Letter

Zero point density fluctuations and electron Brownian motion

L. H. Ford [*]
Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155, USA

G. O. Heymans [†] and N. F. Svaiter [‡]
Centro Brasileiro de Pesquisas Físicas (CBPF), Rio de Janeiro, RJ 22290-180, Brazil

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The fluctuations in the phonon vacuum state can lead to zero point density fluctuations in a material, which in turn lead to local zero point fluctuations of the dielectric properties of the material. We argue that the density fluctuations lead to a fluctuating force on a test charge, such as an electron, located a short distance outside of the material. This force is due to fluctuating dipole moments inside the material and produces Brownian motion of the electron. We calculate the mean squared velocity of the electron in both the normal and transverse directions relative to the boundary of the material. The result is nonzero in both directions, but larger in the normal case. We estimate the magnitude of this quantum Brownian motion and find that, in some cases, it can exceed the effects of both thermal motion and quantum momentum uncertainty. This suggests that the effect may be observable and could constitute a source of quantum noise in nanoscale devices. It also potentially offers a means to remotely sense zero point density fluctuations in a material.

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The observability of zero point or vacuum fluctuations is a subtle issue that has been discussed by several authors [1,2]. Although it is generally agreed that the Lamb shift and the Casimir effect are observable consequences of vacuum fluctuations, it is often not clear where the energy for a detection can come from if both the detector and the quantum system are in their ground states. However, both the Lamb shift and the Casimir effect can be viewed as energy shifts in a ground-state energy that produce observable effects. The effect we will propose can be viewed similarly.

One mechanism for detection of zero point fluctuations can be their effects on the Brownian motion of test particles. The dominant contribution to the Lamb shift can be viewed as due to the effect of electric field fluctuations on the motion of an electron [3]. Several authors have investigated Brownian motion in the modified vacuum fluctuations of a quantum field near a perfect mirror [4–8] or in an expanding universe [9]. Brownian motion has been invoked as a probe for the quantum nature of the gravitational field [10]. Temperature fluctuations in the cosmic microwave background

[*]Contact author: ford@cosmos.phy.tufts.edu
[†]Contact author: olegario@cbpf.br
[‡]Contact author: nfuxsvai@cbpf.br

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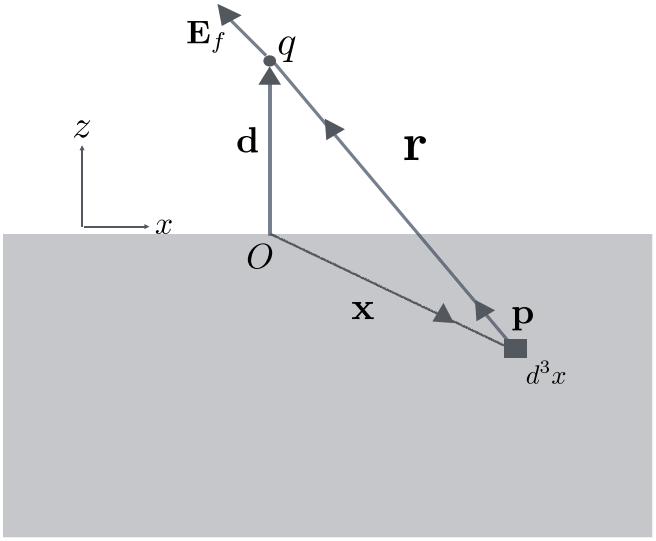


FIG. 1. A test charge q is located a distance d above a dielectric material with dielectric constant κ. Here, the origin is at the point O. The Coulomb field of the charge induces a dipole moment $\mathbf{p}$ in a region with volume d^3x inside the dielectric. The electric field of this dipole exerts a force on the test charge. The spatial integral of the time average of this force is the classical image charge force on the charge. Local density fluctuations produce a fluctuating part of the dipole moment and hence a fluctuating part $\mathbf{E}_f$ of the electric field acting on the charge. This fluctuating field in turn produces force and velocity fluctuations of the charge, resulting in the Brownian motion.

due to gravitons from inflation have not yet been detected, but could be viewed as arising from photon Brownian motion in a bath of gravitons from inflation [11–13].

