Uniqueness of conformal Killing gravity cosmology

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We investigate an extension of Harada gravity, a third-order theory proposed as an alternative to general relativity that preserves energy-momentum conservation and in which the cosmological constant arises as an integration constant. Our aim is to determine whether Harada gravity is unique under the principles that motivated its construction. We show that imposing both the existence of all Einstein solutions and energy-momentum conservation restricts the theory to only two distinct branches: general relativity and Harada gravity. We then relax each of these assumptions independently and demonstrate that, within the class of symmetric theories considered, Harada gravity remains the unique extension of general relativity.

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I. INTRODUCTION

Understanding the nature of dark energy and addressing the cosmological constant problem have motivated the development of numerous modified theories of gravity that aim to explain the observed accelerated expansion of the Universe.

Recently, Harada proposed a gravitational theory in which the cosmological constant neither appears explicitly in the field equations nor is introduced through the energy-momentum tensor. Instead, it emerges naturally as an integration constant [1]. This is achieved by constructing third-order field equations involving the tensors $\nabla_\rho R_{\mu\nu}$, $g_{\mu\nu}\nabla_\rho R$, $\nabla_\rho T_{\mu\nu}$, and $g_{\mu\nu}\nabla_\rho T$. The theory is formulated such that conservation of the energy-momentum tensor follows directly from the field equations, while every solution of Einstein's field equations is also a solution of Harada gravity. Furthermore, unlike the earlier proposal in Ref. [2], conformally flat vacuum spacetimes are not necessarily admitted as solutions.

The theory is defined by the third-rank field equations

$$H_{\mu\nu\rho} = 8\pi G T_{\mu\nu\rho}, \tag{1}$$

where

$$H_{\mu\nu\rho} = \nabla_\rho R_{\mu\nu} + \nabla_\mu R_{\rho\nu} + \nabla_\nu R_{\mu\rho} - \tfrac{1}{3}(g_{\mu\nu}\partial_\rho R + g_{\rho\nu}\partial_\mu R + g_{\mu\rho}\partial_\nu R), \tag{2a}$$

$$T_{\mu\nu\rho} = \nabla_\rho T_{\mu\nu} + \nabla_\mu T_{\rho\nu} + \nabla_\nu T_{\mu\rho} - \tfrac{1}{6}(g_{\mu\nu}\partial_\rho T + g_{\rho\nu}\partial_\mu T + g_{\mu\rho}\partial_\nu T). \tag{2b}$$

These tensors satisfy $g^{\nu\rho}H_{\rho\mu\nu} = 0$ and $g^{\nu\rho}T_{\rho\mu\nu} \propto \nabla^\nu T_{\mu\nu}$, so that the field equations imply conservation of the energy-momentum tensor $\nabla^\nu T_{\mu\nu} = 0$.

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It has been shown that the theory can be recast as a second-rank field equation corresponding to an extension of general relativity (GR) by a conformal Killing tensor [3]:

$$G_{\mu\nu} = T_{\mu\nu} + K_{\mu\nu}, \tag{3a}$$

$$\nabla_\rho K_{\mu\nu} + \nabla_\mu K_{\rho\nu} + \nabla_\nu K_{\mu\rho} = \tfrac{1}{6}(g_{\mu\nu}\partial_\rho K + g_{\rho\nu}\partial_\mu K + g_{\mu\rho}\partial_\nu K). \tag{3b}$$

This formulation motivated the name conformal Killing gravity (CKG). Since its introduction, several aspects of the theory have been investigated. The most general static, spherically symmetric vacuum solution was obtained in Ref. [4]. Black-bounce solutions were studied in Ref. [5], the Tolman-Oppenheimer-Volkoff equations were derived in Ref. [6], charged black-hole solutions were obtained in Ref. [7], and black-hole thermodynamics was investigated in Ref. [8].

