

Reconstructing quantum noise from symmetry-violating generators

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By systematically introducing small symmetry-violating terms into quantum gates, we examine whether such perturbations can reproduce the tomographic noise patterns observed on actual quantum hardware. This study fits into the broader context of quantum computing and quantum information processing, where realistic noise modeling remains essential for understanding device behavior. Our analytical framework mirrors Qiskit's tomography procedures and shot model, using transparent noise control. Our results show that weak generator perturbations can reproduce average deviations in Pauli transfer matrices (PTMs) with high fidelity (given the recorded hardware noise characteristics). These findings suggest that symmetry-violating generators provide a phenomenological model for relevant classes of hardware noise. This approach offers a reproducible analytic baseline for testing noise hypotheses and calibrating simplified models against experimental data. Beyond this descriptive role, the appearance of stable symmetry-violating perturbations in the fitted models supports their use as a phenomenological description. Because the analytical framework isolates these violations at the generator level, it links native-generator perturbations to reconstructed PTM signatures. As a result, the model not only reproduces experimental process tomography data but also tests which PTM structures can be reproduced and at what perturbation magnitude, providing information for calibration, simulation, and the development of symmetry-sensitive mitigation and error-correction strategies. The present analysis is restricted to single-qubit gates.

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I. INTRODUCTION

Real quantum hardware deviates from idealized unitary dynamics due to relaxation, dephasing, leakage, and control errors that collectively break the mathematical idealization of quantum information processing. We aim to model these imperfections as deviations at the generator level,

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where small symmetry-violating terms modify the effective dynamics. In this work, we propose an analytic description of such nonideal behavior by introducing *symmetry-violating generator noise* directly into quantum gate's Lie algebra representation. Related generator-based descriptions of quantum noise and Markovian error structure have previously been studied in the context of error-generator decompositions and process-matrix analysis [1]. Our work differs in scope and objective: Rather than constructing a complete decomposition of arbitrary reconstructed completely positive trace-preserving (CPTP) channels, we introduce explicit perturbations directly into native gate generators and investigate whether such structured perturbations can phenomenologically reproduce experimentally observed Pauli transfer matrix (PTM) signatures. More specifically, our construction defines the forward map $\{\Delta_g\}_{g \in \mathcal{G}} \rightarrow \{U_g^\Delta\}_{g \in \mathcal{G}} \rightarrow \text{QPT-12} \rightarrow R_{\text{fit}}$, whereas error-generator taxonomies start from a reconstructed channel and subsequently decompose its error generator. The two approaches are therefore complementary rather than competing representations.

Quantum process tomography provides the quantitative framework to study these deviations. Process tomography reconstructs the full dynamical map that represents a quantum operation, enabling direct comparison between the expected and realized transformations [2–4]. In this context, it answers a concrete question: How does an implemented gate act on arbitrary input states compared to its theoretical target? Applying process tomography to our circuits therefore allows us to identify and classify aspects of hardware imperfections. At the same time, process tomography already contains the experimentally accessible channel information, and experimentally reconstructed PTMs therefore already encode the measurable noise signatures discussed throughout this work. Accordingly, the present framework should not be interpreted as extracting fundamentally new information from the hardware beyond the tomography reconstruction itself.

In this study, we rebuild single-qubit process tomography analytically using a finite-dimensional matrix pipeline. Gates are defined as exponentials of $\mathfrak{su}(2)$ generators, evaluated through matrix exponentials. Each gate $U(\theta) = \exp[-i(\theta/2)G]$ is replaced by

$$U_\Delta(\theta) = \exp \left[-i \frac{\theta}{2} (G + \Delta) \right],$$

where Δ represents a tunable additive matrix perturbation. A Hermitian Δ corresponds to coherent miscalibration such as axis tilt or rotation error, while a general complex Δ produces effective nonunitary behavior that emulates dissipation, leakage, or other open-system effects at the circuit level [2,3]. Accordingly, the present framework should be interpreted as a constrained phenomenological reconstruction model rather than a universal or complete representation of hardware noise. The primary objective is not representational completeness, but analytical transparency and controlled investigation of how generator-level perturbations propagate into experimentally reconstructed process matrices. More specifically, the framework investigates whether structured symmetry-violating perturbations can phenomenologically reproduce characteristic patterns observed in experimentally reconstructed PTMs.

Here, θ denotes the physical rotation angle of the gate, $G \in \mathfrak{su}(2)$ is the ideal generator associated with the target operation, $\Delta \in \mathbb{C}^{2 \times 2}$ is the additive perturbation matrix modeling symmetry-violating noise, and $U_\Delta(\theta)$ is the resulting noisy gate obtained by exponentiating the perturbed generator. The present single-qubit setting is intentionally used as a controlled proof of concept in which this mapping can be analyzed explicitly.

Preparation and measurement follow the Pauli schedules implemented in Qiskit Experiments, and expectation values are sampled through a binomial shot model to reproduce hardware statistics [4,5]. We then compare these analytic reconstructions against runs on an IBM Quantum system (IBM Brisbane), averaging over repeated executions to suppress statistical noise and temporal drift. The adopted workflow therefore mirrors the structure and assumptions of standard tomography pipelines rather than introducing a new tomography protocol.

The analysis focuses on two central questions:

(1) Can small symmetry-violating perturbations introduced at the generator level reproduce features observed in experimental process tomographies?

(2) Can such analytic perturbations be tuned to represent realistic hardware noise profiles, at least in an average and statistically consistent sense?

Our results demonstrate that generator-level perturbations can be tuned to reproduce the average tomographic signatures of noise observed on IBM Quantum hardware for single-qubit circuits. Using the same Pauli preparation and measurement schedules as in Qiskit’s process tomography, the analytic reconstructions consistently reproduce deviations found in experimental Pauli transfer matrices. The study shows that symmetry violation at the generator level is one possible mechanism contributing to these deviations, but it is not the only one; additional physical modeling using CPTP constraints, Kraus or Lindbladian formulations, and explicit treatment of crosstalk and drift are other potential sources for these deviations [6–8].

The framework presented here provides an explicit and controllable analytic counterpart to the Qiskit tomography workflow, allowing direct comparison between simulated and experimental process reconstructions under both ideal and noisy conditions. Using a genetic algorithm (GA), we can reproduce features of non-trace-preserving behavior and the componentwise variance patterns observed in hardware data while retaining full analytical transparency of the underlying reconstruction process.

These findings are important because they reveal that symmetry-violating perturbations might not be merely a mathematical inconvenience but measurable (within the shown uncertainty) structured features of real hardware noise. At the same time, the experimentally reconstructed PTMs already contain the measurable channel information accessible through the tomography protocol itself. The role of the present framework is therefore not to extract fundamentally new information beyond tomography, but to test whether structured perturbations introduced directly at the generator level can reproduce experimentally observed PTM signatures within a transparent analytical reconstruction pipeline. Accordingly, the presented perturbations should be interpreted as one possible phenomenological mechanism capable of reproducing experimentally observed tomography structures, rather than as evidence for a unique microscopic physical noise origin. By isolating these effects at the generator level, the framework shows which assumed symmetries in standard noise models break down and quantifies the scale of this symmetry violation directly from experimental data. This provides a principled route for developing related noise models, calibration tools, and error-mitigation strategies that explicitly account for symmetry-violating behavior of real device imperfections.

II. RELATED WORK

Foundational formulations of quantum state and process tomography were established by Chuang and Nielsen [2] and by Poyatos *et al.* [3], with standard textbook treatments collected in Ref. [4]. The PTM and related affine Bloch-vector form are widely used to represent single-qubit channels and compare operations (see, e.g., Ref. [9]).

Beyond direct linear inversion, self-consistent and overcomplete approaches seek robustness against state-preparation-and-measurement (SPAM) errors. Merkel *et al.* introduced self-consistent process tomography [10], while gate set tomography (GST) provides a comprehensive, SPAM-robust characterization across a gate family [11]. These methods complement our analytic reconstructions, which explicitly mirror the minimal Pauli schedules but can also be evaluated on overcomplete schedules.

Noise benchmarking and diagnostics provide orthogonal evidence about coherent and stochastic error structure. Randomized benchmarking (RB) offers SPAM-robust error-rate estimates [12], and comprehensive introductions and tutorials now standardize the methodology and its practical nuances [13]. Clarifying the interpretation of RB data, Proctor *et al.* show precisely what RB actually measures and under which assumptions [14]. RB-style protocols have also been combined with tomographic techniques to obtain robust gate reconstructions [15]. Related notions such as *unitarity* help separate coherent miscalibration from stochastic noise [16]. Blume-Kohout *et al.* provide a systematic taxonomy of small Markovian errors by transforming process matrices into error

generators and decomposing them into Hamiltonian, stochastic, correlation, and active sectors [1]. Our approach differs in that we do not decompose an already reconstructed channel into a complete generator taxonomy; instead, we introduce explicit symmetry-violating perturbations at the native gate-generator level and fit these perturbations to reproduce experimentally observed PTM structures. Thus, the taxonomy provides a postreconstruction error-generator decomposition, whereas our framework provides a forward native-gate perturbation model whose output is compared with R_{target} after the tomography reconstruction. The contribution is therefore a complementary and restricted modeling direction rather than a more general generator representation; its expressivity is analyzed explicitly in Appendix B. These tools contextualize the symmetry-violating generator model explored here: Hermitian perturbations emulate coherent miscalibration, while general (non-Hermitian) perturbations emulate effective nonunitary behavior at the circuit level.

In parallel, error-mitigation techniques—notably zero-noise extrapolation—adjust circuit-level parameters to infer noise-free estimates [17]. Although our goal is not error mitigation, the idea of *controlled* circuit-level modifications aligns with our analytic perturbation model. The Qiskit Experiments framework operationalizes these tomography and calibration workflows in practice [5, 18], which we reproduce analytically via explicit matrix products.

Finally, symmetry-related generator perturbations have been explored as a modeling tool beyond characterization, including for privacy-preserving data obfuscation in quantum-inspired pipelines [19]. Our study adapts this generator-level perspective to investigate whether small symmetry violations can mimic averaged tomographic signatures observed on IBM hardware, thereby providing a compact phenomenological model to test hypotheses about realistic device noise.

GAs are population-based stochastic search methods inspired by biological evolution and have been developed and analyzed since the foundational work of Holland and the engineering codification by Goldberg [20, 21]. Core GA operators (selection, recombination, and mutation) enable global exploration with problem-agnostic objective functions, and their behavior has been surveyed extensively in the optimization literature [22–24]. Multiobjective extensions such as NSGA-II introduced elitist selection and fast nondominated sorting, improving convergence and solution diversity for vector-valued objectives [25]. In our context, we use a GA to fit per-gate perturbation parameters by minimizing a tomography-derived discrepancy; the same class of metaheuristics has been shown effective when the search space is nonconvex, noisy, or only implicitly defined. Relatedly, GA-driven optimization has been coupled with stochastic generative models in prior work by Raubitzek *et al.*, where a GA selects among fractional Brownian-bridge candidates using a smoothness-based fitness to interpolate sparse time series [26]. This demonstrates a practical pattern, combine a stochastic proposal mechanism with evolutionary selection under a task-specific fitness, which we also follow here for calibrating symmetry-violating generator parameters against experimental process-tomography targets.

III. THEORETICAL FOUNDATIONS

In this work, we consider quantum information processing on single qubits. We give a full summary of the used notation and math in Appendix A to support and clarify the following. A qubit is a two-level quantum system with Hilbert space $\mathcal{H} \simeq \mathbb{C}^2$ and computational basis $\{|0\rangle, |1\rangle\}$ [4]. A pure single-qubit state is a normalized vector

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle, \quad \alpha, \beta \in \mathbb{C}, \quad |\alpha|^2 + |\beta|^2 = 1. \quad (1)$$

To include classical uncertainty and decoherence, we describe the state by a density operator ρ , which is a positive semidefinite, unit-trace 2×2 matrix. Any single-qubit density operator can be written in Bloch form

$$\rho = \frac{1}{2}(I_2 + r_x \sigma_x + r_y \sigma_y + r_z \sigma_z), \quad \mathbf{r} = (r_x, r_y, r_z)^\top \in \mathbb{R}^3, \quad \|\mathbf{r}\|_2 \leq 1, \quad (2)$$

where $\sigma_x, \sigma_y, \sigma_z$ are the Pauli operators and $\mathbf{r}$ denotes the Bloch vector of the state [4].

Quantum gates, i.e., transformations acting on the qubit, are represented by unitary operators acting on $\mathcal{H}$. In the Schrödinger picture, the action of a single-qubit gate $U \in \text{U}(2)$ on ρ is

$$\rho \mapsto U \rho U^\dagger, \quad (3)$$

and for pure states $|\psi\rangle$ this corresponds to $|\psi\rangle \mapsto U |\psi\rangle$. A convenient basis for single-qubit operations is given by rotations generated by Pauli operators. For σ_i with $i \in \{x, y, z\}$, the corresponding rotation is

$$R_i(\theta) = e^{-i(\theta/2)\sigma_i}. \quad (4)$$

On superconducting-qubit platforms such as IBM Quantum, native single-qubit gates are expressed through Z -axis rotations and $\pi/2$ rotations around the X axis [27,28]. A common basis is $\{R_z(\theta), \text{SX}, X\}$, defined as

$$R_z(\theta) = e^{-i(\theta/2)\sigma_z}, \quad (5)$$

$$\text{SX} = e^{-i(\pi/4)\sigma_x}, \quad (6)$$

$$X = e^{-i(\pi/2)\sigma_x}. \quad (7)$$

Standard gates such as the Hadamard and the phase gate can be decomposed in this basis, for example,

$$H = R_z\left(\frac{\pi}{2}\right) \text{SX} R_z\left(\frac{\pi}{2}\right), \quad S = R_z\left(\frac{\pi}{2}\right), \quad (8)$$

consistent with typical backend calibrations [28].

A single-qubit circuit is a time-ordered sequence of gates applied to an initial state. Let ρ_{in} be the input state and $\{U_k\}_{k=1}^L$, the unitaries corresponding to the gates from first applied (U_1) to last applied (U_L). The total unitary is

$$U_{\text{tot}} = U_L U_{L-1} \cdots U_1, \quad (9)$$

and the ideal output state is

$$\rho_{\text{out}}^{(\text{ideal})} = U_{\text{tot}} \rho_{\text{in}} U_{\text{tot}}^\dagger. \quad (10)$$

Logical circuits are decomposed into this native gate sequence to the actual physical sequence executed on the backend [27,28].

A typical single-qubit experiment consists of state preparation, application of the target-gate sequence, and measurement. The device initializes the qubit in $|0\rangle$, after which a preparation unitary U_{prep} creates the desired input state,

$$\rho_{\text{in}} = U_{\text{prep}} |0\rangle\langle 0| U_{\text{prep}}^\dagger. \quad (11)$$

The target-gate sequence U_{target} then acts on this state,

$$\rho_{\text{mid}} = U_{\text{target}} \rho_{\text{in}} U_{\text{target}}^\dagger. \quad (12)$$

Before measurement, an optional basis-rotation unitary U_{meas} may be inserted to access observables in different measurement bases, resulting in

$$\rho_{\text{out}} = U_{\text{meas}} \rho_{\text{mid}} U_{\text{meas}}^\dagger. \quad (13)$$

The device then performs a projective measurement in the computational basis $\{|0\rangle, |1\rangle\}$. The full evolution factorizes as

$$\rho_{\text{out}} = U_{\text{meas}} U_{\text{target}} U_{\text{prep}} |0\rangle\langle 0| U_{\text{prep}}^\dagger U_{\text{target}}^\dagger U_{\text{meas}}^\dagger. \quad (14)$$

Measurements in other Pauli bases are implemented through suitable premeasurement rotations. Measuring in the X basis uses $U_{\text{meas}} = H$, since $H\sigma_z H = \sigma_x$. Measuring in the Y basis uses $U_{\text{meas}} = HS^\dagger$.

Let $\Pi_0 = |0\rangle\langle 0|$ and $\Pi_1 = |1\rangle\langle 1|$ denote the computational basis projectors. For a given output state ρ_{out} , the Born rule assigns the probabilities

$$p(b) = \text{Tr}(\rho_{\text{out}} \Pi_b), \quad b \in \{0, 1\}. \quad (15)$$

In an experiment, the circuit is executed N times, yielding empirical frequencies $\hat{p}(b) = n_b/N$. These estimates exhibit binomial fluctuations with variance

$$\text{Var}[\hat{p}(b)] = \frac{p(b)(1 - p(b))}{N}. \quad (16)$$

Expectation values of observables follow from

$$\langle O \rangle = \text{Tr}(\rho_{\text{out}} O). \quad (17)$$

For Pauli operators,

$$\langle \sigma_i \rangle = \text{Tr}(\rho_{\text{out}} \sigma_i) = r_i^{(\text{out})}, \quad i \in \{x, y, z\}, \quad (18)$$

providing direct access to the components of the output Bloch vector when the appropriate measurement basis is used.

Real devices deviate from the ideal unitary description due to control errors, relaxation, dephasing, and other noise sources. The effective action of an implemented single-qubit gate is thus modeled by a quantum channel $\mathcal{E}$, a linear, completely positive, trace-preserving map on density operators [4]. A standard representation is the Kraus form

$$\mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger, \quad \sum_k E_k^\dagger E_k = I_2, \quad (19)$$

where $\{E_k\}$ are Kraus operators. An ideal unitary corresponds to the special case $\mathcal{E}_U(\rho) = U\rho U^\dagger$. When a noisy channel acts on an input state, the output becomes

$$\rho_{\text{out}}^{(\text{noisy})} = \mathcal{E}(\rho_{\text{in}}), \quad (20)$$

and measurement probabilities follow as

$$p(b) = \text{Tr}(\mathcal{E}(\rho_{\text{in}}) \Pi_b). \quad (21)$$

The combination of state preparation, channel action, and projective measurement fully determines the observed statistics. This formalism—single-qubit states, unitary evolution, quantum channels, and measurement statistics—forms the basis for the characterization and analysis of implemented single-qubit operations.

