

Reflection, transmission, and trapping of an ultracold atomic soliton at an attractive contact impurity

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Reflection and transmission (tunneling) reveal the wave nature of ultracold matter when scattered from an impurity and are of great importance in atomtronics technology. Using the time-dependent Gross-Pitaevskii equation (GPE), we investigate the trapping, reflection, and transmission of a bright soliton in an ultracold-atom gas propagating through a one-dimensional waveguide with a contact defect. This allows us to analyze the dynamics of the scattering as a function of soliton velocity v , the strength of the contact impurity V_0 , and the intensity of the attractive nonlinear coupling $g < 0$. We evolve the GPE numerically with the contact defect modeled as a δ sequence for the Dirac delta function. We find a universal behavior in terms of v/V_0 and $|g|/V_0$, thus reducing the dimension of the parameter space. Our analysis identifies four distinct regions of nonlinear coupling $|g|/V_0$ that govern scattering dynamics, delimited by $|g|/V_0 \approx 2, 4$, and 6 . Particularly, we observe distinct parameter dependent onsets between the soliton character and highly radiative-dispersive wave packets for the scattering products. To quantify the nature of the scattering products, we use the differential and Fisher information-theoretical measures to determine the degree of localization of the scattering products. The implications of these findings are discussed in the context of both fundamental quantum mechanics and potential experimental setups.

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I. INTRODUCTION

Since the first Bose-Einstein condensates (BECs) were produced in 1995 [1–3], ultracold-atom physics has expanded rapidly and spawned several new research directions. This growth has gone hand in hand with the development of experimental platforms capable of producing, trapping, and manipulating quantum gases with high precision. Among these platforms is the atom chip, which

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has become a cornerstone technology for modern cold-atom experiments and for the emerging field of atomtronics [4,5]. Atomtronics aims to realize atom-based analogs of electronic materials, devices, and circuits by controlling matter-wave currents in engineered potentials [6]. Atom chips support this program by using microfabricated, current-carrying wires to generate magnetic fields that cool, trap, and guide ultracold atoms in compact and potentially portable geometries. In these guided-wave settings, circuit features such as bends, junctions, or localized perturbations can reflect, trap, or transmit portions of an atomic wave packet, thereby shaping the dynamics of subsequent propagation and interference. This level of control is particularly relevant for matter-wave interferometry and precision sensing, including atomic gravimeters that exploit atom interferometry and quantum-coherent manipulation with light, a prime example of atomtronics in practice [7].

Beyond guided transport, atomtronic circuits are being developed as versatile components for quantum technologies, extending well past simple matter-wave currents [8]. Applications include quantum sensing architectures [9] and atom-based circuit elements such as switches, diodes, and universal logic gates, e.g., via Rydberg-atom networks aimed at quantum simulation and information processing [10]. Rapid progress in atom-chip capabilities has also enabled studies of stable quantum mixtures, including impurity dynamics, miscibility, and the emergence of exotic topological structures, as well as investigations of magnetism-superfluidity interplay, artificial molecules, and novel few-body states [11]. Atom-chip and cold-atom platforms have further been explored for quantum inertial sensing in space applications [12] and more recently for probing nonlinear wave dynamics in guided ultracold matter waves [13].

In this work, we revisit soliton scattering in the one-dimensional (1D) Gross-Pitaevskii equation (GPE) in the presence of an attractive, pointlike defect, focusing on the experimentally relevant scenario of an incoming bright soliton. Motivated by atomtronics, we map out transmission, trapping, and reflection across a broad parameter space, providing a systematic characterization of the device-like operating regimes of a localized impurity. While related scattering studies exist, our analysis is, to our knowledge, the most comprehensive survey of these outcomes to date. We also go beyond transmission and reflection coefficients by introducing an information-theoretical (IT) measure of the postscattering state, since population splitting alone does not determine whether the resulting fragments are sufficiently localized for use in applications, e.g., atomtronics.

When describing ultracold atomic gases, one must account for the interactions among the N atoms that compose the system. Within a mean-field approximation, these interactions are incorporated into the Schrödinger equation through the GPE [14,15]. The nonlinear interaction is represented by a term proportional to the s -wave scattering length, a_s , of the bosons. In a tightly confined waveguide, the system may be treated as effectively one dimensional provided that the longitudinal extent of the ultracold BEC is much larger than its transverse size. In units of the transverse harmonic oscillator, the resulting 1D GPE is [16]

$$i \frac{\partial}{\partial t} \Psi(x, t) = \left[-\frac{1}{2} \frac{\partial^2}{\partial x^2} + V(x) + g|\Psi|^2 \right] \Psi(x, t). \quad (1)$$

Here, the nonlinear coupling constant is $g = 4\pi a_s(N-1)/a_\perp$, where $a_\perp = \sqrt{\hbar/m\omega_\perp}$ is the transverse oscillator length, m is the atomic mass, and $\omega_\perp$ is the transverse confinement frequency.

Analytical soliton solutions of Eq. (1) in the absence of an external potential have long been known [17,18]. For attractively interacting condensates ($g < 0$), which are the focus of the present work, bright solitons have been realized experimentally in several ultracold alkali gases [19–23].

The first experimental observation of a bright matter-wave soliton was carried out in 2002, when Khaykovich *et al.* [20] observed a stable, nondispersive wave packet in a quasi-one-dimensional condensate of ^7Li atoms and demonstrated its propagation over macroscopic distances without measurable spreading. Shortly thereafter, Strecker *et al.* [19] reported the formation of soliton trains in ^7Li condensates following a rapid interaction quench from repulsive to attractive interactions, revealing the crucial role of modulational instability in the spontaneous generation of multiple bright solitons. Using condensates of ^{85}Rb , Cornish *et al.* [21] showed that controlled collapse of an

attractive condensate could produce long-lived bright solitary matter waves, thereby extending the study of attractive condensates beyond the original lithium experiments. Subsequently, Marchant *et al.* [24] showed precise control over the formation, propagation, and reflection of bright solitary waves from optical barriers, opening new possibilities for studying nonlinear transport and atom-optical devices based on self-bound matter waves. The first realization of bright solitons in ^{39}K by Lepoutre *et al.* [25] enabled systematic investigations of stability close to the collapse threshold and facilitated quantitative comparisons between experiments and mean-field theory. More recently, experiments with ^{133}Cs by Di Carli *et al.* [26] have further broadened the accessible parameter space of attractive quantum gases. The abovementioned experiments have enabled studies of both intersoliton collisions [22] and scattering from external potentials.

Related theoretical work has examined the splitting of nonlinear Schrödinger breathers by linear and nonlinear localized potentials [27], the scattering of fast bright solitons from narrow Gaussian barriers [28], and the splitting and recombination of bright solitons in low-energy systems governed by the GPE [29].

In this context, the wave nature of quantum mechanics allows the reflection of a particle in a classically allowed region even when there is no classical turning point, which was experimentally demonstrated with ultracold atoms by Pasquini *et al.* [30]. In an attractive condensate ($g < 0$), atoms are held together; interatomic forces make localized states, such as solitons, less likely to split or tunnel, a phenomenon that plays a crucial role in probing nonlinear wave dynamics. Among the first to study this theoretically were Kivshar *et al.*, who analyzed the scattering of solitons by point-defect impurities in 1D systems [31]. Their perturbative treatment details how impurities can cause radiative energy loss and highlights that, unlike linear waves, solitons can propagate through disordered media without experiencing Anderson localization. This was followed by Lee and Brand [32], demonstrating that nonlinear interactions in bright solitons allow for high-efficiency quantum reflection from purely attractive potentials. Their study identifies a sharp velocity-dependent transition, with low-velocity solitons reflecting with minimal loss and offering a potential mechanism for velocity filtering in BECs.

The mean-field Hamiltonian in Eq. (1) operates under the assumption that all N atoms are in the same quantum state characterized by the amplitude $\Psi(x, t)$. After a scattering event, the solution to Eq. (1) can be written as a superposition $\Psi = \sum_n c_n \Psi_n$ provided the overlaps between scattering products Ψ_n are negligible. For a single particle, the corresponding multimodal probability density $\rho(x, t) = |\Psi|^2$ has a clear interpretation. For a condensate, however, it is necessary to distinguish localization in real space and Fock space, as pointed out by Streltsov *et al.* [33]. They explored the many-body dynamics of attractive bosonic clouds, showing that scattering off a narrow potential barrier creates a true quantum superposition of spatially separated wave packets. Weiss and Castin [34] further showed that it should be possible to have small enough decoherence rate for a bright soliton colliding with a potential barrier that results in a coherent mesoscopic quantum superposition state, rather than a statistical mixture, i.e., $\sum_n p_n |\Psi_n\rangle\langle\Psi_n|$.

In view of the different possibilities for the nature of the quantum state resulting from scattering of a bright soliton, and considering that the nonlinear character of the GPE does not allow solutions that are quantum superpositions or mixtures, Gertjerken *et al.* [35] examined the discrepancy between N -particle quantum dynamics and mean-field theory (GPE) when bright matter-wave solitons scatter off a potential barrier at low kinetic energies. They conjectured that the jumps in the transmission behavior of the reflection coefficient using the GPE indicates the formation of quantum superpositions at the N -particle quantum level. Complementary to this, Boissé *et al.* [36] performed experiments showing that matter-wave bright solitons in 1D speckle disorder exhibit collective, all-or-nothing transmission or reflection, rather than individual particle scattering. Their observed bimodal, binary fluctuations result from extreme sensitivity to initial velocity and align very well with Gross-Pitaevskii simulations, highlighting the usefulness of the GPE as an indicator for predicting experimental outcomes.

Much of the previous work based on solving the GPE (1) concentrated on soliton scattering from repulsive external potentials [37,38], including optical-lattice settings. For repulsive barriers,

one may define reflection and transmission coefficients directly, as in recent experiments on BEC tunneling through thin repulsive barriers [39]. In this context, Lorenzi and Salasnich investigated the transmission and collapse dynamics of atomic solitons scattered from a narrow barrier [40]. Watabe and Kato analyzed the transmission and reflection of BEC excitations across barriers separating regions of different condensate density in the low-energy limit, identifying behavior analogous to Snell's law in optics [41]. Lingua *et al.* [42] studied collisions between a bosonic impurity consisting of a few ^{41}K atoms and a quasi-one-dimensional ^{87}Rb condensate under weak harmonic confinement. Holmer *et al.* [43] examined fast soliton scattering in the presence of a repulsive delta-function potential within the GPE framework.

In this work, we consider scattering from attractive potentials, which can support bound states [32,44–58]. Soliton scattering from an attractive defect was first realized in a BEC of ^{85}Rb atoms by Marchant *et al.* [59], with comparison to the theoretical predictions of Ernst and Brand [49] and Goodman *et al.* [47]. Related experiments have also been carried out in nonlinear optical media [60]. As emphasized by Marchant *et al.* [59], the detailed shape and depth of the potential can strongly influence the scattering outcome for highly localized impurity potentials. Wang *et al.* [52] demonstrate that a bright matter-wave soliton acts as a classical particle at low velocities but displays wavelike characteristics at higher speeds during potential barrier tunneling, while Hansen *et al.* [51] investigated 1D matter-wave soliton dynamics, examining reflection and transmission behaviors across localized scattering barriers or wells. The scattering of one-dimensional quantum droplets from a Pöschl-Teller reflectionless potential well has also been investigated, revealing a sharp transition between full reflection and full transmission at a critical incident velocity for small and large droplets (flatop) [61]. Additional studies of transmission have addressed the role of linear and nonlinear inhomogeneities arising from spatial variations in trapping potentials and scattering lengths in BECs [62]. The dynamics of the GPE with a delta-function impurity inside a square well were analyzed by Jensen *et al.* [63].