Here, we will deal with mass density fluctuations of a dielectric material due to zero point motion in the phonon vacuum state. Let the density be a function of space and time of the form $\rho(\mathbf{x}, t) = \rho_0 + \hat{\rho}(\mathbf{x}, t)$. Here, ρ_0 is the mean mass density and $\hat{\rho}(\mathbf{x}, t)$ is an operator representing the quantum fluctuations around this mean value, which can be expanded in terms of phonon creation and annihilation operators and plane wave modes. This operator is analogous to that for a relativistic quantum scalar field, but with the speed of light replaced by the speed of sound. This analogy has been exploited to study quantum field effects near black holes [14,15] and in the early universe [16]. The density fluctuations satisfy a correlation function [17]

$$\langle \hat{\rho}(\mathbf{x}, t)\hat{\rho}(\mathbf{x}', t')\rangle = \frac{\hbar\rho_0}{2\pi^2 c_s}\frac{\Delta\mathbf{x}^2 + 3c_s^2\Delta t^2}{\left(c_s^2\Delta t^2 - \Delta\mathbf{x}^2\right)^3}, \tag{1}$$

where c_s is the speed of sound in the dielectric and the expectation value is taken in the phonon vacuum state. These density fluctuations are large enough to have been observed in light scattering experiments, as described in Refs. [17–21]. See also Ref. [22].

In this Letter, we describe a different effect, in which the density fluctuations cause velocity fluctuations of a test charge, such as an electron, just outside the material. The geometry is illustrated in Fig. 1.

The test charge q located at $z = d$ outside the dielectric induces polarization in the material. The total, time-averaged, electric field of this polarization exerts a force on the charge, which may be described as that due to an image charge $q' = -q(\kappa - 1)/(\kappa + 1)$, located at $z = -d$ [23]. Here, κ is the dielectric constant of the material. The image charge force may equivalently be derived as the net force on the test charge due to induced polarization of the material. Here, we assume that κ is

the static dielectric constant and ignore the effects of dispersion and dissipation in the material. At the frequencies of interest here, which are determined by the sound travel time over a distance d, this should be a good approximation.

The effect of the zero point density fluctuations is to produce fluctuations of κ, and hence of the force on the test charge. Here, we assume small fractional fluctuations $|\hat{\rho}| \ll \rho_0$. In this case, the dielectric constant has a small fluctuating part

$$\kappa_1 = \frac{\hat{\rho}}{\rho_0}\,\kappa_0, \tag{2}$$

where κ_0 is the mean dielectric constant and $\kappa = \kappa_0 + \kappa_1$.

The electric field of q inside the dielectric is equivalent to that of a second image charge [23] $q'' = 2\,\kappa\,q/(\kappa+1)$ at the location of the original charge, resulting in an electric field of $\mathbf{E}_q = -q\,\hat{\mathbf{r}}/[2\pi\epsilon_0\,(\kappa+1)\,r^2]$. This in turn produces a dipole moment density of $\mathbf{P} = \epsilon_0\,\kappa\,\mathbf{E}_q$ and a dipole moment in volume d^3x of $d\mathbf{p} = \mathbf{P}\,d^3x$. The electric field of this dipole at the location of the test charge is

$$d\mathbf{E}_d = \frac{3\,\hat{\mathbf{r}}\,(\hat{\mathbf{r}}\cdot d\mathbf{p}) - d\mathbf{p}}{4\pi\epsilon_0\,r^3} = -\frac{q(\kappa-1)\,\mathbf{r}\,d^3x}{4\pi^2\epsilon_0\,(\kappa+1)\,r^6}. \tag{3}$$

If we set $\kappa = \kappa_0$ and integrate over the volume of the dielectric, the result is the electric field of the image charge q'.

We are interested in the fluctuating electric field $\mathbf{E}_f$ due to the density fluctuations. If we Taylor expand to first order in κ_1, the result is

$$\frac{\kappa-1}{\kappa+1} = \frac{\kappa_0-1}{\kappa_0+1} + \frac{2\,\kappa_1}{(\kappa_0+1)^2} + \cdots. \tag{4}$$

The contribution of region d^3x to $\mathbf{E}_f$ becomes

$$d\mathbf{E}_f = -\frac{q\,\kappa_0\,\hat{\rho}\,\mathbf{r}\,d^3x}{2\pi^2\,\epsilon_0\,\rho_0\,(\kappa_0+1)^2\,r^6}. \tag{5}$$

