

Thermal image velocimetry for rapidly rotating fluid dynamicsRémy Monville ^{1,*}, Céline Guervilly ², Hao Cao ¹ and Jonathan M. Aurnou ¹¹*Department of Earth, Planetary and Space Sciences, [University of California Los Angeles](#), Los Angeles, California, USA*²*School of Mathematics, Statistics and Physics, [Newcastle University](#), Newcastle Upon Tyne NE17RU, United Kingdom*

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Accurate measurement of fluid velocity remains a significant challenge in experimental fluid dynamics, as it requires capturing detailed spatial and temporal information within flows that are frequently turbulent. Obtaining velocity fields requires tracking identifiable features within the flow, such as particles, tracer elements, or coherent flow structures, over time. This typically requires accurate optical visualization of the flow. The most prevalent and effective method for measuring fluid velocity in the laboratory is particle image velocimetry (PIV), which uses the tracking of suspended reflective or fluorescent particles to determine the fluid's velocity. The mathematical method commonly used to track particles is the cross-correlation between two images. PIV relies on a laser sheet to illuminate the particles in a plane and requires the fluid to be transparent to visible light. To overcome these optical constraints, we propose to use the surface temperature field, which is accurately measurable in the context of thermal convection experiments. This provides structures that can be tracked if the temperature field is mainly advected by the flow. We employ the Farneback dense optical flow algorithm [Farneback, Two-frame motion estimation based on polynomial expansion, in *Image Analysis*, edited by J. Bigun and T. Gustavsson (Springer, Berlin, 2003), pp. 363–370] to track the features of the temperature field. Similar methods have proven effective in tracking planetary flows, as demonstrated by measuring Jupiter's surface or Earth's ocean currents. We present here a benchmark study using a simple, synthetic two-dimensional advection-diffusion simulation, as well as a rapidly rotating quasigeostrophic simulation based on the “Coreaboloid” experimental setup [Lonner, Aggarwal, and Aurnou, Planetary core-style rotating convective flows in paraboloidal laboratory experiments, *J. Geophys. Res.: Planets* **127**, e2022JE007356 (2022)]. We investigate the applicability of thermal image velocimetry to measuring the velocity field in rotating convection experiments. The reconstructed velocity field allows for a quantitative physical analysis of the flow dynamics.

DOI: [10.1103/1jkc-jrq8](https://doi.org/10.1103/1jkc-jrq8)**I. INTRODUCTION**

Rotating laboratory experiments can be employed to reproduce and understand the behavior of geophysical fluids. One of the challenges of these experiments is to measure the velocity of the fluid.

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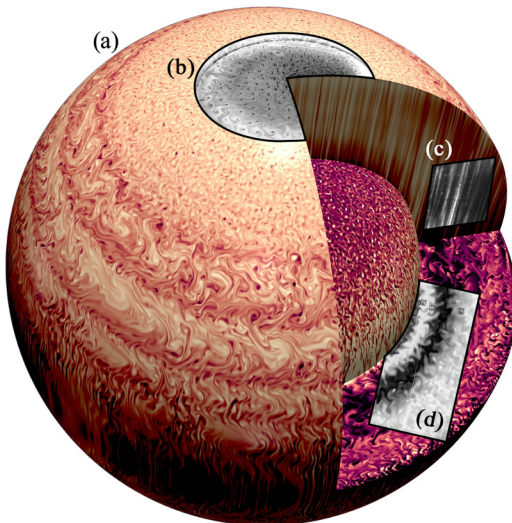


FIG. 1. Visualization of laboratory-numerical models of rapidly rotating planetary core convection [6]. (a) Spherical shell direct numerical simulation [7]. (b) Polar convection dye experiment [8]. (c) Axial slice showing cylindrical laboratory rotating thermal convection experiment [9]. (d) Paraboloidal annular laboratory convection experimental thermal image [16].

The fluid dynamics community employs a diverse range of methods, varying in complexity. These include ultrasound Doppler velocimetry [1,2], laser Doppler velocimetry [3], and shadowgraph imaging [4]. By far the most common visualization method to obtain a two-dimensional high-resolution velocity field remains particle image velocimetry (PIV) [5]. This method is based on the tracking of small suspended particles that reflect the light of a laser sheet. This requires a clear optical path for the laser, necessitating both a transparent fluid and a transparent tank. Even if this is often the case for most of the acrylic tank and water experiments, it can become a strong constraint for certain geometries or if one needs to use opaque metallic boundaries for convection setups.

Most geophysical flows can be considered to be rapidly rotating, such as in Earth’s ocean and atmosphere, when observed at large scale, or in the fluid cores of planets [10]. In rapidly rotating flows, the Coriolis force is principally balanced by the pressure gradient force. Other effects of buoyancy or viscosity, for example, can be considered as perturbations. The result, according to the Proudman-Taylor theorem, is that the fluid motion is mainly two-dimensional and largely invariant along the rotation axis [11]. This is characterized by flows of columnar shape. High rotation rate $\Omega = \Omega \mathbf{1}_x$ implies that the vertical derivative of the velocity is small $\partial_z \mathbf{U} \sim \epsilon$ (with $\epsilon \ll 1$). Then, $\partial_z U_z \sim \epsilon$, which implies that the horizontal divergence is also small, $\nabla_h \cdot \mathbf{U} \sim \epsilon$, in Boussinesq fluids ($\nabla \cdot \mathbf{U} \equiv 0$).

