# Emergence of quantum Darwinism and pointer states for noncommuting evolutions

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(Received 15 October 2025; accepted 1 May 2026; published 22 May 2026)

Quantum systems achieve objectivity by redundantly encoding information about themselves into the surrounding environment, through a mechanism known as quantum Darwinism. When this happens, observers measure the environment and infer the system to be in one of its pointer states. We study the emergence of objectivity whenever the Hamiltonian of the system and the interaction Hamiltonian between system and environment do not commute, a condition where it becomes difficult to identify pointer states, and the use of methods such as the predictability sieve becomes necessary. We show in an explicit qubit model to what degree the noncommuting evolution allows for the emergence of objective states, and show that a definition of pointer states based on spectrum broadcast structure is more natural in this context, and coincides with the one based on the predictability sieve for commuting evolutions.

DOI: 10.1103/75jr-ltct

## I. INTRODUCTION

In the context of quantum Darwinism [1–6], a system is said to be *objective* if many observers can independently infer its state without perturbing it. Let $\rho_{\mathcal{SF}}$ be the joint state of a system $\mathcal{S}$ and an environment $\mathcal{E} = \bigotimes_i \mathcal{F}_i$ partitioned into fractions $\mathcal{F}_i$. We say that $\mathcal{S}$ exhibits objectivity when information about $\mathcal{S}$ is redundantly encoded in many environment fractions, that is, $I_{\text{acc}}(\mathcal{S} : \mathcal{F}_i) \simeq S(\rho_{\mathcal{S}})$ for all $i$ [6,7], where $S(\cdot)$ is the von Neumann entropy and $I_{\text{acc}}$ the accessible mutual information [8]. Most models assume unitary evolution of the joint state under a (often time-independent) Hamiltonian in which the system Hamiltonian $H_{\mathcal{S}}$ commutes with the system-environment interaction $H_{\mathcal{I}}$ [9–12], and all proof-of-principle demonstrations operate within this framework [13–17]. When a dynamic creates objectivity in $\rho_{\mathcal{SE}}$, it has been shown that information about at most one system observable can proliferate and be accessible from the environment [18,19]; the eigenstates of this observable are the *pointer states*, and can be characterized as the states left pure and unperturbed by the evolution, as the eigenbasis of $H_{\mathcal{S}}$, or alternatively as the system basis appearing in the spectrum-broadcast-structure (SBS) decomposition of $\rho_{\mathcal{SE}}$ [4,6,20]. These different notions are equivalent, as long as $[H_{\mathcal{S}}, H_{\mathcal{I}}] = 0$ holds.

However, the condition $[H_{\mathcal{S}}, H_{\mathcal{I}}] = 0$ is quite restrictive and not justified in many real-world settings. In particular, it is to be expected that real measurement devices will involve much more

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complex dynamics and, in particular, noncommuting Hamiltonians. Relaxing the assumption of commutativity has important consequences on the definition and meaning of pointer states: The above definitions of pointer states no longer coincide, and it becomes unclear whether—and how—objectivity and pointer states persist. Here, we investigate dynamics in the presence of noncommutativity, so that $[H_S, H_I] \neq 0$, and study the emergence of objectivity and pointer states in such settings. We show using explicit qubit models how noncommutativity and objectivity can coexist, and how SBS states emerge even in such cases. We then define *SBS-pointer states* as the system states whose information is proliferated into the environment. More specifically, we show quantitatively that (1) noncommutativity hinders—but does not prevent—objectivity; (2) the resulting pointer states depend nontrivially on the interplay between $H_S$ and $H_I$, rather than being defined solely by $H_S$; and (3) the degrading effects of noncommutativity are minimized whenever the interaction dynamic is much faster than the intrinsic system dynamics, a condition that emerges naturally for macroscopic environments [21,22], even though $[H_S, H_I] = 0$ does not generally hold in these scenarios.

Our results provide a controlled case study of quantum objectivity in a noncommuting regime, thereby clarifying how an operative definition of pointer states can be recovered, which offers valuable insight into their physical meaning.