Despite this substantial body of work, studies of scattering from attractive impurities have explored only a limited region of parameter space. The main objective of the present work is therefore to determine the reflection, transmission, and trapping of a bright soliton in a one-dimensional waveguide containing an attractive contact potential $V(x) = -V_0\delta(x)$, with $V_0 \geq 0$. To this end, we survey incident soliton center-of-mass velocities $v > 0$, ranging from low values $v/V_0 \geq 0.01$ to relatively large values $v/V_0 \approx 10$, over a broad interval of nonlinear coupling strengths $0 \leq |g|/V_0 \leq 10$. Particular emphasis is placed on the regime of resonant trapping by the attractive contact impurity, which may serve as an idealized model of a bend or localized defect in an ultracold atomic waveguide. Our aim is to identify the conditions under which efficient transmission sets in and to characterize the accompanying emission of fast radiative wave packets. While it can be argued that a many-body approach in the spirit of Refs. [33–35] is necessary, we find it appropriate to first systematically chart out the mean-field parameter space in detail and leave the full many-body problem out for a future work.

II. THEORY

The normalized bright soliton solution to Eq. (1), i.e., the case corresponding to $V_0 = 0$, is given by

$$\Psi(x, t) = \frac{\sqrt{|g|}}{2} \operatorname{sech}\left(\frac{|g|}{2}(x - vt)\right) e^{i\left(vx - \frac{v^2}{2}t + \frac{g^2}{8}t\right)}. \quad (2)$$

This soliton will collide from the left ($v > 0$) with the contact impurity and a fraction of it will be trapped, another fraction will be reflected, and the remaining will be transmitted.

The contact impurity is described by a Dirac delta function

$$V(x) = -V_0\delta(x). \quad (3)$$

The analytical solution for the bound state in a contact Dirac delta potential was reported by Seaman *et al.* [17] in terms of Jacobi elliptic functions. However, a simpler solution can be obtained by assuming that for the region where there is no potential, the solution is a soliton at rest, and by matching the discontinuity at the contact point on the wavefunction (see also Ref. [47]), one obtains the stationary wavefunction solution

$$\phi(x) = \sqrt{\frac{2|\mu|}{|g|}} \begin{cases} \text{sech}[\sqrt{2|\mu|}(x - x_0)], & x < 0, \\ \text{sech}[\sqrt{2|\mu|}(x + x_0)], & x > 0, \end{cases} \quad (4)$$

with a bound-state chemical potential

$$\mu = -\frac{1}{2} \left(V_0 + \frac{|g|}{2} \right)^2. \quad (5)$$

Here, the shifting position x_0 is given by

$$x_0 = \frac{1}{\sqrt{2|\mu|}} \tanh^{-1} \left(\frac{V_0}{\sqrt{2|\mu|}} \right). \quad (6)$$

Equation (4) is normalized to unity, i.e., $\int |\phi|^2 dx = 1$. In what follows, we will consider partial trapping of a soliton initially normalized to unity. If the trapped fraction is $N_{\text{TR}} = \int |\phi_{\text{TR}}|^2 dx \leq 1$, for a given interaction strength g , the wavefunction ϕ_{TR} of the bound state has a chemical potential

$$\mu_{\text{TR}} = -\frac{1}{2} \left(V_0 + \frac{|g|N_{\text{TR}}}{2} \right)^2. \quad (7)$$

The wavefunction ϕ_{TR} is found by replacing μ in Eqs. (4) and (6) with μ_{TR} .

As the GPE is nonlinear, there is no spectrum in the conventional sense for excitations. However, we may linearize around the trapped state in Eq. (7) with some chemical potential. Thus, for $\epsilon \ll 1$, we make the ansatz

$$\Psi(x, t) = e^{-i\mu t} [\phi(x) + \epsilon \delta\Psi(x, t)]. \quad (8)$$

Inserting this ansatz into the GPE [Eq. (1)] and keeping only terms to first order in ϵ , one obtains

$$i \frac{\partial}{\partial t} \delta\Psi = \left[-\frac{1}{2} \frac{\partial^2}{\partial x^2} + V(x) - \mu + 2g|\phi|^2 \right] \delta\Psi + g\phi^2 \delta\Psi^*. \quad (9)$$

Writing $\delta\Psi = u(x)e^{-i\omega t} + v^*(x)e^{i\omega t}$, one arrives at the Bogoliubov-de Gennes (BdG)-type equation

$$\begin{pmatrix} H_0 & g|\phi|^2 \\ -g|\phi|^2 & -H_0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \omega \begin{pmatrix} u \\ v \end{pmatrix}, \quad (10)$$

where $H_0 = -(1/2)\partial_x^2 + V - \mu + 2g|\phi|^2$. For an attractive potential $V(x) = -V_0\delta(x)$, the potential can be included through the boundary condition at $x = 0^\pm$, as

$$u'(0^+) - u'(0^-) = -2V_0 u(0), \quad v'(0^+) - v'(0^-) = -2V_0 v(0). \quad (11)$$

Separately solving for the odd and even solutions further allows to solve the problem on the half plane $x \geq 0$, with either a homogeneous Dirichlet condition (odd modes) or a mixed boundary condition (even modes).

To obtain the spectrum, we first notice that far away from the potential, $\phi \rightarrow 0$ decouples the system, resulting in plane-plane wave solutions $e^{ikx - i\omega t}$, with dispersion

$$\omega = \pm(k^2/2 - \mu) = \pm \frac{1}{2} \left[k^2 + V_0 \left(1 + \frac{|g|N_{\text{TR}}}{2V_0} \right)^2 \right]. \quad (12)$$

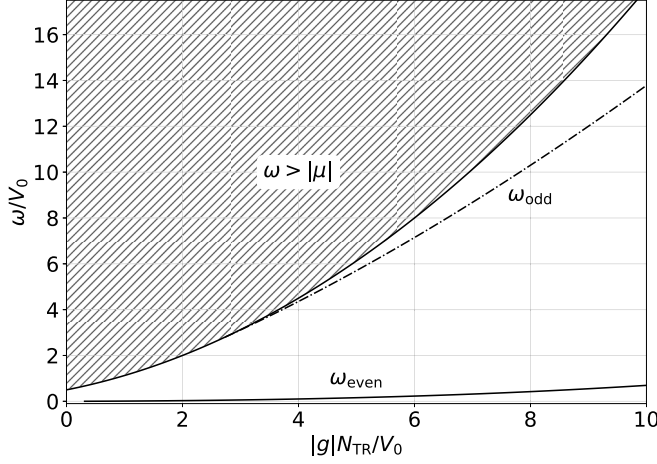


FIG. 1. Excitation spectrum of the bound state in the potential $-V_0\delta(x)$ obtained from linearization and solving Eq. (10) in the region $\omega > 0$. For $\omega > |\mu|$, there is a continuum of states corresponding to radiating solutions. These cause the trapped fraction, N_{TR} , to decrease in time. There are also an odd excitation and an even excitation. These are localized and appear as “sloshing” and breathing modes, respectively.

Hence, the presence of the potential leads to a gap in the spectrum. To obtain the full spectrum, numerical diagonalization is necessary. We do this through solving the eigenvalue problem resulting from applying finite differences (FD) to Eq. (10) with boundary conditions (11).

In Fig. 1, we show the resulting excitation spectrum of the bound states in the potential of the contact impurity for $\omega > 0$. For $|\omega| > |\mu|$, we find a continuum of solutions that correspond to radiating dispersive solutions with dispersion given by Eq. (12). In the same figure, we show also the odd and even excitations, which support the breathing and sloshing modes of the dynamics. These states remain trapped in the potential and do not, to first order in ϵ , cause decay but instead cause a persistent modulation of the trapped density. The existence of the continuum of radiating states together with the nonlinearity will, however, cause up-conversion of bound excited modes, which may lead to a slow higher-order decay mechanism.

The above solution should be contrasted to the bound solutions for the linear case, $g = 0$, which have already been reported in quantum mechanic textbooks, e.g., Griffiths and Schroeter [64]. For the case of a bound state, $E < 0$, the wavefunction is

$$\Phi(x) = \sqrt{V_0} e^{-V_0|x|}, \quad (13)$$

with a bound-state energy $E = -V_0^2/2$. Notice that Eq. (5) reduces to this case for $g = 0$. The solution for the scattering case, $E > 0$, provides the reflection and transmission coefficients [64]

$$R = \frac{1}{1 + 2E/V_0^2} = \frac{1}{1 + (v/V_0)^2}, \quad (14)$$

with $E = v^2/2$ and $T = 1 - R$. Here, there are no excited states and no trapping.

For the nonlinear scattering problem in Eq. (1) with a δ -potential impurity of the form $-V_0\delta(x)$ and an incoming soliton described by Eq. (2), we may reduce the number of free parameters by rescaling the equations. By absorbing the strength of the impurity, V_0 , into the coupling constant as $\tilde{g} = g/V_0$ and the velocity as $\tilde{v} = v/V_0$, the parameter space is reduced. Thus, it suffices to solve the GPE

$$i \frac{\partial \Psi}{\partial \tau} = \left[-\frac{1}{2} \frac{\partial^2}{\partial \xi^2} + \tilde{g} |\Psi|^2 - \delta(\xi) \right] \Psi, \quad (15)$$

with $\xi = V_0 x$ for an initial soliton of the form

$$\Psi(\xi, \tau) = \frac{\sqrt{|\tilde{g}|}}{2} \text{sech}\left(\frac{|\tilde{g}|}{2}[\xi - \tilde{v}\tau]\right) e^{i\tilde{v}\xi}. \quad (16)$$

From Eq. (15), it is clear that the scattering problem is determined only by the parameters $\tilde{g} = g/V_0$ and $\tilde{v} = v/V_0$ for any value of $V_0 \neq 0$. This was recognized by Cao and Malomed [45], as well as by Goodman *et al.* [47] who numerically solved Eq. (15). While Cao and Malomed [45] focused on determining when a soliton would get trapped by the potential, Goodman *et al.* solved for a few select parameter values to demonstrate different types of behavior that may emerge. Importantly, they pointed out that the reflected/transmitted fractions may either remain as solitons or come out as radiation. For low incident velocities, they also put forward that trapping is a resonant phenomena, which was corroborated in simulations by Ernst and Brand [49].

The main argument behind the resonant trapping is based on noticing that the frequency of the bound states containing a fraction $N_{\text{TR}} \in [0, 1]$ is $\Omega_{\text{TR}} = (V_0 + |g|N_{\text{TR}}/2)^2/2$ [see Eq. (5)], while the frequency of the soliton is $\Omega_S = g^2/8 - v^2/2$. Thus, very slow-moving solitons ($v \rightarrow 0$) approaching an initially empty trap ($N_{\text{TR}} \rightarrow 0$) can only be resonantly trapped provided that $|g|/V_0 \gtrsim 2$. In Sec. III B 1, we reproduce this result, but also show that for stronger nonlinearity there is a sequence of such thresholds and onsets where the scattering shows nonanalytic behavior.

To characterize the contact potential within a numerical grid, we use the analytic δ sequence for the Dirac delta function given by the following three representations of it:

$$\delta(\xi) = \frac{\alpha_g}{\sqrt{\pi}} e^{-\alpha_g^2 \xi^2}, \quad (17)$$

as well as

$$\delta(\xi) = \frac{\alpha_s}{\pi} \text{sech}(\alpha_s \xi) \quad (18)$$

and

$$\delta(\xi) = \frac{\alpha_l}{\pi} \frac{1}{1 + \alpha_l^2 \xi^2}, \quad (19)$$

where α_i is the sequence parameter to determine as a function of the grid spacing, $\Delta\xi$. In the limit when $\alpha_i \rightarrow \infty$, these expressions become the Dirac delta function.

