

Machine-learning-assisted characterization of labyrinthine pattern transitions

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Labyrinthine structures are ubiquitous in pattern-forming systems arising from a wide variety of physical, chemical, and biological processes. Yet, their characterization remains challenging due to the lack of long-range order and the resultant complicated morphologies. Here, we present an approach for characterizing labyrinthine patterns based on the identification and the correlation analysis of their topological defects. We employ advanced machine-learning-based-pattern-recognition techniques to identify the types and locations of topological defects of the local stripe ordering. Applying this method to single-crystal Bi-substituted yttrium iron garnet films, we uncover a distinct morphological transition between two zero-field labyrinthine structures. Crucially, the pair distribution functions of the topological defects reveal subtle differences between labyrinthine structures, which are not discernible from conventional global analysis. By systematically analyzing the spatial correlations and geometric properties of these defects, we provide insights into the athermal dynamics that govern the observed morphological transitions. Our work demonstrates that machine-learning-based recognition techniques enable systematic studies of rich and complex labyrinthine-type structures universal to many pattern-formation systems.

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I. INTRODUCTION

Labyrinthine structures are ubiquitous in out-of-equilibrium nonlinear systems ranging from biological and chemical reactions to fluid convection, crystal growth, and magnetic ordering [1–5]. In such pattern-forming systems, the complex structures emerge as a result of nonlinear competing interactions. The labyrinthine patterns are generally characterized by stripe domains of different orientations, sizes, and grain-boundary structures. The predominance of periodic stripes indicates the breaking of translational symmetry locally. Yet, contrary to long-range ordered states in an equilibrium phase transition, labyrinthine patterns are essentially disordered and cannot be described by a well-defined order parameter. Indeed, labyrinthine structures can be viewed as intermediate between a featureless short-range-correlated glassy state and long-range-ordered stripe or crystalline phases [6].

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Despite their prevalence in pattern-forming systems, a complete characterization of labyrinthine structures is still lacking [7]. A defining characteristic of labyrinthine patterns is the ringlike isotropic feature in its structure factor obtained from conventional Fourier analysis [8]. The radius and width of the ring correspond to the wavelength of local stripes and characteristic size of stripe domains, respectively [8–11]. While such global Fourier analysis provides a basic characterization of labyrinths, it fails to capture subtle differences between labyrinthine patterns, which have important structural or dynamical implications. Other useful measures, such as the disorder functions [12–14], have been introduced to quantify deviations from a perfect stripe order. Another important characterization often employed is the density of topological defects of labyrinthine structures [15–18]. Indeed, the distribution and correlation between topological defects, such as disclinations and dislocations, of the stripe order encode important information about the labyrinths [17,18]. However, efficient and accurate identifications of such pointlike defects in large-scale experimental or simulation data remain a challenging task.

In this paper, we present a comprehensive framework for the characterization of labyrinthine structures in nonequilibrium morphological transition of labyrinthine domains in yttrium iron garnet (YIG), a well-known technologically important ferromagnetic material. By leveraging a two-step algorithm that consists of rotation-invariant template matching (TM) followed by convolutional neural network (CNN) filtering, which we call TM-CNN, we precisely identify topological defects called junctions and terminals, and their coordinates in the images. The high-precision real-space configuration data allow us to compute correlation functions of the topological defects by using the information of a sufficiently large number of defects, which provides valuable information about labyrinthine patterns that are complementary to the results from, e.g., conventional Fourier analysis. Through the TM-CNN analysis, we systematically clarify the number, positions, structure factors, radial distribution function, and the effective coordination number of all pairs of topological defects. Furthermore, we successfully quantify the morphological transitions of magnetic labyrinths caused by the demagnetization process by revealing the two-step process: The increase in the number of paired topological defects accompanying the increase in spatial coherence, followed by the denser packing of local stripes to make the labyrinthine structure more compact.

The rest of the paper is organized as follows. In Sec. II, we explain the experimental protocol of the demagnetization process and describe the observed morphological transition of magnetic labyrinthine patterns in YIG. In Sec. III, we provide the result of the conventional global Fourier analysis. In Sec. IV, we provide the analysis based on TM-CNN and the characterization of the observed morphological transition based on the correlation of topological defects. Section V is devoted to the summary.

II. EXPERIMENTS

In the present study, we focus on a pattern transition in experiments performed on Bi-doped YIG films. Magnetic interactions in YIG are dominated by a short-range ferromagnetic exchange and long-range dipolar interactions [19]. YIG films doped with bismuth are known to carry a strong perpendicular magnetic anisotropy and form complex labyrinthine patterns [16–18]. Bismuth doping introduces a number of desirable advantages for magneto-optic isolators and sensors [20,21], enhances the Faraday rotation, improves the perpendicular anisotropy, and lowers the saturation magnetization [19–22]. For films grown under appropriate conditions, the magnetization can be saturated in fields of less than 100 Oe. Thus, the patterns of the labyrinthine stripe domain are easily observed with a small electromagnet in polarized light under a microscope [23]. All measurements in this study were performed at room temperature. The images obtained covered an area of 2 mm × 1.8 mm. Two samples grown under similar conditions were studied.

The pattern transitions in this material are observed by stepping down a perpendicular magnetic field to zero, starting from a fully saturated state following an “annealing” protocol. In this protocol, starting from the sample in the fully magnetized state, we instantly drop the field to zero where it is held for 10 s during which an image is acquired. Further sequential demagnetization is carried out

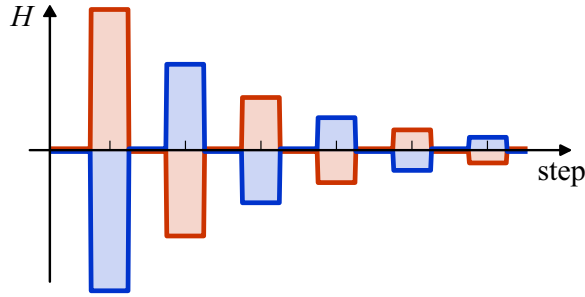


FIG. 1. Schematic illustration of the external magnetic field stepping protocol. The sequence beginning from the positive (negative) magnetic field is represented by the red (blue) line.

but with the magnitude of the field reduced at each step exponentially and with alternate reversal of field directions as indicated in Fig. 1. Experimental images of labyrinthine patterns obtained during the demagnetization process are shown in Fig. 2.

Comparing the patterns at step 0 [Fig. 2(a)] with those at step 12 [Fig. 2(b)] and step 24 [Fig. 2(c)], clear qualitative differences emerge. In the initial state, the boundaries between bright and dark domains exhibit a sinuous, irregular structure, lacking parallel alignment. In contrast, the later state features well defined, highly parallel domains with approximately equal widths of bright and dark regions. Additionally, the areas occupied by these regions remain roughly balanced across any given sample. This approximate equilibrium can manifest through various patterns that appear visually similar. Such morphological transitions have been consistently observed in repeated experiments. Hereafter, we refer to the structures formed in the early stage as the quenched state and those in the later stage as the annealed state, and we proceed to discuss their characterization in detail.