## II. THE MODEL

We consider a single-qubit system $\mathcal{S}$ coupled to $n$ environment qubits $\mathcal{E}_i$ through the standard dephasing-type interaction $H_I = \sum_{i=1}^{n} H_{S\mathcal{E}_i}$, where each $H_{S\mathcal{E}_i} = \gamma \sigma_z^{\mathcal{S}} \otimes \sigma_x^{\mathcal{E}_i}$ describes the interaction between $\mathcal{S}$ and a single environment fraction $\mathcal{E}_i$ [23–26]. With no loss of generality for our purposes, we will neglect the environment free evolution by assuming that its Hamiltonian commutes with the interaction Hamiltonian and thus moving to the interaction picture. The total Hamiltonian is

$$H_{\text{tot}} = H_S + H_I. \tag{1}$$

To study noncommutativity, we choose $H_S = \omega(p\sigma_x^{\mathcal{S}} + (1-p)\sigma_z^{\mathcal{S}})$, $p \in [0, 1]$, and use the parameter $p$ to control the degree of noncommutativity, on account of $[H_S, H_I] \propto p$. Environmental qubits are initialized in $|0\rangle$, and the system in the $\sigma_y$ eigenstate $|\circlearrowleft\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)$. By choosing an initial state that lies in an equal-weight superposition of $|0\rangle$ and $|1\rangle$, one maximizes susceptibility to decoherence induced by $H_I$, and among those the eigenstates of $\sigma_y$ are the most sensitive to the $\sigma_x^{\mathcal{S}}$ term in $H_S$, thereby making the effects of noncommutativity more prominent. Different choices of initial states yield qualitatively similar results. The interaction $H_I$ determines the basis in which the environment "reads" the system: Each environmental qubit $\mathcal{E}_i$ evolves conditionally under either the $\gamma\sigma_x^{\mathcal{E}_i}$ or the $-\gamma\sigma_x^{\mathcal{E}_i}$ Hamiltonian term, undergoing different unitary evolutions conditionally on the system being in the $|0\rangle$ or the $|1\rangle$ state, thus transferring information about the system's $\sigma_z$ eigenvalue into the $|\pm\rangle$ basis of the environment qubits. Meanwhile, if $[H_S, H_I] \neq 0$, the populations of the system in the $\sigma_z^{\mathcal{S}}$ basis are no longer constants of motion, so that the information encoded into the environment will progressively become outdated as time advances.

To study the time evolution of the system-environment state in this model and its objectivity, we will focus on the accessible information $I_{\text{acc}}$ and redundancy Red as quantifiers. Here, $I_{\text{acc}}(\mathcal{S} : \mathcal{F}_i)$ is the accessible mutual information between $\mathcal{S}$ and $\mathcal{F}_i$, defined as the (classical) mutual information of the joint probability distribution obtained measuring $\rho_{\mathcal{S}\mathcal{F}_i}$, maximized over all measurement choices on $\mathcal{S}$ and $\mathcal{F}_i$. This quantity is bounded by the Holevo $\chi$ [27] defined as

$$\chi(\mathcal{S} : \mathcal{F}_i) = \max_{\Pi^{\mathcal{S}}} \left\{ S\left(\sum_a p_a \rho_{\mathcal{F}_i|a}\right) - \sum_a p_a S(\rho_{\mathcal{F}_i|a}) \right\}, \tag{2}$$

where $a$ are the possible measurement outcomes of system measurements $\Pi^{\mathcal{S}}$, with $p_a$ the probability of said outcomes and $\rho_{\mathcal{F}_i|a}$ the corresponding conditioned states of $\mathcal{F}_i$, with the obvious