The net fluctuating field at the location of the test charge, after spatial integration, is

$$\mathbf{E}_f(t) = -\frac{q\,\kappa_0}{2\pi^2\,\epsilon_0\,\rho_0\,(\kappa_0+1)^2}\int d^3x\,\hat{\rho}(\mathbf{x},t)\,\mathbf{g}(\mathbf{x}), \tag{6}$$

where

$$g^j(\mathbf{x}) = \frac{r^j}{r^6} = \frac{d^j - x^j}{|\mathbf{d}-\mathbf{x}|^6}, \tag{7}$$

with $\mathbf{r} = \mathbf{d} - \mathbf{x}$. Note that the electric field changes due to density fluctuations travel to the test charge at the speed of light, which is much larger than the speed of sound. As a result, we may view the response of the test charge to the density fluctuations as instantaneous.

The electric field correlation function becomes

$$\left\langle E_f^j(t)\,E_f^k(t')\right\rangle = \left[\frac{q\,\kappa_0}{2\pi^2\,\epsilon_0\,\rho_0\,(\kappa_0+1)^2}\right]^2\int d^3x\,d^3x'\,g^j(\mathbf{x})\,g^k(\mathbf{x}')\,\langle\hat{\rho}(\mathbf{x},t)\hat{\rho}(\mathbf{x}',t')\rangle, \tag{8}$$

where j and k label Cartesian vector components. The particle will undergo motion in response to the fluctuating field. We will describe this motion using a Langevin equation, which takes the form of Newton's second law with a fluctuating force. Here, we assume that an external force cancels the classical image charge effect, so the particle responds to $\mathbf{E}_f(t)$. This external force could come from an applied electric field, or as a result of confinement in a structure, such as a nanotube. If we set the initial velocity due to the zero point density fluctuations to zero, $v_{\rm zp}^j(0) = 0$, then at time $t = t_1$,

$v^j_{\rm zp}(t_1) = (q/m)\int_0^{t_1} E^j_f(t)\,dt$, where m is the mass of the particle. The mean squared speed in the j direction at time t_1 is

$$\left\langle \left(v^j_{\rm zp}\right)^2 \right\rangle = \frac{q^2}{m^2}\int_0^{t_1} dt \int_0^{t_1} dt'\, \left\langle E^j_f(t)\, E^j_f(t')\right\rangle. \tag{9}$$

The field correlation function depends upon $\tau = t - t'$, so we may use the identity

$$\int_0^{t_1} dt \int_0^{t_1} dt'\, C(t-t') = 2\int_0^{t_1} d\tau (t_1 - \tau)\, C(\tau). \tag{10}$$

Here,

$$\langle \hat\rho(\mathbf{x}, t)\hat\rho(\mathbf{x}', t')\rangle \propto C(\tau) = \frac{3\,\tau^2 + \xi^2}{(\tau^2 - \xi^2)^3}, \tag{11}$$

where $\xi = |\Delta\mathbf{x}|/c_s$. Now,

$$2\int_0^{t_1} d\tau (t_1 - \tau)\, C(\tau) \sim \frac{1}{\xi^2}, \quad t_1 \gg \xi. \tag{12}$$

Here, we have evaluated the integration as an indefinite integral and then evaluated the result at the integration limits. Although $C(\tau)$ has a third-order pole at $\tau = \xi$, this procedure can be justified either as a Hadamard finite part or as the result of analytically continuing ξ into the complex plane before integration, and then back to the real line afterward.

This result tells us that the mean squared velocity components, $\langle (v^j_{\rm zp})^2\rangle$, approach constant values at late times. The dominant contributions to the spatial integrations will come from points separated by $|\Delta\mathbf{x}| \approx d$, so late times mean $t \gtrsim d/c_s$. If the electric field fluctuations were uncorrelated, we would expect to find the particle speed to undergo a random walk where $\langle v^2_{\rm zp}\rangle \propto t$, which would violate energy conservation in the phonon vacuum state. The fact that this does not happen reflects strong anticorrelations in the density fluctuations and hence the electric field fluctuations. In effect, the electron can temporarily acquire energy from a fluctuation, but an anticorrelated fluctuation will soon take back this energy. See Ref. [24] for further discussion of this issue. The situation here is rather different from that in statistical physics, where dissipation is needed to constrain the growth of $\langle v^2\rangle$ in Brownian motion. Note that the density correlation function [Eq. (1)] is negative when $c_s\Delta t < |\Delta\mathbf{x}|$, so fluctuations outside the sound cone are anticorrelated.