Importantly, these labyrinthine patterns consist of a plethora of defects that can be classified into junctions where three domains meet and terminals where domains end. Representative images of a junction and terminal are shown in Figs. 2(d) and 2(e), respectively. Importantly, they are also known as the topological defects associated with the rotational symmetry breaking brought about by the local stripe formation [16]. Due to their topological properties, they introduce constraints to the deformation of labyrinthine structures. For instance, when introducing one of these defects into a stripe, it inevitably gives rise to the appearance of the other. In other words, the disorder manifests as pairs of junctions and terminals. We note that they are also called disclinations, and the topological charges of junctions and terminals are given by $-1/2$ and $+1/2$, respectively (see the Appendix).

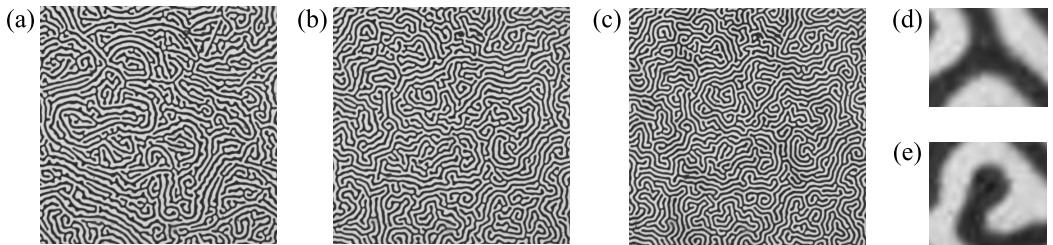


FIG. 2. Evolution of magnetic domain images under the demagnetization process: (a) the quenched state at step 0, (b) the transient state at step 12, and (c) the annealed state at step 24. Representative structures of a (d) junction and (e) terminal.

III. GLOBAL FOURIER ANALYSIS

To characterize the morphologically distinct states shown in Fig. 2, we first employ the conventional global Fourier analysis. For each experimental labyrinthine image, we compute the structure factor defined by

$$\mathcal{S}(\mathbf{q}) = \langle |w_{\epsilon,n,m}(\mathbf{q})|^2 \rangle_{\epsilon,n}, \quad (1)$$

where $w_{\epsilon,n,m}(\mathbf{q})$ is the Fourier transform of the gray-scaled magnetic domain image obtained in experiments at step m ($m = 0, 1, \dots, 36$) of the n th trial ($n = 1, 2, \dots, 6$) under the field stepping protocol starting from the positive ($\epsilon = 1$) and negative ($\epsilon = -1$) field. In Eq. (1), $\mathbf{q} = (q_x, q_y)$ denotes the wave vector and $\langle \dots \rangle_{\epsilon,n}$ represents the average over ϵ and n .

In Fig. 3(a), we show the structure factors for the quenched and annealed states, manifesting circular patterns due to the isotropic nature of the system. The circular pattern is smaller and thicker in the quenched state than in the annealed state. Since the intensities of the structure factor shown in Fig. 3(a) are rotationally symmetric, we further compute the angle-averaged structure factor $I(q)$; we introduce the amplitude of the wave vector q and the polar angle ϕ as $\mathbf{q} = q(\cos \phi, \sin \phi)$, and average $\mathcal{S}(\mathbf{q})$ over ϕ to obtain $I(q)$. The evolution of $I(q)$ across demagnetization steps is shown in Fig. 3(b). As discussed above, the demagnetization leads to the larger radius of the circular pattern, which becomes apparent as the wave number q where $I(q)$ takes the maximum value moves to the right.

To capture the intricate evolution of the peak structure, we fit $I(q)$ by a Gaussian given by $A \exp(-(q - q_0)^2 / (2\sigma^2))$ and extract the peak width σ as well as the peak wave number q_0 . Their step dependencies are shown in Fig. 3(c), exhibiting the decrease of σ and the increase of q_0 as the demagnetization process proceeds. Notably, σ and q_0 exhibit sharp transitions at distinct steps; σ decreases by $\sim 20\%$ between steps 6 and 12 (yellow shaded area), while q_0 increases by $\sim 20\%$ between steps 10 and 20 (green shaded area). It is also clearly seen that between steps 10 and 20 (when the quenching magnetic field falls below 12 gauss), the domains settle down to the annealed state. Considering that $1/\sigma$ and $2\pi/q_0$ correspond to the correlation length and the magnetic modulation period, respectively, we can infer that through the demagnetization process, the labyrinthine structure, which is initially less compact in the quenched state, aligns itself to increase the correlation length and subsequently shortens the magnetic period, transitioning into a more compact annealed state. Further application of the smaller magnetic field reorganizes the domain pattern locally at various points but leaves the overall features intact.

The conventional analysis presented thus far provides a global picture of domain reorganization during the annealing process. However, it fails to yield a detailed understanding of the local response of junctions and terminals. Specifically, the transformation of the labyrinthine pattern is closely associated with the dynamics of local topological defects, which are intrinsically linked to the geometrical properties of labyrinths. This evolution often involves phenomena such as the annihilation of defect pairs. As the dynamics of the labyrinth patterns is mediated by the defects introduced in Fig. 2, the precise identification of their number and coordinates is indispensable for further clarification.

IV. TM-CNN-BASED CHARACTERIZATION

In this section, we briefly introduce the method to detect the positions of junctions and terminals, called template matching-convolutional neural network (TM-CNN), and then use this method to quantify the evolution of these defects by analyzing the experimental data.

A. Template matching-convolutional neural network

For better quantification of the morphological transition observed in the experiments, identifying the positions of topological defects, namely, junctions and terminals, is crucial, as they mediate the local transformation of stripes. However, each image contains a high density of junctions