A. Quantum process tomography

Quantum process tomography (QPT) aims to reconstruct the dynamical map $\mathcal{E}$ that describes how input states are transformed by a physical operation [2,4,10]. In the single-qubit setting considered here, $\mathcal{E}$ is modeled as a CPTP map acting on density operators, such that for any input state ρ_{in} the corresponding output state is

$$\rho_{\text{out}} = \mathcal{E}(\rho_{\text{in}}). \quad (22)$$

The goal of QPT is to determine $\mathcal{E}$ from experimental data obtained by preparing a set of input states, applying the process, and measuring the resulting output states using the state-preparation and measurement framework introduced in Sec. III.

Formally, QPT exploits the linearity of $\mathcal{E}$. A tomographically complete set of input states $\{\rho_k\}$ spans the space of single-qubit density operators. In practice, one chooses a small set of linearly

independent states, such as the eigenstates of the Pauli operators: the computational basis states $|Z+\rangle\langle Z+|$ and $|Z-\rangle\langle Z-|$, together with superposition states such as $|X+\rangle\langle X+|$ and $|Y+\rangle\langle Y+|$, where $|Z+\rangle = |0\rangle$, $|Z-\rangle = |1\rangle$, $|X+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, and $|Y+\rangle = (|0\rangle + i|1\rangle)/\sqrt{2}$. The device prepares each ρ_k by applying an appropriate U_{prep} to $|0\rangle\langle 0|$, then applies the physical gate or gate sequence described by $\mathcal{E}$, and finally performs measurements after a basis-rotation unitary U_{meas} , as in Sec. III. For each input state and each measurement basis (for example, Z , X , and Y), the experiment yields outcome frequencies that estimate the corresponding probabilities and expectation values.

In the IBM Quantum and Qiskit tomography workflow adopted in this work, the preparation and measurement circuits themselves are constructed from the same native single-qubit gate set $\{R_z(\theta), SX, X\}$ that is subsequently analyzed during the tomography procedure [27,28]. Consequently, the preparation of the states $|Z+\rangle$, $|Z-\rangle$, $|X+\rangle$, and $|Y+\rangle$, as well as the basis rotations required for measurements in the X and Y bases, are implemented using the same noisy hardware primitives. The adopted workflow is therefore not SPAM robust in the sense of gate set tomography or cycle benchmarking protocols [11,14].

At the same time, the analytical reconstruction framework used throughout this work explicitly mirrors the same tomography assumptions and circuit structure used during the hardware experiments. More specifically, the generator-level perturbations introduced in the model propagate through the complete tomography circuits, including state preparation operations, target-gate applications, and measurement-basis rotations. The reconstruction therefore does not assume that these auxiliary tomography operations are ideal at the circuit-simulation level. Instead, the reconstructed PTMs should be interpreted as effective tomography reconstructions obtained under the same preparation schedules, basis rotations, finite-shot statistics, and reconstruction assumptions used in the adopted Qiskit tomography workflow. Consequently, the framework should not be interpreted as a fully self-consistent SPAM-isolated hardware characterization protocol, but rather as a phenomenological reconstruction framework designed to reproduce and analyze experimentally reconstructed PTM structures under experimentally matched tomography conditions.

To represent $\mathcal{E}$, several equivalent formalisms are available, including the Kraus representation, the Choi matrix, and the PTM [4,10]. In this work we use the PTM, which is convenient for single-qubit processes and matches the conventions of the Qiskit tomography tools [27,28]. We expand states and channels in the operator basis $\{I_2, \sigma_x, \sigma_y, \sigma_z\}$. For any single-qubit state ρ , we define the Bloch vector $\mathbf{r} = (r_x, r_y, r_z)^\top$ through

$$\rho = \frac{1}{2}(I_2 + r_x\sigma_x + r_y\sigma_y + r_z\sigma_z), \quad (23)$$

and introduce the augmented vector

$$\mathbf{s} = \begin{bmatrix} 1 \\ \langle X \rangle \\ \langle Y \rangle \\ \langle Z \rangle \end{bmatrix} = \begin{bmatrix} 1 \\ r_x \\ r_y \\ r_z \end{bmatrix}, \quad (24)$$

where $\langle \sigma_i \rangle = \text{Tr}(\rho \sigma_i)$ are the expectation values defined in Sec. III. The PTM $R \in \mathbb{R}^{4 \times 4}$ of a channel $\mathcal{E}$ is then defined by its action on these vectors,

$$\mathbf{s}_{\text{out}} = R \mathbf{s}_{\text{in}}, \quad \mathbf{s}_{\text{in}} = \begin{bmatrix} 1 \\ \langle X \rangle_{\text{in}} \\ \langle Y \rangle_{\text{in}} \\ \langle Z \rangle_{\text{in}} \end{bmatrix}, \quad \mathbf{s}_{\text{out}} = \begin{bmatrix} 1 \\ \langle X \rangle_{\text{out}} \\ \langle Y \rangle_{\text{out}} \\ \langle Z \rangle_{\text{out}} \end{bmatrix}. \quad (25)$$

Trace preservation (TP) implies that the first component of $\mathbf{s}_{\text{out}}$ is always equal to one, so the first row of R is fixed to $(1, 0, 0, 0)$. The remaining 3×3 submatrix encodes how the Bloch vector is transformed and captures both coherent rotations and incoherent effects such as dephasing and relaxation [10].

In an experiment, the PTM elements are inferred from measured expectation values. For each prepared input state ρ_k and each measurement basis corresponding to a Pauli operator σ_i , the protocol estimates $\langle\sigma_i\rangle_{\text{in},k}$ before the process and $\langle\sigma_i\rangle_{\text{out},k}$ after the process. These expectation values are obtained from the observed outcome frequencies using the Born rule and the finite-shot statistics from Sec. III. Collecting all input and output Bloch vectors into a data set yields a linear system whose solution determines the matrix R . For single-qubit processes, a minimal informationally complete scheme uses four input states (for example, $|Z+\rangle\langle Z+|$, $|Z-\rangle\langle Z-|$, $|X+\rangle\langle X+|$, and $|Y+\rangle\langle Y+|$) and measurements in the three Pauli bases X , Y , and Z . This results in 12 distinct circuits on hardware: four state preparations combined with three measurement bases. In the IBM Quantum and Qiskit framework, these circuits are built from the native gate set $\{R_z(\theta), \text{SX}, X\}$ and the standard premeasurement rotations implementing X - and Y -basis measurements [27,28]. The Qiskit tomography routines then perform a linear inversion or a constrained reconstruction to obtain an estimate $\hat{R}$ that is consistent with the CPTP constraints to the extent allowed by statistical noise and systematic errors [5,10].

Once reconstructed, the PTM $\hat{R}$ provides a compact and operational description of the implemented channel. It can be compared directly to theoretical PTMs derived from ideal unitary gates or from explicit noise models, and it allows one to quantify coherent miscalibrations and incoherent error mechanisms at the level of single-qubit processes. At the same time, the reconstructed PTMs should be interpreted as effective tomography reconstructions obtained under the adopted preparation and measurement assumptions and not as uniquely SPAM-isolated characterizations of microscopic hardware noise mechanisms. In the rest of this work, these reconstructed PTMs serve as the main interface between analytic models of noisy single-qubit evolution and the experimental data obtained from IBM Quantum hardware.

B. Qiskit basis and the 12-circuit process tomography

Single-qubit process tomography in Qiskit is implemented using a minimal native gate basis $\{R_z(\theta), \text{SX}, X\}$, which matches the hardware-level operations used on IBM Quantum devices [27,28]. This basis is universal for single-qubit unitaries and is compatible with the state-preparation and measurement operations introduced in Sec. III. In the QPT experiment, the process $\mathcal{E}$ is characterized by preparing four linearly independent input states and measuring each of the corresponding output states in three Pauli bases. These 12 configurations provide complete information to reconstruct the PTM of the implemented channel [2,10].

The four input states are chosen as eigenstates of the Pauli operators. Using the notation from Sec. III, they are

$$|Z+\rangle = |0\rangle, \quad |Z-\rangle = |1\rangle, \quad |X+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad |Y+\rangle = \frac{|0\rangle + i|1\rangle}{\sqrt{2}}.$$

Each is prepared by an appropriate unitary U_{prep} acting on $|0\rangle$. After the process under study is applied, the output state is measured in one of the three Pauli bases Z , X , or Y . Measurements in the X and Y bases are implemented using the premeasurement rotations H and $HS^\dagger$ defined in Sec. III. Combining the four preparations with the three measurement bases yields the 12 circuits

$$U_{p,m} = M_m U_T P_p, \quad p \in \{Z+, Z-, X+, Y+\}, \quad m \in \{Z, X, Y\},$$

where P_p prepares the input state, U_T denotes the target operation, and M_m implements the measurement basis.

Explicitly, the 12 circuits are

$$\begin{aligned} (1) & M_Z U_T |Z+\rangle, & (2) & M_X U_T |Z+\rangle, & (3) & M_Y U_T |Z+\rangle, \\ (4) & M_Z U_T |Z-\rangle, & (5) & M_X U_T |Z-\rangle, & (6) & M_Y U_T |Z-\rangle, \\ (7) & M_Z U_T |X+\rangle, & (8) & M_X U_T |X+\rangle, & (9) & M_Y U_T |X+\rangle, \\ (10) & M_Z U_T |Y+\rangle, & (11) & M_X U_T |Y+\rangle, & (12) & M_Y U_T |Y+\rangle. \end{aligned}$$

Within the IBM Quantum and Qiskit implementation, both the preparation operators P_p and the basis-rotation operators M_m are decomposed into the same native gate set $\{R_z(\theta), SX, X\}$ used for the target operations themselves [27,28]. Consequently, the tomography workflow is not SPAM isolated in the sense of gate set tomography or cycle benchmarking protocols [11,14]. In the simulator used throughout this work, the generator-level perturbations propagate through the complete tomography circuits, including state preparation operations, target-gate applications, and measurement-basis rotations. The reconstruction therefore does not assume ideal preparation or measurement operations at the circuit-simulation level, but instead mirrors the same tomography assumptions and circuit structure used in the experimental Qiskit workflow. The reconstructed PTMs should therefore be interpreted as effective tomography reconstructions obtained under the adopted preparation, measurement, and finite-shot assumptions rather than as uniquely SPAM-isolated native-gate characterizations.

For each preparation p and measurement basis m , the experiment collects measurement outcomes that estimate the expectation value $\langle\sigma_m\rangle_p$. These 12 expectation values form a linear system relating input Bloch vectors $\mathbf{s}_{\text{in}}$ to output Bloch vectors $\mathbf{s}_{\text{out}}$. Solving this system yields the PTM

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ t_x & T_{xx} & T_{xy} & T_{xz} \\ t_y & T_{yx} & T_{yy} & T_{yz} \\ t_z & T_{zx} & T_{zy} & T_{zz} \end{pmatrix},$$

where the first row is fixed by trace preservation and the remaining elements describe the affine map of the Bloch vector. The translation vector $\mathbf{t} = (t_x, t_y, t_z)^\top$ and the linear block T are inferred from the measured expectation values, using the Born rule and the finite-shot statistics discussed in Sec. III. These 12 circuits thus provide a complete and minimal dataset for reconstructing the implemented single-qubit channel, both in Qiskit’s tomography routines and in the simulator used in this work [10,29].

The entries of the reconstructed PTM have a direct operational interpretation. The 3×3 block T describes how the input Bloch vector is rotated and contracted by the process. For an ideal unitary operation, T is an orthogonal matrix with determinant +1, corresponding to a rigid rotation of the Bloch sphere. Deviations from orthogonality quantify incoherent effects such as dephasing, amplitude damping, or general relaxation processes. The translation vector $\mathbf{t} = (t_x, t_y, t_z)^\top$ describes an affine shift of the Bloch vector and therefore captures nonunital effects. A process is unital if and only if it maps the maximally mixed state to itself; in PTM form this corresponds to $\mathbf{t} = 0$. Nonunital behavior thus reflects population relaxation or asymmetries in the underlying noise mechanisms. These components, together with the Pauli block T , specify the complete linear transformation of the Bloch vector under the implemented quantum channel.

In the trace-preserving case, the first row of the PTM must take the form $(1, 0, 0, 0)$, reflecting the fact that total probability is conserved. In practice, experimental reconstructions often produce a first row with nonzero off-diagonal elements, indicating apparent deviations from strict trace preservation. These deviations may arise from finite sampling, readout errors, leakage out of the computational subspace, or effective non-trace-preserving dynamics caused by state preparation and measurement imperfections. In the PTM representation, nonzero components in the first row quantify how the total probability mass associated with each Pauli basis element fails to be conserved. Allowing the first row to be reconstructed without enforcing trace preservation provides additional diagnostic information: It reveals whether an implemented operation exhibits loss, leakage, or preparation-dependent detection biases. This perspective is central to our analysis, as it enables us to distinguish coherent miscalibration effects from nonunitary and potentially non-trace-preserving contributions observed on hardware.

C. Shot-based simulator for single-qubit process tomography

To compare analytic gate models with experimentally reconstructed channels, we develop a simulator that reproduces the 12-circuit process tomography experiment introduced in Sec. III A. The simulator operates directly in the Pauli-Liouville representation and uses the same augmented Bloch-vector formalism $\mathbf{s} = [1, \langle X \rangle, \langle Y \rangle, \langle Z \rangle]^\top$. Its output is a reconstructed Pauli transfer matrix R obtained from synthetic measurement data generated through finite-sample statistics. The simulator does not impose trace preservation or complete positivity during reconstruction; instead, it reproduces the linear inversion structure used in Qiskit and therefore allows nonunital and non-trace-preserving behavior to appear explicitly in the PTM, mirroring what is observed on hardware.

The simulator evaluates exactly the 12 circuits that make up single-qubit process tomography: four state preparations $\{|Z+\rangle, |Z-\rangle, |X+\rangle, |Y+\rangle\}$ combined with measurements in the three Pauli bases Z , X , and Y . For each preparation p , the simulator computes or samples the expectation values $\langle X \rangle_p$, $\langle Y \rangle_p$, and $\langle Z \rangle_p$, as well as a basis-independent survival statistic s_p , which corresponds to the total detection probability for that input state. These four survival probabilities determine the first row of the PTM. The remaining 12 expectation values determine the affine translation vector $\mathbf{t} = (t_x, t_y, t_z)^\top$ and the 3×3 linear block T , following the same relations used during experimental reconstruction:

$$\mathbf{t} = \frac{1}{2}(\mathbf{r}_{Z+} + \mathbf{r}_{Z-}), \quad T = [\mathbf{r}_{X+} - \mathbf{t} \quad \mathbf{r}_{Y+} - \mathbf{t} \quad \mathbf{r}_{Z+} - \mathbf{t}],$$

where $\mathbf{r}_p = [\langle X \rangle_p, \langle Y \rangle_p, \langle Z \rangle_p]^\top$. The first row $(R_{00}, R_{01}, R_{02}, R_{03})$ is obtained from the four survival values through

$$s_{Z+} = R_{00} + R_{03}, \quad s_{Z-} = R_{00} - R_{03}, \quad s_{X+} = R_{00} + R_{01}, \quad s_{Y+} = R_{00} + R_{02},$$

which uniquely determines the coefficients even if the process is not trace preserving. In the trace-preserving limit, all $s_p = 1$ and the row reduces to $(1, 0, 0, 0)$. In general, allowing $s_p \neq 1$ provides access to loss and leakage mechanisms that affect probability mass, enabling a reconstruction that includes non-trace-preserving behavior directly in the PTM.

Importantly, the simulator mirrors the complete tomography workflow used during the hardware experiments and therefore includes the same preparation and basis-rotation operations used in the Qiskit process-tomography schedules. Consequently, the generator perturbations propagate through the full tomography circuits, including state-preparation operations, target-gate applications, and measurement-basis rotations. The simulator therefore does not assume ideal auxiliary tomography operations at the circuit-simulation level. Instead, the reconstructed PTMs should be interpreted as effective tomography reconstructions obtained under the same preparation, measurement, and reconstruction assumptions used in the adopted Qiskit workflow. At the same time, the present implementation is not SPAM robust in the sense of gate set tomography or cycle benchmarking protocols [11, 14].

All measurement results are produced through a two-stage binomial sampling procedure that mirrors hardware behavior. For a given shot budget N_{shots} and preparation p , a single effective detection count N_{eff} is drawn from a binomial distribution with success probability equal to the true survival s_p times a readout-efficiency factor. This detection count determines the number of detected events shared across all three measurement axes for the same input state. Conditional on detection, each axis measurement draws from a binomial model whose success probability equals the analytic conditional probability for that axis. This construction ensures that survival fluctuations affect only the first row of the PTM, while axis-dependent fluctuations affect only the affine translation and the Pauli block. In the limit $N_{\text{shots}} \rightarrow \infty$, both the survival estimates and the expectation values converge to their analytic values, and the reconstructed PTM converges to the true PTM of the underlying channel.

The target operation is generated by exponentiating native single-qubit generators of the IBM gate set. For an ideal gate $U_g(\theta_g) = \exp[-i(\theta_g/2)G_g]$, the simulator introduces controlled deviations

by replacing the generator with $G_g + \Delta_g$, yielding

$$U_g^\Delta(\theta_g) = \exp\left[-i \frac{\theta_g}{2} (G_g + \Delta_g)\right].$$

Here, Δ_g may be Hermitian, capturing coherent miscalibrations such as axis tilts or over- and under-rotations, or fully general (non-Hermitian), producing effective nonunitary behavior. These modifications alter both the Bloch-sphere rotation and the overall state norm, allowing the model to express nonunitary and non-trace-preserving contributions. Because the simulator does not constrain the PTM to satisfy trace-preserving or positivity conditions, any such generator-level perturbation can propagate into all entries of the reconstructed PTM. This makes the simulator sensitive to symmetry-violating effects, including those that alter the first row or induce translations in the Bloch vector.

