

Nonlinear quantum transduction of weak topological impurities

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Detecting weak field perturbations at an optical-vortex node is a fundamental challenge across wave physics, as the local signal contrast typically scales quadratically with the impurity amplitude when no reference field is used. Here, we propose a complementary, spatially resolved diagnostic that exploits the strongly nonlinear response of an electromagnetically induced transparency (EIT) medium to transduce weak topological impurities into probe transmission. Using a vortex beam with a weak coherent admixture as a concrete realization, we show theoretically that EIT coherence converts the weak field that lifts the nominal vortex node into a highly sensitive, macroscopic transmission contrast. A resonant weak-probe model with representative cold-⁸⁷Rb parameters predicts a per-exposure detection floor for the impurity amplitude at the level of a few $\times 10^{-3}$, relative to a separately characterized, preaveraged pure-vortex reference. We quantify the intrinsic and experimental limits of this response, analyzing its dependence on coupling strength, ground-state decoherence, optical depth, two-photon detuning, and Maxwell-Boltzmann thermal motion. While not a complete, basis-independent measurement of modal purity, this method provides a calibrated, local all-optical diagnostic for weak coherent field admixtures at a known vortex node.

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I. INTRODUCTION

Optical-vortex beams are a fundamental class of structured light, characterized by an azimuthal phase winding $\exp(im_l\phi)$ and a phase singularity accompanied by a vanishing field amplitude at the vortex core [1–5]. Their orbital angular momentum has enabled applications in high-dimensional photonics [6,7], quantum information [8–11], optical communication [12–14], optical manipulation [15,16], and light-matter interactions [17–20]. These applications rely on the preparation and preservation of well-defined spatial modes. In practice, a nominally pure vortex may contain a weak coherent field admixture, such as a residual Gaussian or nearly copropagating plane-wave component, which lifts the ideal field node at the vortex core. Detecting such a weak admixture is difficult with direct intensity imaging because the on-axis signal scales as the square of the impurity amplitude, $|c_{\text{pl}}|^2$. For example, an on-axis field-amplitude coefficient of 1% corresponds to only

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a 0.01% on-axis intensity fraction, making small deviations from the ideal node susceptible to camera noise, optical aberrations, and alignment errors. This quadratic suppression is specific to direct intensity detection without a reference field; interferometric or homodyne measurements can recover a contribution linear in the impurity amplitude by using a stable local oscillator.

Existing approaches to structured-light characterization include interferometric [21,22] and holographic reconstruction [23], modal projection [24] and sorting [25–27], and correlation-based methods [28]. These techniques can provide detailed information about the complex field or modal composition, but generally require an optical reference, multiple projections, a specified modal basis, or careful alignment. More recent approaches have also explored atomic systems and basis-independent quality factors for characterizing structured optical fields [29,30]. Mallick and De used a phase-sensitive closed-loop atomic system to detect the topological charge of a vortex Bessel beam [31]. Phillips *et al.* used an optical-vortex mask as a spatial filter to observe a weak electromagnetically induced transparency (EIT) probe in the presence of a strong control field [32]. These methods target different observables, including modal content, topological charge, spatial filtering, or local light-matter response. Rather than replacing such approaches, our goal is more specific: to detect a weak coherent field amplitude directly at a known vortex node. For this task, a useful diagnostic should transduce the weak local field associated with the impurity into a readily measurable signal without requiring a separate coherent local oscillator or a complete modal decomposition. Interferometric or homodyne detection can avoid the quadratic suppression of direct intensity imaging by using a stable local oscillator, thereby providing a contribution that is linear in the weak field amplitude. The present scheme instead trades the requirement for such a coherent local oscillator or a complete modal decomposition for the need for a cold, calibrated EIT medium with stable ground-state coherence, controlled two-photon detuning, and accurate spatial registration.

Here, we propose a complementary transduction mechanism based on electromagnetically induced transparency. In a three-level Λ system, the probe absorption depends nonlinearly on the local intensity of a coupling field. At the center of a pure vortex, the coupling field vanishes and EIT is absent, whereas a weak plane-wave admixture lifts the node and locally opens an EIT transmission window. The atomic medium therefore transduces the weak on-axis coupling associated with the admixture into a spatially resolved change in probe transmission. The impurity-bearing test image can be acquired in a single exposure after calibration against a separately characterized pure-vortex reference, without requiring a separate coherent local oscillator or full modal tomography. Its advantage is consequently not a universal measure of modal purity, but a calibrated local diagnostic for a weak coherent amplitude at a known field node.

In this work, we develop a minimal weak-probe model for the EIT transduction of a plane-wave admixture in an optical vortex. We derive the spatially dependent probe transmission, introduce the intrinsic EIT response scale c_{th} separately from the noise-dependent detection floor c_{min} , and quantify the response and detection sensitivity as functions of coupling strength, ground-state decoherence, optical depth, and two-photon detuning. We further evaluate the effect of Maxwell-Boltzmann thermal motion and discuss experimental limitations arising from longitudinal motion, optical pumping, magnetic inhomogeneity, probe saturation, and finite imaging resolution. The resulting scheme provides a local, all-optical complement to interferometric and modal-projection diagnostics rather than a universal or basis-independent measure of modal purity.