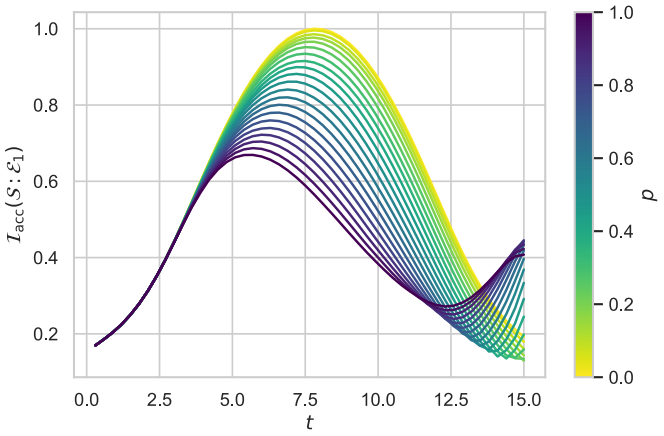



FIG. 1. Accessible information as a function of time. We report the accessible mutual information $I_{\rm acc}(\mathcal{S}:\mathcal{E}_1)$ between system and an individual environmental qubit as a function of time, for different values of $p$. The model parameters are fixed at $\omega=\gamma=0.1$, $n=8$. The reported accessible information is rescaled over $S(\rho_{\mathcal{S}})$.

advantage of needing to maximize over only one set of measurements. On the other hand, the redundancy of a state [28,29] is a standard way to quantify its objectivity. An operative way to define redundancy is as the largest number of fractions $\mathcal{F}_i$ into which we can partition $\mathcal{E}$, still ensuring each fraction has full information about $\mathcal{S}$. More precisely, the redundancy of $\rho_{\mathcal{SE}}$ is defined as [30]

$$\mathrm{Red}(\rho_{\mathcal{SE}}) \equiv \max\left\{ r:\ \mathcal{E}=\bigotimes_{i=1}^{r}\mathcal{F}_i,\ I_{\rm acc}(\mathcal{S}:\mathcal{F}_i)\simeq S(\rho_{\mathcal{S}})\right\}, \tag{3}$$

where $\mathcal{F}_i$ are in general composed of multiple qubits, in the cases where a single environmental qubit does not hold sufficient information about $\mathcal{S}$. Here, $I_{\rm acc}(\mathcal{S}:\mathcal{F}_i)\simeq S(\rho_{\mathcal{S}})$ is to be interpreted here as the condition $I_{\rm acc}(\mathcal{S}:\mathcal{F}_i)\geqslant(1-\delta)S(\rho_{\mathcal{S}})$ for some fixed threshold parameter $\delta>0$. For readability, we will suppress the explicit $\delta$ dependence and only specify its value when needed. The Hamiltonian we consider couples $\mathcal{S}$ symmetrically with each $\mathcal{F}_i$; thus, we do not need to distinguish between redundancy and consensus [30,31], and safely interpret $\mathrm{Red}(\rho_{\mathcal{SE}})$ as the number of observers simultaneously holding near-complete information about $\mathcal{S}$. Although redundancy is a coarse measure of objectivity—especially for small environments—its clear operational interpretation makes it one of the most relevant figures of merit here.

Figure 1 shows the time evolution of the accessible mutual information $I_{\rm acc}(\mathcal{S}:\mathcal{E}_1)$ between $\mathcal{S}$ and one environment qubit $\mathcal{E}_1$, for various values of $p$, for $\gamma=\omega=0.1$ and $n=8$. This clearly shows that for small $p$ one qubit suffices to recover full information about $\mathcal{S}$, but the encoding worsens progressively with increasing $p$, requiring multiple environment qubits to recover the same information. Furthermore, it is clear from these data that the amount of information that leaked into the environment is strongly time dependent—as expected in any unitary dynamics of this kind, due to finite-size effects.

To focus on the maximum degree of objectivity sustained by the dynamic, we then considered the time at which $\mathrm{Red}(\rho_{\mathcal{SE}})$ reaches its first local maximum. Figure 2 shows both $\mathrm{Red}(\rho_{\mathcal{SE}})$ and $I_{\rm acc}(\mathcal{S}:\mathcal{E}_1)$ for different values of $p$, at the time of maximum redundancy. We again use the parameter values $\gamma=\omega=0.1$ and $n=8$; the redundancy is computed using a threshold value of $\delta=0.9$. These results clearly show how objectivity degrades monotonically with increasing $p$, consistent with the intuition that a changing system state causes information acquired at earlier time to become outdated, thus hindering the emergence of objectivity. Similar effects of noncommutativity on quantum objectivity were previously observed in settings involving partially inaccessible environments [32].