We can combine the above results to write an expression for the asymptotic values of $\langle (v^j_{\rm zp})^2\rangle$:

$$\left\langle \left(v^j_{\rm zp}\right)^2 \right\rangle = A\, I^j, \tag{13}$$

where

$$A = \frac{2\hbar}{\pi^4\, c_s^3\, m^2\, \rho_0}\left(\frac{q^2}{4\pi\epsilon_0}\right)^2 \left[\frac{\kappa_0}{(\kappa_0+1)^2}\right]^2 \tag{14}$$

and

$$I^j = \int d^3x\, d^3x'\, \frac{g^j(\mathbf{x})\, g^j(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^2}. \tag{15}$$

The integral I^j is proportional to $1/d^6$, so we can write $I^j = c_j/d^6$, where c_j are dimensionless constants, which may be evaluated numerically, with the results [25]

$$c_z \approx 0.68, \quad c_x = c_y \approx 0.068. \tag{16}$$

Hence, the mean squared speeds are

$$\left\langle \left(v^j_{\rm zp}\right)^2 \right\rangle = \frac{A\, c_j}{d^6}. \tag{17}$$

We see that the mean squared speed in the normal direction is about an order of magnitude larger than that in the transverse direction. Note that we have assumed that the background mass density, ρ_0, is independent of position. The effects of spatial inhomogeneity, dispersion, and dissipation in the dielectric will be topics for future work.

We are now in a position to discuss several conceptual issues related to our model. The first is the question of the energy source for the Brownian motion. It may be viewed as arising from the classical electrostatic energy of the test charge in the image charge field $\mathbf{E}_d$. Recall that the fluctuating electric field $\mathbf{E}_f$ that causes the Brownian motion is much smaller in magnitude. A second question is whether the sudden switching at $t = 0$ assumed in Eq. (9) alters the result, as was found in some other Brownian motion models [6,7] involving relativistic quantum fields. The following thought experiment suggests that this is not the case here: Suppose that the switch-on occurred at a very large value of d, where both $\langle (v^j_{\rm zp})^2 \rangle$ and any effect of the switching are very small. Then, suppose that the electron, or any structure containing it, is slowly lowered to a smaller value of d. Consequently, the result given in Eq. (17) is relatively independent of the details of the switch-on. This result may be viewed as a shift in the ground-state energy of a system, just as in the cases of the Lamb shift and the Casimir effect.

There are two other sources of velocity fluctuations that are distinct from the Brownian motion effect treated here, but that need to be considered in an experiment. The first is the mean squared thermal velocity at temperature T in one direction, $v_T^2 = k_B\, T/m$, where k_B is Boltzmann's constant. The other is the velocity variance, $\langle (v^j_{\rm qu})^2 \rangle$, due to the quantum momentum uncertainty of a localized quantum particle. If the electron is localized in a region of length ℓ in direction j, and the momentum uncertainty is on the order of the uncertainty principle minimum, then

$$\left\langle \left(v^j_{\rm qu}\right)^2 \right\rangle \approx \frac{\hbar^2}{m^2\, \ell^2}. \tag{18}$$

We may compare this effect with that of finite temperature using

$$\frac{\left\langle \left(v^j_{\rm qu}\right)^2 \right\rangle}{v_T^2} \approx 8.8 \left(\frac{1\,{\rm K}}{T}\right) \left(\frac{10\,{\rm nm}}{\ell}\right)^2. \tag{19}$$

It is useful to compare the numerical values of the zero point-fluctuation-induced speeds for an electron with the thermal and momentum uncertainty effects using the above relation and

$$\frac{\left\langle \left(v^j_{\rm zp}\right)^2 \right\rangle}{v_T^2} = \frac{9.1 \times 10^3\, c_j\, \kappa_0^2}{(\kappa_0 + 1)^4} \left(\frac{1\,{\rm K}}{T}\right) \left(\frac{1\,{\rm nm}}{d}\right)^6 \left(\frac{10^3\,{\rm m/s}}{c_s}\right)^3 \left(\frac{10^3\,{\rm kg/m^3}}{\rho_0}\right). \tag{20}$$