II. THEORETICAL MODEL AND RESPONSE SCALE

We consider the standard Λ -type atomic configuration illustrated in Fig. 1. A weak-probe field with Rabi frequency Ω_p drives the transition $|1\rangle \leftrightarrow |3\rangle$, while a strong spatially structured coupling field with Rabi frequency $\Omega_c(\mathbf{r})$ couples $|2\rangle \leftrightarrow |3\rangle$. The probe is assumed to be a plane wave or a gently focused Gaussian beam with a waist much larger than the transverse features of the coupling field, so that its spatial variation can be neglected in the interaction region. The coupling field is treated within the paraxial and slowly-varying-envelope approximations. In the transverse plane of

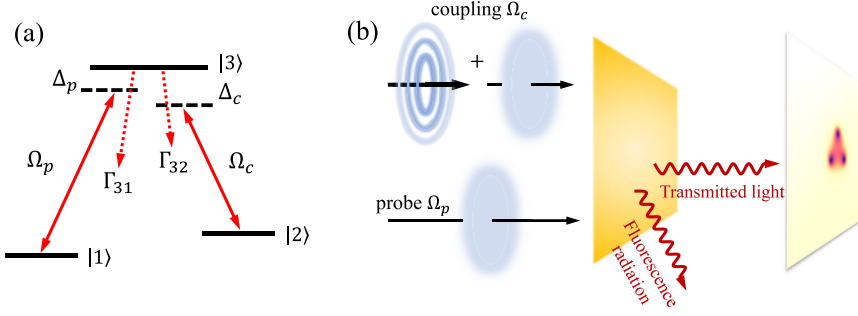


FIG. 1. (a) Three-level Λ scheme. A weak probe interrogates the $|1\rangle \leftrightarrow |3\rangle$ transition, while a vortex-plus-plane-wave coupling field drives $|2\rangle \leftrightarrow |3\rangle$. (b) Transmission geometry. The transverse coupling-field structure is converted into a probe-transmission image by the EIT susceptibility.

the atomic sample, the coupling Rabi frequency is modeled as the superposition of a pure vortex of integer topological charge m_l and a weak copropagating plane-wave admixture:

$$\Omega_c(r, \phi) = c_{\text{tw}} \Omega_{\text{tw}}(r) e^{im_l \phi} + c_{\text{pl}} \Omega_{\text{pl}} e^{i\phi_{\text{pl}}}, \quad (1)$$

where c_{pl} denotes the relative plane-wave field amplitude and $c_{\text{tw}} = \sqrt{1 - c_{\text{pl}}^2}$ is introduced as a convenient peak-amplitude normalization convention. The vortex envelope $\Omega_{\text{tw}}(r)$ is peak normalized (see Sec. IA of the Supplemental Material [33] for the explicit form). Because the twisted-light contribution vanishes completely at the vortex core for any nonzero topological charge, $c_{\text{pl}} = |\Omega_c(0)|/\Omega_{\text{pl}}$ directly represents the on-axis amplitude ratio. Its square is a reference intensity fraction, not an integrated modal-power fraction.

Within the weak-probe regime and under steady-state conditions, the system is described by the density matrix equations. For probe detuning Δ_p and two-photon detuning $\delta = \Delta_p - \Delta_c$, the weak-probe steady-state coherence is

$$\rho_{31} = \frac{i\Omega_p}{2} \frac{\gamma_{21} - i\delta}{(\gamma_{31} - i\Delta_p)(\gamma_{21} - i\delta) + |\Omega_c|^2/4}, \quad (2)$$

and absorption is proportional to $\text{Im } \rho_{31}$. The detailed derivation of the susceptibility is presented in Secs. IB and IC of the Supplemental Material [33]. On exact one- and two-photon resonance, the essential result is that the position-dependent probe absorption coefficient $\alpha(r, \phi)$, normalized to its value α_0 in the absence of the coupling field, takes the compact form

$$\frac{\alpha(r, \phi)}{\alpha_0} = \frac{\Omega_{\text{th}}^2}{|\Omega_c(r, \phi)|^2 + \Omega_{\text{th}}^2}, \quad (3)$$

where $\Omega_{\text{th}} = 2\sqrt{\gamma_{31}\gamma_{21}}$ is the characteristic EIT response scale, γ_{31} the optical coherence decay rate, and γ_{21} the ground-state decoherence rate (see Sec. IB of the Supplemental Material [33] for the explicit definition).

We therefore define the intrinsic EIT amplitude scale as

$$c_{\text{th}} = \frac{\Omega_{\text{th}}}{\Omega_{\text{pl}}}, \quad (4)$$

which gives

$$\frac{\alpha_{\text{core}}}{\alpha_0} = \frac{1}{1 + (c_{\text{pl}}/c_{\text{th}})^2}, \quad (5)$$

where α_{core} is the probe absorption coefficient at the core of the beam. Thus, c_{th} is the intrinsic half-absorption scale, so that $c_{\text{pl}} = c_{\text{th}}$ corresponds to $\alpha_{\text{core}} = \alpha_0/2$. It is not a noise-dependent detection limit.