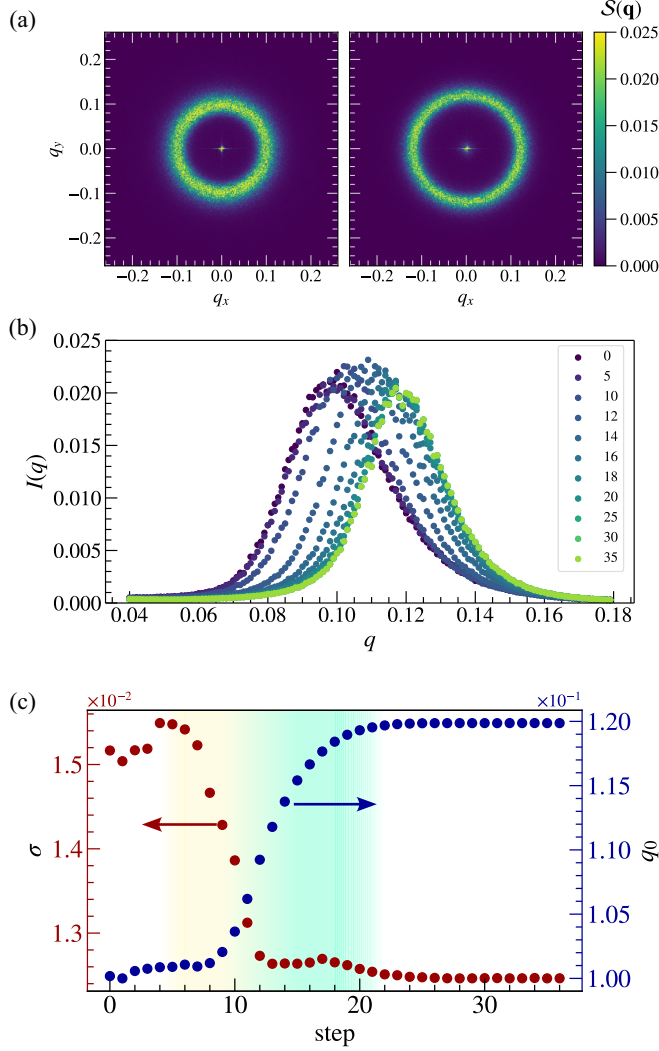


FIG. 3. (a) Structure factor of the experimentally obtained images of labyrinthine patterns $S(\mathbf{q})$ in Eq. (1) for (left) the quenched state at step 0 and (right) the annealed state at step 36. (b) The angle-averaged structure factor, $I(q)$, obtained by averaging $S(\mathbf{q})$ over the polar angle in the Fourier space, for different steps represented by different colors. (c) Evolution of the width of the peak in $I(q)$, σ , and its peak wave number, q_0 , respectively, represented by the red and the blue circles.

and terminals, making it infeasible to identify these defects throughout the entire morphological transition process manually. Recently, we developed a two-step detection algorithm capable of identifying thousands of small objects with minimal manual annotation [24]. In the first step, we employ rotation-invariant TM to identify candidate points for junctions and terminals with a low threshold value to avoid any false negatives, even if it results in several false positives. In the second step, the candidate points from the first stage are processed through the CNN classifier, which differentiates between terminals, junctions, and false detections. The detailed algorithm and the benchmark demonstrating outperforming accuracy for detecting junctions and terminals are presented in Ref. [24].

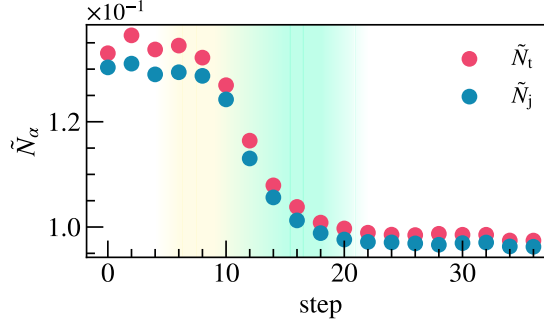


FIG. 4. Evolution of the normalized number of defects $\tilde{N}_\alpha$ defined in Eq. (3). The pink and blue dots show the results for terminals ($\tilde{N}_t$) and junctions ($\tilde{N}_j$), respectively. The results shown are averaged over all trials with $\epsilon = 1$.

For the template matching, we manually created small “template images” representing typical junction and terminal shapes observed in experiments only once during the system’s setup. During the TM process, these templates are systematically matched to regions of the labyrinthine image using the TM algorithm, generating similarity measures stored as correlation maps. We achieve rotation-invariant detection by following the approach detailed in Ref. [25]. For the CNN, we selected some images containing candidate points detected by TM, manually corrected the false positives, and used them to train a CNN classifier. We note that this approach significantly reduces the annotation workload, as we now only need to review the candidate points rather than locating and classifying every defect in the image. By using the trained CNN, the candidate points from the first stage are processed through the classifier in the second step. More details about the proposed algorithm are provided in Ref. [24] and the executable program to demonstrate the automated detection for experimental images is available for download [26].

B. Defect density

First, we compute the defect density

$$\rho_\alpha = \frac{\langle N_\alpha \rangle_n}{V}, \quad (2)$$

where $\langle N_\alpha \rangle_n$ represents the total number of defects averaged over six trials ($n = 1, 2, \dots, 6$), α denotes j for junctions and t for terminals, and V is the image area measured in pixels. When ϵ switches between $+1$ and -1 , the population of black and white domains exhibits slight variation due to the memory of the previous step with the field quench in the opposite direction. Therefore, in the following analysis, we apply the TM-CNN algorithm exclusively to black domains in the images and consider only the case of $\epsilon = +1$, where the magnetic field direction remains unchanged.

In the present system, the period of stripes constituting the labyrinthine patterns varies at each step. To take into account this change of the characteristic length scale, we introduce a normalized quantity utilizing the stripe periodicity $\lambda = 2\pi/q_0$, with peak wave number q_0 obtained in Sec. III, as

$$\tilde{N}_\alpha = \rho_\alpha \lambda^2. \quad (3)$$

Figure 4 shows the evolution of $\tilde{N}_\alpha$ during demagnetization. $\tilde{N}_t$ and $\tilde{N}_j$ remain comparable and exhibit similar behavior throughout the process; $\tilde{N}_\alpha$ initially remains relatively constant for the first 10 steps and gradually decreases between steps 10 and 20. The observed decrease in the number of junctions and terminals is caused by pair creation and annihilation of these defects associated with their topological nature (see the Appendix). The decreasing $\tilde{N}_\alpha$ suggests that the annealed state

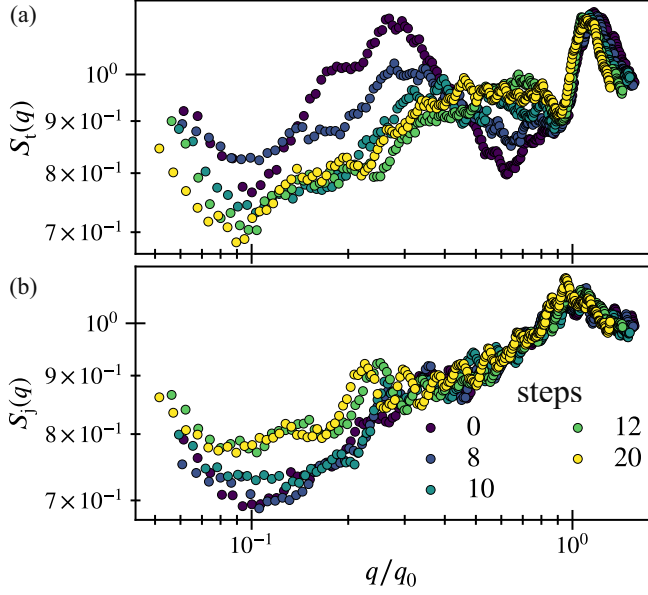


FIG. 5. Structure factor for the positions of (a) terminals $[S_t(q)]$ and (b) junctions $[S_j(q)]$. The wavelength q is rescaled by the ordering wavelength q_0 estimated from the Fourier analysis in Fig. 3(c).

exhibits greater spatial coherence and fewer defects than the quenched state. This observation is consistent with the increased correlation length $1/\sigma$ observed in the Fourier analysis [see Fig. 3(c)].