In the context of cosmology, the CKG model is a one-parameter extension of the Lambda cold dark matter (ΛCDM) model. Observational constraints on this model have shown that CKG provides a description of the late-time cosmic acceleration consistent with current cosmological observations [9,10]. Moreover, recent analyses using DESI Data Release 2 baryon acoustic oscillation (DESI DR2 BAO) data have found CKG to be statistically favoured over the standard ΛCDM model [11].

When proposing a new theory of gravity, an important question is which of its defining features are essential and which can be modified without altering its physical content. In other words, one would like to understand the extent to which the theory is unique. A well-known example is provided by general relativity, whose uniqueness is established by Lovelock's theorem [12]. This theorem states that in four dimensions, the most general theory that can be derived from an action constructed solely from the metric, containing at most second-order derivatives of the metric, is GR with a cosmological constant.

Motivated by this question, we investigate the uniqueness of conformal Killing gravity. We consider an extension of Harada's theory obtained by introducing arbitrary coefficients in the tensors defined in Eq. (2). Our goal is to determine whether theories satisfying the fundamental properties of conformal Killing gravity can exhibit phenomenology distinct from that of the original theory, or whether these requirements uniquely determine its structure. In Sec. II, we investigate the resulting theory under the requirements of energy-momentum conservation and the existence of general relativity solutions. We show that these conditions severely constrain the theory, reducing it either to general relativity or to conformal Killing gravity. In Sec. III, we then relax each of these requirements separately for a simplified version of the theory and study the resulting cosmological dynamics in a spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime. We find that, even in this less restrictive setting, the theory again reduces to conformal Killing gravity.

II. GENERALIZING HARADA GRAVITY

We consider an extension of Harada gravity obtained by assigning independent coefficients to each of the terms appearing in the original third-rank tensors. The resulting field equations are

$$\begin{aligned} &a\nabla_\rho R_{\mu\nu} + b\nabla_\mu R_{\rho\nu} + c\nabla_\nu R_{\mu\rho} + d g_{\mu\nu}\partial_\rho R + e g_{\rho\nu}\partial_\mu R + f g_{\mu\rho}\partial_\nu R \\ &\quad = \alpha\nabla_\rho T_{\mu\nu} + \beta\nabla_\mu T_{\rho\nu} + \gamma\nabla_\nu T_{\mu\rho} + \delta g_{\mu\nu}\partial_\rho T + \epsilon g_{\rho\nu}\partial_\mu T + \lambda g_{\mu\rho}\partial_\nu T. \end{aligned} \tag{4}$$

To impose that every solution of Einstein's field equations should also satisfy this theory, we make the substitution $T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and $T = -R$ in the field equation:

$$\begin{aligned} &a\nabla_\rho R_{\mu\nu} + b\nabla_\mu R_{\rho\nu} + c\nabla_\nu R_{\mu\rho} + d g_{\mu\nu}\partial_\rho R + e g_{\rho\nu}\partial_\mu R + f g_{\mu\rho}\partial_\nu R \\ &\quad = \alpha\nabla_\rho R_{\mu\nu} + \beta\nabla_\mu R_{\mu\nu} + \gamma\nabla_\nu R_{\mu\nu} - (\delta + \alpha/2) g_{\mu\nu}\partial_\rho R - (\epsilon + \beta/2) g_{\rho\nu}\partial_\mu R - (\lambda + \gamma/2) g_{\mu\rho}\partial_\nu R. \end{aligned} \tag{5}$$

The Einstein solutions are guaranteed if

$$a=\alpha,\quad b=\beta,\quad c=\gamma,\quad d=-\left(\frac{\alpha}{2}+\delta\right),\quad e=-\left(\frac{\beta}{2}+\epsilon\right),\quad f=-\left(\frac{\gamma}{2}+\lambda\right). \tag{6}$$