This behavior is observed in both simulations and experiments of planetary fluid layers (Fig. 1). For simulations, this allows the use of quasi-two-dimensional (quasi-2D) numerical methods, such as so-called “quasigeostrophic” (QG) models [12–14].

This invariance along the axis of rotation for laboratory experiments means that measuring the velocity of the fluid at any height is a good proxy for the overall motion of the fluid. In our case, this relies on the assumption that the axializing effect of rapid rotation dominates any effects of local stable stratification or baroclinicity. However, nonaxial flows can become non-negligible for real laboratory experiments, such as when the free surface attains steep slopes (e.g., Ref. [15]).

This article focuses on validating and advancing a thermal image velocimetry method for applications to free surface rapidly rotating convection experiments. To illustrate our method, we will focus on a particular setup named “Coreaboloid” [16]. This experiment aims to reproduce a planetary fluid core in its low latitude zone. A rotating cylindrical shell is filled with hot water. The

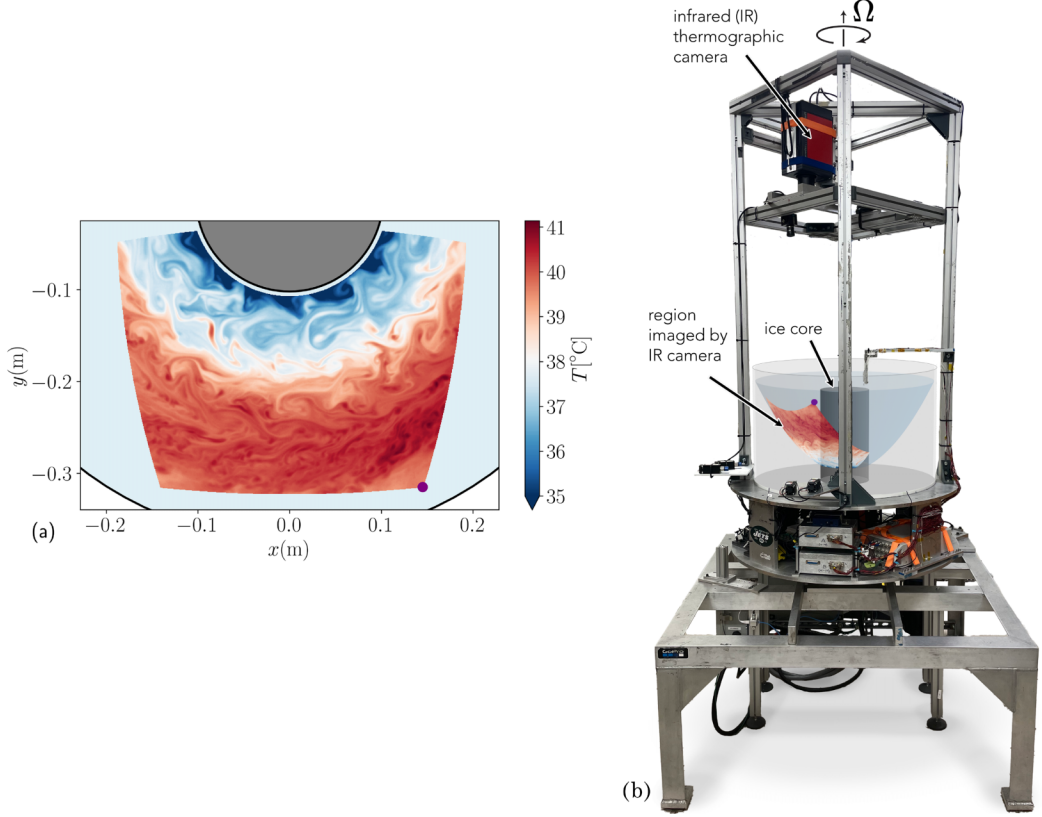


FIG. 2. (a) Instantaneous snapshot of the temperature field, measured on the free surface by the infrared camera in the Coreaboloid experiment [16]. Image is dewarped to correct curved surface distortion. The thermal sensitivity of the camera is 20 mK. (b) Photograph of the Coreaboloid experiment. The annular channel of internal and external radii $R_i = 10.2$ and $R_o = 37.25$ cm is filled to depth $H = 25.5$ cm with $\approx 50^\circ\text{C}$ water. The inner cylinder is filled with a $\approx -60^\circ\text{C}$ propylene glycol-water ice mixture. The table rotates at 72 rpm in this figure. The free surface of the water is bent by the rotation into a paraboloidal shape. Thermal images are captured from above in the rotating frame. (Image courtesy of Cy David.)

tank is rapidly rotating, bending the free surface of water in a paraboloid shape. The inner cylinder is filled with an ice mixture of water and propylene glycol. This thermal forcing combined with the topographic β effect induced by the shape of the free surface [17] generates motions that are characterized at first order by a series of alternating retrograde and prograde azimuthal jets. An infrared camera is located above the tank in the rotating frame, pointing downward. Figure 2 shows the Coreaboloid experiment. Panel (a) shows an example of an infrared thermal field measurement made on the paraboloidal free surface of the experiment. The free surface not imaged by the camera is represented in light blue. Panel (b) presents a photo of the experimental setup, to which is added the infrared image shown in panel (a). For details on this experimental setup, see Ref. [16].