C. Structure factor of defects

While Fig. 4 shows clear drops in the total number of defects at the transition, the dynamics of local structure evolution remain unclear. Since junctions and terminals are topological defects with opposite charges associated with stripe formation, such local dynamics is mediated by defects. To further quantify the morphological transition through characterizing the local dynamics of defects, we compute the structure factor of defects. From TM-CNN, we detect positions of the i th defect of type $\alpha = t$ or j , $\mathbf{r}_{\alpha,i}$, and compute the structure factor

$$S_\alpha(\mathbf{q}) = \left\langle \frac{1}{N_\alpha} \sum_{i,k} e^{i\mathbf{q} \cdot (\mathbf{r}_{\alpha,i} - \mathbf{r}_{\alpha,k})} \right\rangle_n. \quad (4)$$

Since the system is isotropic, we take an angle average of $S_\alpha(\mathbf{q})$ with respect to the wave vector $\mathbf{q}$ and compute $S_\alpha(q)$ with $q = |\mathbf{q}|$ (see Sec. III for details).

Figure 5(a) shows the structure factor for terminals $S_t(q)$. The horizontal axis is rescaled by the ordering wave vector q_0 shown in Fig. 3(c). The structure factor exhibits a peak near $q \simeq q_0$; however, regardless of the demagnetization step, the peak position is slightly above q_0 . The peak height remains nearly unchanged, and the peak width decreases as the step goes on. In the intermediate wavelength region ($0.1q_0 \lesssim q \lesssim q_0$), $S_t(q)$ exhibits a secondary peak, whose position and height show a pronounced step-dependent behavior. Specifically at step 0, the peak is observed around $q \sim 0.3q_0$ with a comparable magnitude to the main peak at q_0 . As the demagnetization step goes on, this secondary peak shifts toward a higher wavelength and diminishes in intensity. After step 10, the peak position is intact from $q \sim 0.6q_0$ and exhibits minimal further change. This pronounced step dependence of the subdominant peak serves as a hallmark of the experimentally observed morphological transition, and we will discuss the interpretation of the changes later. In the lower wavelength regime ($q \lesssim 0.1q_0$), $S_t(q)$ increases with decreasing q .

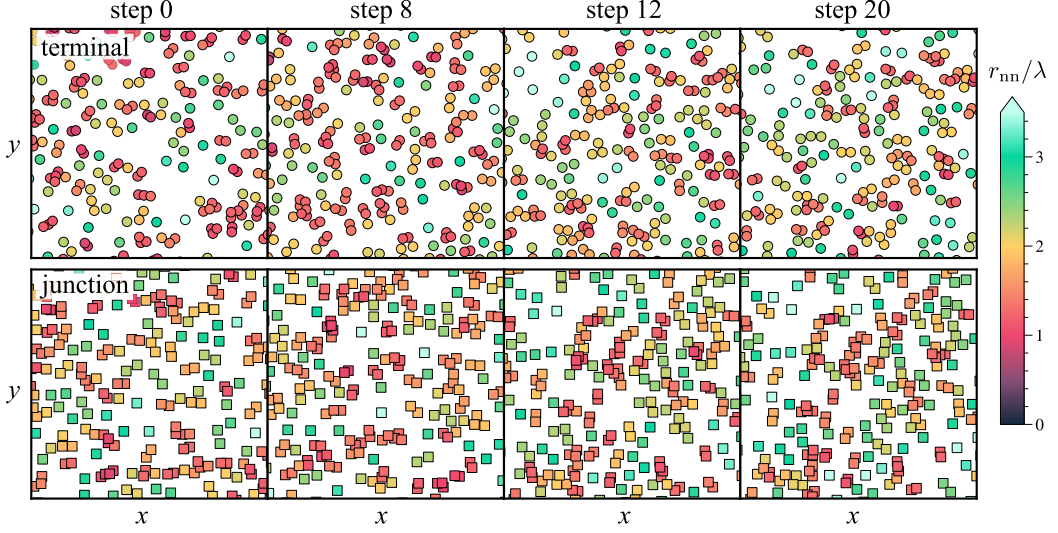


FIG. 6. Real-space distribution of (a) terminals and (b) junctions for several demagnetization steps. Terminals and junctions are represented by circles and squares, respectively. The color represents the distance to the closest different defects r_{nn} rescaled by the stripe period $\lambda = 2\pi/q_0$. The results shown here correspond to a 2400×2400 pixel region obtained from the experiment with $\epsilon = 1$ and $n = 1$.

The structure factor for junctions $S_j(q)$ is shown in Fig. 5(b). Similar to terminals, peaks appear around $q \sim q_0$, though they are broader than those for terminals; junctions show less correlation than terminals. Furthermore, the overall peak profile remains similar across demagnetization steps. In the intermediate wavelength regime, unlike the terminal case, no distinct secondary peak is observed; $S_j(q)$ shows a power-law decrease with q . In the low-wavelength region, it exhibits a similar upturn as seen for terminals. The spatial arrangement of junctions exhibits low correlations due to the broad peak at $q \sim q_0$ and remains largely unaffected by the morphological transition induced by the demagnetization process.

To gain a physical understanding of the structure factor analysis presented above, we examine the real-space configurations of terminals and junctions. Figure 6 displays snapshots of the spatial distribution of terminals and junctions identified by the TM-CNN at different demagnetization steps. To enhance the visibility of spatial correlations, each defect is color coded according to its distance to the nearest neighbor defect of the same type, denoted as r_{nn} .

We first focus on the terminals. The real-space arrangement of terminals exhibits a disordered character. At step 0, a large number of terminals show $r_{nn}/\lambda \sim 1$. Interestingly, terminals with $r_{nn}/\lambda \sim 1$ tend to form clusters together in groups, and clusters containing four or more terminals are readily observed in the figure. This feature undergoes a significant change around step 10. From step 12 onward, the population of terminals with $r_{nn}/\lambda \sim 1$ decreases, while those with $r_{nn}/\lambda \sim 2$ become more prevalent. In the snapshot corresponding to step 20, small clusters composed of two or three terminals are still observed, but larger clusters with four or more terminals become scarce.

Such step dependencies are elucidated by the histograms of the nearest-neighbor and next-nearest-neighbor defect distances, r_{nn} and r_{nnn} , shown in Figs. 7(a) and 7(b), respectively. In the quenched state at step 0, terminals are likely to have their nearest neighbors located at $r_{nn} \sim \lambda$; however, as the morphological transition to the annealed state undergoes, this probability diminishes and becomes comparable to the probability of finding nearest neighbors at $r_{nn} \sim 2\lambda$. This trend is consistent with the reduction in red-colored terminals with $r_{nn} \sim \lambda$ in Fig. 6. Furthermore, the distribution of next-nearest-neighbor distances r_{nnn} reveals a marked decrease in the peak around $r_{nnn} \sim \lambda$, while the structure at $r_{nnn} \gtrsim 2\lambda$ remains largely unchanged. This indicates a reduced