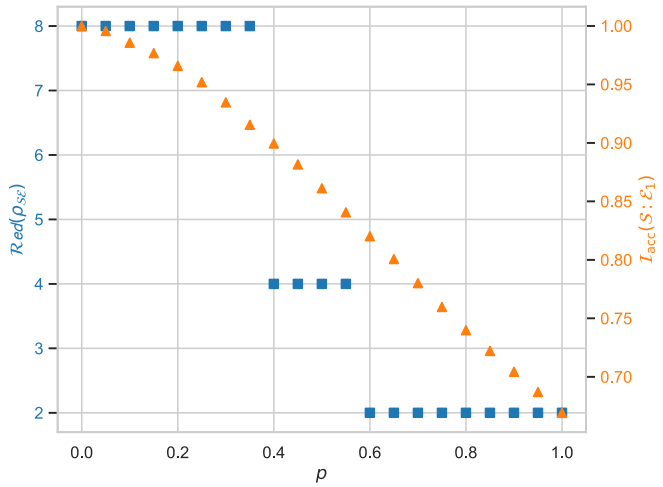



FIG. 2. Maximal redundancy for different values of $\boldsymbol{p}$. Redundancy $\mathrm{Red}(\rho_{\mathcal{SE}})$ and accessible mutual information $I_{\mathrm{acc}}(\mathcal{S}:\mathcal{E}_1)$ as a function of the noncommutativity parameter $p$, at the time maximizing $\mathrm{Red}(\rho_{\mathcal{SE}})$. Parameter values are $\gamma = \omega = 0.1$, $n = 8$, and $\delta = 0.9$. Both $\mathrm{Red}(\rho_{\mathcal{SE}})$ and $I_{\mathrm{acc}}(\mathcal{S}:\mathcal{E}_1)$ are evaluated at the time where they reach their maximum, which depends on $p$ but is the same for both quantities.

To investigate how objectivity depends on the other model parameters, we report in Fig. 3 the time evolution of the accessible mutual information $I_{\mathrm{acc}}(\mathcal{S}:\mathcal{E}_1)$, and in Fig. 4 the behavior of $\mathrm{Red}(\rho_{\mathcal{SE}})$ and $I_{\mathrm{acc}}(\mathcal{S}:\mathcal{E}_1)$ at the time of maximum redundancy, both as a function of $\omega/\gamma$, fixing $p = 1$. The $\omega/\gamma$ ratio characterizes the relative speeds of system and interaction Hamiltonians. For $\gamma \gg \omega$, dephasing outpaces the system's drive: The environment acquires information about $\rho_{\mathcal{S}}$ and induces decoherence before said information had a chance to degrade significantly, making the effects of noncommutativity negligible. On the other hand, when $\gamma \ll \omega$, the system evolves too fast for the environment to have a chance to read it. In this case, noncommutativity becomes highly disruptive and degrades the objectivity. These results are consistent with the fact that macroscopic environments give rise to objectivity, while it is generally unjustified to assume that $[H_{\mathcal{S}}, H_{\mathcal{I}}] = 0$ holds in all such systems. In the limit $\gamma \gg \omega$—expected for large baths—decoherence happens at much smaller timescales compared to the system's dynamics, allowing the emergence of objectivity

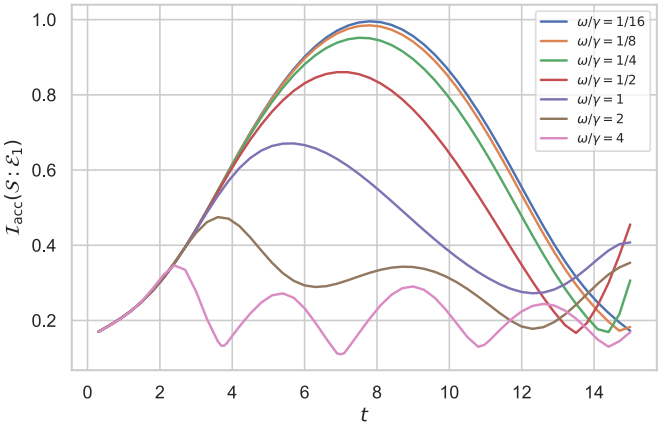



FIG. 3. Accessible information as a function of time. We report the accessible mutual information $I_{\mathrm{acc}}(\mathcal{S}:\mathcal{E}_1)$ between system and an individual environmental qubit as a function of time, for different values of the $\omega/\gamma$ ratio, fixing $p = 1$ and $n = 8$. The reported accessible information is rescaled over $S(\rho_{\mathcal{S}})$.