To impose energy-momentum conservation, we now consider the trace equations. Contracting Eq. (4) with $g_{\mu\nu}$, $g_{\mu\rho}$ and $g_{\rho\nu}$,

$$\partial_\rho R\left(a+\frac{b+c}{2}+4d+e+f\right)=(\beta+\gamma)\nabla^\mu T_{\mu\rho}+(\alpha+4\delta+\epsilon+\lambda)\partial_\rho T, \tag{7a}$$

$$\partial_\rho R\left(b+\frac{a+c}{2}+d+4e+f\right)=(\alpha+\gamma)\nabla^\mu T_{\mu\rho}+(\beta+\delta+4\epsilon+\lambda)\partial_\rho T, \tag{7b}$$

$$\partial_\rho R\left(c+\frac{a+b}{2}+d+e+4f\right)=(\alpha+\beta)\nabla^\mu T_{\mu\rho}+(\gamma+\delta+\epsilon+4\lambda)\partial_\rho T. \tag{7c}$$

Substituting Eq. (6) into the trace equations yields

$$\partial_\rho R\left(a+\frac{b+c}{2}+4d+e+f\right)=(b+c)\nabla^\mu T_{\mu\rho}-\left(a+\frac{b+c}{2}+4d+e+f\right)\partial_\rho T, \tag{8a}$$

$$\partial_\rho R\left(b+\frac{a+c}{2}+d+4e+f\right)=(a+c)\nabla^\mu T_{\mu\rho}-\left(b+\frac{a+c}{2}+d+4e+f\right)\partial_\rho T, \tag{8b}$$

$$\partial_\rho R\left(c+\frac{a+b}{2}+d+e+4f\right)=(a+b)\nabla^\mu T_{\mu\rho}-\left(c+\frac{a+b}{2}+d+e+4f\right)\partial_\rho T. \tag{8c}$$

The constraints imposed by the existence of the Einstein solutions and conservation are not totally independent, because in this case the coefficients of ∂R and ∂T are the same. If we already have imposed the existence of the Einstein solutions, conservation of the energy-momentum tensor requires only one additional relation. Indeed, any one of the following conditions is sufficient:

$$a+\frac{b+c}{2}+4d+e+f=0, \tag{9a}$$

$$b+\frac{a+c}{2}+d+4e+f=0, \tag{9b}$$

$$c+\frac{a+b}{2}+d+e+4f=0. \tag{9c}$$

We now establish the main result of this section.

Theorem 1 (Classification of generalized conformal Killing gravity). Consider the generalized field equations (4) together with the constraints (6) and (9), corresponding, respectively, to the existence of all general relativity solutions and conservation of the energy-momentum tensor. Then, the theory can have only two possible structures: a general-relativity-type structure or a conformal-Killing-gravity-type structure. Furthermore, when restricted to a spatially flat FLRW spacetime, the cosmological dynamics are necessarily those of either general relativity or conformal Killing gravity.

Proof. Concerning the system (9), two qualitatively different situations arise:

(1) At least one of the above conditions is not satisfied.

(2) All three conditions are satisfied.

The proof proceeds by considering these two cases separately, each corresponding to a different structural class of the theory.

Case I. At least one of them is not satisfied.

Without loss of generality, consider

$$c+\frac{a+b}{2}+d+e+4f\neq 0. \tag{10}$$

Since energy-momentum is conserved, the third trace equation

$$\partial_\rho R\left(c+\frac{a+b}{2}+d+e+4f\right)=(a+b)\nabla^\mu T_{\mu\rho}-\left(c+\frac{a+b}{2}+d+e+4f\right)\partial_\rho T \tag{11}$$