In Appendix A, we provide the details of the implementation for the FD approach to carry out the dynamics. In Appendix B, we show the Gauss-Seidel (GS) approach to determine the optimal values of α_i for Eqs. (17)–(19), as well as the details of the implementation of the pseudospectral method to ensure convergence of our results. In Appendix C, we provide the theoretical formulation for the information-theoretical measures motivated by the dynamical results to understand the four threshold regions where the scattering shows nonanalytical behavior. Finally, in Appendix D, we provide the physical parameter values corresponding to the set of values reported in this work for $v/V_0 \in [0.01, 10]$ and $|g|/V_0 \in [0.1, 10]$ and its experimental feasibility.

III. RESULTS AND DISCUSSION

A. Stationary trapped solutions and δ -function representations

In order to describe properly the dynamics of a soliton colliding with a attractive contact impurity, we proceed to optimize the δ sequence. The parameter α_i for a given numerical uniform mesh is obtained by varying it, such that it reproduces the analytical results for the bound state of a contact delta function as a function of $\Delta\xi$. For the linear case $\tilde{g} = 0$, the linear Schrödinger equation analytical results are $\epsilon = -1/2$ for the energy with ground-state wavefunction $\Psi(\xi) = e^{-|\xi|}$. For the GPE case, $\tilde{g} \neq 0$, we use Eqs. (4) and (6), with the energy given by Eq. (5), within the Gauss-Seidel approach described in Appendix B.

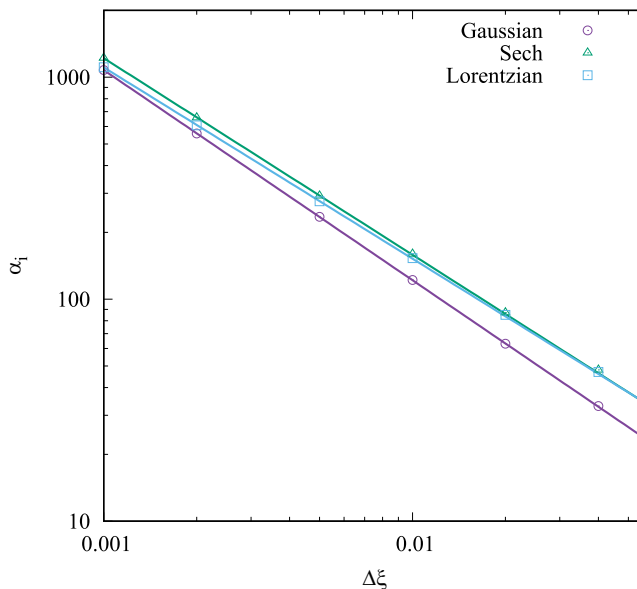


FIG. 2. The α_i parameters computed for the δ -sequence potential of Eqs. (17)–(19), as a function of the grid mesh step size $\Delta\xi$, such that the numerical solution reproduces the analytical results for the ground-state wavefunction of the GPE for the three δ sequences. See text for discussion.

In Fig. 2, we show the behavior of the parameter α_i when varying $\Delta\xi$ in a numerical mesh by requiring the value of $\epsilon = -1/2$ from Eq. (B2) or from Eq. (5). We find that in all the cases the behavior of $\alpha_i = a/\Delta\xi^b$, that is, as $\Delta\xi$ diminishes, α_i increases since the δ -sequence function becomes narrower. The values of α_i for several $\Delta\xi$ are reported in Table I for reference purposes. We find that the same parameter values for α_i hold for any value of $\tilde{g} \lesssim 10$ for the trapped bound states.

In order to evaluate the performance of the parameters α_i fitted in the δ sequence, we compared to the analytical soliton solution when $\tilde{g} \neq 0$, in Fig. 3 for several values of $\tilde{g}$. We also show the GS solution to the GPE when using the δ sequence of Eq. (B2) for several values of $\tilde{g}$. The solid lines are the analytical results of Eq. (4), while the symbols are the results when using Eq. (B2) for $\tilde{g} \neq 0$. We notice excellent agreement for all the values of $\tilde{g}$, which gives us confidence that our numerical approach to describe Dirac's delta function in a mesh is correct for the trapped bound solutions.

TABLE I. Optimized values of the parameter α_i for the analytical δ -sequence functions that reproduce the analytical results of the GPE for the bound ground state using the Gauss-Seidel method.

$\Delta\xi$	α_g	α_s	α_l
0.001	1077	1220	1109
0.002	558	657	607
0.005	235	292	276
0.01	122	160	153
0.02	63	87	85
0.04	33	48	47
0.057	24	37	38

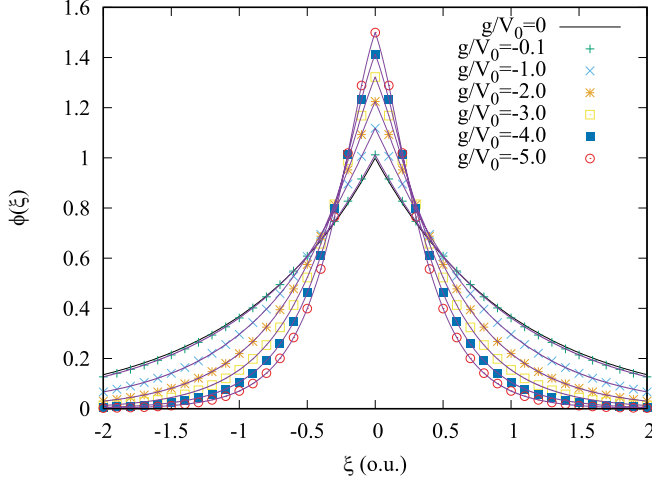


FIG. 3. The analytical (lines) [Eq. (4)] and numerical (symbols) results for the wavefunction of the δ -sequence function potential when represented as a Gaussian function [Eq. (17)] for several values of the nonlinearity $|g|/V_0$ for the case when $\alpha_g = 1077$, corresponding to $\Delta\xi = 0.001$ show very good agreement between them. Notice the excellent description of the cusp at $\xi = 0$.

B. Reflection, transmission, and trapping

For the scattering simulations, we have used two independent methods of integration to solve the dimensionless GPE [Eq. (15)]: the finite-difference method (Appendix A) and the pseudospectral method (Appendix B). For both methods, the impurity was modeled using the Gaussian representation of Eq. (17). We opted to present the Gaussian results as the other two delta sequences show similar outcomes. The spacing value was chosen to reproduce the exact bound states in the interval $0 \leq |\tilde{g}| \leq 10$, while for the scaled velocity values v/V_0 , we chose the interval $0.01 < v/V_0 < 10$. In Appendix E, we provide the corresponding physical range of these values when using ^{85}Rb atoms. A masking function was added to the boundaries to absorb radiation, as implemented in a previous work to study electron ionization [65].

Solitons with a given value of v/V_0 and $|g|/V_0$ were initiated far away from the impurity and then propagated toward it. During the simulation, we keep track of the fractions reflected, N_R , transmitted, N_T , and trapped, N_{TR} , as given by

$$N_R = \int_{\xi_{\min}}^{-\Delta} |\Psi|^2 d\xi, \quad N_{TR} = \int_{-\Delta}^{\Delta} |\Psi|^2 d\xi, \quad N_T = \int_{\Delta}^{\xi_{\max}} |\Psi|^2 d\xi. \quad (20)$$

Here, $\xi_{\min} = -\xi_{\max}$ denote the limits of the simulation cell and lie in the interval $120 \leq \xi_{\max} \leq 480$ depending on the soliton velocity. The value of Δ was chosen such that the trapped component was properly described, in the sense that the trapped solution has fallen off sufficiently. To evaluate N_R , N_T , and N_{TR} , we monitor the total normalization, which is unity until the point where either a reflected or transmitted wave reaches the boundary, causing the normalization to decrease. Transmission, reflection, and trapping are then determined.

In Fig. 4, we show the reflection, transmission, and trapping for all the regions of parameters for the scaled velocity v/V_0 and nonlinear strength $|g|/V_0$. In the same figure, we observe that the behavior of the trapping shows four distinctive regions of behavior, clearly highlighted by the red dashed line, which corresponds to $|g|/V_0 = 2, 4$, and 6 . In what follows, we discuss each region in detail.

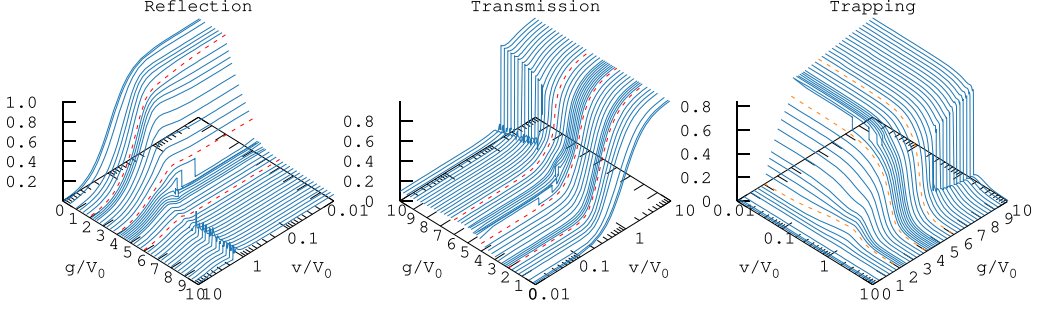


FIG. 4. Reflected, transmitted, and trapped fractions after a soliton is scattered from a contact impurity as a function of scaled initial soliton velocity v/V_0 and the strength of the nonlinear parameter $|g|/V_0$. The red lines separate the four regions of characteristic dynamics. See text for discussion.

1. Small nonlinearity, $|g| \leq 2V_0$

We first consider the regime of small nonlinearity, which we define in the region $0 < |g|/V_0 \leq 2$. In the limit of $V_0 \gg |g|$, we recover the result of the linear Schrödinger equation, i.e., $\tilde{g} = 0$. The corresponding reflection and transmission, as given by Eq. (14) for $\tilde{g} = 0$, are shown in Figs. 5(a) and 5(b) as solid thick gray lines, respectively. For the nearly linear case $|g|/V_0 = 0.1$, we get a nearly perfect agreement. As the strength of nonlinearity increases, we note that in the interval $v/V_0 \lesssim 1$ one gets an initially enhanced reflection accompanied by suppressed transmission. However, there is no discernible trapping until $|g|/V_0 \geq 1$ [see Fig. 5(c)].

An example of the time evolution in this regime is shown in Fig. 6, which shows four scattering events at $|g|/V_0 = 0.5$ for different incident velocities. For low v/V_0 , the incident soliton is perfectly reflected from the impurity and keeps its initial soliton shape. Increasing the velocity leads to transmission in the form of a dispersing wave packet [Fig. 6(b)], while the reflected component remains a soliton. Approaching $v/V_0 = 1$, the soliton nature of both the reflected and the transmitted parts is less well defined, as shown in Fig. 6(c). Increasing the initial velocity even further, one reaches the situation where the forward scattering is in the form of a soliton, while the backscattered packet shows dispersion [Fig. 6(d)].

Upon increasing the nonlinear strength $|g|$, a trapped component appears around $|g|/V_0 = 1$. Figure 7 shows the dynamics for different initial velocities when $|g|/V_0 = 1.75$. The dynamics is similar to that for $|g|/V_0 = 0.5$ with a few exceptions. In Fig. 7(a), the low velocity colliding soliton reflects completely from the contact impurity. In Fig. 7(b), while there is now a trapped component, the nature of the transmitted and reflected parts is also somewhat altered. Particularly, we observe a highly delocalized radiation contribution in the transmission. In panel (c) of Fig. 7, it is observed how the reflected soliton is scattered together with a part that is not a soliton. Note that the color scale is logarithmic and that this part is not contributing significantly to the overall reflection. The same holds for the transmitted soliton in panel (d) of Fig. 7. Also, there is a dispersive background of very low intensity.