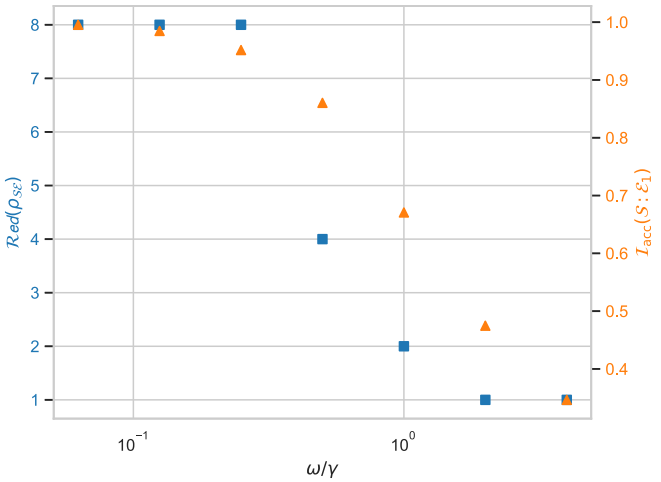



FIG. 4. Maximal redundancy changing model parameters. We report $\mathrm{Red}(\rho_{\mathcal{SE}})$ and $I_{\rm acc}(\mathcal{S}:\mathcal{E}_1)$ computed at the time of maximal redundancy, as a function of $\omega/\gamma$, fixing $p = 1$, $n = 8$, and $\delta = 0.9$.

even for noncommuting interactions. While our results show that noncommutativity is a hindrance to the emergence of objectivity, they also point out that objective states can emerge as a result of noncommutative dynamics, and that in the macroscopic limit the effect of noncommutativity may become negligible.

## III. POINTER STATES

So far, we have shown that objective states may emerge even in the presence of noncommutativity between $H_{\mathcal{S}}$ and $H_{\mathcal{I}}$. We now study whether such dynamics also select pointer states, how these should be defined, and what operational meaning can be attached to them.

In their original conception, pointer states are those least affected by environment-induced decoherence [5]. These can be formalized as $\{|\psi_i^P\rangle\}$ such that, for any environment state $|\psi_{\mathcal{E}}\rangle$ and all $t$, the evolution maps $|\psi_i^P\rangle \otimes |\psi_{\mathcal{E}}\rangle \to |\psi_i^P\rangle \otimes |\psi_{\mathcal{E}}(t)\rangle$. Thus, pointer states remain pure throughout the dynamics, and information about them leaks into the environment. This definition is incompatible with noncommuting $H_{\mathcal{S}}$ and $H_{\mathcal{I}}$ [33–35], as it is not possible to find system states that are simultaneously eigenstates of both $H_{\mathcal{S}}$ and $H_{\mathcal{I}}$. In such cases, one can find approximate pointer states through a mechanism known as the predictability sieve [36–38], which selects the states that suffer the least decoherence due to the interaction with the environment.

Another defining feature of pointer states is that they are the states the environment has the most information about; quoting Zurek [19]: "It is the propagation of the correlations with the pointer basis states which is ultimately responsible for the choice of the pointer observable"—they are the system states that would emerge as objective as a result of measurements on the environment [2,39]. Pointer states were thus originally introduced to refer to the states corresponding to the possible display positions of a measurement apparatus [18].

These definitions are equivalent when $[H_{\mathcal{S}}, H_{\mathcal{I}}] = 0$, where we can identify $|\psi_i^P\rangle$ as the eigenstates of $H_{\mathcal{S}}$ and observe that these states both remain pure and are the ones that the environment gets the most information about. However, in more general cases with $[H_{\mathcal{S}}, H_{\mathcal{I}}] \neq 0$, there may not be a system basis that satisfies both conditions at the same time. We may choose to identify pointer states as the ones suffering the least decoherence (predictability sieve) or we may choose to identify them as the ones whose information has proliferated the most into the environment. Here, we choose the latter approach.