reduces to

$$\partial_\rho R=-\partial_\rho T. \tag{12}$$

Using this identity in the field equations, we obtain

$$\begin{aligned}&a\nabla_\rho R_{\mu\nu}+b\nabla_\mu R_{\rho\nu}+c\nabla_\nu R_{\mu\rho}+dg_{\mu\nu}\partial_\rho R+eg_{\rho\nu}\partial_\mu R+fg_{\mu\rho}\partial_\nu R\\&\quad=a\nabla_\rho T_{\mu\nu}+b\nabla_\mu T_{\rho\nu}+c\nabla_\nu T_{\mu\rho}+\left(d+\frac{a}{2}\right)g_{\mu\nu}\partial_\rho R+\left(e+\frac{b}{2}\right)g_{\rho\nu}\partial_\mu R+\left(f+\frac{c}{2}\right)g_{\mu\rho}\partial_\nu R,\end{aligned} \tag{13}$$

which simplifies to

$$a\nabla_\rho G_{\mu\nu}+b\nabla_\mu G_{\rho\nu}+c\nabla_\nu G_{\mu\rho}=a\nabla_\rho T_{\mu\nu}+b\nabla_\mu T_{\rho\nu}+c\nabla_\nu T_{\mu\rho}. \tag{14}$$

Therefore, every theory belonging to Case I reduces to the third-order form of Einstein's equations. For illustration, consider the FLRW spacetime. The (000) component becomes

$$6H\dot{H}=\dot{\rho}. \tag{15}$$

This integrates to

$$3H^2=\rho+\Lambda, \tag{16}$$

which is precisely the first Friedmann equation of general relativity, with the cosmological constant appearing as an integration constant.

Case II. All of them are satisfied.

If all three trace conditions are imposed simultaneously, the coefficients are uniquely determined from Eq. (9):

$$d=-\frac{4a+b+c}{18},\quad e=-\frac{a+4b+c}{18},\quad f=-\frac{a+b+4c}{18}. \tag{17}$$

Substituting these expressions into the field equations gives

$$\begin{aligned}&a\left(\nabla_\rho R_{\mu\nu}-\frac{1}{6}g_{\mu\nu}\partial_\rho R\right)+b\left(\nabla_\mu R_{\rho\nu}-\frac{1}{6}g_{\rho\nu}\partial_\mu R\right)+c\left(\nabla_\nu R_{\mu\rho}-\frac{1}{6}g_{\mu\rho}\partial_\nu R\right)\\&\quad-\frac{a+b+c}{18}(g_{\mu\nu}\partial_\rho R+g_{\rho\nu}\partial_\mu R+g_{\mu\rho}\partial_\nu R)\\&\quad=a\left(\nabla_\rho T_{\mu\nu}-\frac{1}{3}g_{\mu\nu}\partial_\rho T\right)+b\left(\nabla_\mu T_{\rho\nu}-\frac{1}{3}g_{\rho\nu}\partial_\mu T\right)+c\left(\nabla_\nu T_{\mu\rho}-\frac{1}{3}g_{\mu\rho}\partial_\nu T\right)\\&\quad+\frac{a+b+c}{18}(g_{\mu\nu}\partial_\rho T+g_{\rho\nu}\partial_\mu T+g_{\mu\rho}\partial_\nu T).\end{aligned} \tag{18}$$

The (000) component becomes

$$(a+b+c)\left(\nabla_0 R_{00}-\tfrac{1}{3}g_{00}\partial_0 R\right)=(a+b+c)\left(\nabla_0 T_{00}-\tfrac{1}{6}g_{00}\partial_0 T\right). \tag{19}$$

Provided that $a+b+c\neq 0$, this reduces to

$$\nabla_0 R_{00}-\tfrac{1}{3}g_{00}\partial_0 R=\nabla_0 T_{00}-\tfrac{1}{6}g_{00}\partial_0 T. \tag{20}$$

This is exactly the (000) equation of Harada gravity. Since the cosmological evolution in an FLRW spacetime is completely determined by the continuity equation together with the (000)

field equation, and both are identical to those of Harada gravity, every theory belonging to Case II possesses the same FLRW cosmology as conformal Killing gravity. ■

III. RELAXING THE ASSUMPTIONS

The results of the previous section show that obtaining phenomenology different from either general relativity or conformal Killing gravity requires relaxing at least one of the two key assumptions: conservation of the energy-momentum tensor or the existence of all general relativity solutions.