The common feature for all our simulations in the regime $|g|/V_0 \leq 2$ is that as the initial velocity becomes sufficiently small, the trapping vanishes. This is consistent with the argument that resonant trapping cannot appear unless $|g| > 2V_0$, as put forward by Goodman *et al.* [47]. However, it is important to note that this does not imply that trapping cannot occur at all. The situation is best illustrated by plotting the trapped fraction on a log-log scale, as shown in Fig. 8, where we show the trapping fraction as a function of the incident soliton reduced velocity, v/V_0 , for $|g|/V_0 < 2.25$. Note how the curves on either side of $|g| = 2V_0$ (thick black line) deviate from it, as the velocity is reduced. Furthermore, trapping shows a maximum at a velocity $v \approx V_0$.

An alternative representation of the data shown in Fig. 8 is given in Fig. 9, where the trapped fraction is plotted against the nonlinear strength $|g|/V_0$ for different initial velocities. As v/V_0

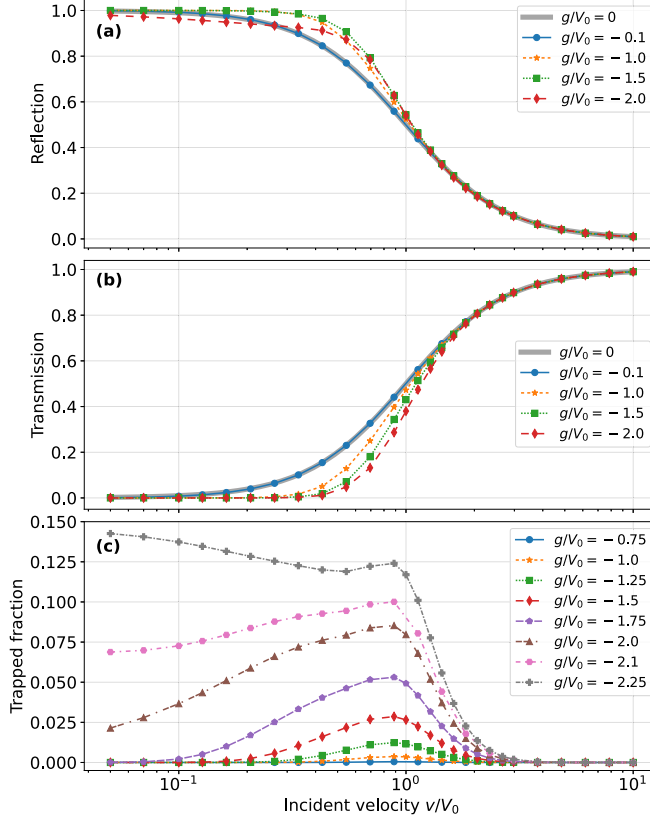


FIG. 5. Reflection, transmission, and trapped fractions after a soliton is scattered from a contact impurity as function of scaled initial soliton velocity (v/V_0) for various strengths of nonlinear parameter $|g|/V_0 \leq 2$. (a) Reflected fraction. Increasing the strength of the nonlinearity will initially increase the reflection compared to the linear case (solid line). (b) Transmitted fraction. Increasing the strength of the nonlinearity causes a suppression of the transmission. (c) Trapped fraction. For velocities $v \sim V_0$, a small portion of the incident soliton is trapped. For $v/V_0 \rightarrow 0$, trapping is minimal or not possible if $|g|/V_0 \leq 2$. See text for discussion.

decreases, the transition to the resonant-trapping regime becomes increasingly sharper. In the inset, we observe that, as $v/V_0 \rightarrow 0$, the onset of resonant trapping is clearly discernible at $|g|/V_0 = 2$. This onset for resonant trapping can also be observed in Fig. 4 at low soliton collision energies discerned by the red dashed line at $|g|/V_0 = 2$. Although the onset does not affect the transmitted component, it is visible in the reflected part that shows a corresponding drop above the threshold $|g|/V_0 = 2$, as seen in Fig. 5. To estimate the trapped fraction in the limit $v \rightarrow 0$, we may use the resonant condition $\Omega_S = \Omega_{TR}$ and solve for the trapped fraction. This leads to the expression

$$N_{TR} \approx \begin{cases} 0, & |g| \leq 2V_0, \\ 1 - 2V_0/|g|, & |g| > 2V_0. \end{cases} \quad (21)$$

This crude estimate for the trapped fraction is shown as the red line in Fig. 9.

2. Intermediate nonlinearity, $2V_0 < |g| \leq 4V_0$

Figure 10 shows the simulation results in the regime $2 < |g|/V_0 \leq 4$. The reflected, transmitted, and trapped fractions differ qualitatively from those found in the weaker-nonlinearity regime, $|g|/V_0 \leq 2$, shown in Fig. 7. In particular, the reflected fraction decreases with increasing $|g|/V_0$,

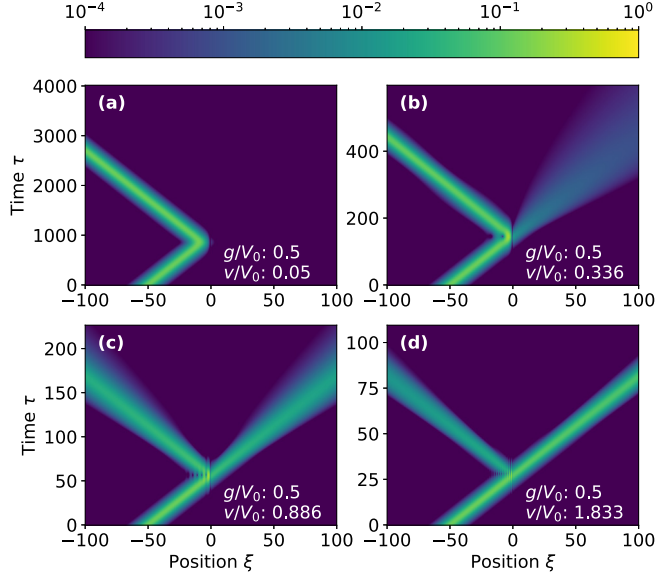


FIG. 6. Soliton scattering for small nonlinearity $|g|/V_0 = 0.5$, for several initial soliton velocities. (a) $v/V_0 = 0.05$. In the limit of small incident soliton velocity, the entire packet is reflected. The reflected part remains a soliton. (b) $v/V_0 = 0.336$. Upon increasing the incident velocity, part of the soliton is transmitted in the form of a dispersing wave packet. (c) $v/V_0 = 0.886$. When $v \sim V_0$, transmission and reflection both show dispersive behavior. (d) $v/V_0 = 1.833$. In the limit of high incident velocities, the initial soliton passes the contact impurity, with the reflected part emitted as a dispersing packet. Notice the logarithmic scale in the intensity mapping to enhance small amplitudes.

as seen in Fig. 10(a). The transmitted fraction, shown in Fig. 10(b), displays a distinct behavior at small v/V_0 , discussed below. At the same time, the trapped fraction in Fig. 10(c) continues to increase at small v/V_0 as the nonlinearity is increased, rather than tending to zero.

The behavior of the transmitted fraction is shown more clearly in Fig. 11. For fixed v/V_0 , the transmission increases with increasing $|g|/V_0$. In particular, for small v/V_0 , the transmitted fraction becomes nonzero at approximately $|g|/V_0 \gtrsim 2$, indicating that transmission sets in once $|g| \gtrsim 2V_0$ in the low-velocity limit. As seen in Fig. 10, the transmission remains nearly constant for $v < V_0$, while increasing for $|g|/V_0 > 2$.

The scattering dynamics in this regime are illustrated in Fig. 12. For small v/V_0 , the system lies in the resonant-trapping regime. Figure 12(a) shows a representative scattering event for an incoming soliton with velocity $v/V_0 = 0.05$ and $|g|/V_0 = 3.25$. As the front of the soliton reaches the δ potential, the wave packet splits rapidly into four components: a bound state, a reflected and transmitted soliton, and reflected dispersive packets. As seen from their slopes in Fig. 12(a), the dispersive packets propagate significantly faster than the reflected soliton and much faster than the incident soliton, while having very small amplitude. Their shape indicates that they correspond to matter-wave radiation. This radiative emission accompanies the trapping process and accounts for the increase in transmission with increasing $|g|/V_0$ in the regime $v/V_0 \ll 1$. This interpretation can be made more explicit using energy conservation arguments. Assume that the outgoing state consists only of a reflected soliton with velocity v_R and a trapped soliton with trapped fraction N_{TR} . Using Eqs. (D8) and (D9) to ensure energy conservation in the scattering process, one obtains

$$v_R^2 = \frac{v^2}{1 - N_{\text{TR}}} + N_{\text{TR}} \left(\frac{2V_0^2 + |g|V_0 N_{\text{TR}}}{2(1 - N_{\text{TR}})} - \frac{g^2}{4} \right). \quad (22)$$

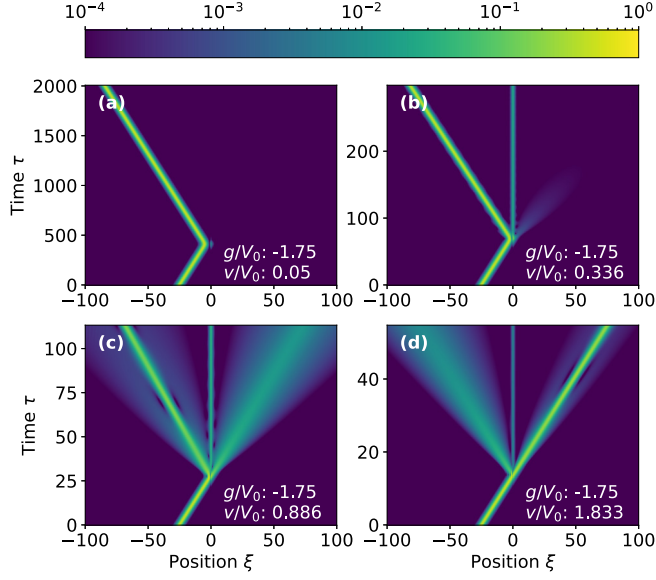


FIG. 7. The same as Fig. 6, but for $|g|/V_0 = 1.75$. (a) $v/V_0 = 0.05$. In the limit of small incident soliton velocities, the entire packet is reflected. The reflected part remains a soliton. (b) $v/V_0 = 0.336$. Upon increasing the incident velocity, part of the soliton is transmitted in the form of a dispersing wave packet, part is reflected as a soliton, and part is trapped in a bound state. (c) $v/V_0 = 0.886$. When $v \sim V_0$, the transmission shows dispersive behavior, while the backscattered part appears superimposed on a dispersive contribution. (d) $v/V_0 = 1.833$. In the limit of high incident velocities, the initial soliton passes the contact impurity, with the reflected part emitted as a dispersive packet.