For a medium of length L with on-resonance optical depth $\text{OD}_0 = \alpha_0 L$, the probe transmission follows the Beer-Lambert law:

$$T(r, \phi) = \exp \left[-\text{OD}_0 \frac{\Omega_{\text{th}}^2}{|\Omega_c(r, \phi)|^2 + \Omega_{\text{th}}^2} \right]. \quad (6)$$

This expression assumes negligible transverse diffraction and a local steady-state response during propagation, conditions expected to be reasonable for the representative short-sample cold-atom regime considered here. For a pure vortex ($m_l \neq 0$), $|\Omega_c(0)| = 0$ and the core transmission is $T_0 = \exp(-\text{OD}_0)$. A finite plane-wave admixture lifts the node, and the core transmission becomes

$$T_{\text{core}}(c_{\text{pl}}) = \exp \left[-\text{OD}_0 \frac{\Omega_{\text{th}}^2}{c_{\text{pl}}^2 \Omega_{\text{pl}}^2 + \Omega_{\text{th}}^2} \right]. \quad (7)$$

For the numerical illustrations throughout this work, we use parameters representative of cold ^{87}Rb atoms (see Sec. ID of the Supplemental Material [33] for the specific values), which yield $c_{\text{th}} \sim 10^{-2}$, enabling detection of very weak plane-wave admixtures through the nonlinear core transmission contrast. The calculation assumes a paraxial coupling field, a weak probe, local steady-state response, and phenomenological coherence rates. It is not a Maxwell-Bloch propagation or multilevel optical-pumping simulation.

Core contrast and spatial signatures. Figure S1 in Sec. II of the Supplemental Material [33] shows how the transverse structure of the coupling field evolves as the plane-wave admixture increases. For a pure vortex ($c_{\text{pl}} = 0$), the intensity exhibits a true dark core and the phase winds by $2\pi m_l$ around the singularity. Adding a small plane-wave component ($c_{\text{pl}} = 0.05$) slightly lifts the on-axis intensity and weakly distorts the annular ring through interference, while the overall vortex structure remains largely intact. As c_{pl} grows further, the annular pattern becomes progressively distorted, although the central feature remains visually inconspicuous until $c_{\text{pl}} \gtrsim 0.5$.

Figure 2 presents the corresponding two-dimensional EIT transmission images at $\text{OD}_0 = 2.0$ for an $m_l = 3$ vortex. The gradual opening of a bright central spot as c_{pl} increases is clearly visible. Already at $c_{\text{pl}} = 0.005$, the core transmission becomes visibly higher than in the pure-vortex case, and the effect strengthens at $c_{\text{pl}} = 0.01$. Because the transmission maps depend on $|\Omega_c|^2$, the on-resonance image does not directly encode the optical phase. Nevertheless, interference between the vortex and plane-wave components produces a characteristic azimuthal modulation. The probe image depends on the coupling intensity:

$$|\Omega_c(r, \phi)|^2 = c_{\text{tw}}^2 |\Omega_{\text{tw}}(r)|^2 + c_{\text{pl}}^2 \Omega_{\text{pl}}^2 + 2c_{\text{tw}} c_{\text{pl}} \Omega_{\text{pl}} \Omega_{\text{tw}}(r) \cos(m_l \phi - \phi_{\text{pl}}). \quad (8)$$

The interference term changes sign azimuthally, producing an asymmetric surrounding profile. For $m_l = 3$, a threefold angular structure appears around the central region, whose orientation rotates with the relative phase ϕ_{pl} [panels (d) and (e)]. At $r = 0$, however, $\Omega_{\text{tw}}(0) = 0$, so the interference term vanishes completely, and the central transmission is fixed symmetrically solely by the plane-wave component and is independent of the azimuthal interference pattern. Thus, the asymmetric structure develops around the core, whereas the on-axis transmission value remains fixed by $c_{\text{pl}}^2 \Omega_{\text{pl}}^2$. The number of modulation features follows $|m_l|$, providing a qualitative visual diagnostic of both the topological charge and the admixture phase. The asymmetry therefore affects the surrounding spatial profile, but not the on-axis transmission value, because the interference term vanishes exactly at the vortex center.

Figure 3(a) quantifies the core transmission as a function of c_{pl} , governed by Eq. (7). For $\text{OD}_0 = 2.0$, the pure-vortex core transmission is $\sim 13.5\%$. With our default parameters, $c_{\text{th}} = 0.01095$, corresponding to an intensity fraction $c_{\text{pl}}^2 \sim 10^{-4}$. Thus, a plane-wave amplitude fraction of just over 1% already produces a substantial change. For $c_{\text{pl}} \ll c_{\text{th}}$, the on-axis coupling is insufficient to