We propose here to obtain the surface velocity field by tracking temperature features using dense optical flow techniques [18], called thermal image velocimetry (TIV [19]). Thermal imaging is commonly used to characterize temperature fields, mixing, and gradients in convection experiments [20]. Given the high quality and sensitivity of current infrared thermal cameras, the temperature field can also be used as a tracer of the convective flow field.

Optical flow (OF), unlike the cross-correlation methods primarily used for PIV, relies on the pattern of apparent motion of objects, particularly the displacement of their edges and surfaces. It is

based on the motion of a brightness/intensity pattern. There are two types of optical flow methods: “sparse” optical flow that tracks isolated features, such as corners or edges. An example of this kind of method is the Lucas-Kanade algorithm [21,22]. The second are the “dense” optical flow algorithms that return the flow vector for each pixel. The two main dense methods are the Farneback [23] and Horn-Schunck [24] algorithms (the original version of Lucas-Kanade can also be applied in a dense version [21]). All the methods mentioned above are based on the assumption of a constant brightness level between images. To overcome this constraint, some have developed variational dense methods based on the fluid continuity equation [25–27]. Several algorithms have also been developed that introduce fluid dynamics, in so-called “physics-based” optical flow methods [28,29].

Dense optical flow methods have been used successfully to track geophysical flows. This has been the case for open channel flow [30] and with satellite data to track flows in the atmosphere [31,32] and in the ocean [33]. These methods have also been applied to planetary data, such as tracking Jupiter’s Great Red Spot [34]. However, these methods are rarely used in experimental studies. They have mainly been applied to PIV images, replacing the traditional cross-correlation method [35], or in algorithms that combine OF and cross-correlation (called hybrid methods) [36,37], but have not yet been used on temperature data. Mendes *et al.* [18] have performed a comparative study of different methods of optical flow on synthetic PIV images. The study shows better performance for the dense Lucas-Kanade and Farneback methods compared to Horn-Schunck. It also shows the improving effects of a physics-based correction, in this case the Liu-Shen variational algorithm [28].

Since heat is not a passive tracer due to thermal diffusion, it may seem nonideal to use thermal data for optical flow. To address this, our first step consists of a benchmark TIV study on synthetic two-dimensional advection-diffusion data to quantify the influence of thermal diffusion on the optical flow error. This is followed by rapidly rotating quasi-2D numerical simulations, drawing inspiration from the Coreaboloid experiment [16]. The Farneback algorithm is used to derive the flow vector from the temperature field, incorporating a horizontal incompressibility correction that is appropriate for rapidly rotating flows for which $\partial_z U_z$ and then $\nabla_h \cdot \mathbf{U}$ are small.

The article is organized as follows. Section II presents the selected optical flow method and the fundamentals of the algorithm. This is followed in Sec. III by the details of the two-dimensional advection-diffusion calculations and the associated results. Section IV presents the numerical simulation that reproduces the Coreaboloid experiment. This is followed by an analysis of the TIV results obtained. The article concludes with a discussion of our findings and future applications in laboratory experimental TIV.

II. METHOD

The TIV scheme is described to invert the velocity field from two consecutive temperature images. The algorithm used to calculate the fluid’s displacement is introduced, followed by the numerical treatment of the images used in the optical flow algorithm. The numerical tool developed in this article can all be found in Ref. [38]

A. Optical flow

The Farneback algorithm [23] reconstructs the velocity field from gray scale images. In this study, the Python implementation used is included in the OpenCV computer vision library [39]. Comparisons between several standard optical flow formulations (not shown) confirmed that the chosen approach offered the best performance for our specific problem.

Farneback is one of the so-called “dense” optical flow methods [23,24]. Its principle is based on approximating the intensity in the neighborhood of pixel via a quadratic polynomial expansion. The displacement field is then determined by identifying the transformation that best matches the polynomial approximations from the first frame to the second as

$$f_1(\mathbf{x}) = \mathbf{x}^T \mathbf{A}_1 \mathbf{x} + \mathbf{b}_1^T \mathbf{x} + c_1, \quad (1)$$

$$f_2(\mathbf{x}) = \mathbf{x}^T \mathbf{A}_2 \mathbf{x} + \mathbf{b}_2^T \mathbf{x} + c_2 = f_1(\mathbf{x} - \mathbf{d}), \quad (2)$$

with f_1 and f_2 the polynomials of the first and second image pixel neighborhood, $\mathbf{x}$ is the position of the pixel, $\mathbf{A}$ is a (2×2) matrix, $\mathbf{b}$ is a vector, c is a scalar, and $\mathbf{d}$ is the displacement. The displacement is then obtained as

$$\mathbf{d} = \frac{1}{2} \mathbf{A}_1^{-1} (\mathbf{b}_2 - \mathbf{b}_1). \quad (3)$$

This approach allows one to calculate the best fit displacement vector at every pixel. In the algorithm, we call Ps the parameter corresponding to the pixel neighborhood radius used to find the polynomial expansion for each pixel, and the standard deviation of the Gaussian that is used to smooth the derivatives used as a basis for the polynomial expansion σ_p .

This displacement $\mathbf{d}$ can be solved pointwise, but the result would be too noisy. Instead, with the assumption that the displacement field slowly varies, $\mathbf{d}$ is considered constant in a neighborhood window of size Ws . The displacement is then calculated by minimizing the equations for all pixels within this window, increasing the stability of the method.