For $|g| > 2V_0$, the right-hand side of Eq. (22) can become negative if the velocity v of the incoming soliton is below a critical velocity

$$v \leq v_c = \frac{g^2 - 4V_0^2}{4\sqrt{g^2 + 2|g|V_0}}. \quad (23)$$

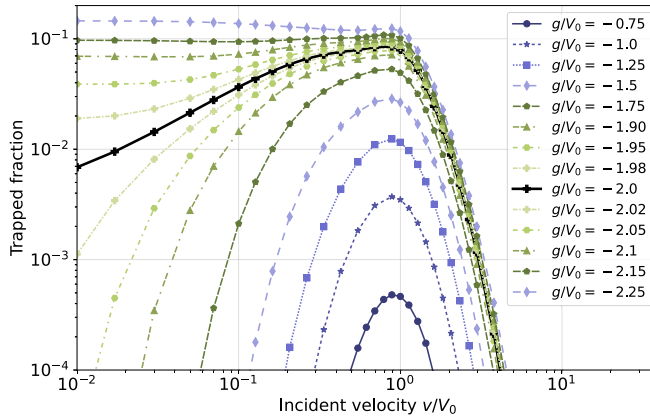


FIG. 8. Trapped fraction for $|g| \leq 2.25V_0$ illustrating resonant trapping. Below $|g| = 2V_0$, the trapped fraction goes to zero as $v \rightarrow 0$. For $|g| > 2V_0$, resonant trapping is possible and the trapped fraction remains nonzero as $v \rightarrow 0$.

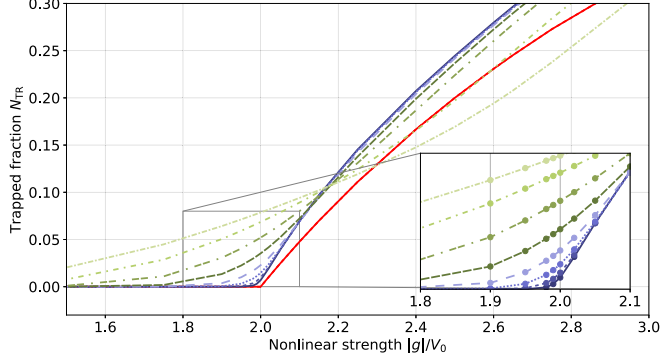


FIG. 9. Transition to resonant trapping. In the limit $v/V_0 \rightarrow 0$, trapping is only possible if $|g| > 2V_0$. Using the simple estimate in Eq. (21) results in the red line.

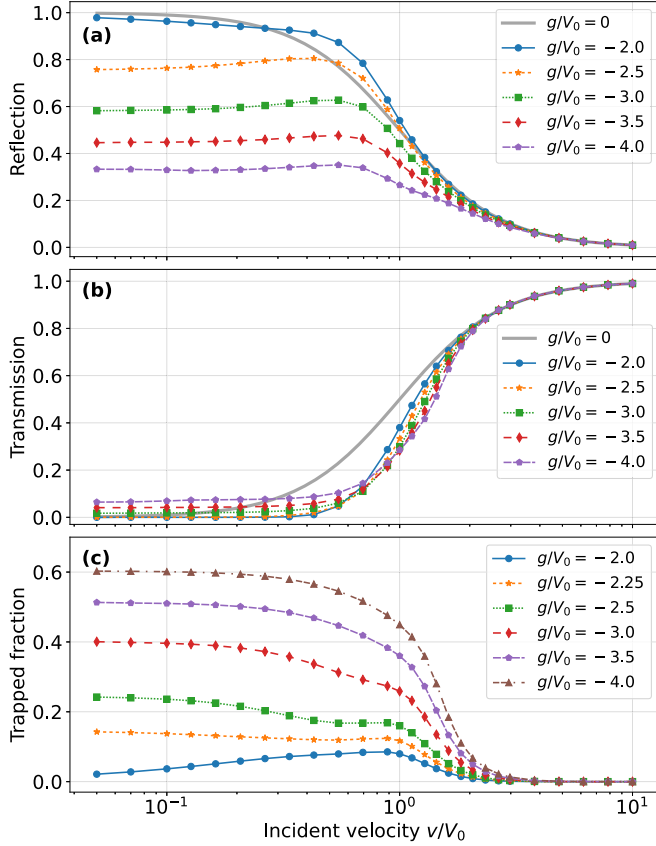


FIG. 10. Reflection, transmission, and trapping fractions after soliton scattering from a contact impurity as a function of scaled initial soliton velocity v/V_0 for various strengths of the nonlinear parameter $2 \leq |g|/V_0 \leq 4$. (a) Reflected fractions. While increasing the strength of the nonlinearity, the reflection decreases. The linear case ($g = 0$) has been added as the solid gray line, for reference. (b) Transmitted fractions. Note that for $v/V_0 \lesssim 0.5$, the transmission increases as the nonlinearity increases. (c) Trapped fraction. The trapped fraction increases as $|g|$ increases in the interval $2 \leq |g|/V_0 \leq 4$.

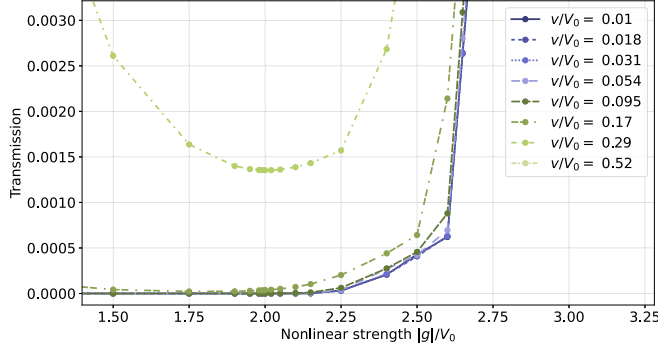


FIG. 11. Transmission probability fraction as a function of the nonlinear strength $|g|/V_0$ for several initial soliton velocities. Note that as $v/V_0 \rightarrow 0$, transmission occurs for $|g|/V_0 \gtrsim 2$.

Thus, for $v < v_c$ scattering into only a reflected soliton is no longer possible. The excess energy must therefore be carried away by additional channels, namely, the radiative components seen in Fig. 12(a). This conclusion is also consistent with the larger reflected-soliton velocities observed at low incident velocity when comparing Fig. 7(a) with Fig. 12(a).

In Fig. 12(b), we see the same behavior for $v/V_0 = 0.695$, for the trapped and reflected solitons, as well as for the dispersive (radiative) contribution. Here, it should be noted that the reflected soliton shows breathing as it propagates. Proceeding to still larger initial velocities, one eventually reaches a regime where we identify two solitons emerging from the scattering event, as observed in Fig. 12(c).

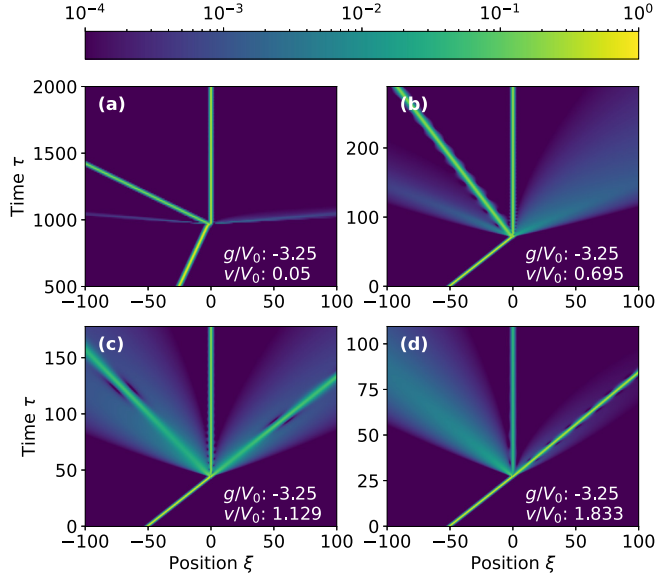


FIG. 12. Soliton scattering from an attractive contact impurity for nonlinear strength $|g|/V_0 = 3.25$. (a) $v/V_0 = 0.050$. In the limit of small incident soliton velocities, the soliton is reflected together with reflected and transmitted dispersive packets. Note the high velocities of the reflected and transmitted parts. (b) $v/V_0 = 0.695$. The behavior is qualitatively similar to panel (a) but with altered magnitudes and velocities. In addition, the reflected soliton is now breathing. (c) $v/V_0 = 1.129$. Here, there are both transmitted and reflected solitons superimposed on a dispersive contributions. The trapped part shows damped oscillatory behavior. (d) $v/V_0 = 1.833$. In the limit of high incident velocities, the initial soliton passes the contact impurity, with the reflected part emitted as a dispersive packet.

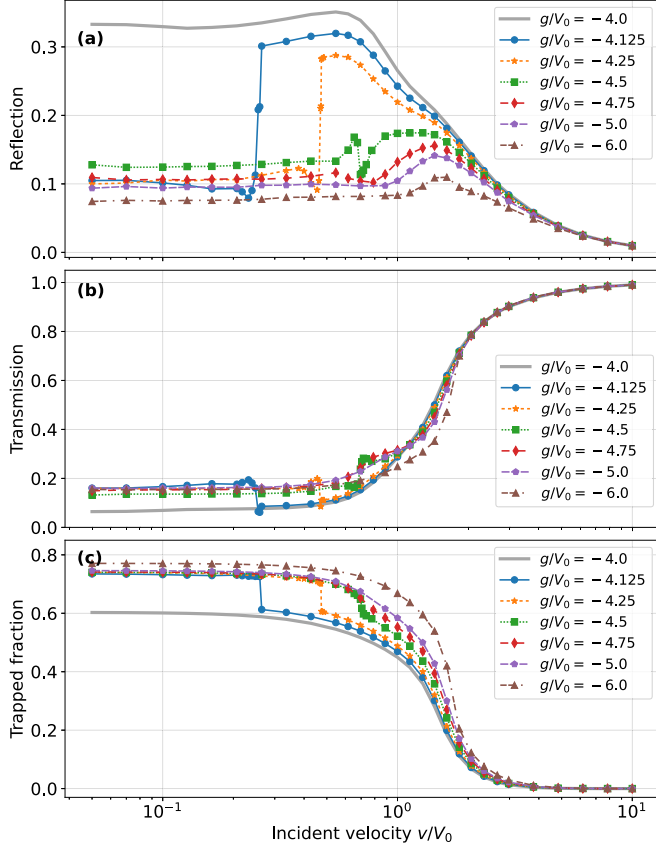


FIG. 13. Reflection, transmission, and trapped fractions after soliton is scattered by the impurity as function of scaled initial soliton velocity v/V_0 for various strengths of nonlinear parameter $4 \leq |g|/V_0 \leq 6$. (a) Reflected fraction. In the interval $4 \leq |g|/V_0 \leq 6$, sharp transitions and oscillations appear. These features also appear in the transmitted fraction, shown in panel (b) and the trapped fraction shown in panel (c). In this regime, when v/V_0 is small, resonant trapping of solitons is not accompanied by reflected or transmitted solitons, but instead the created defect mode radiates in both directions. See text for discussion.

Close inspection reveals that it takes some time for the trapped state to settle. During this time ($50 < \tau < 100$), the scattering process continues to radiate in both directions. This corresponds to the excitation of a radiation oscillating excited mode of the bound state, belonging to the continuum of excited states as shown in Fig. 1.

3. Strong nonlinearity, $4V_0 < |g| < 6V_0$

When the nonlinearity enters the regime $4 < |g|/V_0 < 6$, the scattering dynamics change qualitatively. As shown in Fig. 13, the reflected, transmitted, and trapped fractions are no longer smooth functions of v/V_0 . The most prominent feature is the appearance of a threshold in the reflected fraction: As $|g|/V_0$ increases, the onset velocity at which the reflection rises abruptly shifts to larger values, up to approximately $|g|/V_0 \sim 4.5$. A similar feature is also visible in the transmitted fraction, but it is most pronounced in the trapped fraction.