The SBS formalism is particularly well suited to describe states where the environment holds redundant information about the system state. From this perspective, a system-environment state is

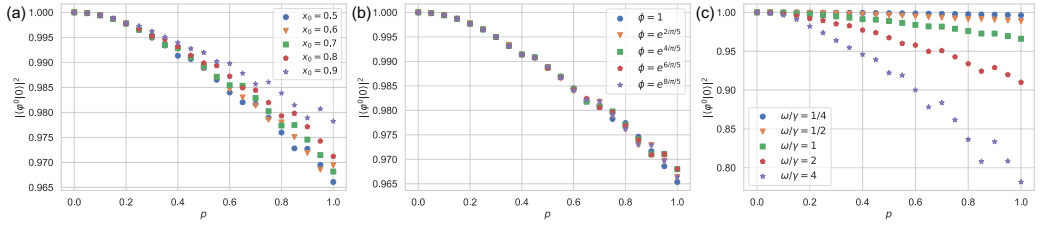



FIG. 5. Fidelity, with respect to the computational basis, for the pointer states. Fidelity between one of the SBS-pointer states emerging from the dynamics $|\varphi^0\rangle$ and $|0\rangle$ as a function of the $p$ parameter. (a) The initial state is of the form $\sqrt{x_0}\,|0\rangle + i\sqrt{1-x_0}\,|1\rangle$, with $\gamma = \omega = 0.1$. (b) The initial state is of the form $\frac{1}{\sqrt{2}}\,|0\rangle + \phi\frac{1}{\sqrt{2}}\,|1\rangle$, with $\gamma = \omega = 0.1$. (c) The initial state is $|\circlearrowleft\rangle$, with different degrees of the $\omega/\gamma$ ratios. For all panels, $n = 6$ and $\delta = 0.9$. The plots show the dependence of pointer states with respect to the dynamical parameters $p$ and $\omega/\gamma$; they also show that SBS-pointer states depend not only on the dynamical parameters but also on the initial state of the system.

objective when it can be written as [20]

$$\rho_{\mathrm{SBS}} = \sum_i p_i |\psi_i\rangle\langle\psi_i| \bigotimes_j^N R_i^j, \tag{4}$$

with $\{|\psi_i\rangle\}$ a set of orthogonal system states and $R_i^j$ states of environmental fractions such that $R_i^j R_{i'}^j = 0$ when $i \neq i'$. The $\{|\psi_i\rangle\}$ states define the basis in which we need to measure the system in order to maximize the mutual information between the system and the environmental fraction, and are therefore the states that the environment has the most information about.

Thus, whenever the joint state is close to SBS form, the SBS decomposition provides a natural operational definition of pointer basis. In the case where commutativity is restored, SBS-pointer states will also be the ones most resilient to decoherence. It is worth noting that in the noncommuting scenario, the SBS-pointer basis can no longer be determined *a priori* from the Hamiltonian alone, due to the resulting complex dynamics; rather, it must be obtained by casting the joint system-environment state in SBS form. Indeed, SBS-pointer states are uniquely defined once the system-environment state is known, even if the underlying dynamic that created such state is not known. The predictability sieve, on the other hand, requires knowledge of the open dynamic of the system, which is needed to identify the states that remain the most pure throughout the evolution. The concept of SBS-pointer states was also explored in Ref. [40], where it was referred to as the generalized pointer.

With this alternative definition of pointer states, we can now analyze the pointer states that emerge as a result of the system and environment evolving under the Hamiltonian in Eq. (1). We consider three different scenarios. In the first one, the initial state of the system is of the form $\sqrt{x_0}\,|0\rangle + i\sqrt{1-x_0}\,|1\rangle$, with $x_0$ ranging from $x_0 = 0.5$ to $x_0 = 0.9$. In the second scenario, the initial state of the system lies on the equatorial line of the Bloch sphere, and is of the form $\frac{1}{\sqrt{2}}\,|0\rangle + \phi\frac{1}{\sqrt{2}}\,|1\rangle$, with $\phi$ a complex number of unit norm. In both first and second scenarios, $\gamma = \omega = 0.1$. Finally, in the third case, the initial state of the system is $|\circlearrowleft\rangle$, but we have different ratios between the interaction strength $\gamma$ and the system frequency $\omega$. In all scenarios, the environment consists in $n = 6$ qubits all initially in the $|0\rangle$ state.