The Farneback algorithm [23] can include *a priori* knowledge of the displacement field, comparing the polynomial at the $\mathbf{x}$ position in the first image to the polynomial at $\mathbf{x} + \tilde{\mathbf{d}}$ in the second image, with $\tilde{\mathbf{d}}$ the *a priori* displacement. This prior displacement can be used iteratively, refining the estimate. The number of iterations is denoted by n_i here.

This *a priori* displacement can also be used to handle large motions. For motions exceeding approximately the polynomial neighborhood radius Ps , the local displacement estimate breaks down. To handle this, a coarse-to-fine pyramid is built by progressively downsampling the image. The flow is first estimated at the coarsest scale, where large displacements appear small, and then propagated to finer scales, initialized by the coarser estimates. The number of pyramid levels is denoted by N_p . For further details, see OpenCV's Farneback documentation [39].

The method assumes that what is visible in the image is a passive tracer (with no diffusion) and that the motion is purely two dimensional. The first assumption is fulfilled if the advection timescale is shorter than the diffusive one (large Péclet number). In rapidly rotating applications, the flow is dynamically constrained to be in a quasi-two-dimensional flow state ($\nabla_h \cdot \mathbf{U} \sim \epsilon$), thus approximately fulfilling the second assumption.

In experiments, water can be considered as an incompressible fluid. This implies divergence free flow. The nearly invariance along the vertical rotation axis as a result rapid rotation also requires that the horizontal divergence is low. To improve the accuracy of the algorithm and correct the result obtained with a physics-based approach, zero horizontal divergence of the flow is imposed ($\nabla_h \cdot \mathbf{U} = 0$, referred to as a horizontally incompressible or horizontally solenoidal flow). To do that, the TIV calculation of the velocity field $\mathbf{U}$, which is not necessarily solenoidal, is decomposed into an irrotational potential flow $\mathbf{U}_{\text{irrot}} = \nabla \psi$ and a solenoidal component $\mathbf{U}_{\text{sol}}$ that is divergence-free. This allows us to write the TIV inverted velocity field as $\mathbf{U} = \mathbf{U}_{\text{sol}} + \nabla \psi$ [40,41]. We calculate the potential field component of the flow by solving the Poisson equation

$$\nabla \cdot \mathbf{U} = \nabla \cdot (\nabla \psi + \mathbf{U}_{\text{sol}}) = \nabla^2 \psi \quad (4)$$

through a Fourier decomposition as

$$\hat{\psi} = -\frac{i\mathbf{k} \cdot \hat{\mathbf{U}}}{k_x^2 + k_y^2}, \quad (5)$$

with the wave vector $\mathbf{k} = [k_x, k_y, 0]$, and $\hat{\cdot}$ representing Fourier transforms. Taking the inverse Fourier transform, we can then subtract the potential flow from $\mathbf{U}$ to obtain the solenoidal component of the TIV flow $\mathbf{U}_{\text{sol}} = \mathbf{U} - \nabla \psi$. Although the spectral projection could not be strictly consistent with the boundary conditions of the image, it acts as an approximate divergence-free projection, effectively removing divergence while preserving the main flow structures. As shown in our results, this remarkably fast, Fourier-based method significantly improves the final velocity field.

B. Filtering of temperature images

To enhance the accuracy of the optical flow algorithm, temperature field images need to be modified to reduce noise. In addition, improving the contrast of the images ensures that the maximum amount of information is extracted.

First, we remove the variations that are at the scale of the image. For the thermal image of the Coreaboloid experiment, the thermal forcing from the ice core induces a mean radial temperature gradient. The azimuthal average of the temperature is then subtracted from the temperature field images.

A bilateral filtering method [42] is then applied to reduce the noise in each image. The filtered image I_f has the mathematical form

$$I_f(\mathbf{x}) = \frac{1}{W(\mathbf{x}_i, \mathbf{x})} \sum_{(\mathbf{x}_i) \in \Omega_g} I(\mathbf{x}_i) K(\mathbf{x}_i, \mathbf{x}), \quad (6)$$

with $W(\mathbf{x}_i, \mathbf{x}) = \sum_{(\mathbf{x}_i) \in \Omega_g} K(\mathbf{x}_i, \mathbf{x})$. The original image to be filtered is I , $\mathbf{x}$ is the coordinate of the current pixel with Ω_g the window around $\mathbf{x}$, and $\mathbf{x}_i$ denotes the pixels inside this window. The kernel $K(\mathbf{x}_i, \mathbf{x})$ is written as

$$K(\mathbf{x}_i, \mathbf{x}) = G_s(\|\mathbf{x}_i - \mathbf{x}\|) G_r(\|I(\mathbf{x}_i) - I(\mathbf{x})\|), \quad (7)$$

where G_s and G_r can be different functions. Here, $G_s(d) = \exp(-d^2/2\sigma_s^2)$ is the spatial Gaussian kernel with $d = \|\mathbf{x}_i - \mathbf{x}\|$, the spatial distance between pixels $\mathbf{x}_i$ and $\mathbf{x}$ in our case, and $G_r(r) = \exp(-r^2/2\sigma_r^2)$ is the intensity range Gaussian kernel with $r = \|I(\mathbf{x}_i) - I(\mathbf{x})\|$ being here the intensity difference between the two pixels. σ_r and σ_s are smoothing parameters. This method has the advantage of filtering the noise of the image, while conserving the sharpness of the edges [42].