To understand the origin of this behavior, it is useful to examine the scattering events shown in Fig. 14 for several incident velocities and compare them with the reflected fraction N_R for $|g|/V_0 = 4.125$ in Fig. 13(a) (blue circles). For $v/V_0 \lesssim 0.25$, Fig. 14(a) shows that the reflected soliton does not have sufficient kinetic energy to separate from the trapped component localized

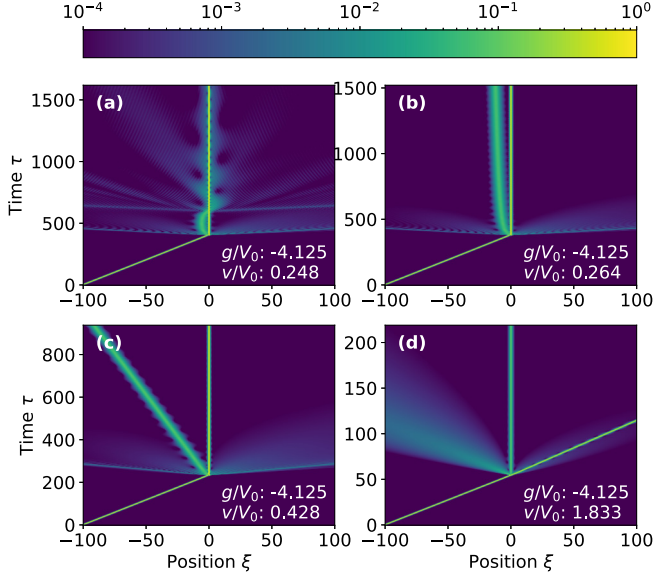


FIG. 14. Soliton scattering from an impurity for large nonlinearity, $|g|/V_0 = 4.125$. (a) $v/V_0 = 0.248$. Below a critical incidence velocity, the reflected soliton does not have sufficient speed to escape the trapped fraction at the location of the contact impurity. Instead it oscillates around the trapped fraction, emitting radiation in the process. (b) $v/V_0 = 0.264$. Above the critical velocity, the reflected soliton can escape the contact impurity. (c) $v/V_0 = 0.428$ and (d) $v/V_0 = 1.833$. Increasing v/V_0 , the qualitative behavior is the same as for the intermediate regime $2V_0 < |g| \leq 4V_0$.

at the impurity. Instead, it turns around and collides repeatedly with the trapped mode, giving rise to an oscillatory dynamics. During these repeated interactions, radiation is emitted and contributes to both the reflected and transmitted fractions. These oscillations differ from those reported for nonlocal solitons [45,57] and for solitons scattered from extended potentials such as square wells [48,49]. In the present case, the impurity-bound state remains continuously populated, while only a smaller fraction oscillates around it.

Figure 14(b) shows the scattering just above the discontinuity in the reflected fraction, at $v/V_0 = 0.264$. Here, the reflected soliton has just enough kinetic energy to detach from the trapped component and escape the impurity region, producing the abrupt increase in reflection seen in Fig. 13(a). For larger incident velocities, the scattering returns to the same qualitative behavior found in the intermediate regime $2 < |g|/V_0 \leq 4$, as illustrated in Figs. 14(c) and 14(d). For $v/V_0 < 1$, trapping is accompanied by the emission of a fast radiative component. As the collision velocity increases, a more clearly defined transmitted soliton emerges, while the reflected part becomes increasingly dispersive.

4. Very strong nonlinearity, $|g|/V_0 > 6$

Finally, we address the regime $|g|/V_0 > 6$, specifically focusing on the domain $6 \leq |g|/V_0 \leq 10$. Figures 15(a)–15(c) show the reflection, transmission, and trapping characteristics in this regime, respectively. The most striking feature here is the appearance of a critical velocity $v/V_0 \approx 1.7$, where there is an abrupt change in the scattering dynamics that appears for $|g|/V_0 > 6$.

We note that both above and below this value of v , there is a nonmonotonic behavior of the reflection, transmission, and trapping. This is most notable in the reflection where we obtain a sharp peak, which is very pronounced for $|g|/V_0 = 7.0$. To gain insight into the scattering process, Fig. 16 shows the scattering for four different initial velocities for $|g|/V_0 = 7.0$. For small velocities, panel (a) shows that most of the incoming soliton becomes trapped, with some radiation scattered in both

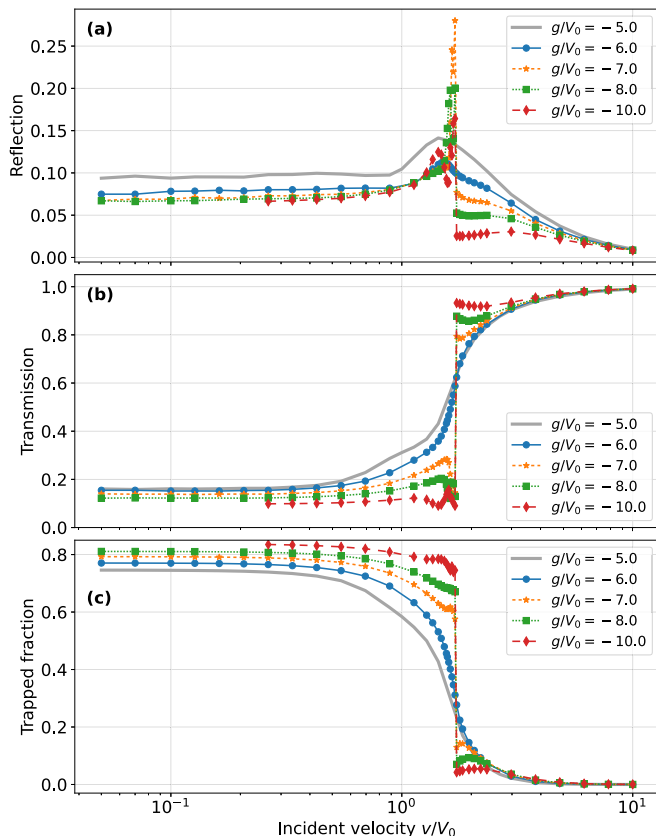


FIG. 15. Reflection, transmission, and trapped fractions after a soliton is scattered by the impurity as a function of scaled initial soliton velocity v/V_0 for various strength of nonlinear parameter $6 \leq |g|/V_0 \leq 10$. (a) Reflected fraction. For large nonlinearities, $6 < |g|/V_0 \leq 10$, a sharp transition occurs at $v \approx 1.7V_0$. It is characterized by a peak in the reflection [panel (a)] and a steep rise (drop) in transmission [panel (b)] and trapping [panel (c)], respectively. This singular behavior is associated with the appearance of a transmitted soliton solution appearing when $v/V_0 \gtrsim 1.7$, as seen in Fig. 16.

directions. Just below the threshold, panel (b) shows that there is a dramatic increase in scattering that enhances the reflection and transmission. This increased radiation is associated with the trapped state being excited in an oscillatory motion that keeps radiating until it has stabilized. Above the transition, panel (c) shows that an outgoing slow soliton is permitted, resulting in a drastic increase in overall transmission accompanied by a drop in the trapped fraction of the wave packet.

Thus far, we have focused on quantifying the scattering process through reflection, transmission, and trapping. As we have seen, the changes in the scattering behavior are connected to the nature of the wavefunctions of the scattering products. In particular, we seek to quantify if localized solutions in terms of solitons are present, or whether the scattering products are in the form of more delocalized waves (dispersive wave packets) and/or radiation. For this purpose, it is of interest to also use other measures that can quantify this qualitative differences between the scattering products.

C. Information-theoretical measures analysis

An essential concept in quantum mechanics is the study of IT measures, which provides key insights into the uncertainty and spatial distribution of a quantum system [66]. IT measures allow to study how localized or delocalized the probability distribution of the system is. This can be

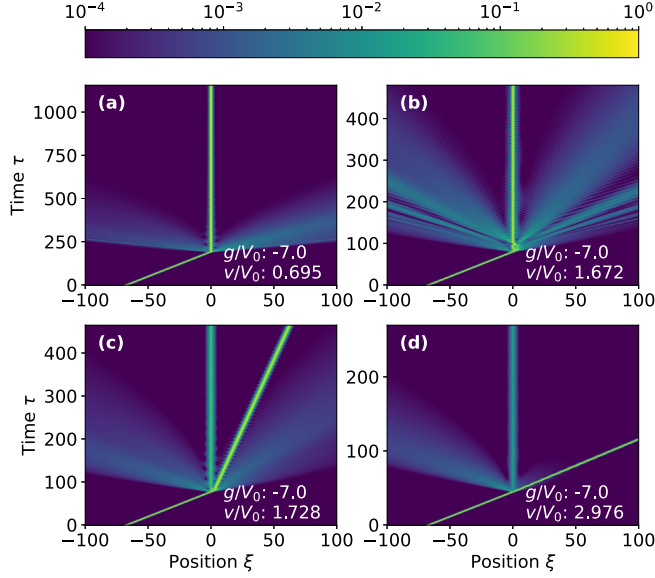


FIG. 16. Soliton scattering for very large nonlinearity $|g|/V_0 = 7.0$. (a) $v/V_0 = 0.695$. Below a critical incidence velocity, a significant fraction is trapped, while reflection and transmission radiate. (b) $v/V_0 = 1.672$. Just below the threshold velocity, the transmitted and reflected radiation covers a broad spectrum of frequencies. (c) $v/V_0 = 1.728$. Just above the threshold velocity, slow propagating soliton solutions are possible for the transmitted wave. (d) $v/V_0 = 2.976$ a full soliton transmission and resonant trapping is observed.

quantified using various entropic measures, such as differential and Fisher information entropy, each offering a distinct viewpoint on the system's behavior [67–70].

We wish to emphasize that, within the framework of the GPE, all N atoms are by construction in the same pure quantum state, $|N\rangle$, with density matrix $\hat{\rho} = |N\rangle\langle N|$. Thus, the corresponding physical entropy, or von Neumann entropy, vanishes: $S \equiv -k_B \text{Tr} \hat{\rho} \ln \hat{\rho} = 0$. Equivalently, since only one state is occupied, the Shannon entropy $-\sum_n p_n \ln p_n$ is likewise zero. This is distinct from the IT differential entropy $-\int dx p(x) \ln p(x)$ that can be meaningfully defined for any probability distribution $p(x)$ and is the continuum analog of the Shannon entropy.

The differential entropy, in particular, serves as a global IT measure and reflects the degree of delocalization within the system. A higher differential entropy measure indicates a more delocalized wave-packet distribution, meaning a greater uncertainty, randomness, and information content in the system, whereas low entropy signifies more order or compactness.

The differential entropy, S , is defined as

$$S = - \int_{-\infty}^{\infty} \rho(\xi, \tau) \ln \rho(\xi, \tau) d\xi, \quad (24)$$

where

$$\rho(\xi, t) = \Psi^\dagger(\xi, \tau) \Psi(\xi, \tau) \quad (25)$$

is the system probability density distribution to find the system in ξ at time τ and $\Psi(\xi, \tau)$ is the particle wavefunction. In contrast, Fisher information entropy serves as a local measure of information entropy. It behaves in a way that is complementary to the differential entropy, being more sensitive to local fluctuations in the probability distribution.