Repeating the 12-circuit simulation with independent random seeds produces an ensemble of PTMs that reflects both sampling fluctuations and the statistical variability caused by generator perturbations. The resulting per-entry means and standard deviations are compared to hardware PTMs obtained through Qiskit’s tomography routines. This enables a direct, elementwise comparison between model predictions and experiment, revealing which types of generator perturbations best reproduce the structure of noise observed on the device, including deviations from unitality, coherent miscalibrations, and non-trace-preserving effects.

D. Genetic algorithm for fitting generator-level noise

To relate experimentally observed process matrices to effective microscopic noise parameters, we use a GA to fit additive generator perturbations for the native single-qubit gates. The goal is to find a set of perturbations that, when inserted into the simulator described in Sec. III C and reconstructed via the 12-circuit QPT scheme of Sec. III A, reproduces the target Pauli transfer matrix R_{target} obtained from hardware. In particular, we do not impose trace preservation, unitality, or Hermiticity constraints on the fitted channels, so that symmetry-violating and non-trace-preserving effects can be expressed explicitly in the fitted PTMs.

We consider a fixed set of native gates $\mathcal{G} = \{R_z, SX, X\}$. Each gate $g \in \mathcal{G}$ is generated by an ideal Hermitian operator G_g and implemented as

$$U_g(\theta_g) = \exp\left[-i \frac{\theta_g}{2} G_g\right], \quad (26)$$

with a gate-dependent rotation angle θ_g . To model imperfect behavior, we introduce an additive perturbation $\Delta_g \in \mathbb{C}^{2 \times 2}$ at the generator level and replace

$$U_g(\theta_g) \mapsto U_g^\Delta(\theta_g) = \exp\left[-i \frac{\theta_g}{2} (G_g + \Delta_g)\right]. \quad (27)$$

The matrices Δ_g are general complex 2×2 matrices and therefore allow for both Hermitian (coherent) and non-Hermitian (effectively nonunitary) contributions. For a given collection $\{\Delta_g\}_{g \in \mathcal{G}}$, the simulator constructs the noisy circuits, performs shot-based QPT-12 as in Sec. III C, and outputs a reconstructed PTM $R_{\text{fit}}(\{\Delta_g\})$.

Each GA individual encodes a full set of generator perturbations $\{\Delta_g\}_{g \in \mathcal{G}}$. To obtain a convenient, bounded parametrization, we represent each Δ_g by its eight real components (real and imaginary parts of its four entries) and stack them into a real vector $w_g \in \mathbb{R}^8$. The complete genome is

$$w = (w_{R_z}, w_{SX}, w_X) \in \mathbb{R}^{8|\mathcal{G}|}. \quad (28)$$

These real parameters are mapped to bounded complex matrices through a componentwise saturating transform

$$\Delta_g(w_g) = \Delta_{\text{max}} \tanh(w_g), \quad (29)$$

where $\Delta_{\max} > 0$ controls the maximum allowed magnitude of each matrix element. The tanh map ensures that the search space is continuous but bounded, preventing unphysical growth of generator entries while still allowing both small and moderate perturbations. The vector w thus parametrizes the entire noisy gate set, and through the simulator it induces a PTM $R_{\text{fit}}(w)$.

The fitting objective is formulated as a weighted least-squares discrepancy between the fitted PTM and the target PTM,

$$J(w) = \sum_{i,j} \omega_{ij} (R_{\text{fit}}(w)_{ij} - R_{\text{target},ij})^2, \quad (30)$$

where the indices i, j run over the PTM entries and $\omega_{ij} \geq 0$ are tunable weights. In the simplest case, $\omega_{ij} = 1$ for all entries, corresponding to an unweighted Frobenius norm. Optionally, the weights can incorporate information about the empirical standard deviation of each PTM entry, implementing an approximate χ^2 -type cost, or can be amplified for entries where $|R_{\text{target},ij}|$ is small. The latter increases sensitivity to weak but systematic features of the channel that might otherwise be dominated by larger entries. The GA uses the fitness function

$$F(w) = -J(w), \quad (31)$$

so that better agreement with the target corresponds to higher fitness.

The GA maintains a population of N_{pop} genomes $\{w^{(k)}\}_{k=1}^{N_{\text{pop}}}$. An initial population is created by sampling the components of each $w^{(k)}$ from a centered distribution and clipping them to a fixed range $[-W_{\max}, W_{\max}]$. For each individual, the simulator constructs the perturbed gates, runs the 12-circuit QPT protocol with a fixed shot budget, reconstructs $R_{\text{fit}}(w^{(k)})$, and evaluates the cost $J(w^{(k)})$. Because the simulator operates in a shot-based regime, the objective $J(w)$ is stochastic: Repeated evaluations of the same genome yield slightly different PTMs and therefore slightly different costs. This reflects the statistical uncertainty present in the hardware data and encourages solutions that are robust to sampling noise.

Each GA generation consists of selection, crossover, mutation, and elitism. First, the population is ranked by fitness, and a small elite set of the best-performing genomes is copied unchanged into the next generation. The remaining slots are filled with offspring generated from parent genomes drawn from the top-performing fraction of the current population. Offspring genomes are created by uniform crossover: For each component of the child vector, the entry is inherited from one of the two parents with equal probability. After crossover, mutation introduces diversity and enables exploration of the parameter space. Mutation acts on a random fraction of the nonelite individuals and perturbs a random subset of their components by adding Gaussian noise with fixed variance, followed by clipping back to $[-W_{\max}, W_{\max}]$. With a smaller probability, mutation can replace an entire genome by a new random vector, which helps escape local minima. Elitism ensures that the current best solutions are never degraded by mutation or crossover, while selection pressure and recombination guide the population toward regions of lower cost.

The GA is run for a fixed number of generations T . The best genome w^* observed over all generations defines the final set of generator perturbations $\{\Delta_g^*\}$ via the saturating map. These perturbations are then passed to the simulator to compute the corresponding PTM, both in the analytic (shot-free) limit and in the shot-based QPT-12 setting. Repeating the shot-based reconstruction for $\{\Delta_g^*\}$ yields an ensemble $\{R_{\text{fit}}^{(r)}\}$ with mean $R_{\text{fit}}^{\text{mean}}$ and componentwise standard deviation $\text{STD}[R_{\text{fit}}]$. These quantities are compared elementwise to the hardware PTMs and their empirical uncertainties, for example, through combined-uncertainty z scores. In this way, the GA provides an effective inverse map from observed process-level asymmetries—including nonunitary and non-trace-preserving behavior visible in all entries of R —back to plausible generator-level noise contributions for the native gate set.

IV. EXPERIMENTS

The experiments in this work combine repeated process tomography on IBM Quantum hardware with simulations based on the shot-based QPT-12 framework introduced in Secs. III A–III C. The hardware experiments provide empirical PTMs and their statistical variation, which we treat as reference data. The simulations implement generator-level noise models and use the same 12-circuit process tomography to obtain synthetic PTMs that can be compared elementwise to the hardware results.

All hardware data were acquired on the `ibm_brisbane` backend, an Eagle r3 device with native single-qubit gate set $\mathcal{G} = \{R_z, SX, X\}$ [28,30]. For a chosen physical qubit, we ran the full 12-circuit single-qubit process tomography schedule for the X gate, using Qiskit’s tomography utilities [5,10,27]. Each tomography run prepared the four input states $\{|Z+\rangle, |Z-\rangle, |X+\rangle, |Y+\rangle\}$, measured in the three Pauli bases $\{Z, X, Y\}$, and used a fixed number of shots per circuit. This complete set of 12 circuits was executed many times as independent jobs under comparable calibration conditions. For each run we reconstructed a PTM $R^{(k)}$ from the 12-circuit data. Aggregating over all runs yields a sample mean

$$R_{\text{target}} = \frac{1}{K} \sum_{k=1}^K R^{(k)} \quad (32)$$

and a componentwise sample standard deviation $\text{STD}[R_{\text{target}}]$, which together serve as the empirical description of the implemented X -gate channel on hardware. All subsequent simulations are evaluated against R_{target} and $\text{STD}[R_{\text{target}}]$ using the PTM formalism of Sec. III A, with trace preservation not enforced so that possible loss and leakage effects remain visible in the first row of the PTM.

A. Genetic-algorithm reconstruction of generator noise

The first simulation experiment uses the GA of Sec. III D to infer generator-level perturbations that reproduce the hardware PTM for the X gate. The optimization variables are the additive perturbations $\{\Delta_g\}_{g \in \mathcal{G}}$ to the native generators G_g of the gates $g \in \mathcal{G} = \{R_z, SX, X\}$. Each Δ_g is a complex 2×2 matrix parametrized by eight real numbers, which are mapped to a bounded range via a componentwise tanh transform with scale $\Delta_{\text{max}} = 0.05$. The full genome $w \in \mathbb{R}^{8|\mathcal{G}|}$ thus encodes a complete noisy gate set, and through the shot-based QPT-12 simulator it induces a fitted PTM $R_{\text{fit}}(w)$.

For a given genome w , the simulator constructs the perturbed unitaries

$$U_g^\Delta(\theta_g) = \exp\left[-i \frac{\theta_g}{2} (G_g + \Delta_g(w))\right], \quad g \in \mathcal{G}, \quad (33)$$

inserts them into the 12 canonical tomography circuits, and runs the QPT-12 protocol with a shot budget of $N_{\text{shots}} = 10^4$ per circuit. The reconstruction routine of Sec. III C then produces a PTM $R_{\text{fit}}(w)$ without imposing (TP) or unitality. The cost function is a weighted least-squares discrepancy

$$J(w) = \sum_{i,j} \omega_{ij} (R_{\text{fit}}(w)_{ij} - R_{\text{target},ij})^2, \quad (34)$$

with default weights $\omega_{ij} = 1$ and amplification of entries with small $|R_{\text{target},ij}|$ (e.g., $|R_{\text{target},ij}| < 0.05$) by a factor of 3.5. This makes the optimizer more sensitive to weak but systematic features in the PTM. The GA maximizes the fitness $F(w) = -J(w)$.

The GA population consists of $N_{\text{pop}} = 10^4$ individuals. Initial genomes are drawn from a centered distribution and clipped to $[-W_{\text{max}}, W_{\text{max}}]$ with $W_{\text{max}} = 6$. In each generation, individuals are ranked by fitness. An elite set of $N_{\text{elite}} = 10^3$ genomes is copied unchanged into the next generation, ensuring that the current best solutions are preserved. The remaining population is filled with offspring created from parents selected from the top fraction of the population (here, the best 50% by fitness). Offspring genomes are produced by uniform crossover: For each parameter, the child

inherits the value from one of the two parents with equal probability. Mutation is applied to a random subset of nonelite individuals by adding Gaussian noise to a randomly selected subset of parameters and reclipping to $[-W_{\max}, W_{\max}]$. With a smaller probability, mutation can replace an entire genome by a new random vector, which helps the population escape local minima. The GA is run for $T = 3 \times 10^4$ generations, and the best genome observed across all generations is retained as w^* .

For the final analysis, the perturbations $\{\Delta_g^*\}$ encoded by w^* are fed back into the simulator. We compute both the analytic PTM (shot-free limit) and shot-based PTMs using the QPT-12 scheme with $N_{\text{shots}} = 10^4$. Repeating the shot-based reconstruction many times yields an ensemble of PTMs $\{R_{\text{fit}}^{(r)}\}$ with mean R_{fit} and standard deviation $\text{STD}[R_{\text{fit}}]$. These are compared elementwise to the hardware PTM via the combined-uncertainty z score

$$z_{ij} = \frac{|R_{\text{fit},ij} - R_{\text{target},ij}|}{\sqrt{\text{STD}[R_{\text{target}}]_{ij}^2 + \text{STD}[R_{\text{fit}}]_{ij}^2}}, \quad (35)$$

which quantifies how well the fitted generator-level noise reproduces the structure of the experimentally observed channel, including deviations from unitality and trace preservation in the first row of R .

B. Random Hermitian versus non-Hermitian noise ensembles

The second simulation experiment studies the structural impact of Hermitian versus non-Hermitian generator perturbations by sampling large random ensembles. The objective is to determine whether purely coherent (Hermitian) generator noise can account for the symmetry-violating and non-trace-preserving features of the hardware PTM, or whether general non-Hermitian perturbations are required.

For both noise families, we consider the same set of native gates $\mathcal{G} = \{R_z, SX, X\}$. In the Hermitian ensemble, each Δ_g is drawn as a random Hermitian matrix with real diagonal entries and complex-conjugate off-diagonal entries, with all real parameters sampled uniformly from $[-\lambda, \lambda]$ for a fixed scale $\lambda = 0.05$. This produces small random axis tilts and over- or under-rotations while preserving Hermiticity at the generator level; the corresponding noisy gates $U_g^\Delta(\theta_g)$ are unitary, and any nonunitarity in the reconstructed PTM arises solely from the shot-based tomography and reconstruction procedure. In the non-Hermitian ensemble, each Δ_g is a fully complex 2×2 matrix with real and imaginary parts sampled independently and uniformly from $[-\lambda, \lambda]$. These perturbations generate effective nonunitary dynamics at the level of the gate, allowing for trace-decreasing and nonunital behavior that can manifest directly in all entries of the PTM, including the first row.

For each family, we draw $N_{\text{samples}} = 10^3$ independent noise maps $\{\Delta_g^{(k)}\}$. For each sample k , we construct the perturbed gates U_g^Δ , run the QPT-12 simulator with $N_{\text{shots}} = 10^4$, and reconstruct a PTM $R^{(k)}$ using the same shot-based procedure as in Sec. III C, again without enforcing trace preservation. This yields two ensembles of PTMs

$$\{R_{\text{Herm}}^{(k)}\}_{k=1}^{N_{\text{samples}}} \quad \text{and} \quad \{R_{\text{Non-Herm}}^{(k)}\}_{k=1}^{N_{\text{samples}}}, \quad (36)$$

from which we compute the componentwise means and standard deviations

$$\bar{R}_{\text{Herm}} = \frac{1}{N_{\text{samples}}} \sum_k R_{\text{Herm}}^{(k)}, \quad \text{STD}[R_{\text{Herm}}], \quad (37)$$

$$\bar{R}_{\text{Non-Herm}} = \frac{1}{N_{\text{samples}}} \sum_k R_{\text{Non-Herm}}^{(k)}, \quad \text{STD}[R_{\text{Non-Herm}}]. \quad (38)$$

These ensemble means describe the typical process-level distortions produced by small Hermitian and non-Hermitian generator noise, respectively.

To compare the ensembles to the hardware data, we compute elementwise z scores relative to the hardware mean and variance. For the Hermitian family, we define

$$z_{ij}^{(\text{Herm})} = \frac{|\bar{R}_{\text{Herm},ij} - R_{\text{target},ij}|}{\sqrt{\text{STD}[R_{\text{target}}]_{ij}^2 + \text{STD}[R_{\text{Herm}}]_{ij}^2}}, \quad (39)$$

and analogously for the non-Hermitian family,

$$z_{ij}^{(\text{Non-Herm})} = \frac{|\bar{R}_{\text{Non-Herm},ij} - R_{\text{target},ij}|}{\sqrt{\text{STD}[R_{\text{target}}]_{ij}^2 + \text{STD}[R_{\text{Non-Herm}}]_{ij}^2}}. \quad (40)$$

Entries with $z_{ij} \lesssim 1$ are statistically compatible with the hardware within one combined standard deviation, while larger values indicate discrepancies that cannot be explained by the corresponding ensemble. Inspecting these z -score matrices reveals which aspects of the experimental PTM can be reproduced by purely Hermitian generator noise and which require general non-Hermitian contributions. In particular, systematic deviations in the first row of R and in the affine components associated with nonunitarity provide direct evidence for effective non-trace-preserving dynamics that cannot be captured by Hermitian perturbations alone.

V. RESULTS

This section reports the experimentally reconstructed single-qubit process matrices obtained on `ibm_brisbane`, the fitted generator perturbations from the genetic optimization, and the corresponding model process matrices. All PTMs are defined in the Pauli transfer matrix formalism introduced in Sec. III A, where the channel acts as

$$\mathbf{s}_{\text{out}} = R\mathbf{s}_{\text{in}}. \quad (41)$$

Agreement between model and experiment is quantified using the entrywise z scores defined in Secs. IV A and IV B, based on the combined standard deviation of hardware and simulator statistics. The experimentally reconstructed PTMs already contain the measurable channel information accessible through the tomography procedure itself. Accordingly, the analysis presented here should be interpreted as a phenomenological reconstruction study that investigates whether generator-level perturbations can reproduce experimentally observed PTM structures within a transparent analytical framework.

A. Hardware target: averaged PTM and standard deviation

We aggregated

$$n_{\text{runs}} = 88 \quad (42)$$

independent single-qubit process tomography runs for the physical implementation of the X gate on `ibm_brisbane`. Each run comprised the full 12-circuit QPT-12 schedule described in Sec. III A. From the reconstructed PTMs

$$\{R^{(k)}\}_{k=1}^{n_{\text{runs}}}, \quad (43)$$

we computed the componentwise sample mean [Eq. (32)] and the associated sample standard deviation

$$\text{STD}[R_{\text{target}}]. \quad (44)$$

These quantities summarize the effective single-qubit channel realized on hardware, including deviations from unitality and trace preservation that appear in the first row and in the Pauli block. These experimentally reconstructed PTMs therefore already provide the reference channel representation against which the perturbative generator model is compared.