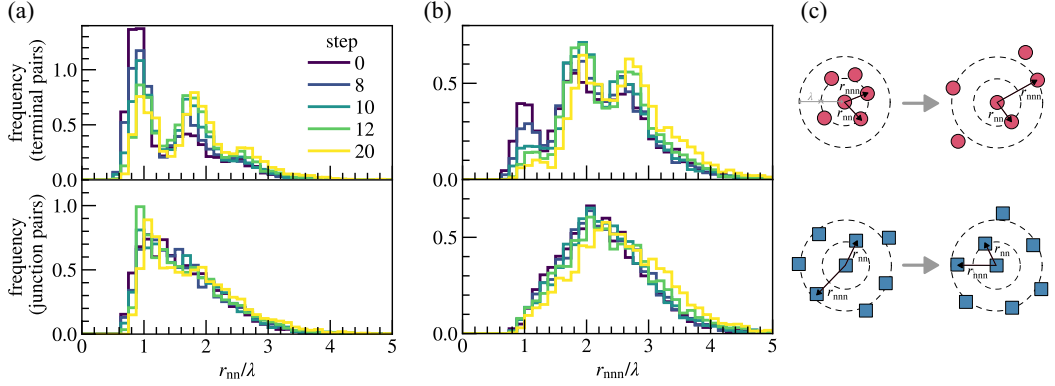


FIG. 7. (a) Normalized histogram of the nearest-neighbor defect distance r_{nn} for terminal-terminal pairs (top) and junction-junction pairs (bottom). The results for different steps are presented. (b) Similar figure for the next-nearest-neighbor defect distance r_{nnn} . (c) Schematic illustrations for the spatial evolution of terminals (top) and junctions (bottom).

probability of finding additional terminals within a radius λ from a given terminal. This trend is consistent with the reduction in the size and the number of terminal clusters composed of many terminals in Fig. 6.

These observations provide insight into the characteristic behavior of $S_t(q)$ in the intermediate wave number region, which serves as a hallmark of the morphological transition. Terminal clusters allow a given terminal to find others not only at a distance λ , but also at 2λ , 3λ , and beyond. Although each cluster is relatively small, their large number contributes to the emergence of a secondary peak at $q \sim 0.3q_0$ in the intermediate- q regime of $S_t(q)$ in Fig. 5(a). As the demagnetization process proceeds, the size and the number of such clusters decrease. This, in turn, lowers the probability of finding other terminals at distances such as 3λ or 4λ , resulting in a shift of the secondary peak of $S_t(q)$ to higher wave numbers and a concomitant decrease in its intensity.

We now turn to the analysis of junctions. The spatial distributions of the positions of junctions for several steps are presented in Fig. 6. Similar to terminals, their spatial configuration appears disordered; however, it exhibits minimal variation across the morphological transition. The histogram of the nearest-neighbor distance r_{nn} , shown in Fig. 7(a), displays a broad peak centered around $r_{nn} \sim \lambda$, in contrast to the multipeak structure observed for terminals. Moreover, this distribution remains nearly unchanged throughout the transition. The next-nearest-neighbor distance r_{nnn} shows a similarly broad peak around $r_{nnn} \sim 2\lambda$, which also exhibits negligible change. These broad and intact peak features in the histogram indicate that junctions maintain only weak, short-range correlations during the entire demagnetization process. Such behavior is compatible with the broad peak and its negligible step dependence of $S_j(q)$ in Fig. 5(b). In Fig. 7(c), we show the schematic illustration of the real-space distribution of junctions. Although the detailed arrangement varies from step to step, the qualitative behavior remains unchanged: The defects exhibit a disordered configuration with mutual separations $\sim \lambda$.

D. Radial distribution function

Through the structure factor analysis in Fig. 5, we found that while the morphological transition induces a pronounced change in the spatial correlations among terminals, the correlations among junctions remain largely unaffected. This result suggests that terminals and junctions make qualitatively distinct contributions to the experimental morphological transition. To quantify these differences and uncover the underlying local dynamics emerging during the transition, we conducted a real-space-based correlation analysis. In Fig. 8, we display the evolution of the spatial distribution

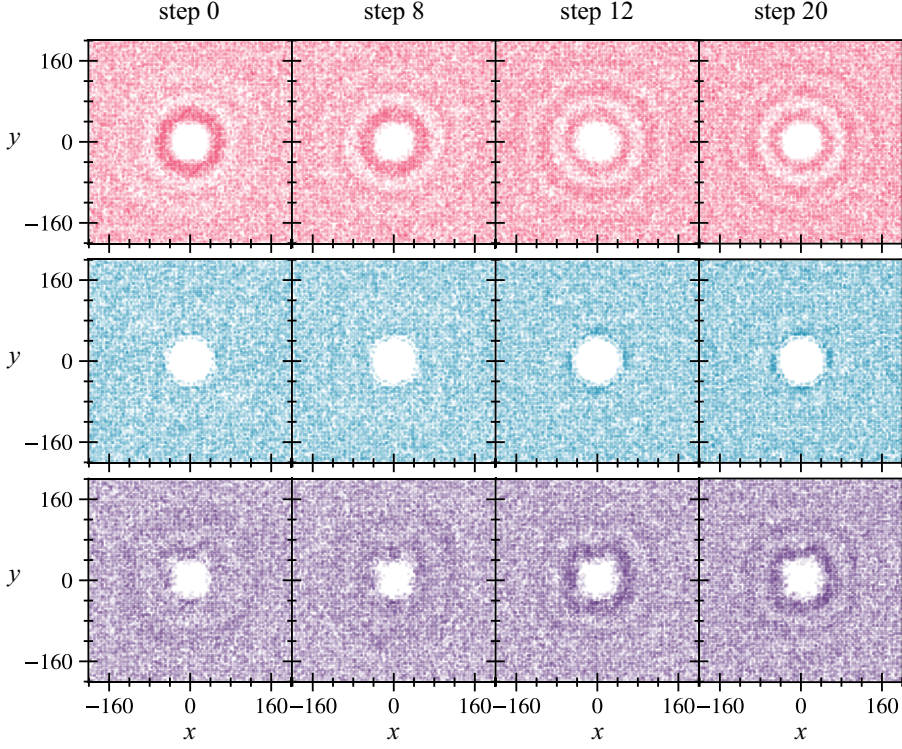


FIG. 8. A scatter plot representing the relative positions $\mathbf{r}_{\alpha,i} - \mathbf{r}_{\beta,k}$ for all pairs of defects: (top) terminal-terminal pairs with $\alpha = \beta = t$, (middle) junction-junction pairs with $\alpha = \beta = j$, and (bottom) terminal-junction pairs with $\alpha = t$ and $\beta = j$.

of the relative positions of other defects, which are represented as point clouds, for steps 0, 8, 12, and 20. In addition to the results for the same type pairs (terminal-terminal and junction-junction), the results for junction-terminal pairs are also shown. For instance, the point cloud in the top panel shows $\mathbf{r}_{t,i} - \mathbf{r}_{t,k}$ for all possible pairs of terminals i and k . The angularly isotropic distribution observed in all defect pairs suggests that, regardless of the pair type, the spatial arrangement of defects around a given defect remains isotropic. Meanwhile, the relative positional distributions of defect pairs in the radial direction exhibit characteristic oscillatory patterns. A faint fourfold modulation may be perceived in the junction-junction point clouds at steps 12 and 20. However, this weak feature is not consistently observed at other steps or for other defect pairs, and its apparent axes coincide with the Cartesian axes used to construct the two-dimensional histograms. It may therefore reflect effects associated with the finite square observation window and binning effect rather than an intrinsic fourfold anisotropy of the defect distribution.