To simplify the analysis, we restrict our attention to the symmetric class of theories defined by

$$a = b = c, \quad d = e = f, \quad \alpha = \beta = \gamma, \quad \delta = \epsilon = \lambda. \tag{21}$$

A discussion of nonconservation in the case $a = \alpha = 1$ was recently presented in Ref. [13].

A. Conservation without Einstein solutions

The field equations for our theory are

$$\begin{aligned} &a(\nabla_\rho R_{\mu\nu} + \nabla_\mu R_{\rho\nu} + \nabla_\nu R_{\mu\rho}) + d(g_{\mu\nu}\partial_\rho R + g_{\rho\nu}\partial_\mu R + g_{\mu\rho}\partial_\nu R) \\ &\quad = \alpha(\nabla_\rho T_{\mu\nu} + \nabla_\mu T_{\rho\nu} + \nabla_\nu T_{\mu\rho}) + \delta(g_{\mu\nu}\partial_\rho T + g_{\rho\nu}\partial_\mu T + g_{\mu\rho}\partial_\nu T). \end{aligned} \tag{22}$$

In this case, the trace equations are all the same:

$$\partial_\rho R(2a + 6d) = 2\alpha\nabla^\mu T_{\mu\rho} + (\alpha + 6\delta)\partial_\rho T, \tag{23}$$

and we recover energy-momentum conservation by the conditions $2a + 6d = 0$ and $\alpha + 6\delta = 0$. This gives the field equations

$$\begin{aligned} &(\nabla_\rho R_{\mu\nu} + \nabla_\mu R_{\rho\nu} + \nabla_\nu R_{\mu\rho}) - \frac{1}{3}(g_{\mu\nu}\partial_\rho R + g_{\rho\nu}\partial_\mu R + g_{\mu\rho}\partial_\nu R) \\ &\quad = \frac{\alpha}{a}(\nabla_\rho T_{\mu\nu} + \nabla_\mu T_{\rho\nu} + \nabla_\nu T_{\mu\rho}) - \frac{\alpha}{6a}(g_{\mu\nu}\partial_\rho T + g_{\rho\nu}\partial_\mu T + g_{\mu\rho}\partial_\nu T). \end{aligned} \tag{24}$$

This is precisely the field equation of conformal Killing gravity, differing only by the overall factor α/a, which amounts to a rescaling of the coupling constant. Therefore, within this symmetric subclass, relaxing the requirement that Einstein solutions exist while preserving energy-momentum conservation does not lead to a new phenomenology.

B. Einstein solutions without energy-momentum conservation

For the symmetric theory, the conditions for the existence of Einstein solutions (6) reduce to

$$a = \alpha, \quad d = -\left(\frac{\alpha}{2} + \delta\right), \tag{25}$$

so the field equation is

$$\begin{aligned} &(\nabla_\rho R_{\mu\nu} + \nabla_\mu R_{\rho\nu} + \nabla_\nu R_{\mu\rho}) + x(g_{\mu\nu}\partial_\rho R + g_{\rho\nu}\partial_\mu R + g_{\mu\rho}\partial_\nu R) \\ &\quad = (\nabla_\rho T_{\mu\nu} + \nabla_\mu T_{\rho\nu} + \nabla_\nu T_{\mu\rho}) - \left(x + \tfrac{1}{2}\right)(g_{\mu\nu}\partial_\rho T + g_{\rho\nu}\partial_\mu T + g_{\mu\rho}\partial_\nu T), \end{aligned} \tag{26}$$

where $x \equiv \frac{d}{a}$.