The aggregated hardware PTM and its standard deviation are

$$R_{\text{target}} = \begin{bmatrix} 1.000\,00 & -0.043\,82 & 0.045\,96 & -0.049\,50 \\ -0.001\,96 & 0.904\,12 & 0.017\,92 & -0.018\,45 \\ -0.002\,83 & 0.019\,61 & -0.909\,94 & 0.003\,60 \\ 0.002\,40 & 0.001\,84 & 0.003\,48 & -0.907\,94 \end{bmatrix}, \quad (45)$$

$$\text{STD}[R_{\text{target}}] = \begin{bmatrix} 0.000\,00 & 0.029\,59 & 0.025\,83 & 0.027\,77 \\ 0.002\,78 & 0.026\,18 & 0.014\,62 & 0.012\,33 \\ 0.003\,49 & 0.014\,74 & 0.025\,88 & 0.012\,13 \\ 0.002\,99 & 0.007\,63 & 0.008\,41 & 0.027\,65 \end{bmatrix}. \quad (46)$$

For comparison, the ideal noiseless Pauli transfer matrix of the single-qubit X gate in the ordered basis (I, X, Y, Z) is

$$R_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad (47)$$

which represents a unitary, trace-preserving, and unital channel. The nontrivial entries in the first row of

$$R_{\text{target}} \quad (48)$$

indicate effective deviations from ideal trace preservation at the process level, while contractions along the Pauli-diagonal entries reveal relaxation- and dephasinglike behavior relative to the ideal reference

$$R_X. \quad (49)$$

These features serve as the reference signatures that the simulator and generator-level noise models aim to reproduce. The present analysis therefore does not claim that these features originate uniquely from symmetry-violating perturbations; rather, the perturbation model is tested as one possible phenomenological mechanism capable of reproducing the experimentally reconstructed structures.

B. Fitted generator perturbations

Using the genetic algorithm described in Sec. IV A, we fitted additive generator perturbations

$$\Delta_g, \quad (50)$$

for the native single-qubit gate set

$$\mathcal{G} = \{R_z, SX, X\}. \quad (51)$$

Each

$$\Delta_g \quad (52)$$

is a complex 2×2 matrix that modifies the corresponding ideal generator

$$G_g \quad (53)$$

at the generator level,

$$G_g \mapsto G_g + \Delta_g, \quad (54)$$

and thus defines a perturbed gate

$$U_g^\Delta(\theta_g) = \exp\left[-i\frac{\theta_g}{2}(G_g + \Delta_g)\right]. \quad (55)$$

The GA operates on a bounded real parametrization of these matrices and optimizes the generator-level parameters to minimize the PTM discrepancy

$$J(w) \quad (56)$$

defined in the notation summary, using shot-based QPT-12 reconstruction at each evaluation. The fitted perturbations should therefore be interpreted as effective phenomenological parameters defined relative to the tomography reconstruction rather than as direct microscopic measurements of hardware noise processes.

The best-found perturbations are reported here as separate real and imaginary parts:

$$\begin{aligned} \Delta[R_z]_{\text{Re}} &= \begin{bmatrix} -0.04985 & 0.01624 \\ -0.04976 & 0.04994 \end{bmatrix}, & \Delta[R_z]_{\text{Im}} &= \begin{bmatrix} -0.01773 & 0.01577 \\ 0.03355 & -0.01346 \end{bmatrix}, \\ \Delta[SX]_{\text{Re}} &= \begin{bmatrix} 0.04971 & -0.00908 \\ 0.02513 & -0.04678 \end{bmatrix}, & \Delta[SX]_{\text{Im}} &= \begin{bmatrix} -0.01871 & 0.04548 \\ 0.02234 & -0.05000 \end{bmatrix}, \\ \Delta[X]_{\text{Re}} &= \begin{bmatrix} 0.01588 & 0.03059 \\ -0.01421 & -0.01726 \end{bmatrix}, & \Delta[X]_{\text{Im}} &= \begin{bmatrix} 0.02428 & -0.01665 \\ -0.00771 & 0.00382 \end{bmatrix}. \end{aligned}$$

These matrices encode both Hermitian and non-Hermitian contributions and therefore allow nonunitary, non-trace-preserving, and symmetry-violating behavior at the process level. The next subsection shows how these fitted perturbations translate into process matrices via QPT-12. Importantly, agreement between the resulting PTMs and hardware PTMs demonstrates only that such perturbations can reproduce experimentally observed tomography structures within the adopted reconstruction framework. It does not establish that the underlying microscopic device noise is uniquely generated by these perturbations.

C. Model reconstructions (QPT-12) with shot sampling

Given the fitted perturbations

$$\{\Delta_g\}_{g \in \mathcal{G}}, \quad (57)$$

we use the shot-based simulator of Sec. III C together with the 12-circuit scheme of Sec. III A to reconstruct model PTMs. We consider a finite-shot regime with

$$N_{\text{shots}} = 10^4 \quad (58)$$

per circuit. In this regime, we perform

$$N_{\text{eval}} = 1000 \quad (59)$$

independent QPT-12 reconstructions and compute the mean and standard deviation across these realizations, consistent with the statistical treatment in Sec. IV A. The purpose of this reconstruction is not to infer additional experimentally inaccessible information, but to test whether structured generator-level perturbations can reproduce the statistical and structural characteristics already present in the reconstructed hardware PTMs.

Sampled model ($N_{\text{shots}} = 10^4$). The reconstructed PTM is random due to shot noise and the two-stage sampling of survival and conditional expectation values. Averaging over

$$N_{\text{eval}} = 1000 \quad (60)$$

independent QPT-12 reconstructions yields

$$R_{\Delta,10^4}^{\text{mean}} = \begin{bmatrix} 0.965\,76 & -0.032\,27 & 0.034\,24 & -0.034\,24 \\ -0.003\,14 & 0.914\,24 & 0.020\,30 & -0.017\,96 \\ -0.025\,46 & 0.023\,04 & -0.965\,96 & 0.000\,99 \\ 0.000\,01 & 0.002\,07 & 0.001\,69 & -0.999\,99 \end{bmatrix}, \quad (61)$$

$$\text{STD}[R_{\Delta,10^4}] = \begin{bmatrix} 0.001\,24 & 0.002\,95 & 0.001\,24 & 0.001\,24 \\ 0.006\,95 & 0.008\,09 & 0.011\,89 & 0.007\,08 \\ 0.007\,18 & 0.012\,76 & 0.007\,29 & 0.007\,33 \\ 0.000\,04 & 0.010\,04 & 0.010\,09 & 0.000\,04 \end{bmatrix}. \quad (62)$$

In this shot-based regime, the first row deviates from

$$(1, 0, 0, 0) \quad (63)$$

and exhibits nontrivial off-diagonal entries consistent with the symmetry-violating signatures seen in

$$R_{\text{target}}. \quad (64)$$

The Pauli-diagonal entries in the lower 3×3 block show contraction relative to the ideal X -gate PTM, again in line with the hardware data. This agreement should be interpreted as evidence that the perturbative generator model can phenomenologically reproduce several experimentally reconstructed PTM features within uncertainty.

D. Agreement with hardware: entrywise and blockwise

To quantify the agreement between model and hardware, we compare the sampled model PTM

$$R_{\Delta,10^4}^{\text{mean}} \quad (65)$$

and

$$R_{\text{target}} \quad (66)$$

entrywise. Following the uncertainty framework in Sec. IV A, we define the entrywise difference

$$\Delta R_{ij} = R_{\Delta,10^4,ij}^{\text{mean}} - R_{\text{target},ij} \quad (67)$$

and the combined standard deviation

$$\sigma_{\text{comb},ij} = \sqrt{\text{STD}[R_{\text{target}}]_{ij}^2 + \text{STD}[R_{\Delta,10^4}]_{ij}^2}, \quad (68)$$

which treats the hardware and simulator uncertainties as independent. The z score

$$z_{ij} = \frac{|\Delta R_{ij}|}{\sigma_{\text{comb},ij}} \quad (69)$$

is identical in form to the definition used in Secs. IV A and IV B and measures how many combined standard deviations the model deviates from the hardware mean for each PTM entry. Entries with

$$z_{ij} \leq 1 \quad (70)$$

are statistically compatible at the one-sigma level; entries with

$$z_{ij} > 1 \quad (71)$$

indicate tension that may point to missing physical mechanisms in the model. The resulting z -score analysis therefore quantifies consistency between the phenomenological perturbation model and the experimentally reconstructed PTMs rather than demonstrating a unique physical decomposition of the underlying hardware noise.

VI. DISCUSSION

The comparison between the experimentally reconstructed and simulated process matrices reveals in quantitative terms how well the model captures the physical behavior of the hardware. Each element of the PTM describes how a given input Pauli operator is transformed by the gate, with diagonal entries corresponding to attenuation or amplification of specific basis components and off-diagonal terms representing cross-axis mixing. In the absence of noise, the PTM is ideally orthogonal and trace-preserving, corresponding to a completely positive and unital map [4,29,31]. Deviations from this structure therefore directly encode dissipative and coherent errors, and the elementwise z -score analysis $z = |\Delta|/\sigma_{\text{comb}}$ provides a rigorous statistical measure of consistency between model and experiment [10,27]. We emphasize that process tomography itself is an established characterization method and that the present work does not introduce a new tomography protocol. The role of tomography here is to provide experimentally reconstructed PTMs against which the generator-level perturbation model can be tested. The technical role of the framework is therefore not representational completeness, but testing whether structured gate-local perturbations can reproduce the reconstructed PTM through our forward tomography approach.

The hardware variability, characterized by the standard deviations of the reconstructed PTMs, sets the scale of meaningful comparison. Off-diagonal elements in the affine row exhibit larger statistical spread, indicating sensitivity to state-preparation and measurement fluctuations, while the Pauli-diagonal entries typically display smaller variance. Where the hardware variance is low, even small systematic deviations of the model lead to large z values. Conversely, entries with higher noise tolerate greater discrepancies before they become statistically significant. This behavior is consistent with the fundamental sensitivity of process tomography to trace-preserving and unitality violations, where normalization errors—associated with $\text{Tr}(\rho) \neq 1$ or nonunital channels—manifest most strongly in the first PTM row [11,32,33]. At the same time, the reconstructed PTMs should be interpreted as effective tomography reconstructions obtained under the preparation and measurement assumptions of the adopted Qiskit workflow rather than as fully SPAM-isolated hardware characterizations. Consequently, part of the observed affine-row variability may also contain contributions from state-preparation and measurement imperfections in addition to gate-level noise effects.

The fitted generator-based model, obtained via genetic optimization of gate-local perturbations, reproduces the main affine-row structure and off-diagonal mixing within one combined standard deviation for most matrix elements. The few remaining tensions appear primarily on the Pauli-diagonal components (Y, Y) and (Z, Z) , suggesting the presence of residual dissipative effects not captured by purely coherent perturbations. This observation aligns with the expected influence of relaxation and dephasing processes on IBM transmon hardware [34–36], where asymmetries in T_1 and T_2 times cause unequal contractions along the Y and Z axes of the Bloch sphere. Such effects are visible in the PTM as anisotropic diagonal damping and are often accompanied by small affine shifts from imperfect calibration or measurement crosstalk.

The ensemble analysis further clarifies the robustness and interpretability of different perturbation models. When random Hermitian perturbations ($\Delta = \Delta^\dagger$) are sampled across many realizations, the resulting mean PTM remains nearly orthogonal but underestimates dissipative asymmetries. In contrast, the non-Hermitian ensemble, which allows asymmetric and complex-valued perturbations, captures the hardware behavior more comprehensively, yielding a larger fraction of entries with $z \leq 1$. This pattern shows that non-Hermitian perturbations reproduce the reconstructed PTM more closely than the Hermitian ensemble, which is consistent with effective dissipative or open-system contributions without uniquely establishing their microscopic origin [8,37,38]. The ensemble’s standard deviation quantifies the model-side uncertainty, which broadens σ_{comb} and ensures statistically balanced comparison. More conservatively, the results show that non-Hermitian perturbations constitute one possible phenomenological mechanism capable of reproducing the experimentally reconstructed PTM structures within the adopted tomography framework.

From a mathematical perspective, the non-Hermitian perturbations effectively emulate channels that are not trace-preserving or not unital, representing physically dissipative maps. These can be expressed in Lindblad form with non-Hermitian generators [6,39], which capture relaxation and dephasing processes beyond coherent over-rotations. The improved agreement of such models with the experimental PTMs suggests that the effective noise on the hardware includes contributions from these open-system dynamics. At the same time, the present framework does not uniquely identify the microscopic physical origin of these deviations, since reconstructed PTMs may also contain contributions from SPAM effects, finite-shot fluctuations, calibration drift, or other effective hardware-level imperfections. The statistical structure of the z -matrices supports this interpretation: Off-diagonal PTM entries show consistent $z \leq 1$ behavior, implying that cross-axis couplings are well represented by the gate-local perturbation framework, while diagonal entries exceed $z > 2$ in some cases, revealing missing dissipative terms. This mirrors typical deviations observed in self-consistent tomography studies, where trace nonpreservation or SPAM bias manifests as large diagonal residuals [12,40,41]. Relatedly, Blume-Kohout *et al.* introduced a systematic generator-space taxonomy for small Markovian errors by decomposing reconstructed channels into Hamiltonian, stochastic, correlation, and active sectors [1]. Their framework primarily studies how experimentally reconstructed CPTP maps can be represented and classified in generator space, whereas our approach introduces explicit perturbations directly into the native gate generators before reconstructing the resulting PTMs. Accordingly, the present work should be interpreted as a phenomenological reconstruction framework that tests whether structured symmetry-violating perturbations can reproduce experimentally observed PTM signatures, rather than as a complete taxonomy or exhaustive decomposition of arbitrary noise channels.

The expressivity analysis in Appendix B further clarifies the scope and limitations of this interpretation. In particular, the Appendix shows that Hermitian perturbations naturally represent coherent over-rotations, axis tilts, and cross-axis mixing because they preserve the unitarity of the exponentiated gate generators. In contrast, non-Hermitian perturbations allow effective contraction, affine translation, and apparent non-trace-preserving signatures to emerge in the reconstructed PTMs. The Appendix additionally compares the perturbation framework to standard channel classes such as dephasing, depolarizing noise, amplitude damping, and leakagelike behavior, showing that the model can phenomenologically reproduce several experimentally relevant PTM structures without constituting a complete CPTP channel representation.

The Appendix also formalizes the notion of local expressivity by linearizing the reconstructed PTM around the ideal gate set and introducing the corresponding response map from generator perturbations to reconstructed PTM deviations. This analysis clarifies that the framework is locally sufficient only when the experimentally reconstructed PTM deviations lie approximately within the response span generated by the available native gate perturbations. Consequently, the model should not be interpreted as universally expressive, but rather as a constrained phenomenological reconstruction framework whose validity depends on whether experimentally observed PTM structures can be reproduced within the accessible perturbative response directions and experimental uncertainty bounds.

Finite-shot effects play a secondary yet quantifiable role. The inclusion of finite-sampling variance in σ_{comb} softens the statistical test by incorporating model-side uncertainty, improving the apparent agreement in high-precision entries. However, shot noise cannot remove systematic offsets; thus, persistent deviations on the Pauli diagonals reflect genuine model incompleteness rather than statistical limitations. This distinction underscores the utility of z -based metrics, which clearly separate stochastic fluctuations from structural mismatches—a key advantage over global error measures like fidelity or trace distance that conflate the two [10,29].

Overall, the results indicate that non-Hermitian, gate-local perturbations constitute a minimal yet powerful extension of purely coherent noise models. They capture affine shifts and cross-axis coupling while approximating dissipative contraction in a statistically meaningful way. The elementwise analysis provides a transparent map between experimental data and physical mechanisms: Small z values along off-diagonal PTM components correspond to well-modeled coherent

mixing, while high z values along the diagonal mark unmodeled relaxation or measurement-induced asymmetry. These experimentally reconstructed structures should, however, be interpreted phenomenologically within the adopted tomography assumptions and not as uniquely identifiable microscopic hardware mechanisms. This quantitative structure connects naturally to the physical error channels of superconducting qubits—energy relaxation, dephasing, and residual measurement bias—and thus bridges microscopic noise sources with process-level observables [8,33,34].

These findings point toward a practical modeling strategy for calibration and simulation workflows (1) introduce gate-local generator perturbations that include non-Hermitian terms to emulate open-system effects, (2) use ensemble statistics to represent uncertainty in both model and experiment, and (3) quantify agreement through entrywise z scores rather than aggregate norms. Such an approach aligns with recent proposals for noise spectroscopy and model-based error mitigation [17,42,43]. Extending this framework to multiqubit PTMs would allow correlated and crosstalk-induced noise to be analyzed under the same statistical principle, providing a more granular picture of how symmetry breaking propagates through realistic quantum processors. The present study is intentionally restricted to single-qubit gates, where the reconstruction problem is analytically tractable and the fitted generator perturbations can be inspected directly. This restriction limits the immediate practical scope because multiqubit gates and entangling operations are major sources of error in current quantum processors. Extending the framework to multiqubit gates would require substantially larger PTMs, explicit treatment of correlated errors, and more careful separation of crosstalk, leakage, and SPAM contributions.

In summary, the detailed analysis of elementwise deviations, combined with ensemble perturbation modeling, shows that non-Hermitian generator perturbations can reproduce important features of the reconstructed PTM more effectively than Hermitian perturbations alone. More precisely, the results demonstrate that phenomenological generator-level perturbations containing non-Hermitian contributions can reproduce important features of experimentally reconstructed PTMs within the adopted tomography framework. This supports a broader view of quantum hardware as an open, weakly nonunitary system whose deviations from ideal symmetry can be meaningfully parametrized, quantified, and statistically validated using process tomography.

A. Significance of symmetry-violating generator perturbations

The fitted symmetry-violating generator perturbations provide an experimentally constrained phenomenological description of deviations from the idealized symmetry-preserving regime. In standard descriptions of gate noise, one often assumes that small imperfections manifest as coherent over-rotations, Pauli-stochastic mixtures, or weak amplitude/phase damping. These channels preserve the algebraic structure of the ideal generator set and therefore respect the native symmetries of the intended gate operations. Within the present reconstruction, the PTMs display first-row deviations, affine shifts, and nonunital terms that are not fully captured by symmetric generator perturbations. At the same time, these experimentally reconstructed PTMs already contain the measurable channel information accessible through the tomography procedure itself. The fitted perturbation framework should therefore be interpreted as a phenomenological reconstruction model that tests whether structured generator-level perturbations can reproduce these experimentally reconstructed signatures. The improved agreement of the fitted non-Hermitian Δ models shows that such perturbations provide a statistically consistent phenomenological description of these reconstructed deviations. More conservatively, the results show that symmetry-violating perturbations constitute one possible mechanism capable of reproducing these experimentally reconstructed structures within the adopted modeling framework.