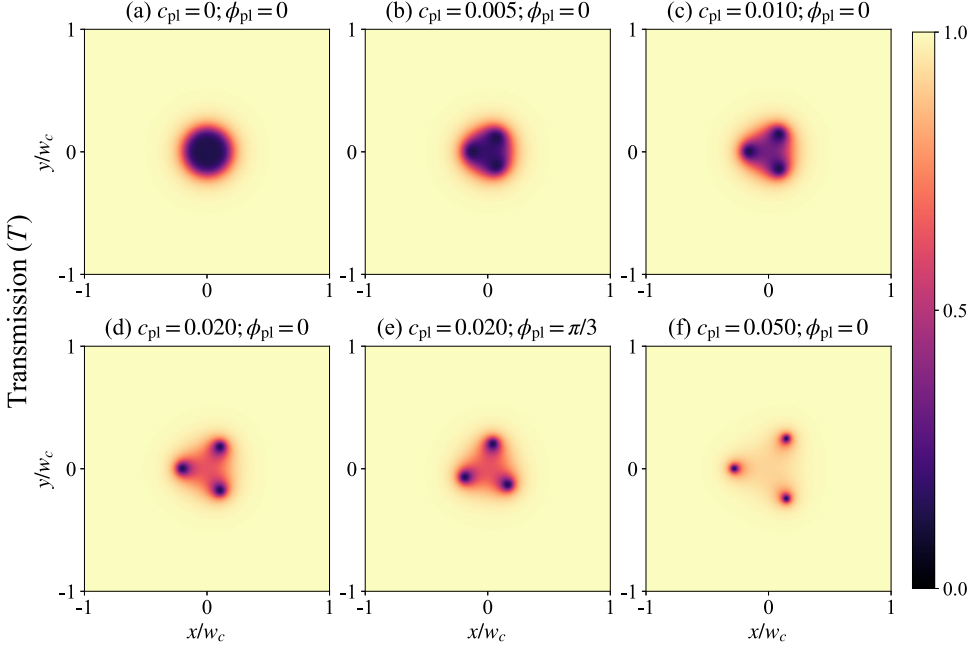


FIG. 2. Calculated resonant probe-transmission maps at $OD_0 = 2.0$ from Eq. (6) for an $m_l = 3$ vortex with increasing plane-wave admixture. The pure vortex leaves an absorptive core. A weak plane-wave admixture opens a transparent central feature and produces azimuthal modulations whose number and orientation are controlled by m_l and ϕ_{pl} .

induce significant EIT transparency, and the core transmission remains near the pure-vortex value. When c_{pl} approaches c_{th} , the denominator term $c_{pl}^2 \Omega_{pl}^2$ becomes comparable to Ω_{th}^2 , and absorption is suppressed. This explains the sharp transition in Fig. 3(a): For $OD_0 = 2.0$, the transmission rises from $\sim 13\%$ to about 67% as c_{pl} increases from zero to twice c_{th} .

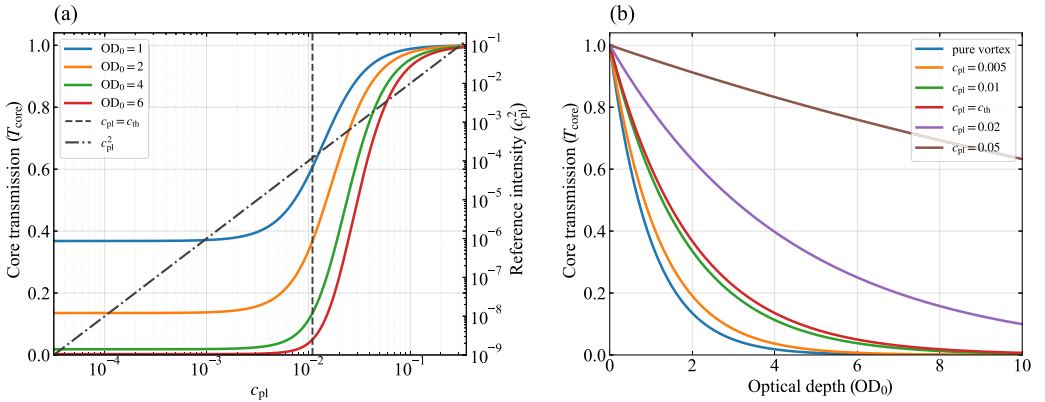


FIG. 3. Resonant nonlinear core contrast. (a) Core transmission vs plane-wave field-amplitude coefficient for several resonant optical depths. The vertical dashed line marks the characteristic EIT response scale $c_{th} = \Omega_{th}/\Omega_{pl}$ for the default parameters. The dot-dashed curve on the right axis shows the corresponding reference intensity fraction c_{pl}^2 . (b) Core transmission vs optical depth OD_0 for six representative admixture amplitudes spanning the characteristic EIT response scale.