For terminal-terminal pairs, at step 0, the prominent ring with a radius of ~ 60 , corresponding to the stripe period, is observed. Faint outer rings at roughly 2 and 3 times this radius are also visible. As the demagnetization step progresses, the intensity of the inner ring diminishes, while the outer rings become more pronounced. Due to the decrease in the size and the number of terminal clusters, terminals that tend to be located close in the early stages increasingly distance themselves from one another through the demagnetization process.

In contrast, for junction-junction pairs, no clear spatial correlations akin to those seen in terminal-terminal pairs are observed throughout the demagnetization process, as expected from the discussion in Sec. IV C. Beyond the radius corresponding to the stripe period (~ 60), the point cloud appears nearly uniform. In the later stages of demagnetization, the intensity along this ring slightly increases.

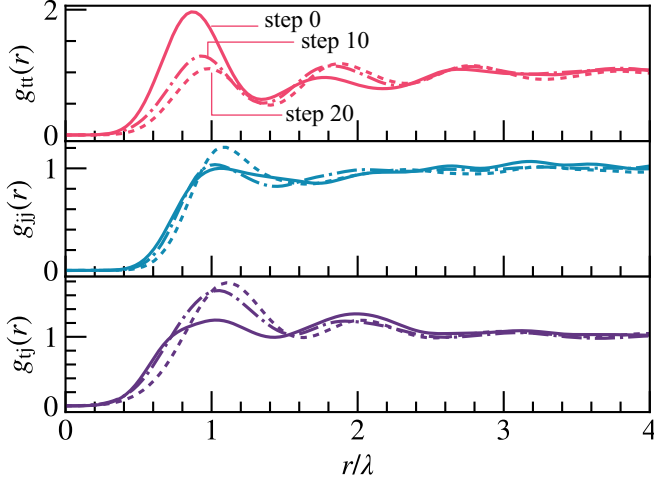


FIG. 9. Radial distribution functions, $g_{\alpha\beta}(r)$, calculated using Eq. (5) for terminal-terminal [$g_{tt}(r)$, top], junction-junction [$g_{jj}(r)$, middle], and terminal-junction [$g_{tj}(r)$, bottom] pairs at demagnetization steps 0 (solid lines), 10 (dash-dotted lines), and 20 (dashed lines). A Gaussian convolutional kernel with a variance of 4 was used for smoothing. Results are averaged over all trials with $\epsilon = 1$ (see main text).

For junction-terminal pairs, weak intensity enhancements are visible along the rings with radii corresponding to the stripe period and its multiples during the initial stages of demagnetization. Up to step 8, no strong correlation is observed, but by step 12, a sharp increase in intensity appears on the inner ring, forming a thick and dark circle. By step 20, this ring becomes somewhat thinner. These observations suggest that junction-terminal pairs tend to cluster more closely together as demagnetization progresses, which is an opposite trend to that of terminal-terminal pairs.

The isotropic distribution in Fig. 8 motivates us to compute the radial distribution function (RDF) $g_{\alpha\beta}(r)$ to investigate the real-space correlation for all three possible pairs of defects, where α and β denote the types of defects (terminal or junction). Figure 9 shows the RDF $g_{\alpha\beta}(r)$ at several demagnetization steps for terminals and junctions, defined by

$$g_{\alpha\beta}(r) = \left\langle \frac{V}{2\pi r N_{\alpha} N_{\beta}} \sum_{ik} \delta(r - |\mathbf{r}_{\alpha,i} - \mathbf{r}_{\beta,k}|) \right\rangle_n, \quad (5)$$

with the detected position $\mathbf{r}_{\alpha,i}$ ($\mathbf{r}_{\beta,k}$) of the i th (k th) defect α (β). The results for all three possible pairs—terminal-terminal, junction-junction, and terminal-junction—are provided. Except for junction-junction pairs, $g_{\alpha\beta}(r)$ reveals broad peaks at integer multiples of the stripe period λ during the whole demagnetization process. For the first and second peaks in $g_{\alpha\beta}(r)$, we compute the peak height and peak width and plot their step dependencies in Fig. 10. The peak width ξ is defined as the full width at half maximum of each peak and normalized by the stripe period λ in Fig. 10. Below, we discuss the features of $g_{\alpha\beta}(r)$ for each pair in detail.

We begin by discussing the RDF for terminal pairs, $g_{tt}(r)$, shown in the top panel of Fig. 9. It exhibits a dominant first peak in the quenched state at step 0 with a liquidlike correlation. The height of the first peak is reduced by half during step 0 to step 10, denoted by the yellow region, and then remains nearly constant after step 10, as shown in Fig. 10(a). The peak width remains nearly unchanged with $\xi/\lambda \approx 0.5$. In addition to the peak at the stripe period $r \sim \lambda$, the second peak at $r \sim 2\lambda$ is visible during the demagnetization process. The peak height exhibits the gradual increase from step 0 to step 10, denoted by the yellow region, followed by a slight decrease after step 10, thereby leaving a peak at step 12, as shown in Fig. 10(b). The peak width ξ/λ decreases slightly, but no clear change is observed.

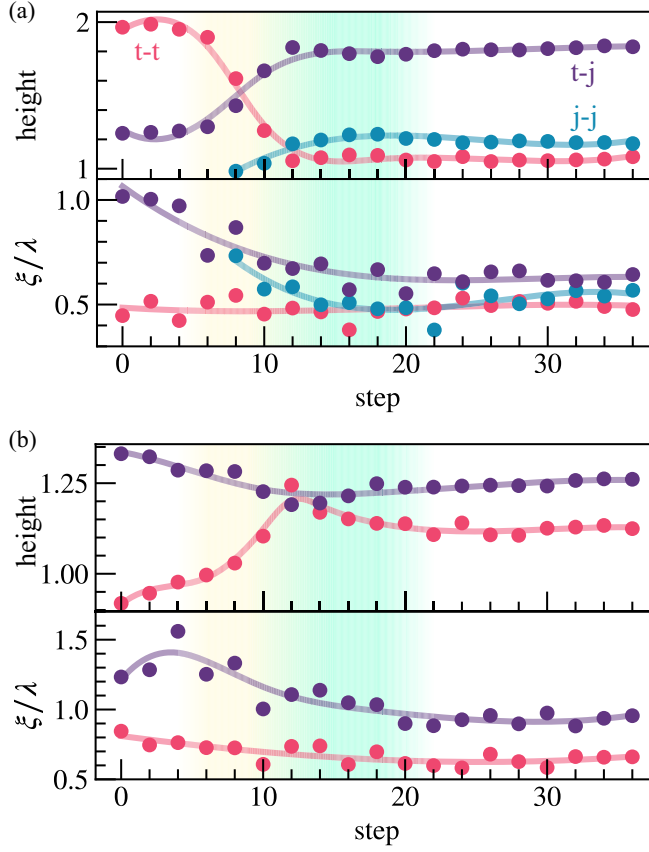


FIG. 10. Step dependencies of the peak height (top) and peak width (bottom) of (a) the first peak and (b) the second peak in $g_{\alpha\beta}(r)$. The peak width ξ is normalized by the stripe period λ . Red, blue, and purple symbols represent the results for terminal-terminal, junction-junction, and terminal-junction pairs, respectively. Pale lines are guides for the eye. In panel (a), the results of junction-junction pairs with steps below 8 are not shown due to the ambiguous peak structure (see main text). In panel (b), the second peak for junction-junction pairs is not shown because it is too broad to define the peak width.