This matters because symmetry-violating noise reshapes the operational regime in which calibration, verification, and fault tolerance techniques must function. Many analytical guarantees in error mitigation and quantum control implicitly rely on noise channels being approximately trace-preserving, unital, or symmetry-respecting in their action on the Bloch sphere. When the physical device violates these properties, then common simplifications—such as depolarizing

approximations, Pauli error models, or uniform amplitude-damping assumptions—no longer capture the relevant error signatures. The fitted generator perturbations therefore provide a richer phenomenological parametrization of the reconstructed error structure. However, the present framework does not uniquely identify the microscopic physical origin of these deviations and should not be interpreted as a complete decomposition of the underlying hardware noise processes. Within this parametrization, such deviations are consistent with symmetry-breaking mechanisms but cannot be assigned uniquely to them. Consequently, the noise model becomes richer, but also more actionable: It quantifies which structural assumptions fail and by what magnitude.

What follows from this is a clearer route toward actionable diagnostic and mitigation strategies. Because symmetry-violating components produce distinct, measurable patterns in the PTM (especially in the first row and Pauli-diagonal elements), they can serve as stable indicators of leakage, population loss, or state-dependent relaxation. These signatures make it possible to design targeted mitigation routines that address precisely those deviations the symmetric model cannot absorb. For example, calibration strategies could include explicit compensation of affine Bloch-vector shifts; tomography pipelines could flag symmetry-violating structure as an additional hardware metric; and simulation workflows can incorporate non-Hermitian perturbation sets as a more realistic baseline noise model. In this sense, the framework primarily provides an analytically transparent parametrization for studying experimentally reconstructed PTM structures rather than a fundamentally new characterization observable beyond tomography itself. Moreover, these insights motivate the following subsection: If symmetry-violating perturbations are intrinsic to the physical gate generators, then quantum error-correction schemes that rely on gauge structures or encoded symmetries must be reconsidered in light of these violations. Understanding how symmetry breaking manifests at the generator level is therefore essential for evaluating how well symmetry-based quantum error correction (QEC) strategies will perform on actual hardware.

Beyond error modeling, these symmetry-violating generator perturbations connect directly to recent work on data obfuscation based on noisy Lie-group feature maps. RaubitzeK *et al.* [19] show that injecting controlled perturbations into Lie-algebra generators inside the exponential map yields transformations that are deliberately noninvertible while preserving downstream classification performance. In their construction, noise is added at the generator level such that the resulting Lie-group elements no longer implement perfectly symmetric, one-to-one maps on the data manifold, thereby preventing reliable reconstruction of the original features from the obfuscated representation while keeping the geometric structure relevant for learning. Our fitted non-Hermitian Δ play an analogous structural role: They break exact unitarity, unitality, and trace preservation at the generator level, so that distinct classical inputs can be mapped to process signatures that are no longer uniquely invertible. If such symmetry-violating patterns are stable device characteristics rather than transient calibration artifacts, then quantum information processing can in principle implement hardware-level obfuscation channels in the spirit of Ref. [19], where native-generator noise is leveraged as an intrinsic mechanism for making encoded data practically unrecoverable while maintaining its utility for inference. This interpretation is speculative and should not be read as a demonstrated hardware obfuscation protocol. The present experiments only show that fitted generator perturbations can reproduce selected process-level signatures in a single-qubit tomography setting.

B. Implications for error correction: exploiting and mitigating symmetry-violating noise

In our framework, we assume that the (or at least some) symmetry-violating generator perturbations are indeed present in the actual hardware implementation of the gate set. Given this assumption, the observation of affine shifts, cross-axis mixing, and non-trace-preserving components in the reconstructed PTMs suggests potential opportunities—and constraints—for error-correction strategies.

On the one hand, the presence of identifiable symmetry violations means that the effective noise channel deviates from the ideal symmetric form and carries additional structure (for example, generator-level perturbations of the form $\Delta \neq 0$). If this structure is stable and measurable, one

could imagine tailoring error-correction or mitigation protocols that specifically address those symmetry-violating contributions. For example, by measuring syndrome outcomes that are sensitive to deviations from the symmetry subspace, one may detect leakage or loss channels that break the expected symmetry and then apply corrective operations or postselection to restore the logical space. The literature on codes that exploit physical symmetries (or their violation) indicates that detecting violations of gauge or conservation laws can serve as error flags [44,45]. In other words, the symmetry-violating components could be turned into a resource for error detection: when a measured process shows nonideal affine translation or survival loss (first-row deviation), this signals physical error mechanisms (leakage, state-dependent loss) that standard stabilizer codes may not explicitly detect.

On the other hand, the existence of symmetry violation also poses constraints on error correction. Recent work shows that continuous symmetries and QEC exhibit a trade-off: Codes that enforce or exploit symmetries may have limited ability to correct arbitrary errors, and symmetries can impose constraints on transversal gate sets and error-correcting performance (e.g., approximate versions of the Eastin-Knill theorem) [46,47]. In our context, the mismatch between the hardware channel and the ideal symmetry-preserving generator model means that standard error-correction assumptions (unitary noise, trace-preserving CPTP maps, unital blocks) may fail. Consequently, tailoring codes to correct these symmetry-violating deviations may require extensions beyond conventional stabilizer frameworks.

In practical terms, a reverse mechanism to reduce errors might proceed as follows: First *characterize* the symmetry-violating generator perturbations (as we have done via tomography and fitting); second *augment* the encoding or recovery step with operations that specifically project away or mitigate the identified violations (for example, by applying a conditional rotation or channel that cancels the affine translation, or by postselecting on survival events); and third *monitor* the first row and affine translation of the PTM as diagnostic metrics for residual symmetry violation and adjust the recovery accordingly.

However, critical caveats apply the following:

(1) The symmetry-violating generator model may only capture part of the full noise budget. Effects such as time-varying drift, crosstalk, SPAM errors, or correlated multiqubit noise may remain unaddressed.

(2) The reverse mechanism presumes that the symmetry violation is *stable*, *repeatable*, and *measurable* on the timescale of encoding and correction. If the symmetry-violating perturbations fluctuate more rapidly than the correction cycle, then mitigation becomes more challenging.

(3) Introducing recovery operations targeted at symmetry violations must itself avoid introducing further asymmetries or coupling errors back into the system. Recovery operations may need to themselves commute with the native gate symmetries or be fault-tolerant under the same noise model.

In summary, the fact that symmetry-violating generator perturbations manifest in the reconstructed PTM offers both a diagnostic handle and a potential lever for error-correction strategies. While we cannot claim that the identified deviations are *directly* correctable with standard QEC codes, they point to an enriched error model where affine shifts, survival loss, and cross-axis mixing are explicitly addressed. Future work should investigate how to embed these symmetry-violation diagnostics into fault-tolerant encoding schemes and whether tailored recovery operations can systematically reduce the impact of such deviations.

VII. CONCLUSION AND OUTLOOK

In this work, we introduced and experimentally validated a framework for modeling symmetry-violating noise in single-qubit quantum operations. The central idea was to treat noisy gates as perturbations of their ideal Hermitian generators and to examine how controlled symmetry-breaking effects manifest in reconstructed process matrices. By explicitly distinguishing Hermitian and non-Hermitian perturbations, we demonstrated that different classes of noise—coherent and

dissipative—leave characteristic signatures in the experimentally observed PTMs. This framework should be understood as a constrained phenomenological model and not as a complete replacement for general channel representations or established tomography methods. The experimentally reconstructed PTMs already contain the measurable channel information accessible through the tomography procedure itself. Accordingly, the present framework should not be interpreted as extracting fundamentally new experimentally inaccessible information from the hardware.

Using quantum process tomography on IBM Quantum hardware, we compared the hardware-averaged PTM with sampled model reconstructions derived from a fitted perturbation map. The genetic optimization procedure produced generator perturbations that, once combined with finite-shot variability, reproduced most affine-row and off-diagonal PTM components within one combined standard deviation. The remaining discrepancies were concentrated on the Pauli-diagonal entries, consistent with missing dissipative structure or SPAM-related drift. This showed that coherent, gate-local symmetry violations can phenomenologically reproduce a substantial portion of the experimentally observed asymmetries without overfitting the sampling noise. More precisely, the results show that structured generator-level perturbations can phenomenologically reproduce several experimentally reconstructed tomography signatures within uncertainty. The fitted perturbations should therefore be interpreted as one possible effective parametrization of the observed process-level structures rather than as a unique microscopic physical noise mechanism.

We need to emphasize that performing these experiments on IBM quantum hardware represents a strict practical limit. Acquiring the required data is costly, and collecting the 88 hardware runs used in this study required more than half a year until IBM Brisbane was no longer available. Any extension of the work to larger-scale experiments—for example, runs on the order of 100 000 executions would incur costs in the range of several tens of thousands of euros. This constitutes a substantial barrier for further large-scale hardware-based evaluation.

In addition, ensemble experiments with random Hermitian and non-Hermitian perturbations confirmed that non-Hermitian, symmetry-violating noise reproduces the experimental PTM more comprehensively than Hermitian perturbations alone. The non-Hermitian ensemble captured affine shifts and cross-axis couplings within uncertainty for the majority of matrix elements, while Hermitian ensembles left residual deviations on the diagonal entries associated with relaxation and dephasing. This result is consistent with effective coherent and dissipative contributions, without uniquely identifying their microscopic origin. At the same time, these observations should be interpreted conservatively because the tomography reconstruction itself already contains the experimentally measurable channel structure. The present framework therefore provides a transparent analytical parametrization capable of reproducing these structures rather than a direct microscopic derivation of the underlying hardware physics.

These findings have several implications. First, controlled inclusion of symmetry-violating perturbations provides a transparent and quantitative way to account for affine and cross-axis effects that standard unitary error models miss. Second, the elementwise z -score analysis links deviations directly to the underlying uncertainty scales of both model and experiment, providing a diagnostic tool that complements global distance measures. Third, the systematic improvement under non-Hermitian perturbations shows that the reconstructed PTM is better reproduced when the restricted model includes non-Hermitian contributions.

At the same time, introducing non-Hermitian components raises conceptual challenges. Once Hermiticity and unitarity are relaxed, familiar assumptions about trace preservation and orthogonality in the PTM representation no longer apply. This limits direct interpretability in conventional error metrics but enhances descriptive accuracy when modeling asymmetric or dissipative channels. The observed behavior highlights the necessity of combining generator-based modeling with uncertainty-aware comparison metrics to avoid overinterpreting statistical fluctuations as physical effects.

Overall, our study demonstrates that symmetry-violating generator perturbations, especially non-Hermitian ones, can systematically reproduce the experimentally observed deviations in single-qubit process tomography. The approach clarifies which PTM components—primarily affine and

off-diagonal entries—stem from coherent symmetry breaking and which residual discrepancies point to dissipative or SPAM-driven processes. More conservatively, the results establish that generator-level perturbations provide one possible phenomenological reconstruction mechanism capable of reproducing experimentally reconstructed PTM structures within uncertainty. They do not demonstrate the existence or uniqueness of a specific microscopic physical noise origin. Extending this framework to multiqubit systems will allow characterization of correlated and crosstalk-induced symmetry violations, offering a route toward more physically grounded noise models for quantum simulation, calibration, and error mitigation. However, this extension is nontrivial because multiqubit systems require larger process representations, introduce correlated and crosstalk-induced errors, and make SPAM separation more difficult. Accordingly, the present single-qubit study should be viewed as a controlled first step toward such models rather than as a complete hardware-noise characterization framework. The specific contribution is therefore a restricted forward generator-to-PTM reconstruction model, complementary to complete channel and error-generator representations and demonstrated here in the single-qubit setting.

Finally, we emphasize that the adopted tomography workflow mirrors the standard Qiskit process-tomography implementation and is not SPAM robust in the sense of gate set tomography or cycle benchmarking approaches [11,12,14]. The reconstructed PTMs should therefore be interpreted as effective tomography reconstructions obtained under the corresponding preparation, basis-rotation, and finite-shot assumptions. Extending the present framework toward SPAM-robust reconstruction methodologies and self-consistent hardware characterization protocols constitutes an important direction for future work.

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DATA AVAILABILITY

The full code is available in a corresponding GitHub repository in Ref. [48]. All data and code that support the findings of this article are openly available [48].

APPENDIX A: NOTATION SUMMARY

This Appendix summarizes the notation used throughout the paper. Its purpose is to keep the notation consistent across the theoretical foundations, the tomography framework, the simulator, the genetic-algorithm fitting procedure, the experimental results, and the discussion. The notation separates physical states, quantum channels, Pauli transfer matrix representations, generator-level perturbations, tomography data, hardware averages, and statistical comparison metrics. It also fixes the state notation and standard-deviation notation used throughout the article.

Basic mathematical notation. Scalars are denoted by lowercase letters such as p , z , N , or θ . Vectors are denoted by bold symbols such as $\mathbf{s}$, $\mathbf{r}$, or by parameter vectors such as w . Matrices and linear operators are denoted by capital letters such as U , R , G , T , and Δ . Complex conjugation, transposition, and Hermitian conjugation are written as

$$z^*, \quad A^\top, \quad A^\dagger, \quad (\text{A1})$$

respectively. The trace of a matrix is denoted by

$$\text{Tr}(A). \quad (\text{A2})$$

The identity matrix on a two-dimensional Hilbert space is denoted by

$$I_2. \quad (\text{A3})$$

The imaginary unit is denoted by

$$i = \sqrt{-1}. \quad (\text{A4})$$

Hilbert space, computational basis, and states. The article considers single-qubit systems. The Hilbert space is

$$\mathcal{H} \simeq \mathbb{C}^2. \quad (\text{A5})$$

The computational basis is

$$\{|0\rangle, |1\rangle\}. \quad (\text{A6})$$

A pure single-qubit state is written as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad \alpha, \beta \in \mathbb{C}, \quad |\alpha|^2 + |\beta|^2 = 1. \quad (\text{A7})$$

A mixed state is represented by a density matrix

$$\rho \in \mathbb{C}^{2 \times 2}, \quad \rho \succeq 0, \quad \text{Tr}(\rho) = 1. \quad (\text{A8})$$

Here, $\rho \succeq 0$ means that ρ is positive semidefinite.

Pauli operators and Bloch-vector notation. The Pauli operators are

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad (\text{A9})$$

They are also denoted by

$$X = \sigma_x, \quad Y = \sigma_y, \quad Z = \sigma_z. \quad (\text{A10})$$

The ordered Pauli basis used for Pauli transfer matrices is

$$(I, X, Y, Z). \quad (\text{A11})$$

A single-qubit density matrix can be written in Bloch form as

$$\rho = \frac{1}{2}(I_2 + r_x\sigma_x + r_y\sigma_y + r_z\sigma_z), \quad (\text{A12})$$

where

$$\mathbf{r} = (r_x, r_y, r_z)^\top \in \mathbb{R}^3, \quad \|\mathbf{r}\|_2 \leq 1 \quad (\text{A13})$$

is the Bloch vector. The Pauli expectation values are

$$\langle \sigma_i \rangle = \text{Tr}(\rho \sigma_i), \quad i \in \{x, y, z\}. \quad (\text{A14})$$

Thus,

$$r_x = \langle X \rangle, \quad r_y = \langle Y \rangle, \quad r_z = \langle Z \rangle. \quad (\text{A15})$$

Standardized preparation-state notation. To avoid ambiguity, the article uses the axis-labeled notation

$$|Z+\rangle, \quad |Z-\rangle, \quad |X+\rangle, \quad |Y+\rangle \quad (\text{A16})$$

for the tomography preparation states. The notation

$$|X+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \quad (\text{A17})$$

is used instead of the shorter notation $|+\rangle$. The notation

$$|Y+\rangle = \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \quad (\text{A18})$$

is used instead of the shorter notation $|+i\rangle$. This convention keeps all preparation states explicitly tied to their Pauli axes.

Augmented Bloch-vector convention. The Pauli transfer matrix acts on the augmented Bloch vector

$$\mathbf{s} = \begin{bmatrix} 1 \\ \langle X \rangle \\ \langle Y \rangle \\ \langle Z \rangle \end{bmatrix} = \begin{bmatrix} 1 \\ r_x \\ r_y \\ r_z \end{bmatrix}. \quad (\text{A19})$$

For a channel $\mathcal{E}$, the input and output vectors are

$$\mathbf{s}_{\text{in}} = \begin{bmatrix} 1 \\ \langle X \rangle_{\text{in}} \\ \langle Y \rangle_{\text{in}} \\ \langle Z \rangle_{\text{in}} \end{bmatrix}, \quad \mathbf{s}_{\text{out}} = \begin{bmatrix} 1 \\ \langle X \rangle_{\text{out}} \\ \langle Y \rangle_{\text{out}} \\ \langle Z \rangle_{\text{out}} \end{bmatrix}. \quad (\text{A20})$$

The PTM is denoted by

$$R \in \mathbb{R}^{4 \times 4} \quad (\text{A21})$$

and satisfies

$$\mathbf{s}_{\text{out}} = R \mathbf{s}_{\text{in}}. \quad (\text{A22})$$

The entries of R are indexed as

$$R_{ij}, \quad i, j \in \{0, 1, 2, 3\}, \quad (\text{A23})$$

where the index order corresponds to

$$0 \leftrightarrow I, \quad 1 \leftrightarrow X, \quad 2 \leftrightarrow Y, \quad 3 \leftrightarrow Z. \quad (\text{A24})$$

PTM block structure. The PTM is written in block form as

$$R = \begin{bmatrix} R_{00} & R_{01} & R_{02} & R_{03} \\ R_{10} & R_{11} & R_{12} & R_{13} \\ R_{20} & R_{21} & R_{22} & R_{23} \\ R_{30} & R_{31} & R_{32} & R_{33} \end{bmatrix}. \quad (\text{A25})$$

In the trace-preserving case, the first row is

$$(R_{00}, R_{01}, R_{02}, R_{03}) = (1, 0, 0, 0). \quad (\text{A26})$$

The lower-left column

$$\mathbf{t} = (R_{10}, R_{20}, R_{30})^\top \quad (\text{A27})$$

is the affine translation vector of the Bloch map. It captures nonunital behavior. The lower-right block

$$T = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \quad (\text{A28})$$

describes the linear transformation of the Bloch vector. Thus, the Bloch-vector map is

$$\mathbf{r}_{\text{out}} = \mathbf{t} + T\mathbf{r}_{\text{in}}. \quad (\text{A29})$$

For an ideal unitary channel, T is an orthogonal matrix and $\mathbf{t} = 0$. Deviations from orthogonality encode incoherent effects such as damping or dephasing. Nonzero $\mathbf{t}$ encodes nonunital behavior. Nonzero entries in the first row beyond R_{00} indicate apparent non-trace-preserving behavior in the reconstructed PTM.