A histogram equalization filter [43] is then applied to enhance image contrast. This method provides a remapping of the intensity of the pixels, making the histogram of pixel intensity values more uniform (approximately yielding the same number of pixels for each intensity value). This mapping is calculated through the cumulative function $\text{cdf}_X(i) = \sum_{j=0}^i p_X(j, L)$ with p_X the histogram discretized into L bins. The new pixels values are computed as

$$s_X = (L - 1) \text{cdf}_X. \quad (8)$$

For $L = 2^8 = 256$, the output fits 8-bit intensities, for $L = 2^{16} = 65\,536$, it fits 16-bit intensities.

The intensity range can be heterogeneous within the same image (e.g., between the inner cylinder and the outer boundary in the Coreaboloid experiment). Thus, each image is divided into a grid of small square windows and a histogram is computed within each one. The filter is then applied based on these local histograms. We denote by W_c the number of windows on each axis. The histogram equalization is applied to the images using the ‘‘CLAHE’’ algorithm (contrast limited adaptive histogram equalization [44]) implemented in the OpenCV library [39], which also limits the maximum contrast variation, with the clip contrast limit noted c_l .

Figure 3 shows an example of the application of this filter to a thermal image from one of our quasigeostrophic simulations. This filtering does not preserve the relative values of the temperature, but improves the contrast of the structures, even if this means altering the raw image. Note that our filter is particularly effective on OF reconstruction for images with a low color/intensity range, such as 8-bit images, and becomes less significant in calculating the flow when this range is larger, with gradients becoming better resolved. The influence of the filter on the final results is discussed later (Fig. 6).

III. IDEALIZED ADVECTION-DIFFUSION SIMULATIONS

Our TIV algorithm is first tested via a simplified case study. The analysis is performed on a two-dimensional advection-diffusion simulation with an idealized initial pattern of temperature and

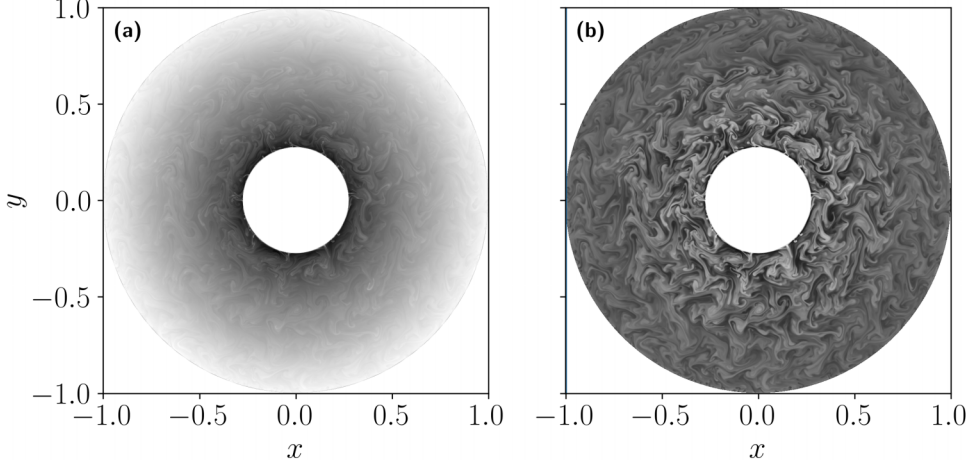


FIG. 3. Example of original temperature field (left) and filtered with bilateral filter and histogram equalization (right) for a quasi-2D Coreaboloid simulation.

a steady velocity field. The initial dimensional temperature field T_0 is periodic in both directions with a wave number k , such that

$$T_0 = \sin(2\pi kx) \sin(2\pi ky). \quad (9)$$

An example for $k = 9$ is shown in Fig. 4(a). The governing equation for temperature is

$$\partial_t T + (\mathbf{U} \cdot \nabla)T = \kappa \nabla^2 T, \quad (10)$$

with κ the thermal diffusivity.

The background velocity field is prescribed and is not modified by the temperature field evolution. The velocity field is a pattern of Taylor-Green vortices, which are well-known solutions of the incompressible Navier-Stokes equations [45]. The velocity field,

$$\mathbf{U} = U_0 \begin{bmatrix} \sin(2\pi x) \cos(2\pi y) \\ -\sin(2\pi y) \cos(2\pi x) \end{bmatrix}, \quad (11)$$

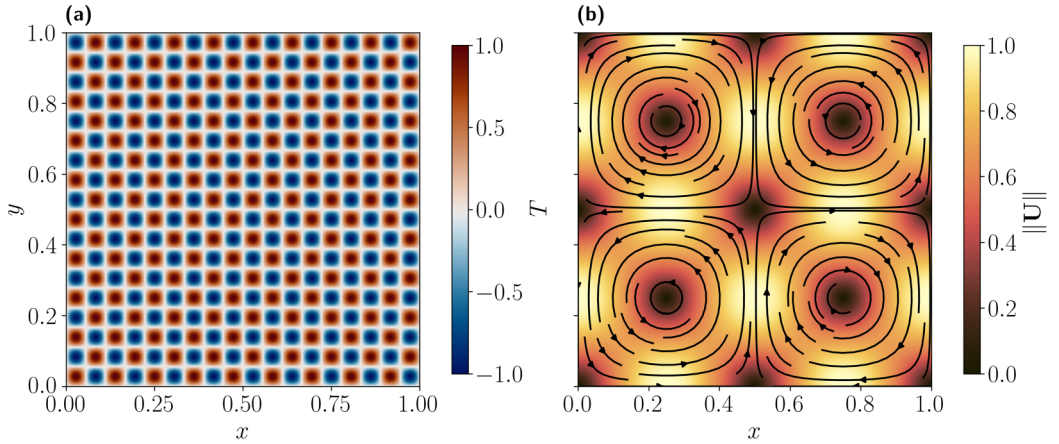


FIG. 4. (a) Initial temperature field for $k = 9$. (b) Imposed Taylor-Green velocity field. Colors indicate the magnitude of the velocity, black curves mark sections of streamlines of the flow.