The variation in the first-peak height reflects the tendency of terminals to be densely clustered in the quenched state, while gradually dispersing over the demagnetization process (see also Sec. IV C). As a result, the peak height at $r \sim 2\lambda$ increases. It is likely that further homogenization of these terminals continues beyond step 10, and the height of the second peak gradually decreases as the system transitions into the annealed state. The gradual dissolution of terminal clusters observed in the quenched state can be attributed to the repulsive force between the terminals, namely, between the topological defects with the same charges [see also Fig. 7(c)]. Due to the repulsive force, terminals that initially resided in the shell at $r \sim \lambda$ are pushed outward to the shell at $r \sim 2\lambda$, resulting in an increased peak height of $g_{tt}(r)$ at $r \sim 2\lambda$ in the later stages. We note that, since the width of the first peak ξ/λ remains nearly unchanged throughout the demagnetization process, the correlation length among terminals within the nearest-neighbor shell remains almost constant.

Next, we discuss the RDF for junction pairs, $g_{jj}(r)$, in the middle panel of Fig. 9. In the quenched state at step 0, $g_{jj}(r)$ remains nearly flat for $r \gtrsim \lambda$, exhibiting an RDF more characteristic of a gas than a liquid. From step 8 onward, a peak begins to emerge at $r \sim \lambda$, with its intensity gradually increasing and its width slightly narrowing as the demagnetization process proceeds. The peak height slightly increases and ξ/λ slightly decreases from step 8 to 20, as shown in Fig. 10(a).

Unlike terminals, a second peak at $r \sim 2\lambda$ does not appear even in the later stages, indicating that the spatial correlations among junctions remain significantly weaker compared to those among terminals. Although terminals and junctions are topological defects with the opposite topological charge, they exhibit distinct correlations.

These differences presumably arise from their distinct geometric configurations and the corresponding differences in their energy profiles or interaction between defects. Junctions and terminals can be viewed as disclinations of opposite topological charge. Under the standard mapping of the intensity of the labyrinthine image $\psi(\mathbf{r})$ to a nematic phase field $\mathbf{n}(\mathbf{r})$, such disclinations map onto vortices of the phase field (see the Appendix). In this simplified picture, one would expect their correlations to follow the familiar Coulomb-gas interaction between vortices, implying no qualitative distinction between different types of oppositely charged defects. Meanwhile, the accompanying local stripe patterns and symmetries differ markedly (see the Appendix). Indeed, junctions can exhibit threefold rotational symmetry around the defect core, whereas terminals possess no such rotational symmetry. This distinction indicates that these defects differ in their rank structure beyond their topological charge alone, suggesting corresponding differences in their interaction profiles, e.g., directional anisotropy in the interaction. Recent studies in liquid-crystal systems have highlighted the significance of such rank-dependent structures in determining defect-defect interactions beyond simple Coulomb-gas picture for the interaction between topological defects [27,28], and a similar mechanism may plausibly arise in magnetic labyrinthine patterns. Nevertheless, in magnetic systems, long-range and intrinsically complex interactions, such as magnetic dipolar interactions, as well as pinning effects, are expected to play non-negligible roles [29]. Consequently, the observed differences in the correlations exhibited by junctions and terminals are likely influenced by these additional factors as well.

Finally, we discuss the RDF for terminal-junction pairs, $g_{tj}(r)$, shown in the bottom panel of Fig. 9. It exhibits a dominant first peak and a weaker second peak like $g_{tt}(r)$. Meanwhile, in contrast to $g_{tt}(r)$, the height of the first peak in $g_{tj}(r)$ increases in steps, as shown in Fig. 10(a). Furthermore, the peak width ξ/λ shows a $\sim 30\%$ reduction from step 0 to step 20, indicating the increased spatial coherence between terminals and junctions. These behaviors can be attributed to the presence of an attractive force between junctions and terminals, namely, topological defects with opposite charges. In the quenched state, junctions and terminals were distributed with little correlation; however, the presence of an attractive interaction between them likely caused these defects to cluster more closely, leading to an enhanced spatial coherence of such pairs. This reduction in ξ/λ is correlated with the increase in the overall coherence marked by the reduction of σ in Fig. 3(c). We note that the second peak in $g_{tj}(r)$ shows slight reduction in height and width during the demagnetization process, as shown in Fig. 10(b).

E. Coordination number

From the RDF computed in Sec. IV D, we further calculate the effective coordination number $n_{\alpha\beta}$, which counts the number of defects α within the nearest-neighboring shell of defects β defined by

$$n_{\alpha\beta} = \int_0^{r_0} 2\pi r \rho_\alpha g_{\alpha\beta}(r) dr. \quad (6)$$

Here, r_0 is the first local minimum of $g_{\alpha\beta}(r)$ around $r/\lambda \sim 1.5$. The step dependence of $n_{\alpha\beta}$ for all three possible pairs is presented in Fig. 11. Intriguingly, $n_{\alpha\beta}$ exhibits different trends in the demagnetization process depending on the pairs and characterizes the transition from the quenched state to the annealed state by the following two-step process. In the initial stage of the transition from step 6 to step 12, highlighted in yellow, terminals significantly reduce the number of neighboring terminals while increasing the number of adjacent junctions. Since terminals can pair annihilate with junctions, this stage can be considered a precursor to reducing the total number of defects in the system (see Fig. 4). The increase in paired topological defects enhances the system's coherence,

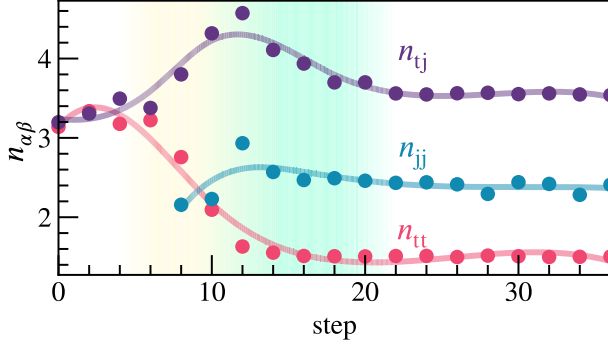


FIG. 11. Evolution of the coordination number, $n_{\alpha\beta}$, calculated using Eq. (6) for terminal-terminal (red), junction-junction (blue), and terminal-junction (purple) pairs. Pale lines are guides for the eye.

which is consistent with the decrease in σ shown in Fig. 3(c). After the transition indicated by the peak of n_{tj} , terminal-junction pairs annihilate by reducing n_{tj} from step 12 to step 20, highlighted in green. In this region, although n_{tt} and n_{jj} are intact, the stripe wavelength q_0 (the stripe period λ) increases (decreases) as shown in Fig. 3(c). These results can be understood by noting that the space vacated by the eliminated defects is subsequently occupied by a denser packing of stripes, thereby giving rise to a more compact labyrinthine pattern.