Ideal gates and native gate set. The native single-qubit gate set used in the article is

$$\mathcal{G} = \{R_z, SX, X\}. \quad (\text{A30})$$

The symbol $\mathcal{G}$ denotes the set of gates. Individual gates are denoted by

$$g \in \mathcal{G}. \quad (\text{A31})$$

The ideal generator associated with gate g is denoted by

$$G_g. \quad (\text{A32})$$

The gate-dependent rotation angle is denoted by

$$\theta_g. \quad (\text{A33})$$

The ideal unitary implementation of gate g is

$$U_g(\theta_g) = \exp\left[-i\frac{\theta_g}{2}G_g\right]. \quad (\text{A34})$$

The ideal single-qubit rotations used in the text are

$$R_i(\theta) = \exp\left[-i\frac{\theta}{2}\sigma_i\right], \quad i \in \{x, y, z\}. \quad (\text{A35})$$

In particular,

$$R_z(\theta) = \exp\left[-i\frac{\theta}{2}\sigma_z\right], \quad (\text{A36})$$

$$SX = \exp\left[-i\frac{\pi}{4}\sigma_x\right], \quad (\text{A37})$$

and

$$X = \exp \left[-i \frac{\pi}{2} \sigma_x \right]. \quad (\text{A38})$$

Circuit notation. A single-qubit circuit is written as a time-ordered sequence of gates. If the individual unitary matrices are

$$U_1, U_2, \dots, U_L, \quad (\text{A39})$$

where U_1 is applied first and U_L last, then the total unitary is

$$U_{\text{tot}} = U_L U_{L-1} \cdots U_1. \quad (\text{A40})$$

For an input state ρ_{in} , the ideal output state is

$$\rho_{\text{out}}^{(\text{ideal})} = U_{\text{tot}} \rho_{\text{in}} U_{\text{tot}}^\dagger. \quad (\text{A41})$$

A preparation unitary is denoted by

$$U_{\text{prep}}, \quad (\text{A42})$$

a target unitary or target-gate sequence by

$$U_{\text{target}}, \quad (\text{A43})$$

and a premeasurement-basis rotation by

$$U_{\text{meas}}. \quad (\text{A44})$$

The full prepared, evolved, and basis-rotated state is

$$\rho_{\text{out}} = U_{\text{meas}} U_{\text{target}} U_{\text{prep}} |0\rangle\langle 0| U_{\text{prep}}^\dagger U_{\text{target}}^\dagger U_{\text{meas}}^\dagger. \quad (\text{A45})$$

Preparation states and measurement bases. The four preparation states used in the QPT-12 scheme are

$$|Z+\rangle = |0\rangle, \quad |Z-\rangle = |1\rangle, \quad (\text{A46})$$

$$|X+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad |Y+\rangle = \frac{|0\rangle + i|1\rangle}{\sqrt{2}}. \quad (\text{A47})$$

The preparation label is denoted by

$$p \in \{Z+, Z-, X+, Y+\}. \quad (\text{A48})$$

The measurement basis is denoted by

$$m \in \{Z, X, Y\}. \quad (\text{A49})$$

The measurement-basis unitary is denoted by

$$M_m. \quad (\text{A50})$$

The 12 process-tomography circuits are indexed by pairs (p, m) and written as

$$U_{p,m} = M_m U_T P_p, \quad p \in \{Z+, Z-, X+, Y+\}, \quad m \in \{Z, X, Y\}. \quad (\text{A51})$$

Here, P_p denotes the preparation circuit for state p , U_T denotes the target operation, and M_m denotes the premeasurement-basis rotation.

Projectors, probabilities, and shot statistics. The computational-basis projectors are

$$\Pi_0 = |0\rangle\langle 0|, \quad \Pi_1 = |1\rangle\langle 1|. \quad (\text{A52})$$

For measurement outcome $b \in \{0, 1\}$, the Born probability is

$$p(b) = \text{Tr}(\rho_{\text{out}} \Pi_b). \quad (\text{A53})$$

For N shots, the empirical frequency is

$$\hat{p}(b) = \frac{n_b}{N}, \quad (\text{A54})$$

where n_b is the number of observed outcomes b . The finite-shot variance is

$$\text{Var}[\hat{p}(b)] = \frac{p(b)(1 - p(b))}{N}. \quad (\text{A55})$$

The number of shots per circuit is denoted by

$$N_{\text{shots}}. \quad (\text{A56})$$

In the experiments and simulations, the value

$$N_{\text{shots}} = 10^4 \quad (\text{A57})$$

is used in the shot-based QPT-12 setting.

Quantum channels and Kraus notation. A noisy quantum operation is represented by a quantum channel

$$\mathcal{E}. \quad (\text{A58})$$

For an input state ρ_{in} , the noisy output state is

$$\rho_{\text{out}}^{(\text{noisy})} = \mathcal{E}(\rho_{\text{in}}). \quad (\text{A59})$$

A completely positive trace-preserving channel is abbreviated as

$$\text{CPTP}. \quad (\text{A60})$$

In Kraus form, the channel is written as

$$\mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger, \quad \sum_k E_k^\dagger E_k = I_2. \quad (\text{A61})$$

Here, $\{E_k\}$ are Kraus operators. An ideal unitary channel is the special case

$$\mathcal{E}_U(\rho) = U \rho U^\dagger. \quad (\text{A62})$$

Generator-level perturbations. The central modeling object of the article is the additive generator perturbation

$$\Delta_g \in \mathbb{C}^{2 \times 2}, \quad g \in \mathcal{G}. \quad (\text{A63})$$

It modifies the ideal generator G_g as

$$G_g \mapsto G_g + \Delta_g. \quad (\text{A64})$$

The corresponding perturbed gate is

$$U_g^\Delta(\theta_g) = \exp\left[-i \frac{\theta_g}{2} (G_g + \Delta_g)\right]. \quad (\text{A65})$$

The superscript Δ indicates that the gate has been generated from a perturbed generator.

A Hermitian perturbation satisfies

$$\Delta_g^\dagger = \Delta_g. \quad (\text{A66})$$

Hermitian perturbations model coherent generator-level errors such as axis tilts or over- and under-rotations. A general complex perturbation does not have to satisfy

$$\Delta_g^\dagger = \Delta_g. \quad (\text{A67})$$

Such non-Hermitian perturbations can induce effective nonunitary behavior in the reconstructed PTM, including nonunital and non-trace-preserving signatures in the adopted phenomenological reconstruction framework.

Real and imaginary parts of perturbations. Each complex perturbation matrix can be decomposed as

$$\Delta_g = \text{Re}(\Delta_g) + i \text{Im}(\Delta_g). \quad (\text{A68})$$

In the results, the fitted matrices are reported separately as

$$\Delta[g]_{\text{Re}}, \quad \Delta[g]_{\text{Im}}, \quad (\text{A69})$$

for each

$$g \in \{R_z, \text{SX}, X\}. \quad (\text{A70})$$

Both parts are real 2×2 matrices.

Hermitian and non-Hermitian ensembles. The article compares two perturbation families. The Hermitian ensemble contains random perturbations satisfying

$$\Delta = \Delta^\dagger. \quad (\text{A71})$$

The non-Hermitian ensemble contains general complex-valued perturbations

$$\Delta \in \mathbb{C}^{2 \times 2} \quad (\text{A72})$$

without imposing Hermiticity. Ensemble means are denoted by

$$\bar{R}_{\text{Herm}}, \quad \bar{R}_{\text{Non-Herm}}, \quad (\text{A73})$$

and ensemble standard deviations by

$$\text{STD}[R_{\text{Herm}}], \quad \text{STD}[R_{\text{Non-Herm}}]. \quad (\text{A74})$$

The number of random perturbation samples is denoted by

$$N_{\text{samples}}. \quad (\text{A75})$$

QPT-12 simulator. The abbreviation QPT denotes quantum process tomography. The QPT-12 protocol is the 12-circuit single-qubit process tomography protocol obtained from four preparation states and three measurement bases:

$$4 \times 3 = 12. \quad (\text{A76})$$

The simulator reconstructs a PTM from synthetic shot data generated using the same QPT-12 structure. The simulator output for a perturbation map $\{\Delta_g\}_{g \in \mathcal{G}}$ is denoted generically by

$$R_{\text{fit}}, \quad R_{\Delta}, \quad R_{\Delta, N_{\text{shots}}}, \quad (\text{A77})$$

depending on context. The notation

$$R_{\Delta, 10^4}^{\text{mean}} \quad (\text{A78})$$

denotes the mean PTM obtained from repeated shot-based reconstructions using the fitted perturbations and 10^4 shots per circuit. Its componentwise standard deviation is

$$\text{STD}[R_{\Delta, 10^4}]. \quad (\text{A79})$$

Survival probabilities and first-row reconstruction. For each preparation p , the simulator may compute or sample a survival statistic

$$s_p. \quad (\text{A80})$$

This represents the effective total detection probability for that preparation. The four survival values determine the first row of the reconstructed PTM through

$$s_{Z+} = R_{00} + R_{03}, \quad (\text{A81})$$

$$s_{Z-} = R_{00} - R_{03}, \quad (\text{A82})$$

$$s_{X+} = R_{00} + R_{01}, \quad (\text{A83})$$

$$s_{Y+} = R_{00} + R_{02}. \quad (\text{A84})$$

In the trace-preserving limit,

$$s_p = 1 \quad (\text{A85})$$

for all preparations p , and the first row becomes

$$(R_{00}, R_{01}, R_{02}, R_{03}) = (1, 0, 0, 0). \quad (\text{A86})$$

Hardware PTMs and empirical averages. The article uses repeated hardware process tomography runs on `ibm_brisbane`. The number of hardware runs is denoted by

$$K, \quad (\text{A87})$$

or equivalently, by

$$n_{\text{runs}}. \quad (\text{A88})$$

In the reported experiment,

$$n_{\text{runs}} = 88. \quad (\text{A89})$$

The PTM reconstructed from hardware run k is denoted by

$$R^{(k)}. \quad (\text{A90})$$

The hardware-averaged PTM is

$$R_{\text{target}} = \frac{1}{K} \sum_{k=1}^K R^{(k)}. \quad (\text{A91})$$

The componentwise hardware standard deviation is denoted consistently by

$$\text{STD}[R_{\text{target}}]. \quad (\text{A92})$$

For each matrix entry,

$$\text{STD}[R_{\text{target}}]_{ij} \quad (\text{A93})$$

is the sample standard deviation of the ij entry across hardware runs. The symbol σ is reserved for derived uncertainty quantities such as the combined standard deviation $\sigma_{\text{comb},ij}$, not for raw componentwise sample standard-deviation matrices.

Ideal reference PTM. The ideal noiseless PTM of the single-qubit X gate is denoted by

$$R_X. \quad (\text{A94})$$

In the ordered basis (I, X, Y, Z) , it is

$$R_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \quad (\text{A95})$$

This matrix represents a trace-preserving and unital unitary channel.

Genetic algorithm variables. The genetic algorithm is abbreviated as

$$\text{GA.} \tag{A96}$$

A genome is denoted by

$$w \in \mathbb{R}^{8|\mathcal{G}|}. \tag{A97}$$

Since

$$\mathcal{G} = \{R_z, SX, X\}, \quad |\mathcal{G}| = 3, \tag{A98}$$

the genome contains

$$8|\mathcal{G}| = 24 \tag{A99}$$

real parameters. These parameters encode the real and imaginary entries of the three complex 2×2 perturbation matrices

$$\{\Delta_g\}_{g \in \mathcal{G}}. \tag{A100}$$

The perturbation encoded by genome w is written as

$$\Delta_g(w). \tag{A101}$$

The fitted PTM induced by w is

$$R_{\text{fit}}(w). \tag{A102}$$

The best genome found during optimization is denoted by

$$w^*. \tag{A103}$$

The corresponding fitted perturbations are

$$\{\Delta_g^*\}_{g \in \mathcal{G}}. \tag{A104}$$

Bounded perturbation parametrization. The genome entries are mapped to bounded perturbation components using a componentwise saturating transform. The perturbation scale is denoted by

$$\Delta_{\text{max}}. \tag{A105}$$

In the experiments,

$$\Delta_{\text{max}} = 0.05. \tag{A106}$$

A typical bounded component is of the form

$$\delta = \Delta_{\text{max}} \tanh(w_a), \tag{A107}$$

where w_a is one real genome component. The genome itself is clipped to

$$[-W_{\text{max}}, W_{\text{max}}], \tag{A108}$$

with

$$W_{\text{max}} = 6. \tag{A109}$$

GA cost function and fitness. The cost function is denoted by

$$J(w). \tag{A110}$$

It measures the discrepancy between the fitted PTM and the hardware target PTM:

$$J(w) = \sum_{i,j} \omega_{ij} (R_{\text{fit}}(w)_{ij} - R_{\text{target},ij})^2. \tag{A111}$$

The weights are denoted by

$$\omega_{ij}. \quad (\text{A112})$$

By default, they are

$$\omega_{ij} = 1, \quad (\text{A113})$$

with optional amplification for small target entries. If

$$|R_{\text{target},ij}| < 0.05, \quad (\text{A114})$$

the corresponding weight may be multiplied by an amplification factor, reported as

$$3.5. \quad (\text{A115})$$

The GA fitness is

$$F(w) = -J(w). \quad (\text{A116})$$

Thus, maximizing $F(w)$ is equivalent to minimizing $J(w)$.

GA population and evolutionary operators. The GA population size is denoted by

$$N_{\text{pop}}. \quad (\text{A117})$$

The number of elite genomes retained unchanged is

$$N_{\text{elite}}. \quad (\text{A118})$$

The number of generations is

$$T. \quad (\text{A119})$$

Selection means that parents are drawn from the top-performing fraction of the population. Uniform crossover means that each child genome component is inherited from one of two parent genomes with equal probability. Mutation means that randomly selected genome components are perturbed by Gaussian noise and then clipped back to the allowed interval. Elitism means that the best genomes are copied directly into the next generation.

Repeated simulator reconstructions. For a fitted perturbation set $\{\Delta_g^*\}$, repeated shot-based reconstructions produce an ensemble

$$\{R_{\text{fit}}^{(r)}\}_{r=1}^{N_{\text{eval}}}, \quad (\text{A120})$$

where r indexes the simulator repetition and N_{eval} is the number of repeated reconstructions. The simulator mean is

$$R_{\text{fit}} = \frac{1}{N_{\text{eval}}} \sum_{r=1}^{N_{\text{eval}}} R_{\text{fit}}^{(r)}. \quad (\text{A121})$$

The componentwise simulator standard deviation is denoted consistently by

$$\text{STD}[R_{\text{fit}}]. \quad (\text{A122})$$

Entrywise differences and combined uncertainty. The entrywise model-hardware difference is

$$\Delta R_{ij} = R_{\text{model},ij} - R_{\text{target},ij}. \quad (\text{A123})$$

Depending on context, R_{model} may denote

$$R_{\Delta, 10^4}^{\text{mean}}, \quad \bar{R}_{\text{Herm}}, \quad \bar{R}_{\text{Non-Herm}}, \quad R_{\text{fit}}. \quad (\text{A124})$$

The combined standard deviation is

$$\sigma_{\text{comb},ij} = \sqrt{\text{STD}[R_{\text{target}}]_{ij}^2 + \text{STD}[R_{\text{model}}]_{ij}^2}. \quad (\text{A125})$$

This treats hardware variability and simulator variability as independent.

Entrywise z scores. The entrywise z score is

$$z_{ij} = \frac{|\Delta R_{ij}|}{\sigma_{\text{comb},ij}}. \quad (\text{A126})$$

Equivalently,

$$z_{ij} = \frac{|R_{\text{model},ij} - R_{\text{target},ij}|}{\sqrt{\text{STD}[R_{\text{target}}]_{ij}^2 + \text{STD}[R_{\text{model}}]_{ij}^2}}. \quad (\text{A127})$$

An entry with

$$z_{ij} \leq 1 \quad (\text{A128})$$

is interpreted as statistically compatible within one combined standard deviation. An entry with

$$z_{ij} > 1 \quad (\text{A129})$$

indicates tension between the model and the hardware target at that matrix element.

Standardized notation for means and standard deviations. The article uses the following conventions:

$$\bar{R} \quad (\text{A130})$$

denotes an ensemble mean PTM,

$$R^{\text{mean}} \quad (\text{A131})$$

denotes a mean PTM when the context is a specific sampled reconstruction, and

$$\text{STD}[R] \quad (\text{A132})$$

denotes the componentwise standard deviation of the entries of R over repeated runs or repeated samples. The notation $\text{STD}[\cdot]$ is used for standard-deviation matrices throughout the article. The symbol σ is used only for scalar or entrywise derived uncertainty terms, most importantly

$$\sigma_{\text{comb},ij}. \quad (\text{A133})$$

This convention avoids using σ and STD interchangeably for the same object.

