative agreement with those predicted by the analytical approximation. Overall, the analytical approximation reliably reproduces the DHW results for

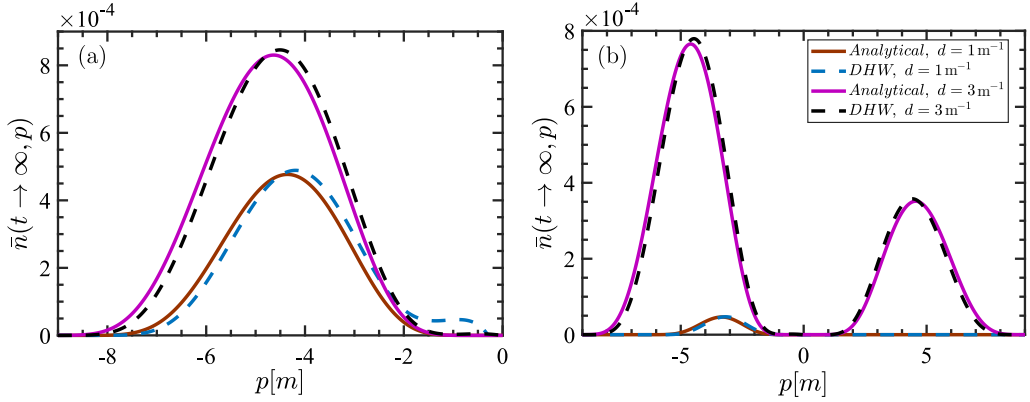


FIG. 6. Comparison of the DHW results (dashed curves) with the analytical approximation (solid curves): (a) for the two-field case, which employs the electric field profile given in Eq. (15) and (b) for the three-field case, which employs the electric field profile given in Eq. (16), both at fixed $\lambda = 50 \text{ m}^{-1}$ and under varying values of $d = 1 \text{ m}^{-1}$ and $d = 3 \text{ m}^{-1}$.

spatially antisymmetric fields in the large- λ limit, regardless of whether the separation distance d is small or comparatively large.

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