In the case of liquid ^{3}He, where $T = 3.2\,{\rm K}$, $\rho_0 = 83\,{\rm kg/m^3}$, $c_s = 100\,{\rm m/s}$, and $\kappa_0 = 1.026$, we have

$$\left\langle \left(v^j_{\rm zp}\right)^2 \right\rangle \approx 8.6\, c_j \left(\frac{10\,{\rm nm}}{d}\right)^6 v_T^2 \approx 3.1 \times 10^4\, c_j \left(\frac{1\,{\rm nm}}{d}\right)^6 \left(\frac{\ell}{1\,{\rm nm}}\right)^2 \left\langle \left(v^j_{\rm qu}\right)^2 \right\rangle. \tag{21}$$

In the case of liquid ^{2}N, where $T = 77\,{\rm K}$, $\rho_0 = 808\,{\rm kg/m^3}$, $c_s = 850\,{\rm m/s}$, and $\kappa_0 = 1.5$, we find

$$\left\langle \left(v^j_{\rm zp}\right)^2 \right\rangle \approx 57\, c_j \left(\frac{1\,{\rm nm}}{d}\right)^6 v_T^2 \approx 5.0\, c_j \left(\frac{1\,{\rm nm}}{d}\right)^6 \left(\frac{\ell}{1\,{\rm nm}}\right)^2 \left\langle \left(v^j_{\rm qu}\right)^2 \right\rangle. \tag{22}$$

We see that there are ranges of the parameters where the effects of zero point density fluctuations can dominate over both thermal motion and quantum momentum uncertainty. In the case of motion transverse to the dielectric interface, it is possible to have $\ell \gg d$, in which case we can make $\langle (v^j_{\rm qu})^2 \rangle$ very small. It is also possible to distinguish $\langle (v^j_{\rm zp})^2 \rangle$ from $\langle (v^j_{\rm qu})^2 \rangle$ by their associated probability distributions. That for $\langle (v^j_{\rm zp})^2 \rangle$ is Gaussian, as these are vacuum fluctuations arising from the ground state of the quantum harmonic oscillator. However, the distribution for $\langle (v^j_{\rm qu})^2 \rangle$ depends upon the quantum state of the localized electron.

Note the very strong dependence of $\langle (v_{zp}^2)^2 \rangle$ upon the distance of the electron from the boundary of the dielectric. Our analysis, based upon Eq. (1), strictly assumes a continuum description of the material, and hence d large compared to the interatomic spacing. However, Eq. (20) may give a fair order of magnitude estimate even when d is on the order of the interatomic spacing. Another characteristic of our results is the marked anisotropy of the Brownian motion, in contrast to thermal motion.

Note that here we treat the fluctuations of the linear phonon operator $\hat{\rho}$, with the consequence that the electron velocity fluctuations will be described by a Gaussian probability distribution. This need not be the case if we include the effects of higher-order terms in Eq. (4), which will be a topic for future work. A related topic is the possible correlation between light scattering by a localized density fluctuation [22] and the electron velocity fluctuations.

If the electron is confined within a structure near the dielectric, such as a thin film or a nanotube, then our analysis can apply so long as the electron Coulomb field and the induced dipole fields can penetrate the structure. In this case, the Brownian motion can act as a source of noise that may exceed the thermal noise and the effects of quantum momentum uncertainty. Indeed, modern nanofabrication techniques routinely achieve electron confinement in ultrathin films, semiconductor heterostructures, and carbon nanotubes with characteristic dimensions of only a few nanometers, and in some cases approaching the 1 nm scale [26,27]. Since the velocity fluctuations induced by zero point density fluctuations scale as d^{-6}, this regime is particularly favorable for their observation. As shown by Eq. (20), for distances of order 1 nm these fluctuations can become comparable to, or even exceed, both the thermal velocity fluctuations and those associated with quantum momentum uncertainty. Consequently, the effect discussed here may constitute an experimentally relevant source of quantum noise.

Thus, the effect treated here may be observable and have implications both for nanotechnology and for fundamental physics, including as an analog model.

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Data availability. The data that support the findings of this article are available in the Supplemental Material [25].

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