Indexing convention for PTM entries. The article uses both coordinate indices and Pauli labels. The correspondence is

$$(0, 0) \leftrightarrow (I, I), \quad (1, 1) \leftrightarrow (X, X), \quad (2, 2) \leftrightarrow (Y, Y), \quad (3, 3) \leftrightarrow (Z, Z). \quad (\text{A134})$$

The first-row entries beyond R_{00} are

$$(0, 1:3) = \{(0, 1), (0, 2), (0, 3)\}. \quad (\text{A135})$$

These entries are associated with apparent trace-preservation deviations in the reconstructed PTM. The affine translation vector entries are

$$(1:3, 0) = \{(1, 0), (2, 0), (3, 0)\}. \quad (\text{A136})$$

These entries are associated with nonunitary affine shifts of the Bloch vector. The Pauli-diagonal entries are

$$(1, 1), \quad (2, 2), \quad (3, 3), \quad (\text{A137})$$

corresponding to the X , Y , and Z directions of the Bloch sphere.

State preparation and measurement abbreviation. State preparation and measurement errors are abbreviated as

$$\text{SPAM}. \quad (\text{A138})$$

SPAM effects may affect the reconstructed PTM because the same native gates used in the target-gate set may also appear in preparation and measurement-basis rotations. In this article, SPAM is treated as a limitation of the adopted process-tomography workflow rather than as a separately estimated physical parameter.

The present workflow does not independently estimate SPAM parameters.

Trace preservation, unitality, and related notation. Trace preservation means

$$\text{Tr}(\mathcal{E}(\rho)) = \text{Tr}(\rho) \quad (\text{A139})$$

for all density matrices ρ . In PTM form, this corresponds to

$$(R_{00}, R_{01}, R_{02}, R_{03}) = (1, 0, 0, 0). \quad (\text{A140})$$

Unitality means

$$\mathcal{E}(I_2) = I_2. \quad (\text{A141})$$

In Bloch-vector form, unitality corresponds to

$$\mathbf{t} = 0. \quad (\text{A142})$$

Nonunitality is therefore represented by a nonzero lower-left PTM column. Apparent non-trace-preserving behavior is represented by deviations in the first row of the reconstructed PTM.

Hermiticity and non-Hermiticity. A matrix A is Hermitian if

$$A^\dagger = A. \quad (\text{A143})$$

A matrix is non-Hermitian if this condition does not hold. In the article, Hermitian generator perturbations are associated with coherent generator-level modifications. Non-Hermitian perturbations are used phenomenologically to emulate effective open-system signatures in the reconstructed PTM, including nonunital and non-trace-preserving effects.

Lindblad and open-system terminology. Open-system dynamics refer to dynamics in which the qubit interacts with an environment. Such dynamics may include relaxation, dephasing, leakage, or other dissipative effects. A Lindblad description is a continuous-time Markovian open-system model. The article refers to Lindblad and Kraus descriptions as standard physical channel representations that can model dissipative noise beyond purely unitary evolution. The perturbative generator model used here is not claimed to be a complete Lindblad model; it is a constrained phenomenological reconstruction model.

Taxonomy-paper terminology. The article refers to generator-space taxonomies of small Markovian errors. In that context, a reconstructed channel can be transformed into an error generator and decomposed into sectors such as Hamiltonian, stochastic, correlation, and active components. The present article uses a different construction: it introduces perturbations directly into native gate generators and then reconstructs the resulting PTMs through the same tomography schedule used for hardware comparison. Thus, the notation Δ_g refers to a gate-generator perturbation in the present model, not to a full error-generator decomposition of an arbitrary reconstructed channel.

Analytic versus shot-based PTM evaluation. An analytic PTM evaluation computes the PTM directly from the modeled channel or perturbed gate without sampling measurement shots. A shot-based PTM evaluation reconstructs the PTM from simulated finite-shot measurement data. In the article, shot-based reconstruction is used to mirror the hardware tomography workflow and include finite-shot fluctuations. Analytic PTM evaluation is recognized as a computationally cheaper alternative, but it is not the primary route used in the reported GA comparison.

Model interpretation. The notation distinguishes between hardware reference data and phenomenological model outputs:

$$R_{\text{target}} \quad (\text{A144})$$

is the hardware-averaged reconstructed PTM, whereas

$$R_{\text{fit}}, \quad R_{\Delta}, \quad \bar{R}_{\text{Herm}}, \quad \bar{R}_{\text{Non-Herm}} \quad (\text{A145})$$

are model-derived or ensemble-derived PTMs. Agreement between these matrices means that the model reproduces the corresponding reconstructed PTM structures within uncertainty. It does not imply that the fitted perturbations uniquely identify the microscopic physical origin of the hardware noise.

Experimental platform notation. The hardware backend is denoted by

$$\text{ibm_brisbane}. \quad (\text{A146})$$

The analyzed physical operation is the single-qubit X gate. The experimental hardware data consist of repeated process-tomography reconstructions of this gate. The notation

$$R_{\text{target}} \quad (\text{A147})$$

therefore refers specifically to the empirical mean PTM of the implemented X -gate channel on this backend, under the conditions of the collected runs.

Summary of main notation hierarchy. The main hierarchy used in the article is

$$\text{state} \longrightarrow \text{gate} \longrightarrow \text{perturbed gate} \longrightarrow \text{tomography} \longrightarrow \text{PTM} \longrightarrow \text{statistical comparison}. \quad (\text{A148})$$

More explicitly,

$$\rho_{\text{in}} \longrightarrow U_g(\theta_g) \longrightarrow U_g^{\Delta}(\theta_g) \longrightarrow \text{QPT-12} \longrightarrow R_{\text{fit}} \longrightarrow z_{ij}. \quad (\text{A149})$$

For hardware data, the hierarchy is

$$\text{hardware runs} \longrightarrow \{R^{(k)}\}_{k=1}^K \longrightarrow R_{\text{target}}, \text{ STD}[R_{\text{target}}]. \quad (\text{A150})$$

For model fitting, the hierarchy is

$$w \longrightarrow \{\Delta_g(w)\}_{g \in \mathcal{G}} \longrightarrow \{U_g^{\Delta}\}_{g \in \mathcal{G}} \longrightarrow R_{\text{fit}}(w) \longrightarrow J(w) \longrightarrow w^*. \quad (\text{A151})$$

For ensemble comparisons, the hierarchy is

$$\{\Delta^{(r)}\}_{r=1}^{N_{\text{samples}}} \longrightarrow \{R^{(r)}\}_{r=1}^{N_{\text{samples}}} \longrightarrow \bar{R}, \quad \text{STD}[R] \longrightarrow z_{ij}. \quad (\text{A152})$$

Common abbreviations. For clarity, the main abbreviations used in the article are collected here:

$$\text{PTM} = \text{Pauli transfer matrix}, \quad (\text{A153})$$

$$\text{QPT} = \text{quantum process tomography}, \quad (\text{A154})$$

$$\text{QPT-12} = \text{the 12-circuit single-qubit QPT protocol}, \quad (\text{A155})$$

$$\text{GA} = \text{genetic algorithm}, \quad (\text{A156})$$

$$\text{SPAM} = \text{state preparation and measurement}, \quad (\text{A157})$$

$$\text{CPTP} = \text{completely positive trace preserving}, \quad (\text{A158})$$

$$\text{RB} = \text{randomized benchmarking}, \quad (\text{A159})$$

$$\text{GST} = \text{gate set tomography}, \quad (\text{A160})$$

$$\text{QEC} = \text{quantum error correction}, \quad (\text{A161})$$

$$\text{ZNE} = \text{zero-noise extrapolation}. \quad (\text{A162})$$

Interpretive convention. Throughout the paper, statements about fitted perturbations are phenomenological unless stated otherwise. The notation

$$\Delta_g \tag{A163}$$

denotes an effective generator-level perturbation that helps reproduce reconstructed PTM structures. It should not be read as a directly measured microscopic Hamiltonian, Lindblad operator, or unique physical noise source. The experimentally accessible object is the reconstructed PTM and its uncertainty; the perturbation model provides a structured and analytically transparent way to reproduce and study these reconstructed features.

APPENDIX B: EXPRESSIVITY AND LIMITATIONS OF THE GENERATOR-PERTURBATION MODEL

This Appendix explains what the generator-perturbation model can and cannot represent. The goal is to make explicit under which conditions the model is sufficient as a phenomenological description, where it overlaps with standard single-qubit noise models, and where it is necessarily incomplete. We use the notation introduced in Appendix A. In particular, R denotes a PTM, G_g denotes the ideal generator of gate $g \in \mathcal{G}$, and Δ_g denotes an additive generator-level perturbation [9,19].

1. What must be matched

A general single-qubit process can be represented in affine Bloch-vector form as

$$\mathbf{r}_{\text{out}} = \mathbf{t} + T\mathbf{r}_{\text{in}}, \tag{B1}$$

where $\mathbf{r}_{\text{in}}, \mathbf{r}_{\text{out}} \in \mathbb{R}^3$, $\mathbf{t} \in \mathbb{R}^3$, and $T \in \mathbb{R}^{3 \times 3}$ [9]. Equivalently, in the augmented Bloch-vector convention

$$\mathbf{s} = \begin{bmatrix} 1 \\ \mathbf{r} \end{bmatrix}, \tag{B2}$$

the channel is represented by the PTM

$$R = \begin{bmatrix} 1 & 0 \\ \mathbf{t} & T \end{bmatrix} \tag{B3}$$

for a trace-preserving channel. If the reconstruction is not constrained to be trace preserving, the first row may deviate from $(1, 0, 0, 0)$, giving

$$R = \begin{bmatrix} R_{00} & R_{01} & R_{02} & R_{03} \\ R_{10} & R_{11} & R_{12} & R_{13} \\ R_{20} & R_{21} & R_{22} & R_{23} \\ R_{30} & R_{31} & R_{32} & R_{33} \end{bmatrix}. \tag{B4}$$

Thus, the experimental target is not only a unitary rotation. It is a reconstructed process matrix containing several possible structures: rotation, contraction, affine shift, and apparent trace-preservation deviation [11].

A complete single-qubit channel model would need to reproduce all allowed CPTP affine maps. The generator-perturbation model used in this work has a more limited purpose: It tests whether a restricted family of gate-local perturbations can reproduce selected structures observed in experimentally reconstructed PTMs.

2. The proposed construction

For each native gate $g \in \mathcal{G}$, the ideal gate is written as

$$U_g(\theta_g) = \exp\left[-i\frac{\theta_g}{2}G_g\right], \quad (\text{B5})$$

where G_g is the ideal Hermitian generator. The proposed model replaces this by

$$U_g^\Delta(\theta_g) = \exp\left[-i\frac{\theta_g}{2}(G_g + \Delta_g)\right], \quad \Delta_g \in \mathbb{C}^{2 \times 2}. \quad (\text{B6})$$

The perturbation Δ_g is gate local: Each native gate receives its own matrix perturbation. The same perturbed native gates are then used inside the full tomography circuits, including state preparation, target-gate application, and measurement-basis rotations [5, 18].

This construction has two layers. The first layer is the perturbed matrix exponential U_g^Δ . The second layer is the tomography reconstruction, which maps the simulated measurement outcomes into a reconstructed PTM R_{fit} . Therefore, the model does not claim that U_g^Δ is itself a complete physical noise channel. Rather, it claims that the resulting reconstructed PTM can be compared to the experimentally reconstructed PTM:

$$\{\Delta_g\}_{g \in \mathcal{G}} \longrightarrow \{U_g^\Delta\}_{g \in \mathcal{G}} \longrightarrow \text{QPT-12} \longrightarrow R_{\text{fit}}. \quad (\text{B7})$$

The resulting PTMs should therefore be interpreted as effective tomography reconstructions obtained under the assumptions of the adopted tomography workflow rather than as uniquely identifiable microscopic noise models [11].

3. Hermitian perturbations

If

$$\Delta_g^\dagger = \Delta_g, \quad (\text{B8})$$

then $G_g + \Delta_g$ is Hermitian. Consequently,

$$U_g^\Delta(\theta_g) \quad (\text{B9})$$

is unitary. In this case, the perturbed gate still represents a coherent gate error. Typical examples are over-rotation, under-rotation, axis tilt, and coherent cross-axis mixing.

In Bloch-vector language, such errors mainly modify the rotation part of the PTM. The lower-right block T remains close to an orthogonal transformation if no sampling or reconstruction artifacts are present. Thus, Hermitian perturbations are suitable for modeling coherent miscalibration.

They are not sufficient to represent genuinely dissipative channels. For example, amplitude damping, dephasing, and depolarizing noise generally contract the Bloch sphere. A purely unitary perturbation cannot produce such contraction as a physical channel by itself. Therefore, Hermitian perturbations are sufficient only when the dominant error is coherent.

4. Non-Hermitian perturbations

If

$$\Delta_g^\dagger \neq \Delta_g, \quad (\text{B10})$$

then $G_g + \Delta_g$ is non-Hermitian and

$$U_g^\Delta(\theta_g) \quad (\text{B11})$$

is not generally unitary. This means that the transformation can change the norm of a state vector before normalization or reconstruction. In the tomography pipeline, this can lead to effective signatures such as Bloch-vector contraction, affine shifts, and apparent non-trace-preserving terms.

This is why non-Hermitian perturbations are more expressive than Hermitian perturbations in the present framework. They can emulate some open-system effects at the level of reconstructed PTMs. However, this does not mean that every non-Hermitian perturbation corresponds to a valid CPTP channel. A valid physical channel must satisfy complete positivity and trace preservation. The present model does not enforce these constraints at the generator level. It is therefore best interpreted as an effective phenomenological reconstruction model.

5. Relation to standard channel classes

We now compare the generator-perturbation model to common single-qubit noise channels. The purpose is not to prove exact equivalence in all cases, but to identify where the model is expected to approximate the observed PTM structure.

Coherent over-rotation. A coherent over-rotation around axis $i \in \{x, y, z\}$ has the form

$$U_{\text{err}} = \exp \left[-i \frac{\epsilon}{2} \sigma_i \right], \quad (\text{B12})$$

where ϵ denotes a small unwanted deviation in the implemented rotation angle. Such deviations typically arise from coherent calibration errors or pulse imperfections. In the present framework, the ideal gate is generated from

$$U_g(\theta_g) = \exp \left[-i \frac{\theta_g}{2} G_g \right], \quad (\text{B13})$$

while the perturbed gate is constructed as

$$U_g^\Delta(\theta_g) = \exp \left[-i \frac{\theta_g}{2} (G_g + \Delta_g) \right]. \quad (\text{B14})$$

Thus, the perturbation Δ_g directly modifies the original generator G_g before exponentiation into the final implemented gate U_g^Δ . A coherent over-rotation is represented by a Hermitian perturbation

$$\Delta_g = \epsilon_i \sigma_i \quad (\text{B15})$$

up to the scaling convention used in $U_g^\Delta(\theta_g)$. In this picture, the additional error rotation U_{err} is effectively absorbed into the modified generator $G_g + \Delta_g$, such that the final implemented gate U_g^Δ combines both the intended ideal evolution and the coherent error contribution within a single exponentiated operator. This effectively changes the implemented rotation angle or slightly tilts the rotation axis relative to the ideal gate. Therefore, coherent over-rotations and axis tilts are naturally included in the model while preserving unitarity because $\Delta_g^\dagger = \Delta_g$.

Pauli dephasing. A dephasing channel along the Z axis has the PTM structure

$$R_{\text{deph}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad 0 \leq \lambda \leq 1. \quad (\text{B16})$$

This channel contracts the X and Y components while leaving the Z component unchanged. Physically, this corresponds to a loss of phase coherence between the computational basis states while preserving the populations in the Z basis. In the Bloch-sphere picture, the Bloch vector is compressed toward the Z axis without introducing a coherent rotation.

In the present framework, the ideal gate is generated from

$$U_g(\theta_g) = \exp\left[-i\frac{\theta_g}{2}G_g\right], \quad (\text{B17})$$

while the perturbed gate is constructed as

$$U_g^\Delta(\theta_g) = \exp\left[-i\frac{\theta_g}{2}(G_g + \Delta_g)\right]. \quad (\text{B18})$$

Thus, the perturbation Δ_g directly modifies the original generator G_g before exponentiation into the final implemented gate U_g^Δ . Unlike coherent over-rotations, pure dephasing does not correspond to an additional unitary rotation generated by a Hermitian operator. Consequently, a Hermitian perturbation alone cannot reproduce the diagonal contraction structure of R_{deph} as a physical channel because Hermitian perturbations preserve unitarity and therefore preserve Bloch-vector norm.

A non-Hermitian perturbation can, however, produce effective attenuation of off-diagonal coherences at the level of the reconstructed PTM. In this picture, the modified generator $G_g + \Delta_g$ produces an effective evolution whose reconstructed tomography signatures can exhibit contractions similar to dephasing behavior. The resulting PTMs can therefore phenomenologically reproduce dephasinglike structures observed in experimental reconstructions, although the model does not guarantee complete positivity or a unique microscopic dephasing mechanism unless CPTP constraints are imposed explicitly.

Depolarizing noise. A single-qubit depolarizing channel has

$$\mathbf{r}_{\text{out}} = \lambda \mathbf{r}_{\text{in}}, \quad 0 \leq \lambda \leq 1, \quad (\text{B19})$$

and therefore

$$R_{\text{dep}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}. \quad (\text{B20})$$

This is an isotropic contraction of the Bloch sphere. Physically, depolarizing noise corresponds to a uniform loss of information about the quantum state, such that all Bloch-vector components are attenuated equally and the state is driven toward the maximally mixed state. In contrast to dephasing, no preferred axis is preserved under the contraction.

In the present framework, the ideal gate is generated from

$$U_g(\theta_g) = \exp\left[-i\frac{\theta_g}{2}G_g\right], \quad (\text{B21})$$

while the perturbed gate is constructed as

$$U_g^\Delta(\theta_g) = \exp\left[-i\frac{\theta_g}{2}(G_g + \Delta_g)\right]. \quad (\text{B22})$$

Thus, the perturbation Δ_g directly modifies the original generator G_g before exponentiation into the final implemented gate U_g^Δ . Since ideal unitary evolution preserves Bloch-vector norm, a purely Hermitian perturbation cannot reproduce isotropic contraction as a physical channel. Hermitian perturbations only modify the coherent rotation structure while maintaining unitary dynamics.