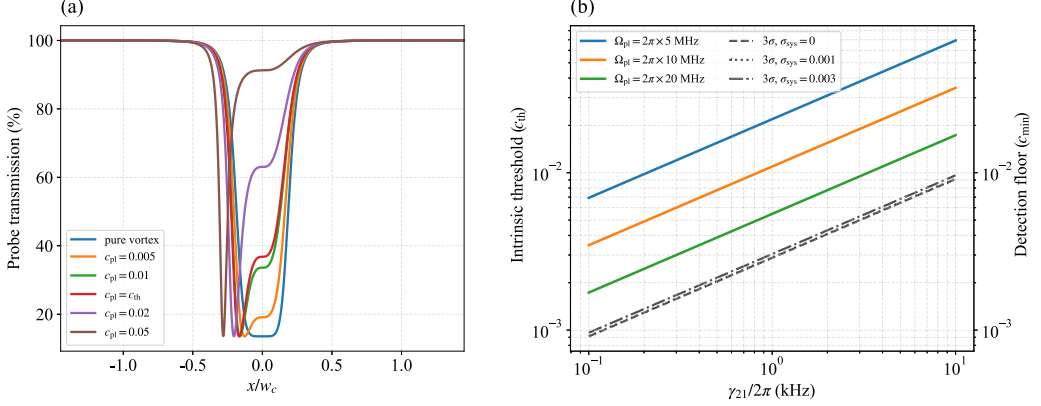


FIG. 4. Spatial response and sensitivity estimates. (a) One-dimensional central line transmission cuts through the center of the image for six representative admixture amplitudes at $OD_0 = 2.0$. (b) Intrinsic EIT response scale c_{th} (solid curves, left axis) and illustrative three-sigma detection floors c_{min} (dashed/dotted curves, right axis) for $N = 4000$ incident probe photons per resolution element.

The resonant optical depth also influences the contrast. A larger OD_0 makes the pure-vortex core darker, increasing the absolute contrast $\Delta T = T_{core}(c_{pl}) - T_0$. For $OD_0 = 4.0$, the baseline transmission drops to 1.8%, so a modest c_{pl} can yield changes of several tens of percent. However, an excessively large OD_0 may render the core nearly completely dark, requiring a larger c_{pl} for observable recovery. Conversely, $OD_0 = 1.0$ gives a brighter baseline, reducing the dynamic range. An intermediate value $OD_0 \approx 2-3$ represents a practical compromise. The right axis of Fig. 3(a) shows c_{pl}^2 , highlighting the key distinction: While the reference intensity fraction is only 10^{-4} at $c_{pl} = 0.01$, the corresponding change in core transmission is already large. Figure 3(b) further illustrates the dependence on OD_0 . For c_{pl} below c_{th} (e.g., 0.005, 0.010), the transmission decays rapidly with OD_0 , leading to complete absorption in optically thick media. Once c_{pl} exceeds c_{th} (e.g., 0.020, 0.050), the stronger local coupling produces increased EIT transparency and slows the decay of the core transmission with optical depth. For $c_{pl} = 0.050$, substantial transparency remains even at $OD_0 = 10$.

III. SENSITIVITY ESTIMATE AND CALIBRATION

To assess the experimental detectability, we adopt a noise model that accounts for both photon shot noise and technical imperfections. We define the signal relative to a separately characterized pure-vortex reference, $\Delta T = T_{core}(c_{pl}) - T_0$, and use the illustrative per-resolution-element transmission uncertainty

$$\sigma_T = \sqrt{\frac{T_{core}(c_{pl})}{N} + \sigma_{sys}^2}, \quad \text{SNR} = \frac{|\Delta T|}{\sigma_T}, \quad (9)$$

where N is the number of incident probe photons per spatial resolution element (pixel) and σ_{sys} represents the effective root-mean-square technical noise level (e.g., camera readout, power fluctuations, and normalization errors). Equation (9) assumes that the reference has been averaged sufficiently that its statistical uncertainty is negligible; otherwise its variance must be added.

For a typical probe pulse with a power of $1 \mu\text{W}$ and a duration of $10 \mu\text{s}$ at 780 nm , the total photon budget is $\sim 4 \times 10^7$. Assuming a moderate spatial resolution where the probe field is mapped onto a 100×100 pixel region to fully resolve the transverse profile of the core, each resolution element receives approximately $N = 4000$ photons, the value used in Fig. 4(b). As an illustrative technical-noise level, we take $\sigma_{sys} = 10^{-3}$. Notably, for $OD_0 = 2.0$ ($T_0 \approx 0.135$), the photon shot

noise is $\sqrt{T_0/N} \approx 5.8 \times 10^{-3}$, meaning that, under the assumed preaveraged reference, the single impurity-bearing test exposure operates primarily in the shot-noise-limited regime rather than being technical-noise dominated.

We define the minimum detectable admixture amplitude $c_{\min}$ by requiring a three-sigma threshold ($\text{SNR} = 3$). Figure 4(a) displays the one-dimensional transmission cuts along $y = 0$ for several admixture amplitudes. As c_{pl} increases, the EIT-induced central transparency fills the absorptive dip. The detection floor $c_{\min}$ is analyzed in Fig. 4(b) as a function of the ground-state decoherence rate γ_{21} . For our default parameters ($\Omega_{\text{pl}} = 2\pi \times 10$ MHz, $\gamma_{21} = 2\pi \times 1$ kHz), the intrinsic EIT response scale is $c_{\text{th}} \approx 1.1 \times 10^{-2}$. Specifically, the three-standard-deviation floor is $c_{\min} = 2.87 \times 10^{-3}$ for shot noise alone and 2.90×10^{-3} for $\sigma_{\text{sys}} = 10^{-3}$. Crucially, $c_{\min}$ remains well below the intrinsic scale c_{th} across the entire relevant parameter range, indicating that admixtures below the characteristic EIT response scale can still produce a macroscopically resolvable transmission contrast. These are model-dependent amplitude floors, not demonstrated experimental sensitivities and not a generic “ 10^{-3} limit.”