is shown in Fig. 4(b). In the following, $U_0 = 1$. Four vortices exist in the domain so that the velocity field is spatially periodic.

We define the local Péclet number as $Pe = U_0/(\kappa k)$, which represents the ratio between the diffusion timescale $[1/(\kappa k^2)]$ and the advection timescale $[1/(U_0 k)]$.

Simulations are conducted employing a Fourier spectral method coupled with a Crank-Nicholson Adams-Bashford (CNAB2) semi-implicit time scheme. The code used for these simulations is accessible at Ref. [46]. The spectral resolution of 256×256 in Fourier space corresponds to a final image of 512×512 pixels. The snapshots used for optical flow are at the times $t = 0$ and $t = 5 \times 10^{-3} = \delta t$. All results in this section are calculated without any filtering of the 16-bit images and use the following parameters for the Farneback algorithm: $N_p = 2$, $Ws = 25$, $n_i = 2$, $Ps = 9$, and $\sigma_p = 1.4$.

Figure 5(a) compares the velocity $\mathbf{U}_{OF}$ from optical flow with the true velocity $\mathbf{U}_{true}$ from Eq. (11). The figure shows the magnitude of $\mathbf{U}_{true}$ (red line) and the magnitude of $\mathbf{U}_{OF}$ for different Péclet numbers (in color), along a line at $y = 0.25$ (center of a vortex). In this case, the temperature field has a wave number $k = 15$.

A periodic deviation from the true velocity field is observed, with a period depending on the gradient of the temperature distribution. This is observed for all Péclet numbers. The velocity decreases almost uniformly as the Péclet number decreases.

The normalized root mean square error (NRMSE) quantifies the error between $\mathbf{U}_{true}$ and $\mathbf{U}_{OF}$ as

$$\text{NRMSE} = \sqrt{\frac{\frac{1}{N} \sum_{i=1}^N \|\mathbf{U}_{true,i} - \mathbf{U}_{OF,i}\|^2}{\frac{1}{N} \sum_{i=1}^N \|\mathbf{U}_{true,i}\|^2}}, \quad (12)$$

with i denoting a pixel in the image and N the total number of pixels. Figure 5(b) shows NRMSE as a function of Pe and k . The black zone corresponds to simulations in which the temperature pattern after the considered time step $T(\delta t)$ has variations below the computer precision ($\max(\|T\|) < 10^{-15}$). This limit is superimposed on the theoretical limit given by the exponential decay of temperature diffusion. The white dashed line is $\exp(-8\pi^2 k^2 \kappa \delta t) = 10^{-15}$. This image shows that the error decreases as the number of temperature structures increases (larger k). The errors became almost invariant above $k \gtrsim 10$. We see that for images containing a small-scale temperature field, as is the case with real rotating convection measurements, our method can be reliably used for $Pe > 1$. This is because, unlike PIV cross-correlation, velocity is calculated here using a diffusing scalar field, which implies a potentially larger null-space where features cannot be tracked. The null-space corresponds to pixels where the displacement is too small ($\ll 1$ pixel between two frames), where the velocity is parallel to the temperature iso values, or where temperature features are too smooth. This depends greatly on the intensity range of the image. Low range cannot ensure accurate discretization of the gradients, and thus increases the size of this null-space. Applying a contrast filter improves this discretization keeping the same color range.

Despite this, a window large enough to contain mostly pixels outside of this null-space allows the Farneback algorithm to smooth the result and partially alleviate this issue.

The observed error is also greatly affected by diffusion. It can be seen that the error decreases with increasing Pe , with an asymptotic limit reached for $Pe \gtrsim 1$.

Figure 5(c) shows the equivalent of three cross sections of Fig. 5(b). This corresponds to the error as a function of Pe for three different wave numbers, $k = 5, 11$, and 21 . The solid lines show the error with solenoidal projection, whereas the dashed lines show the result without. The figure shows that applying the solenoidal correction consistently reduces the error for all tested values of k and Pe , with a more pronounced improvement observed for smaller length scale patterns (larger k). This improvement becomes less efficient for lower Péclet numbers ($Pe \lesssim 1$). The thermal diffusion timescale, if similar to or greater than the advective timescale, will generate an apparent velocity indistinguishable for the optical flow algorithm. Since apparent velocity is not necessarily solenoidal, it can be partially eliminated by an incompressibility correction.