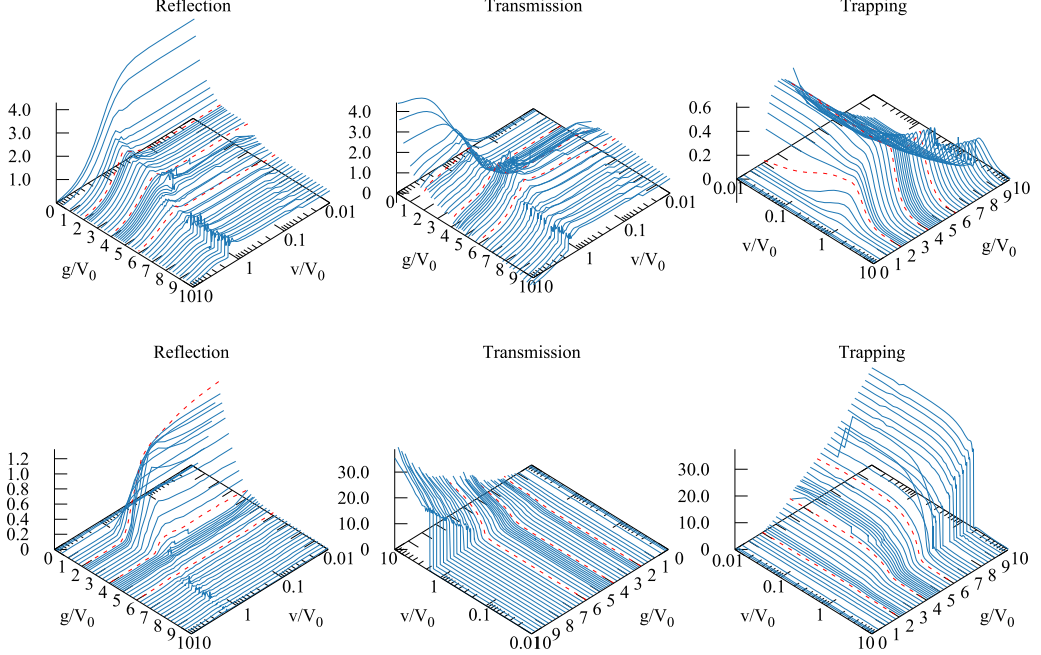


FIG. 17. First row, the differential entropy as a function of the scaled soliton velocity, v/V_0 , and nonlinear strength g/V_0 for the reflected, transmitted, and trapped regions. Second row, the same for the Fisher entropy. See text for discussion.

The Fisher entropy, F , is defined as

$$F = \int_{-\infty}^{\infty} \frac{\left| \frac{d}{d\xi} \rho(\xi, \tau) \right|^2}{\rho(\xi, \tau)} d\xi. \quad (26)$$

Fisher information entropy quantifies the sharpness of the probability density, often correlating with a more localized distribution. These IT measures offer considerable potential for gaining deeper insights into scattering processes. To the best of our knowledge, no previous studies have examined the role of IT measures in scattering processes within a nonlinear regime.

In Fig. 17, first row, we show the results for the differential entropy for the reflected, transmitted, and trapped components of the wavefunction, as discussed in Appendix C. For the reflection, we observe that for small collision energies $v/V_0 < 1$ and small nonlinear strength $|g|/V_0 < 2$, corresponding to the small nonlinearity region, the differential entropy is large. Thus, a first interpretation of the differential entropy suggests that the reflection at low collision velocities shows a large delocalization. However, this is partly a consequence of the incident soliton at low $|g|/V_0$ being relatively wide, in conjunction with a contribution $N_R \ln N_R$ to the differential entropy [see Eq. (C7)]. A similar behavior is seen for the differential entropy of the transmitted fraction at large incident velocities v/V_0 and low nonlinear strength $|g|/V_0$. In both cases, we observe a ridge at $v = V_0$ for all the nonlinear strength where mostly half of the soliton has tunneled and half has bounced back (reflected). Even more interesting is the behavior of the differential entropy for the trapped component. For all the collision velocities, and small nonlinear strength, there is no contribution to the differential entropy, as there is no trapping, in agreement with our results shown in Fig. 10. At large trapping, within the region $4 < g/V_0 < 6$, the differential entropy reaches its higher values, but it is still small compared to the reflected and transmitted values at maximum. Thus, the trapping is well localized.

This discussion is reinforced by the Fisher entropy. In Fig. 17, it is shown in the second row. As the Fisher entropy is more sensitive to local fluctuations, one observes that for low values of the collision velocity, $v/V_0 < 1$ and $2 < g/V_0 < 4$, the reflection is more dispersed, while the transmitted component shows dispersion at large values of the collision energy and nonlinear strength. Interestingly, the trapping shows a larger sharpness of the probability density as the trapping probability increases. Furthermore, the onsets observed in the dynamics are clearly enhanced in both IT measure results.

IV. CONCLUSIONS

In this work, we have studied the scattering of a bright matter-wave soliton from an attractive contact impurity within the one-dimensional time-dependent Gross-Pitaevskii equation. By expressing the problem in terms of the reduced parameters v/V_0 and g/V_0 , we showed that the scattering dynamics can be organized into a universal two-parameter map, allowing a systematic characterization of reflection, transmission, and trapping over a broad range of incident velocities and nonlinear couplings. Our numerical treatment of the contact defect, based on optimized δ -sequence representations and independently verified time-propagation schemes, reproduces the expected bound-state properties and provides a robust framework for dynamical analysis.

The central result is the emergence of four distinct dynamical regimes separated by characteristic thresholds in the nonlinear coupling for $0 < |g|/V_0 < 10$. For $|g|/V_0 \leq 2$, the low-velocity dynamics is dominated by reflection and the trapped fraction vanishes as $v \rightarrow 0$. At $|g|/V_0 \approx 2$, resonant trapping sets in, and for $2 < |g|/V_0 \leq 4$, the trapped fraction increases while low-velocity transmission becomes possible through the emission of fast radiative components. In the interval $4 < |g|/V_0 < 6$, the scattering becomes nonsmooth, with threshold behavior associated with whether the reflected soliton can detach from the impurity-bound component. For still stronger nonlinearity, $|g|/V_0 > 6$, an additional transition appears near $v \approx 1.7V_0$, where the opening of a transmitted soliton channel leads to a sharp redistribution among reflection, transmission, and trapping. These results show that the attractive defect does not simply split the incoming packet but can induce qualitatively different output states depending on the interplay between kinetic energy, nonlinearity, and defect-induced trapping.

Beyond population fractions, our information-theoretical analysis provides a complementary characterization of the scattering products. The differential entropy and the Fisher entropy confirm that while the trapped component remains comparatively well localized, reflected and transmitted components can become highly delocalized when radiative and dispersive emission is important. In this way, the entropy measures help distinguish genuinely solitonic output channels from highly excited, nonsolitonic wave packets, and thereby clarify the physical meaning of the threshold structures identified in the scattering map and that are of interest in atomtronics currents.

Together, our results identify the parameter regimes in which an attractive point defect may function as a controllable atomtronic element, acting as a reflector, transmitter, or trap for guided bright solitons. The threshold structure reported here should be useful for future experimental studies of localized defects in quasi-one-dimensional Bose-Einstein condensates, and it also suggests several natural extensions, including the role of finite-width impurities, and multidefect geometries.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: NONUNIFORM MESH

To implement the FD approach, we follow the procedure reported in Ref. [71]. Let us consider the energy functional

$$E = \frac{\int \left(\frac{1}{2} \frac{d\psi^*}{d\xi} \frac{d\psi}{d\xi} + V \psi^* \psi \right) d\xi}{\int \psi^* \psi d\xi}. \quad (\text{A1})$$

We choose quadrature and differencing rules for the integrals and derivatives at the midpoint of the mesh as follows:

$$\left. \frac{d\psi}{d\xi} \right|_{\xi_i} \rightarrow \frac{\psi_{i+1/2} - \psi_{i-1/2}}{\xi_{i+1/2} - \xi_{i-1/2}} \quad (\text{A2})$$

and

$$\int \psi d\xi \rightarrow \sum_{i=1}^N \psi_i (\xi_{i+1/2} - \xi_{i-1/2}), \quad (\text{A3})$$

where ξ_i is the i th point in a general nonuniform grid.

Thus, introducing Eqs. (A2) and (A3) in Eq. (A1) and minimizing with respect to ψ_j^* , one obtains

$$\frac{1}{2} \left[\frac{\psi_j - \psi_{j-1}}{\xi_j - \xi_{j-1}} - \frac{\psi_{j+1} - \psi_j}{\xi_{j+1} - \xi_j} \right] + V_j \psi_j (\xi_{j+1/2} - \xi_{j-1/2}) = E \psi_j (\xi_{j+1/2} - \xi_{j-1/2}), \quad (\text{A4})$$

which in matrix form becomes

$$\mathbf{H} \vec{\psi} = E \mathbf{S} \vec{\psi}. \quad (\text{A5})$$

Here, $\mathbf{H}$ is a tridiagonal Hamiltonian matrix, as for the usual second-order finite differencing of the Schrödinger equation, while the overlap matrix $\mathbf{S}$ is diagonal. This generalized eigenvalue problem can be put in somewhat more convenient form using the usual transformation $\mathbf{S} = \mathbf{L}^T \mathbf{L}$. Since $\mathbf{S}$ is diagonal, then $L_i = \sqrt{S_i}$, such that $\tilde{\mathbf{H}} = \mathbf{L}^{-1} \mathbf{H} \mathbf{L}^{-T}$ and $\vec{\phi} = \mathbf{L}^T \vec{\psi}$, obtaining the standard eigenvalue problem, $\tilde{\mathbf{H}} \vec{\phi} = E \vec{\phi}$.

The nonzero matrix elements of $\tilde{\mathbf{H}}$ are

$$\tilde{H}_{ii} = \frac{1}{2} \left[\frac{1}{\xi_i - \xi_{i-1}} + \frac{1}{\xi_{i+1} - \xi_i} \right] \frac{1}{\xi_{i+1/2} - \xi_{i-1/2}} + V_i \quad (\text{A6})$$

and

$$\tilde{H}_{i,i+1} = -\frac{1}{2} \frac{1}{\xi_{i+1} - \xi_i} \frac{1}{\sqrt{(\xi_{i+1/2} - \xi_{i-1/2})(\xi_{i+3/2} - \xi_{i+1/2})}}. \quad (\text{A7})$$

These results are for the case of Dirichlet boundary conditions, i.e., $\psi_0 = \psi_{N+1} = 0$.

APPENDIX B: GAUSS-SEIDEL AND PSEUDOSPECTRAL METHOD

In order to characterize the δ function within a given numerical lattice with point spacing $\Delta\xi$, we have implemented the GS method [72] to find the ground-state solution of the GPE. In this case, the energy is evaluated at an *improved* solution on each point of the numerical grid through

$$\Psi_i \rightarrow \Psi'_i = (1 - \omega) \Psi_i + \frac{\omega(\Psi_{i+1} + \Psi_{i-1})}{2[1 - (\delta(\xi_i) - g|\Psi_i|^2 + \mu)(\Delta\xi)^2]}, \quad (\text{B1})$$

where ω is the relaxation parameter that ensures convergence to the lowest-energy state. In our case, we find that $\omega = 3/4$ provides a fast convergence.

For the time evolution, we have independently verified the results using two different methods. The first employs a finite-differences scheme, together with the Crank-Nicolson (CN) method [72,73] as mentioned in Appendix A, while the other is the pseudospectral split-operator method discussed here. Thus, numerical solutions to the GPE can be found for a given value of $\tilde{g} = g/V \neq 0$ without approximations and they are compared with available analytical results. These methods evolve the time-dependent GPE in time, requiring that the wavefunction at a given time to be discretized in space on a numerical grid, viz., $\Psi(\xi_k, \tau_n) \rightarrow \Psi_k^n$.