For small signed parameter drifts, the intrinsic scale obeys the first-order calibration relation:

$$\frac{\delta c_{\text{th}}}{c_{\text{th}}} = \frac{1}{2} \frac{\delta \gamma_{31}}{\gamma_{31}} + \frac{1}{2} \frac{\delta \gamma_{21}}{\gamma_{21}} - \frac{\delta \Omega_{\text{pl}}}{\Omega_{\text{pl}}}. \quad (10)$$

For a fixed transverse mode profile, $\Omega_{\text{pl}} \propto \sqrt{I_{\text{pl}}}$, so that $\delta \Omega_{\text{pl}}/\Omega_{\text{pl}} = (1/2)\delta I_{\text{pl}}/I_{\text{pl}}$. Thus, coupling intensity and ground-state decoherence rate must be monitored. Optical depth does not shift c_{th} , but it changes contrast and shot noise. An uncertainty in the calibrated optical depth therefore propagates directly into the inferred transmission contrast and hence into $c_{\min}$, even though it does not alter the intrinsic scale c_{th} . Detailed parameter scans are provided in Table S1 and Fig. S2 of the Supplemental Material [33].

Equation (2) also clarifies the off-resonant behavior. Two-photon detuning weakens the core EIT feature, while one-photon detuning changes both absorption and dispersion. Figure S2(c) of the Supplemental Material [33] shows that the smallest illustrated amplitude produces a narrow feature around $\delta = 0$. Detection therefore requires two-photon stabilization within the local EIT linewidth; this requirement becomes more restrictive as c_{pl} decreases. The numerical scan in Fig. S2(c) of the Supplemental Material [33] is restricted to two-photon detuning at $\Delta_p = 0$. The effect of finite one-photon detuning is contained in Eq. (2), but a systematic optimization over both detunings is beyond the scope of the present local model.

IV. THERMAL MOTION AND EXPERIMENTAL LIMITATIONS

Several practical factors must be considered before implementing the proposed method. The local-response approximation in Eq. (6) assumes that atomic motion does not blur the transverse structure during the probe exposure. To quantitatively assess this assumption for a laser-cooled ^{87}Rb ensemble, we consider ballistic transverse motion with a Maxwell-Boltzmann velocity distribution. The exposure-averaged image is

$$\bar{T}_\tau(\mathbf{r}) = \frac{1}{\tau} \int_0^\tau dt \int d^2v_\perp \frac{e^{-v_\perp^2/(2\sigma_v^2)}}{2\pi\sigma_v^2} T(\mathbf{r} - \mathbf{v}_\perp t), \quad \sigma_v = \sqrt{\frac{k_B T_a}{M_{\text{Rb}}}}. \quad (11)$$

This kinematic convolution adopts a frozen-response approximation and does not constitute a velocity-resolved solution of the convective optical Bloch equations. This average includes the high-velocity tail rather than assigning all atoms one mean speed. At $T_a = 100$ μK and $\tau = 10$ μs , the one-component rms displacement remains below 1 μm , much smaller than the representative $w_c = 50$ μm scale. For $c_{\text{pl}} = 0.005$, the Maxwell-Boltzmann average changes the exact-core transmission by less than 10^{-5} , while larger differences occur only near steep off-axis spatial features (Fig. S3 of the Supplemental Material [33]). Motional blurring is therefore negligible for the representative cold-atom parameters considered here, although tighter focusing, longer

exposures, or warmer ensembles require a new average [37]. This contrasts with warm vapors, where thermal diffusion and velocity-changing collisions introduce nonlocal dynamics that reshape the EIT transmission window [38].

Instrumental and optical factors also affect performance. The probe field must remain in the linear-response regime; optical pumping and probe saturation would severely distort the absorption profile and must be avoided by reducing probe intensity and verifying linearity. Finite imaging resolution broadens the apparent core and reduces peak contrast, an effect that can be modeled by convolving the theoretical transmission map with the measured point-spread function. Residual absorptive or refractive backgrounds (e.g., stray light, imperfect polarization) can mimic a weak core signal, making a pure-vortex reference measurement essential for background subtraction. Finally, while Eq. (1) considers a simple plane-wave admixture, the framework can naturally be extended to more general transverse-mode contaminations.

Longitudinal motion and internal-state dynamics impose separate constraints. In an experiment, the ground-state coherence should be interpreted as an effective rate

$$\gamma_{21}^{\text{eff}} = \gamma_{21}^{(0)} + \gamma_{\text{tr}} + \gamma_{\text{op}} + \gamma_{\text{D}} + \gamma_{\text{B}}, \quad (12)$$

where the added terms phenomenologically account for transit, optical-pumping/depolarization, two-photon Doppler, and magnetic-inhomogeneity contributions. For nearly copropagating fields, $\gamma_{\text{D}} \sim |\mathbf{k}_p - \mathbf{k}_c| \sigma_v$; a crossing angle increases this scale. Longitudinal atomic motion can additionally change the interaction time and produce a velocity-dependent Doppler shift; these effects are included phenomenologically in γ_{tr} and γ_{D} . A realistic implementation therefore requires a closed or repumped Λ system, weak-probe linearity checks, magnetic shielding and bias-field stabilization, coupling-power monitoring, optical-depth calibration, temporal stationarity over the exposure, and registration of the atomic image to a measured pure-vortex reference. These requirements are not contained in the ideal local susceptibility model.