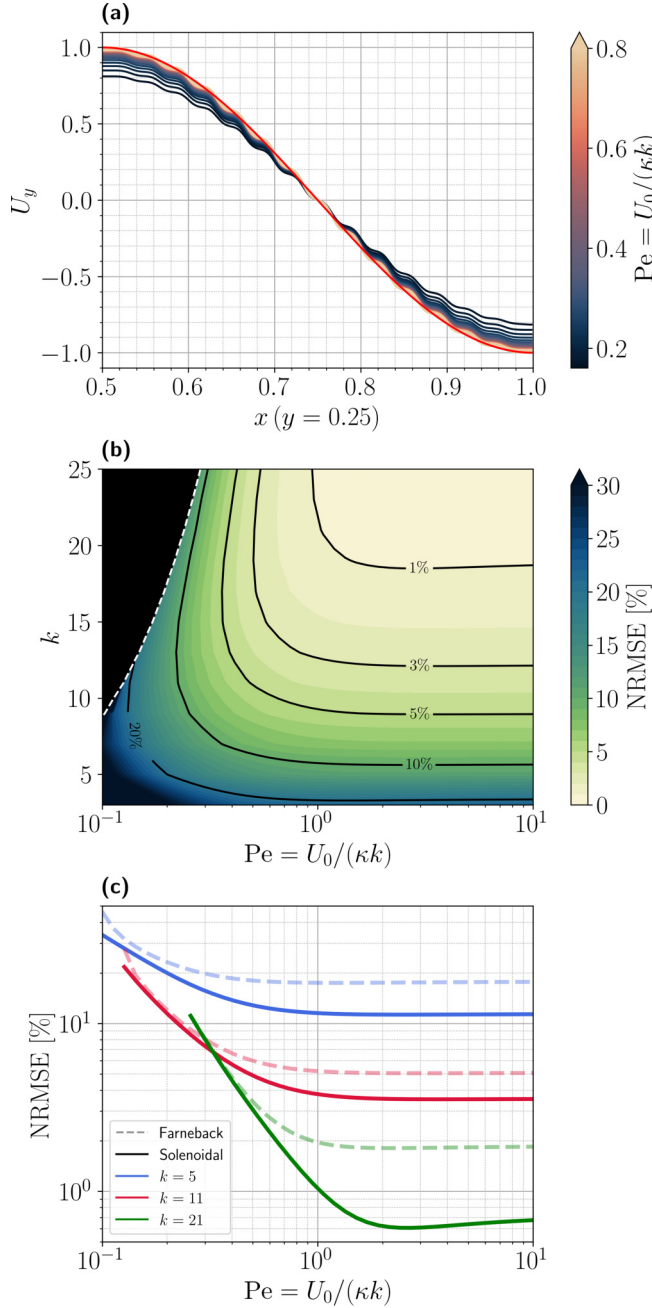


FIG. 5. (a) Velocity magnitude as function of x at $y = 0.25$. Red line is the true velocity. Each color is a calculated velocity from simulations at different Péclet number. The wave number of the temperature field is $k = 15$. (b) Normalized root mean square error (%) between true velocity and reconstructed velocity as a function of Péclet number Pe and initial temperature field wavelength k . The black zone corresponds to where $T(\delta t)$ is lower than the computer precision ($< 10^{-15}$). This limit is superimposed on the theoretical limit given by the exponential decay of temperature diffusion [white dashed line is $\exp(-8\pi^2 k^2 \kappa \delta t) = 10^{-15}$]. (c) Normalized root mean square error (%) between true velocity and calculated velocity as a function of Pe , with (solid lines) and without (dotted) solenoidal correction and for different temperature field wavelength $k = 5$ (blue), 11 (red), and 21 (green).

Overall, the results of these idealized advection-diffusion simulations show that the TIV accuracy is improved with increasing temperature spatial wave number and Péclet number. The error is dominated by diffusion at low Pe. The solenoidal constraint consistently reduces the error, particularly for small-scale temperature patterns, which is relevant for rapidly rotating convection experiments [6].

IV. QUASI-2D SIMULATION OF THE COREABOLOID

In order to further benchmark, our TIV method and take the next step toward rapidly rotating laboratory applications, we performed numerical quasigeostrophic simulations [12], which are modified here to reproduce the experimental setup of the Coreaboloid.

A. Method

When the characteristic timescale of the fluid velocity is much lower than the one of rotation rate (low Rossby number), the quasigeostrophic approximation may be used to simulate fluid dynamics problems more efficiently [12]. Quasigeostrophic simulation consists in calculating the flow on a two-dimensional plane perpendicular to the rotation vector. The vertical velocity is parametrized in the equations to include the effect of top and bottom boundaries.

The geometry of this quasi-2D problem is an annulus of outer radius R_o and inner radius R_i . The bottom is flat and $z = 0$. The following equations are in a cylindrical frame of coordinates (s, θ, z) . In our problem, lengths are scaled with R_o , times with R_o^2/ν with ν the kinematic viscosity, and temperatures with ΔT , where $\Delta T > 0$ is the temperature difference between the inner and outer boundaries. The setup spins at an angular rotation rate $\Omega \mathbf{1}_z$ and we write the acceleration due to radial gravity as $\mathbf{g} = -\gamma \tilde{s} \mathbf{1}_s$ in the two-dimensional framework, where $\tilde{s}$ is the dimensional cylindrical radius.

The top boundary is a free surface of (dimensional) height

$$\tilde{h}(\tilde{s}) = \tilde{h}_0 + \frac{\Omega^2 \tilde{s}^2}{2g}, \quad (13)$$

with g the uniform vertical gravity in the laboratory, $\tilde{h}_0 = H - \Omega^2(R_i^2 + R_o^2)/(4g)$ and H being the average height [16].