For a spatially flat FLRW spacetime, the two independent equations are obtained from the 000 and 011 components of the field equations:

$$-3(1 + 2x)\ddot{H} - 6(1 + 4x)H\dot{H} = \left(\tfrac{1}{2} - x\right)\dot{\rho} + 3\left(\tfrac{1}{2} + x\right)\dot{p}, \tag{27a}$$

$$(1 + 6x)\ddot{H} + (10 + 24x)H\dot{H} = \left(\tfrac{1}{2} + x\right)\dot{\rho} - \left(\tfrac{1}{2} + 3x\right)\dot{p} - 2H(\rho + p). \tag{27b}$$

To close the system, we assume the equation of state $p = w\rho$. It is worth noting that this freedom is a consequence of the symmetric ansatz adopted here. In the general theory, the three trace equations are independent and provide additional constraints relating ρ and p. Consequently, the equation of state would no longer be an independent input, but would instead be determined by the field equations themselves.

Integrating the first equation allows us to eliminate ρ from the second, yielding a differential equation for the Hubble parameter:

$$\left[1+6x+3(1+2x)\frac{f_1}{f_0}\right]\ddot{H}+\left[10+24x+6(1+4x)\frac{f_1}{f_0}-6(1+w)(1+2x)\frac{1}{f_0}\right]H\dot{H} - 6(1+w)(1+4x)\frac{1}{f_0}H^3-2H(1+w)\frac{\Lambda}{f_0}=0, \tag{28}$$

where we have defined $f_0(w,x)\equiv(\frac{1}{2}-x)+3w(\frac{1}{2}+x)$ and $f_1(w,x)\equiv(\frac{1}{2}+x)-w(\frac{1}{2}+3x)$.

Remarkably, for $x\neq -1/4$, all dependence on the parameter x cancels identically, reducing the cosmological evolution equation to

$$\ddot{H}+(1+3w)H\dot{H}-3(1+w)H^3-H\frac{(1+w)\Lambda}{1+4x}=0. \tag{29}$$

This can be rewritten as

$$H^2=C_1a^{-3(1+w)}+C_2a^2+C_3. \tag{30}$$

The three contributions have a clear physical interpretation. The first term corresponds to the standard general relativity evolution of a perfect fluid satisfying $\rho\propto a^{-3(1+w)}$, the second is the characteristic contribution already present in conformal Killing gravity, and the third is an effective cosmological constant arising as an integration constant. Therefore, although the generalized theory does not conserve the energy-momentum tensor, its homogeneous and isotropic cosmology is indistinguishable from that of conformal Killing gravity.

IV. CONCLUSIONS

In this work, we investigated an extension of Harada gravity obtained by introducing arbitrary coefficients in the third-rank tensors that define the field equations. Our aim was to determine whether the particular structure of conformal Killing gravity is unique or whether physically distinct generalizations can be constructed while preserving its main features.

We first considered the general theory under the requirements of energy conservation and the existence of all general relativity solutions. We showed that these conditions strongly constrain the free coefficients, leaving only two possible branches. In the case of a homogeneous and isotropic spacetime, the first branch reduces to general relativity, while the second reproduces conformal Killing gravity, demonstrating that the latter is the unique nontrivial extension of general relativity within this framework.

We then relaxed each of these assumptions separately in a symmetric subclass of the generalized theory. When energy-momentum conservation was maintained while the requirement of Einstein solutions was abandoned, the theory was shown to reduce exactly to conformal Killing gravity up to a rescaling of the gravitational coupling constant. Conversely, when Einstein solutions were preserved but energy-momentum conservation was relaxed, the resulting cosmological equations for a spatially flat FLRW spacetime became independent of the additional free parameter and again coincided with those of conformal Killing gravity.

Overall, these results reveal a remarkable rigidity of Harada gravity. Despite introducing additional degrees of freedom in the field equations, the physically viable theories consistently collapse to either general relativity or conformal Killing gravity under the assumptions considered. This

suggests that genuinely new phenomenology within this type of theories, if it exists, is likely to require extensions beyond the symmetric class considered here.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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