V. CONCLUSION

We have developed a theoretical scheme for transducing a weak coherent plane-wave admixture in an optical vortex into a spatially resolved probe-transmission signal using electromagnetically induced transparency. For the representative cold- ^{87}Rb parameters considered here, the intrinsic EIT response scale is $c_{\text{th}} = 1.10 \times 10^{-2}$, while the illustrative three-standard-deviation detection floor is $c_{\text{min}} \approx 2.9 \times 10^{-3}$ for $N = 4000$ incident probe photons per spatial resolution element. The predicted sensitivity depends on the coupling strength, ground-state decoherence, optical depth, and two-photon detuning, while Maxwell-Boltzmann averaging indicates negligible modification of the exact-core transmission and only limited spatial blurring for the stated 100 μK , 10 μs , and 50 μm example. In practice, the achievable sensitivity will additionally be limited by longitudinal motion, optical pumping, magnetic inhomogeneity, probe saturation, imaging resolution, and calibration errors. The proposed method should therefore be viewed as a calibrated local diagnostic of a weak coherent field amplitude at a known vortex node, complementary to interferometric and modal-projection techniques rather than as a universal measure of modal purity.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

- [1] J. F. Nye and M. V. Berry, Dislocations in wave trains, [Proc. R. Soc. London A](#) **336**, 165 (1974).
- [2] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes, [Phys. Rev. A](#) **45**, 8185 (1992).
- [3] M. S. Soskin and M. V. Vasnetsov, Singular optics, in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 2001), Vol. 42, pp. 219–276.
- [4] G. Molina-Terriza, J. P. Torres, and L. Torner, Twisted photons, [Nat. Phys.](#) **3**, 305 (2007).
- [5] A. M. Yao and M. J. Padgett, Orbital angular momentum: Origins, behavior and applications, [Adv. Opt. Photon.](#) **3**, 161 (2011).
- [6] H. Rubinsztein-Dunlop *et al.*, Roadmap on structured light, [J. Opt.](#) **19**, 013001 (2017).
- [7] A. Forbes, M. de Oliveira, and M. R. Dennis, Structured light, [Nat. Photon.](#) **15**, 253 (2021).
- [8] A. Mair, A. Vaziri, G. Weihs, and A. Zeilinger, Entanglement of the orbital angular momentum states of photons, [Nature \(London\)](#) **412**, 313 (2001).
- [9] A. C. Dada, J. Leach, G. S. Buller, M. J. Padgett, and E. Andersson, Experimental high-dimensional two-photon entanglement and violations of generalized Bell inequalities, [Nat. Phys.](#) **7**, 677 (2011).
- [10] M. Krenn, M. Huber, R. Fickler, R. Lapkiewicz, S. Ramelow, and A. Zeilinger, Generation and confirmation of a (100×100)-dimensional entangled quantum system, [Proc. Natl. Acad. Sci. USA](#) **111**, 6243 (2014).
- [11] M. Mirhosseini, O. S. Magaña-Loaiza, M. N. O’Sullivan, B. Rodenburg, M. Malik, M. P. J. Lavery, M. J. Padgett, D. J. Gauthier, and R. W. Boyd, High-dimensional quantum cryptography with twisted light, [New J. Phys.](#) **17**, 033033 (2015).
- [12] J. Wang, J.-Y. Yang, I. M. Fazal, N. Ahmed, Y. Yan, H. Huang, Y. Ren, Y. Yue, S. Dolinar, M. Tur, and A. E. Willner, Terabit free-space data transmission employing orbital angular momentum multiplexing, [Nat. Photon.](#) **6**, 488 (2012).
- [13] N. Bozinovic, Y. Yue, Y. Ren, M. Tur, P. Kristensen, H. Huang, A. E. Willner, and S. Ramachandran, Terabit-scale orbital angular momentum mode division multiplexing in fibers, [Science](#) **340**, 1545 (2013).
- [14] A. E. Willner, H. Huang, Y. Yan, Y. Ren, N. Ahmed, G. Xie, C. Bao, L. Li, Y. Cao, Z. Zhao, J. Wang, M. P. J. Lavery, M. Tur, S. Ramachandran, A. F. Molisch, N. Ashrafi, and S. Ashrafi, Optical communications using orbital angular momentum beams, [Adv. Opt. Photon.](#) **7**, 66 (2015).
- [15] D. G. Grier, A revolution in optical manipulation, [Nature \(London\)](#) **424**, 810 (2003).
- [16] K. Dholakia and T. Čížmár, Shaping the future of manipulation, [Nat. Photon.](#) **5**, 335 (2011).
- [17] M. Babiker, W. L. Power, and L. Allen, Light-induced torque on moving atoms, [Phys. Rev. Lett.](#) **73**, 1239 (1994).
- [18] M. Babiker, C. R. Bennett, D. L. Andrews, and L. C. Dávila Romero, Orbital angular momentum exchange in the interaction of twisted light with molecules, [Phys. Rev. Lett.](#) **89**, 143601 (2002).
- [19] C. T. Schmiegelow, J. Schulz, H. Kaufmann, T. Ruster, U. G. Poschinger, and F. Schmidt-Kaler, Transfer of optical orbital angular momentum to a bound electron, [Nat. Commun.](#) **7**, 12998 (2016).
- [20] S. A.-L. Schulz, A. A. Peshkov, R. A. Müller, R. Lange, N. Huntemann, C. Tamm, E. Peik, and A. Surzhykov, Generalized excitation of atomic multipole transitions by twisted light modes, [Phys. Rev. A](#) **102**, 012812 (2020).
- [21] J. Leach, M. J. Padgett, S. M. Barnett, S. Franke-Arnold, and J. Courtial, Measuring the orbital angular momentum of a single photon, [Phys. Rev. Lett.](#) **88**, 257901 (2002).
- [22] J. Leach, J. Courtial, K. Skeldon, S. M. Barnett, S. Franke-Arnold, and M. J. Padgett, Interferometric methods to measure orbital and spin, or the total angular momentum of a single photon, [Phys. Rev. Lett.](#) **92**, 013601 (2004).
- [23] E. Bolduc, N. Bent, E. Santamato, E. Karimi, and R. W. Boyd, Exact solution to simultaneous intensity and phase encryption with a single phase-only hologram, [Opt. Lett.](#) **38**, 3546 (2013).
- [24] A. Forbes, A. Dudley, and M. McLaren, Creation and detection of optical modes with spatial light modulators, [Adv. Opt. Photon.](#) **8**, 200 (2016).
- [25] M. Mirhosseini, M. Malik, Z. Shi, and R. W. Boyd, Efficient separation of the orbital angular momentum eigenstates of light, [Nat. Commun.](#) **4**, 2781 (2013).
- [26] G. C. G. Berkhout, M. P. J. Lavery, J. Courtial, M. W. Beijersbergen, and M. J. Padgett, Efficient sorting of orbital angular momentum states of light, [Phys. Rev. Lett.](#) **105**, 153601 (2010).