We show in Fig. 5 the fidelity between one of the two pointer states emerging from the dynamics, $|\varphi^0\rangle$ and $|0\rangle$, for different values of the noncommutativity parameter $p$. The basis $\{|0\rangle, |1\rangle\}$, being the eigenstates of the interaction Hamiltonian $H_{\mathcal{I}}$, would be the pointer states emerging from the dynamics in the presence of commutativity; however, when $H_{\mathcal{S}}$ and $H_{\mathcal{I}}$ do not commute, the system evolves in an objective state but the resulting pointer states are different.

Figure 5(a) shows that the resulting pointer basis depends not only on $H_{\mathcal{S}}$ and $H_{\mathcal{I}}$ but also on the initial state of the system, further showing how pointer states cannot be determined *a priori*

from the Hamiltonian in the presence of noncommutativity. This dependence on the initial state seems, however, minimized whenever the system is initially in an equal-weight superposition of $|0\rangle$ and $|1\rangle$, i.e., one of the states that suffers the most decoherence from the environment, as shown in Fig. 5(b). Finally, Fig. 5(c) shows how different driving strengths have a large influence on the resulting pointer states, with pointer states being minimally altered by noncommutativity for $\gamma \gg \omega$, while they change significantly for $\omega \gg \gamma$. This is expected for the same reasoning outlined in the previous section. The mere fact that SBS-pointer states depend also on the initial state of the system is proof enough that pointer states selected via the SBS criterion and those selected via the predictability sieve do not coincide, meaning that the states that suffer the least decoherence are in general not the ones that maximize system-environment correlations.

If we interpret the interaction with the environment as a measurement operation carried out by an apparatus, what we can see here is that in the presence of noncommutativity the apparatus is measuring something different from what was initially intended, in that the apparatus was designed to measure in the $\{|0\rangle, |1\rangle\}$ basis and is instead measuring in a different one, making the measurement operation imperfect. This, however, cannot be avoided if we want to measure a system in a basis that does not commute with the system's own Hamiltonian, making this one of the many systematic errors that real measurements are subject to. What we show here is that the fidelity between the intended measurement and the effective one can be high in a wide range of parameters.

## IV. CONCLUSIONS

In this work, we analyzed the emergence of objectivity in the case where the system Hamiltonian and the system-environment interaction Hamiltonian do not commute, something that would naturally happen in many realistic situations. We quantified in a simple qubit model to what degree noncommutativity hinders the emergence of objectivity, and how objective states may still emerge as a result of such noncommuting evolutions.

The studied model provided an interesting platform to further explore the concept of pointer states, which are understood as the ones whose information has proliferated into the environment, as well as the states most resilient to environmental decoherence. We showed that if we relax the condition of resilience from decoherence, and simply require proliferation of information, we can define a class of pointer states that are always well defined whenever the system is objective according to quantum Darwinism, and can be identified within the SBS framework. Such states are not the same states selected by the predictability sieve. It remains an open question whether one should use one definition or the other in noncommuting scenarios, but we argue that since SBS-pointer states are rooted in the information proliferated into the environment, they may be the more appropriate choice in the context of the objectivity of quantum systems.

Our work is a further step into exploring the emergence of objectivity and the quantum-to-classical transition for nontrivial, nonideal scenarios, and offers a different perspective on the meaning of pointer states. The analysis of generic properties of these SBS-pointer states remains an interesting research avenue for future work.

## ACKNOWLEDGMENTS

D.A.C. acknowledges support from the Italian National Quantum Science and Technology Institute (NQSTI) (PE0000023)—SPOKE 2 through project ASpEQCt and from Taighde Éireann-Research Ireland under Grant No. GOIPD/2025/1353. G.M.P. acknowledges support by MUR under PRIN Project No. 2022FEXLYB "Quantum Reservoir Computing" (QuReCo).

## DATA AVAILABILITY

The data that support the findings of this article are openly available [41].

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