V. SUMMARY

To summarize, we have identified a morphological transition of magnetic labyrinthine patterns and characterized their evolution by using the topological argument and pattern-recognition analysis combined with modern machine learning, TM-CNN. The magnetic labyrinthine patterns are notoriously difficult to quantify due to the lack of clear long-range order. Although the canonical Fourier analysis captures some global features, the local dynamics of labyrinthine structures cannot be captured. Among several local characters of labyrinths, topological defects associated with symmetry breaking play a pivotal role due to their geometrical properties that restrict structural transformation.

Recently, we developed a powerful technique combining TM and CNN to efficiently detect and classify topological defects as either terminals or junctions. This approach enables rapid identification of numerous defects with superior accuracy and reliability, as demonstrated in Ref. [24]. By detecting the positions of junctions and terminals, we compute the correlations of topological defects as well as their number and elucidate how these defects mediate the morphological transition from the quenched state to the annealed state. We clarify the role of topological defects in the transition and reveal the details of the two-step transition through the systematic analysis.

To the best of our knowledge, this systematic detection and correlation analysis of thousands of topological defects has been achieved for the first time through the algorithm we developed. The method developed in this study is generalizable and extends beyond magnetism, offering applicability to a wide range of systems, including labyrinthine patterns observed in ecological systems and chemical reactions. This approach enables the evaluation of local correlations of topological defects, which can serve as a promising metric for characterizing labyrinths, thereby paving the way of characterizing such disordered systems. The present analysis identifies the types and positions of the defects and focuses primarily on direction-averaged correlations, but it does not assign an orientational descriptor to each defect or retain information about the surrounding local stripe director. Incorporating the connectivity of defects through the intervening stripe segments [30], together with the orientational degrees of freedom of the background stripes, into correlation

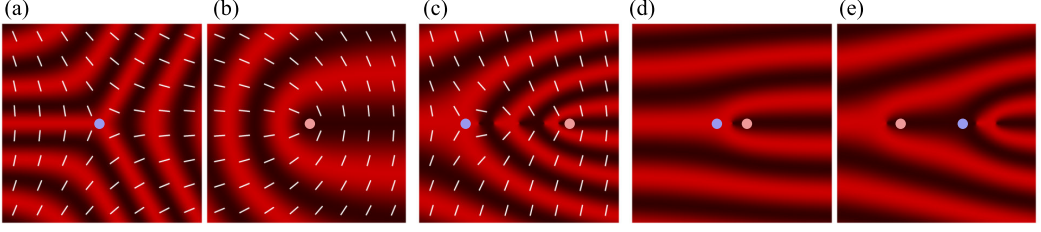


FIG. 12. Schematic illustrations of disclinations in the labyrinthine patterns: (a) $-1/2$ disclination, (b) $+1/2$ disclination, and (c) pair of $-1/2$ and $1/2$ disclinations. The color represents the intensity of the magnetic domain image $\psi(\mathbf{r})$ and the white bars represent the director field $\mathbf{n}(\mathbf{r})$ defined by Eq. (A1). (d) Schematic illustration of a dislocation represented by a tightly bounded pair of a $-1/2$ disclination for the red stripe and a $1/2$ disclination for the black stripe. (e) A pair of $-1/2$ and $1/2$ disclinations for the black stripe, which can be pair annihilated.

analyses would be an important next step toward a more complete characterization of labyrinthine structures and their dynamics.

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DATA AVAILABILITY

The TM-CNN package is publicly available on GitHub [26]. Requests for further information or data should be directed to the corresponding authors.

APPENDIX: TOPOLOGICAL DEFECTS

In ordered phases, topological defects corresponding to broken symmetries emerge. Specifically, in the case of stripes, both spatial translational and rotational symmetries are broken, leading to the corresponding topological defects, dislocations and disclinations, respectively [31]. In magnetic domain images, these defects are quantified by the director field $\mathbf{n}(\mathbf{r})$ given by

$$\mathbf{n}(\mathbf{r}) = \frac{\nabla \psi(\mathbf{r})}{|\nabla \psi(\mathbf{r})|}, \quad (\text{A1})$$

with the intensity of the image $\psi(\mathbf{r})$. Then, $\mathbf{n}(\mathbf{r})$ denotes the direction for stripes to propagate. Since the propagation direction is equivalent under the transformation $\mathbf{n}(\mathbf{r}) \rightarrow -\mathbf{n}(\mathbf{r})$, directors do not have either heads or tails; namely, $\mathbf{n} \in S^1/\mathbb{Z}_2$. Figures 12(a) and 12(b) show real-space configurations of the intensity $\psi(\mathbf{r})$ and the director field $\mathbf{n}(\mathbf{r})$, exhibiting the winding textures of $\mathbf{n}(\mathbf{r})$ around the defects. These defects belong to the class of topological defects called disclinations, arising from rotational symmetry breaking, and they are classified by the topological charge $\pm 1/2$ depending on the winding texture [16,31]. A disclination with the topological charge of $-1/2$

and $+1/2$ are shown in Figs. 12(a) and 12(b), respectively. By circling around the disclination represented by the blue and red dots in the counterclockwise direction, $\mathbf{n}(\mathbf{r})$ rotates clockwise and counterclockwise by half a turn, and thus the topological charges $-1/2$ and $+1/2$ are assigned, respectively. Interestingly, these topological defects differ not only in terms of their topological charge but also in their symmetry properties. For example, the disclination shown in Fig. 12(a) exhibits a threefold rotational symmetry in the surrounding stripe pattern, whereas no such rotational symmetry is present in Fig. 12(b). This distinction reflects a difference in the rank structure of the disclinations, and the impact of such rank differences on defect-defect interactions has been discussed in the context of liquid crystals [27,28].

When a single disclination exists, the stripes at the system's boundaries are oriented in different directions even at a significant distance from the defect, resulting in substantial energy loss due to exchange interactions. However, positioning these defects close to each other can mitigate this energy loss. As illustrated in Fig. 12(c), placing disclinations with opposite topological charges near each other aligns the director field at the system's edges approximately in the $\pm y$ direction. Bringing two disclinations even closer results in another defect shown in Fig. 12(d), corresponding to an end point of the black stripes, or equivalently, a branching point of the red stripes. This defect is a dislocation, associated with the translational symmetry breaking, and it is understood as a tightly bounded pair of disclinations with opposite topological charges. These disclinations and dislocations cannot be removed alone by continuous deformations of the stripe pattern due to the topological protection. Meanwhile, these topological defects can be removed in pairs. In Fig. 12(d), we considered a bound pair of disclinations in stripes of different colors, while Fig. 12(e) depicts a pair of disclinations in stripes of the same color. In this case, bringing two disclinations with opposite topological charges closer together enables their annihilation, transforming the stripes into a topologically trivial configuration. Such a pair annihilation is inevitable to remove these defects due to the topological protection.

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