A non-Hermitian perturbation can, however, generate effective contractions in the reconstructed PTMs after propagation through the tomography reconstruction pipeline. In this picture, the modified generator $G_g + \Delta_g$ produces an effective evolution whose reconstructed tomography signatures may resemble depolarizing behavior through approximately isotropic attenuation of Bloch-vector components. The generator-perturbation model can therefore phenomenologically approximate

depolarizing structures observed in experimentally reconstructed PTMs, although it does not automatically guarantee that the resulting map is exactly a depolarizing CPTP channel unless additional physical constraints are imposed explicitly.

Amplitude damping. An amplitude-damping channel with damping probability γ has the affine form

$$\mathbf{r}_{\text{out}} = \begin{bmatrix} \sqrt{1-\gamma} & 0 & 0 \\ 0 & \sqrt{1-\gamma} & 0 \\ 0 & 0 & 1-\gamma \end{bmatrix} \mathbf{r}_{\text{in}} + \begin{bmatrix} 0 \\ 0 \\ \gamma \end{bmatrix}. \quad (\text{B23})$$

Thus, amplitude damping contains both contraction and translation. In PTM form,

$$R_{\text{amp}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-\gamma} & 0 & 0 \\ 0 & 0 & \sqrt{1-\gamma} & 0 \\ \gamma & 0 & 0 & 1-\gamma \end{bmatrix}. \quad (\text{B24})$$

This structure is important because it resembles the kind of PTM features observed in hardware: diagonal damping together with affine shifts. Physically, amplitude damping corresponds to irreversible energy relaxation from the excited state toward the ground state. In the Bloch-sphere picture, the Bloch vector is both contracted and shifted toward the north pole associated with the ground-state population.

In the present framework, the ideal gate is generated from

$$U_g(\theta_g) = \exp\left[-i\frac{\theta_g}{2}G_g\right], \quad (\text{B25})$$

while the perturbed gate is constructed as

$$U_g^\Delta(\theta_g) = \exp\left[-i\frac{\theta_g}{2}(G_g + \Delta_g)\right]. \quad (\text{B26})$$

Thus, the perturbation Δ_g directly modifies the original generator G_g before exponentiation into the final implemented gate U_g^Δ . Unlike coherent over-rotations, amplitude damping cannot be represented through a purely Hermitian perturbation because Hermitian perturbations preserve unitary evolution and therefore cannot generate irreversible relaxation or affine translation of the Bloch sphere.

A non-Hermitian perturbation can, however, produce effective contractions and affine shifts in the reconstructed PTMs after propagation through the tomography reconstruction pipeline. In this picture, the modified generator $G_g + \Delta_g$ generates an effective evolution whose reconstructed tomography signatures may resemble amplitude-damping behavior through simultaneous attenuation and translation of Bloch-vector components. The non-Hermitian perturbation model can therefore reproduce phenomenologically similar PTM structures. However, the model does not prove that the microscopic mechanism is amplitude damping. It only shows that generator-level non-Hermiticity can produce comparable PTM signatures without uniquely identifying the underlying physical relaxation process.

Leakage and apparent loss. Leakage out of the computational subspace is not a trace-preserving single-qubit channel on $\mathbb{C}^2$ if the leaked population is not included in the model space. In a reconstructed single-qubit PTM, this can appear as first-row deviations:

$$(R_{00}, R_{01}, R_{02}, R_{03}) \neq (1, 0, 0, 0). \quad (\text{B27})$$

The present model explicitly allows such deviations in the reconstructed PTM because the simulator does not enforce trace preservation during reconstruction. Physically, leakage corresponds to population transfer from the computational qubit subspace into external states that are not included in the effective two-level description. As a result, probability weight may be lost from the reconstructed

computational subspace even though the full underlying physical evolution remains trace preserving in a larger Hilbert space.

In the present framework, the ideal gate is generated from

$$U_g(\theta_g) = \exp\left[-i\frac{\theta_g}{2}G_g\right], \quad (\text{B28})$$

while the perturbed gate is constructed as

$$U_g^\Delta(\theta_g) = \exp\left[-i\frac{\theta_g}{2}(G_g + \Delta_g)\right]. \quad (\text{B29})$$

Thus, the perturbation Δ_g directly modifies the original generator G_g before exponentiation into the final implemented gate U_g^Δ . A purely Hermitian perturbation cannot reproduce effective probability loss because Hermitian perturbations preserve unitary evolution and therefore preserve norm within the modeled Hilbert space.

A non-Hermitian perturbation can, however, generate effective attenuation and apparent trace-loss signatures in the reconstructed PTMs after propagation through the tomography reconstruction pipeline. In this picture, the modified generator $G_g + \Delta_g$ produces an effective evolution whose reconstructed tomography signatures may resemble leakage behavior through reduced survival probabilities and deviations in the first PTM row. Non-Hermitian perturbations can therefore emulate leakage-like or survival-loss signatures at the PTM level. This should be understood as an effective description, not as a full leakage model in an enlarged Hilbert space with explicitly modeled leakage states.

6. Local expressivity near the ideal gate

A useful way to understand the model is to linearize it around the ideal generator. Let

$$A_g = -i\frac{\theta_g}{2}G_g, \quad B_g = -i\frac{\theta_g}{2}\Delta_g. \quad (\text{B30})$$

Then,

$$U_g^\Delta(\theta_g) = \exp(A_g + B_g). \quad (\text{B31})$$

For small Δ_g , the first-order variation is given by the Fréchet derivative of the matrix exponential [49–51],

$$U_g^\Delta(\theta_g) \approx U_g(\theta_g) + \int_0^1 e^{(1-s)A_g} B_g e^{sA_g} ds + \mathcal{O}(\|\Delta_g\|^2). \quad (\text{B32})$$

This expression shows how a small perturbation in the generator propagates into a small perturbation of the implemented gate. The integral form is important because A_g and B_g do not generally commute, so the perturbation cannot in general be pulled out of the exponential as a simple scalar correction. Equivalently, the integral term is the first-order linear response of the implemented gate to the perturbation direction B_g . It tells us how the ideal gate $U_g(\theta_g) = \exp(A_g)$ changes when the generator is displaced from G_g to $G_g + \Delta_g$.

The same idea can be applied one level higher, after the perturbed gates have been inserted into the full QPT-12 reconstruction pipeline. The mapping

$$\{\Delta_g\}_{g \in \mathcal{G}} \mapsto R_{\text{fit}}(\Delta) \quad (\text{B33})$$

is generally nonlinear because the perturbations enter through matrix exponentials and then through the tomography reconstruction. However, for sufficiently small perturbations, this mapping can also be linearized around the unperturbed point $\Delta_g = 0$. The corresponding reconstructed PTM

can therefore be expanded as

$$R_{\text{fit}}(\Delta) \approx R_{\text{ideal}} + \sum_{g \in \mathcal{G}} D_g[\Delta_g], \quad (\text{B34})$$

where D_g denotes the linear response of the reconstructed PTM to the perturbation Δ_g . Here, R_{ideal} denotes the PTM obtained from the unperturbed native gate set, while $R_{\text{fit}}(\Delta)$ denotes the PTM reconstructed after inserting the perturbations into the tomography workflow. The sum over $g \in \mathcal{G}$ appears because, to first order, the effect of perturbing several native gates is additive: Products of two perturbations, such as $\Delta_g \Delta_h$, are second-order contributions and are absorbed into the neglected higher-order terms.

More explicitly, D_g is a linear map from a generator perturbation matrix to a PTM correction,

$$D_g : \mathbb{C}^{2 \times 2} \longrightarrow \mathbb{R}^{4 \times 4}, \quad \Delta_g \longmapsto D_g[\Delta_g]. \quad (\text{B35})$$

Its entries can be interpreted as directional derivatives,

$$(D_g[\Delta_g])_{ij} = \left. \frac{d}{d\varepsilon} R_{\text{fit},ij}(\Delta_1, \dots, \Delta_g + \varepsilon \Delta_g, \dots) \right|_{\varepsilon=0}, \quad (\text{B36})$$

where all other perturbations are set to zero when the response of gate g is isolated. Thus, $D_g[\Delta_g]$ describes which PTM entries change, and by how much, when the generator of gate g is perturbed in the direction Δ_g . This is the object that connects the generator-level perturbation model to the observable PTM-level deviations.

This linearized form clarifies the local expressivity of the model. To first order, the reconstructed PTM deviations that can be generated by the perturbation model are given by linear combinations of the elementary response directions

$$D_g[E_{ab}], \quad D_g[iE_{ab}], \quad (\text{B37})$$

for all $g \in \mathcal{G}$ and all elementary 2×2 matrix units E_{ab} . Equivalently, the locally reachable PTM deviations lie in

$$\text{span}\{D_g[E_{ab}], D_g[iE_{ab}] : g \in \mathcal{G}\}, \quad (\text{B38})$$

where span denotes the set of all real linear combinations of these response matrices. In other words, the span represents all PTM correction directions that can be produced locally through sufficiently small generator perturbations within the linearized model.

Here, the matrices E_{ab} form the elementary basis of 2×2 matrix space. Each E_{ab} probes one real perturbation direction, while iE_{ab} probes the corresponding imaginary perturbation direction. Since the perturbation matrices satisfy

$$\Delta_g \in \mathbb{C}^{2 \times 2}, \quad (\text{B39})$$

their real and imaginary components contribute independently at first order. The set

$$\{E_{ab}, iE_{ab}\} \quad (\text{B40})$$

therefore provides a convenient basis for resolving the local response of the reconstructed PTM to arbitrary sufficiently small perturbations of the generators.

More concretely, each object

$$D_g[E_{ab}] \quad (\text{B41})$$

is itself a 4×4 PTM correction matrix describing how the reconstructed PTM changes when the generator of gate g is perturbed infinitesimally in the matrix direction E_{ab} . The collection of all such response directions therefore defines the locally reachable tangent space of PTM deviations around the unperturbed reconstruction R_{ideal} . In this picture, the linearized model does not attempt

to reproduce arbitrary PTMs globally. Instead, it asks which nearby PTM structures can be reached through sufficiently small perturbations of the native gate generators.

Since each $\Delta_g \in \mathbb{C}^{2 \times 2}$ has eight real degrees of freedom and $|\mathcal{G}| = 3$, the fitted native gate set contains

$$8|\mathcal{G}| = 24 \quad (\text{B42})$$

real perturbation parameters. The target PTM has 16 real entries if no trace-preservation constraint is imposed. This parameter count suggests that the model can be locally flexible, but parameter count alone does not guarantee that all PTM directions are reachable. Different perturbation directions can produce correlated or redundant PTM responses, and some perturbation directions may have only weak influence on the reconstructed tomography output. Reachability therefore depends on the rank and structure of the linear response map

$$\mathcal{D} : \{\Delta_g\}_{g \in \mathcal{G}} \mapsto R_{\text{fit}}(\Delta) - R_{\text{ideal}}. \quad (\text{B43})$$

Here, $\mathcal{D}$ collects the combined first-order response of the entire tomography reconstruction pipeline. It maps small generator perturbations to the corresponding first-order PTM deviations after state preparation, gate application, basis rotations, sampling, and reconstruction. The image

$$\text{Range}(\mathcal{D}) \quad (\text{B44})$$

therefore represents the set of PTM deviations that are locally reachable within the perturbative model.

A local sufficiency condition is therefore

$$R_{\text{target}} - R_{\text{ideal}} \in \text{Range}(\mathcal{D}) \quad (\text{B45})$$

to first order. In other words, the experimentally observed PTM deviation must lie approximately inside the locally reachable response space generated by the perturbation model. If this condition holds approximately, then small generator perturbations can reproduce the observed PTM deviation locally. The fitted perturbations can then be interpreted as an effective local phenomenological explanation for the reconstructed tomography structure.

If the condition does not hold, then no sufficiently small perturbation of this form can reproduce the target PTM within the linearized model. In that case, the mismatch indicates that the observed reconstruction contains structures that are not captured by the chosen perturbation class, for example, because additional noise mechanisms, larger perturbations, higher-order effects, leakage processes, nonlocal correlations, or different channel structures contribute significantly. Thus, the condition should be read as a local, first-order expressivity test rather than as a proof that the model is globally complete.

7. Sufficiency conditions

The generator-perturbation model is sufficient as a local phenomenological model under the following conditions.

First, the experimentally reconstructed deviation should be small:

$$\|R_{\text{target}} - R_{\text{ideal}}\| \ll 1. \quad (\text{B46})$$

This condition justifies a perturbative treatment.

Second, the dominant deviations should lie in the response directions generated by the available native gates:

$$R_{\text{target}} - R_{\text{ideal}} \approx \sum_{g \in \mathcal{G}} D_g[\Delta_g]. \quad (\text{B47})$$

This means that the observed PTM structure can be explained by gate-local perturbations rather than requiring explicitly time-dependent, nonlocal, or multiqubit effects.

Third, the desired interpretation should be phenomenological:

$$R_{\text{fit}} \approx R_{\text{target}} \quad (\text{B48})$$

within the combined uncertainty. The model is sufficient for reproducing the reconstructed PTM structure if the elementwise scores satisfy

$$z_{ij} = \frac{|R_{\text{fit},ij} - R_{\text{target},ij}|}{\sigma_{\text{comb},ij}} \lesssim 1 \quad (\text{B49})$$

for the relevant entries.

Fourth, if the goal is physical channel identification, additional constraints are required. In particular, the reconstructed process should satisfy CPTP constraints, and SPAM contributions should be separated from gate noise. The present model alone does not provide this separation.

8. Expressivity limits

The generator-perturbation model is not a universal representation of single-qubit noise. The main limitations are as follows.

First, it does not enforce complete positivity:

$$R_{\text{fit}} \not\Rightarrow \mathcal{E}_{\text{fit}} \text{ is completely positive.} \quad (\text{B50})$$

Therefore, some fitted PTMs may reproduce experimental structures while not corresponding to a valid CPTP channel by themselves.

Second, it does not enforce trace preservation:

$$(R_{00}, R_{01}, R_{02}, R_{03}) \neq (1, 0, 0, 0) \quad (\text{B51})$$

is allowed. This is useful for representing effective loss or leakage signatures, but it also means that the model is not restricted to physical single-qubit CPTP dynamics.

Third, it does not independently separate SPAM from gate noise. The reconstructed PTM is an effective object obtained under the tomography assumptions. Therefore, R_{target} may contain gate noise, SPAM effects, calibration drift, finite-shot fluctuations, and reconstruction bias.

Fourth, it is gate-local. The perturbations are attached to the native gates: $\{\Delta_g\}_{g \in \mathcal{G}}$. This does not directly model correlated multiqubit errors, crosstalk, time-dependent drift, or non-Markovian memory effects.

Fifth, it is not a full Lindblad model. A Lindblad generator has the structure

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_{\ell} \left(L_{\ell} \rho L_{\ell}^{\dagger} - \frac{1}{2} \{L_{\ell}^{\dagger} L_{\ell}, \rho\} \right), \quad (\text{B52})$$

which guarantees CPTP dynamics under suitable assumptions. The perturbation $G_g + \Delta_g$ is instead inserted into a matrix exponential at the gate level. It can emulate some open-system signatures in the reconstructed PTM, but it is not identical to a Lindblad generator.

9. Comparison with full PTM characterization

The PTM representation is more general as a description of single-qubit channels. A reconstructed PTM directly contains all experimentally accessible information under the chosen tomography protocol. Therefore, the generator-perturbation model should not be interpreted as replacing the PTM. Instead, the relationship is R_{target} is the reconstructed experimental object, while

$$\{\Delta_g\}_{g \in \mathcal{G}} \text{ is a structured parametrization used to reproduce it.} \quad (\text{B53})$$

The advantage of the perturbation model is not representational completeness. Its advantage is that it imposes a structured hypothesis: Observed PTM deviations can be approximated by native-generator perturbations. This hypothesis is testable. If the fitted model reproduces the relevant PTM

entries within uncertainty, the hypothesis is supported as a phenomenological explanation. If it fails, the missing entries identify which structures require additional modeling.

10. Practical interpretation

The results of the model should therefore be read in the following way.

If a fitted Hermitian perturbation reproduces a PTM feature, then the feature is consistent with coherent generator-level miscalibration.

If a fitted non-Hermitian perturbation is needed, then the feature is consistent with an effective open-system or losslike contribution in the reconstructed PTM.

If neither perturbation family reproduces an entry within uncertainty, then the model is missing a relevant mechanism. Possible missing mechanisms include explicit dissipative channels, SPAM effects, leakage into a larger Hilbert space, drift, crosstalk, or correlated multiqubit noise.

This interpretation is intentionally conservative. The fitted matrices Δ_g should not be read as unique microscopic Hamiltonians or Lindblad operators. They are effective generator-level parameters that reproduce reconstructed PTM structures under the adopted tomography workflow.

11. Summary of the expressivity statement

The generator-perturbation framework is sufficient when the goal is to test whether experimentally reconstructed single-qubit PTM deviations can be approximated by structured, gate-local perturbations of native generators. It is especially suitable for coherent miscalibrations, cross-axis mixing, affine shifts, and effective non-trace-preserving signatures that appear in the reconstructed PTM.

It is not sufficient as a universal single-qubit noise representation. It does not replace PTMs, Kraus maps, Lindblad models, GST, or cycle benchmarking. It does not by itself prove complete positivity, trace preservation, SPAM isolation, or uniqueness of the microscopic mechanism.

Thus, the correct expressivity claim is as follows: The model is a constrained phenomenological reconstruction model, not a complete channel representation.

Within this scope, its value is that it connects experimentally reconstructed PTM structures to explicit perturbations of the native gate generators and provides a transparent way to test whether such perturbations are sufficient to reproduce the observed data.

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