- [27] M. P. J. Lavery, D. J. Robertson, G. C. G. Berkhout, G. D. Love, M. J. Padgett, and J. Courtial, Refractive elements for the measurement of the orbital angular momentum of a single photon, *Opt. Express* **20**, 2110 (2012).
- [28] D. Flamm, D. Naidoo, C. Schulze, A. Forbes, and M. Duparré, Mode analysis with a spatial light modulator as a correlation filter, *Opt. Lett.* **37**, 2478 (2012).
- [29] M. Bahl, G. M. Koutentakis, M. Maslov, T. Jungnickel, T. Gaßen, M. Lemeshko, and O. H. Heckl, The R-index: A universal metric for evaluating OAM content and mode purity in optical fields, *J. Phys. Photon.* **8**, 015071 (2026).
- [30] R. P. Schmidt, S. Ramakrishna, A. A. Peshkov, N. Huntemann, E. Peik, S. Fritzsche, and A. Surzhykov, Atomic photoexcitation as a tool for probing purity of twisted light modes, *Phys. Rev. A* **109**, 033103 (2024).
- [31] N. S. Mallick and S. De, Phase-controlled efficient detection of topological charge of vortex Bessel beam, *Appl. Phys. B* **131**, 168 (2025).
- [32] N. B. Phillips, G. V. Romanov, W. F. Ames, and I. Novikova, Optical vortex filtering for the detection of electromagnetically induced transparency, *J. Opt. Soc. Am. B* **28**, 2129 (2011).
- [33] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/8tqj-y91q> for the coupling-field parametrization, detailed derivation of the EIT susceptibility, sensitivity calibration, detuning dependence, and transverse-motion analysis, which includes Refs. [34–36,38].
- [34] R. S. N. Moreira, J. C. de Aquino Carvalho, and J. W. R. Tabosa, Amplification and storage of light carrying orbital angular momentum via magnetically assisted gain in cold atoms, *J. Opt.* **25**, 024003 (2023).
- [35] Y. Wang, M. Guo, J. Wu, J. Liu, X. Yang, and J. Li, Propagation of Gaussian vortex beams in electromagnetically induced transparency media, *Opt. Express* **30**, 43426 (2022).
- [36] J. Gea-Banacloche, Y.-Q. Li, S.-Z. Jin, and M. Xiao, Electromagnetically induced transparency in ladder-type inhomogeneously broadened media: Theory and experiment, *Phys. Rev. A* **51**, 576 (1995).
- [37] M. Shuker, O. Firstenberg, R. Pugatch, A. Ron, and N. Davidson, Storing images in warm atomic vapor, *Phys. Rev. Lett.* **100**, 223601 (2008).
- [38] O. Firstenberg, M. Shuker, R. Pugatch, D. R. Fredkin, N. Davidson, and A. Ron, Theory of thermal motion in electromagnetically induced transparency: Effects of diffusion, Doppler broadening, and Dicke and Ramsey narrowing, *Phys. Rev. A* **77**, 043830 (2008).