Using the CN method, the temporal evolution can be well approximated by separating the kinetic and potential energy terms by the split-operator method [72]

$$\Psi(\xi, \tau_0 + \Delta\tau) \cong e^{-i\Delta\tau\hat{T}} e^{-i\Delta\tau\hat{V}} \Psi(\xi, \tau_0). \quad (\text{B2})$$

The nonlinear GPE is then

$$\Psi_k^{n+1} - \nu(\Psi_{k+1}^{n+1} - 2\Psi_k^{n+1} + \Psi_{k-1}^{n+1}) = f_k + \nu(f_{k+1} - 2f_k + f_{k-1}), \quad (\text{B3})$$

where $\nu = i\Delta\tau/(4\Delta\xi^2)$ for the case of a uniform mesh and the potentials are in $f_k = \exp(-i\Delta\tau[V(\xi_k) + g|\Psi_k^n|^2])\Psi_k^n$.

Equation (B3) is written in matrix form as $\mathbf{A}^+ \vec{\Psi}^{n+1} = \mathbf{A}^- \vec{f}$, where

$$\mathbf{A}^\pm = \begin{pmatrix} 1 \pm 2\nu & \mp\nu & 0 & \dots & 0 \\ \mp\nu & 1 \pm 2\nu & \mp\nu & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \mp\nu & 1 \pm 2\nu & \mp\nu \\ 0 & \dots & 0 & \mp\nu & 1 \pm 2\nu \end{pmatrix}. \quad (\text{B4})$$

Note that $\mathbf{A}^\pm$ is a constant matrix for fixed $\Delta\tau$ and $\Delta\xi$. This method preserves the norm of the wavefunction and can be unconditionally stable for long times. The solution of these tridiagonal linear equations is obtained via a lower-upper decomposition method [73] through a FORTRAN 90 code.

The second method is the pseudospectral method. We use the same discretization in time, but propagate the solution as

$$\Psi(\xi, \tau_0 + \Delta\tau) = e^{-i\Delta\tau\hat{V}(\tau_0)/2} \mathcal{F}^{-1} \left[e^{-i\Delta\tau\hat{T}} \mathcal{F} \left[e^{-i\Delta\tau\hat{V}(\tau_0)/2} \Psi(\xi, \tau) \right] \right], \quad (\text{B5})$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the spatial Fourier transform and its inverse, respectively. The time dependence of the potential term comes from the inclusion of self-interaction, i.e., $\hat{V}(\tau_0) = V(\xi) + g|\Psi(\xi, \tau_0)|^2$. The implementation of this algorithm leveraged CUDA through the PyTorch library with 64-bit floating point representation.

APPENDIX C: INFORMATION-THEORETIC MEASURES

Motivated by the numerical results shown in Sec. III, we construct the wavefunction, at the end of the dynamics, as a linear combination of reflected contribution on the left of the contact impurity, the trapped wave packet, and the transmitted wave packet on the right as

$$\psi(\xi) = \psi_R(\xi) + \psi_{\text{TR}}(\xi) + \psi_T(\xi). \quad (\text{C1})$$

Here, ψ_R is the reflected wave packet, ψ_T the transmitted wave packet, and the trapped $\psi_{\text{TR}}(\xi)$ wave packet remains at the origin where the impurity is located. By assuming that there is no overlap between these three parts, one can determine the differential information entropy from Eq. (24) and

Fisher information entropy from Eq. (26). With a little bit of algebra one obtains from Eq. (24) that

$$S = S_R + S_{TR} + S_T, \quad (C2)$$

with

$$S_i = - \int \rho_i \ln \rho_i d\xi, \quad (C3)$$

where $\rho_i = \psi_i^* \psi_i$ is the wave-packet density for $i = R, TR, \text{ or } T$. However, ψ_i is not normalized; thus, ρ_i does not represent a distribution of probability. Renormalizing [Eq. (C1)], this becomes

$$\psi(\xi) = \sqrt{N_R} \tilde{\psi}_R(\xi) + \sqrt{N_{TR}} \tilde{\psi}_{TR}(\xi) + \sqrt{N_T} \tilde{\psi}_T(\xi), \quad (C4)$$

such that when introducing it in Eq. (24) gives

$$S = \sum_{i \in R, TR, T} N_i \ln N_i + N_i \bar{S}_i, \quad (C5)$$

where

$$\bar{S}_i = - \int \bar{\rho}_i \ln \bar{\rho}_i d\xi, \quad (C6)$$

with $\bar{\rho}_i = |\tilde{\psi}_i|^2$ being the normalized wavefunction from the reflected, trapped, and transmitted contributions.

Comparing Eq. (C2) to Eq. (C5), we identify that

$$S_i = N_i \ln N_i + N_i \bar{S}_i, \quad (C7)$$

connecting the non-normalized differential information entropy to the normalized one through the reflection, trapping, or transmitted coefficient, N_i .

Following a similar procedure for the Fisher entropy [Eq. (26)] gives

$$F = F_R + F_{TR} + F_T, \quad (C8)$$

where

$$F_i = \int \frac{\left| \frac{d\rho_i}{d\xi} \right|^2}{\rho_i} d\xi. \quad (C9)$$

After normalization, this becomes

$$F = \sum_{i \in R, TR, T} N_i \bar{F}_i, \quad (C10)$$

with

$$\bar{F}_i = \int \frac{\left| \frac{d\bar{\rho}_i}{d\xi} \right|^2}{\bar{\rho}_i} d\xi. \quad (C11)$$

Similarly to the differential information entropy, we identify that

$$F_i = N_i \bar{F}_i. \quad (C12)$$

In the discussion of our results, we show the results of Eqs. (C7) and (C12).

APPENDIX D: ENERGY CONSERVATION ARGUMENT

In order to understand the onset behavior of our results, we resort to the energy conservation of the system. For the GPE, the total energy is given by

$$E = \int \left(\frac{1}{2} \left| \frac{d\psi}{d\xi} \right|^2 + V(\xi) |\psi(\xi)|^2 + \frac{\tilde{g}}{2} |\psi(\xi)|^4 \right) d\xi. \quad (D1)$$

At the end of the dynamics, we assume that

$$\psi(\xi) = \psi_R(\xi) + \psi_{TR}(\xi) + \psi_T(\xi) \quad (D2)$$

for the reflected, trapped, and transmitted wave packets. Since they do not overlap at the end of the dynamics, we find that

$$E = E_R + E_{TR} + E_T, \quad (D3)$$

where the energy contribution from each region is determined from the same Eq. (D1), but replacing $\psi \rightarrow \psi_i$, with $i = R, TR$, or T , depending on which contribution we consider.

From the dynamics, we found that we have three types of wave packets at the end of the dynamics. The soliton, given by Eq. (2), but normalized to the reflected or transmitted contribution, i.e.,

$$\psi_i(\xi, \tau) = \frac{\sqrt{2|\mu_i|}}{|\tilde{g}|} \text{sech}(\sqrt{2|\mu_i|}(\xi - \tilde{v}\tau)) e^{i\left(\tilde{v}\xi - \frac{\tilde{v}^2}{2}\tau - \mu_i\tau\right)}, \quad (D4)$$

with μ_i given by

$$|\mu_i| = \frac{|\tilde{g}|^2}{8} N_i^2 \quad (D5)$$

and i is the reflected or transmitted component. The trapped component is given by Eq. (4) with μ given by Eq. (7), and finally, the radiative component. In the dynamics, the radiative component resembles an Airy train with a high velocity, but to simplify the analysis, we assume that it is a plane wave traveling with a high velocity, in agreement with our analysis on the BdG-type equation [Eq. (12)]. In this case, we propose that

$$\psi_{\text{rad}}(\xi, \tau) = \psi_o e^{i(\tilde{v}_{\text{rad}}\xi - \mu_{\text{rad}}\tau)}. \quad (D6)$$

Under these assumptions, we find that the energy contribution from each component is given by

$$E_{\text{rad}} = \left(\tilde{v}_{\text{rad}}^2 - |\tilde{g}| \frac{N_{\text{rad}}}{L_{\text{rad}}} \right), \quad (D7)$$

where L_{rad} is the length of the radiative wave packet within our numerical box, i.e., $N_{\text{rad}} = |\psi_o|^2 L_{\text{rad}}$. Thus, L_{rad} is the width of the radiative wave packet.

For the trapped component, we find that

$$E_{TR} = \frac{1}{3|\tilde{g}|} (1 - (2|\mu_{TR}|)^{3/2}) = -\frac{N_{TR}}{2} \left(1 + \frac{|\tilde{g}|N_{TR}}{2} + \frac{|\tilde{g}|^2 N_{TR}^2}{12} \right). \quad (D8)$$

Finally, for the reflected or transmitted soliton, its energy is

$$E_i = \frac{1}{2} N_i v^2 - \frac{1}{24} |g|^2 N_i^3. \quad (D9)$$

Thus, for instance, the onset found at $|\tilde{g}| = |g|/V_0 = 2$ is obtained by assuming that the initial energy of the incoming soliton is equal to the trapped soliton plus the reflected soliton, i.e.,

$$\frac{1}{2} \left(\tilde{v}^2 - \frac{|\tilde{g}|^2}{12} \right) = \frac{N_R}{2} \left(\tilde{v}^2 - \frac{|\tilde{g}|^2 N_R^2}{12} \right) + \frac{1}{3|\tilde{g}|} \left(1 - \left[1 + \frac{|\tilde{g}|N_{TR}}{2} \right]^3 \right). \quad (D10)$$

In this case, resonant trapping is achieved when $N_R = 1 - \epsilon$ and $N_{TR} = \epsilon$ in the limit when $\epsilon \rightarrow 0$. Fulfilling the previous equation to first order in ϵ requires that $|\tilde{g}| \geq 2$, as discussed in Secs. III B 1 and III B 2.

The remaining onsets observed in our numerical results are inferred by associating them with the corresponding changes in the character of the resulting wave packet.

APPENDIX E: BRIGHT SOLITON EXPERIMENTAL VALUES

In order to compare the range of parameters that we are using in this work, we use the experimental data on bright ^{85}Rb atoms soliton in a scattering process with a potential barrier. Marchant *et al.* [24,59] created bright solitons of ^{85}Rb with 2000 atoms in an elongate optical trap with frequency $\omega_r = 2\pi \times 27$ Hz in the transverse direction, $\omega_x = 2\pi$ Hz in the longitudinal one, and $a_s = -11a_0$. In this case, a potential barrier with $V_0/k_B = 760$ nK is used. The range of velocities that they explore is from 1 to 5 mm/s approximately. Therefore, this corresponds to a value of $|g|/V_0 = 1.7 \times 10^{-3}$ and the range of $v/V_0 \in [0.06, 0.12]$. Marchant *et al.* find that in this range of values, the bright soliton is reflected, in agreement with our work.

Furthermore, Wales *et al.* [38] create bright solitons with 2500 atoms of ^{85}Rb , where $a_s \in [-13, -7]a_0$. They report kinetic energies for the bright soliton just before it interacts with the potential barrier in the range $E_k/k_B \in [2.2, 41]$ nK. For two potential widths, they consider $V_0 \approx 1.01E_k$ in the wide case and $V_0 \approx 1.11E_k$ in the narrow case. So, the range of velocities correspond to $v/V_0 \in [0.1, 1]$ and for the nonlinear constant $|g|/V_0 \in [0.5, 2.0]$. In this case, the velocity is high enough for transmission to occur, in agreement with our results.

Within these experimental value conditions, the large values for $|g|/V_0 \approx 10.0$ are achieved for a bright soliton with $N = 10^3$ – 10^4 , $|a_s| = 3a_0$ – $20a_0$, $\omega_\perp = 2\pi \times 20$ – 80 Hz, $V_0 = 20$ – 500 nK, and velocities in the range 0.3–30 mm/s. These are parameters that are all accessible with current atomchip technology.

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