The dimensionless control parameters for this system are the Ekman Ek, Froude Fr, Rayleigh Ra, and Prandtl Pr numbers, as well as the radius ratio χ :

$$\begin{aligned} \text{Ek} &= \frac{\nu}{\Omega R_o^2} = \frac{H^2}{R_o^2} \text{Ek}^*, \quad \text{where} \quad \text{Ek}^* = \frac{\nu}{\Omega H^2}, \\ \text{Fr} &= \frac{\Omega^2 R_o}{g}, \quad \text{Ra} = \frac{\alpha \Delta T \gamma R_o^4}{\nu \kappa}, \quad \text{Pr} = \frac{\nu}{\kappa}, \quad \chi = R_i/R_o. \end{aligned}$$

The dimensionless height is then

$$h(s) = \frac{\tilde{h}}{R_o} = h_0 + \frac{\text{Fr}}{2} s^2, \quad (14)$$

where $s = \tilde{s}/R_o$ and

$$h_0 = \frac{\tilde{h}_0}{R_o} = \frac{H}{R_o} - \frac{\text{Fr}(\chi^2 + 1)}{4}. \quad (15)$$

The dimensionless governing equation for the z component of the vorticity, $\zeta = (\nabla \times \mathbf{U}) \cdot \mathbf{1}_z$, is

$$\frac{\partial \zeta}{\partial t} + (\mathbf{U}_h \cdot \nabla_h) \zeta = \left(\frac{2}{\text{Ek}} + \zeta \right) \left\langle \frac{\partial U_z}{\partial z} \right\rangle + \nabla_h^2 \zeta - \frac{\text{Ra}}{\text{Pr}} \frac{\partial \Theta}{\partial \theta} \quad (16)$$

with

$$\left\langle \frac{\partial U_z}{\partial z} \right\rangle = \beta U_s + \frac{1}{h} (\mathbf{U}_h \cdot \hat{\mathbf{n}}|_{z=h} - \mathbf{U}_h \cdot \hat{\mathbf{n}}|_{z=0}), \quad (17)$$

where $\mathbf{U}$ is the velocity field, Θ the temperature, $\mathbf{U}_h = (U_s, U_\theta, 0)$, $\nabla_h f \equiv (\partial_s f, \partial_\theta f/s, 0)$, and $\nabla_h^2 f \equiv \partial_s^2 f + s^{-1} \partial_s f + s^{-2} \partial_\theta^2 f$. The angle brackets denote an axial average between 0 and $h(s)$, and $\hat{\mathbf{n}}$ is the surface normal vector at the boundary considered. The effects of the bottom no-slip boundary condition are modeled through a parametrization of fluid pumping in and out of the linear Ekman layer [47]: $\mathbf{U}_h \cdot \hat{\mathbf{n}}|_{z=0} = (Ek^{1/2}/2)\zeta$. At the free surface, we impose $\mathbf{U}_h \cdot \hat{\mathbf{n}}|_{z=h} = 0$. The β effect, arising from axial variation in fluid height, is controlled by the parameter β as

$$\beta = \frac{1}{h} \frac{dh}{ds} = \text{Fr} \frac{s}{h}. \quad (18)$$

The equation for the 2D temperature is

$$\frac{\partial \Theta}{\partial t} + \mathbf{U}_h \cdot \nabla_h \Theta + U_s \frac{dT_s}{ds} = \frac{1}{\text{Pr}} \nabla_h^2 \Theta, \quad (19)$$

where the background temperature gradient (obtained by solving $\nabla_h^2 T_s = 0$) is

$$\frac{dT_s}{ds} = \frac{1}{\ln \chi} \frac{1}{s}. \quad (20)$$

On the side walls, the boundary conditions are fixed temperature, no-slip, and impenetrable.

The code uses a pseudospectral code with a Fourier decomposition in the azimuthal direction and a second-order finite-difference scheme in radius with irregular spacing. Further details about the formulation of the model are available in Ref. [12].

B. Results

Quasigeostrophic simulations have been performed over 20 rotation periods after reaching statistical equilibrium. We collected temperature snapshots and the corresponding velocity fields at intervals of $dt = 5 \times 10^{-7}$, which corresponds to approximately the frame rate of the Coreabolid's current infrared camera (≈ 10 Hz). The dimensionless parameters used in these calculations are

$$\begin{aligned} H/R_o &= 0.68, & \chi &= R_i/R_o = 0.274, \\ \text{Fr} &= 2.16, & \text{Pr} &= 5, & \text{Ra} &= 5.47 \times 10^{10}, \\ \text{Ek}^* &= 1.48 \times 10^{-6} \Rightarrow \text{Ek} &= 6.84 \times 10^{-7}, \end{aligned}$$

and correspond to typical Coreabolid experiment (e.g., Ref. [16]). The numerical resolution is 1320 radial grid points and 1024 Fourier modes in azimuth. All TIV results in this section are calculated with a filter using parameters $\sigma_s = \sigma_r = 2$, Ω_g is a window of size 3×3 pixels, $Wc = 9$, and $c_l = 7$, with the Farneback algorithm using $N_p = 2$, $Ws = 19$, $n_i = 2$, $Ps = 9$, and $\sigma_p = 1.4$.

Figure 6 shows the effect of various filters and corrections on the quality of the TIV inversion. The figure shows the different velocity fields obtained for images quantized in 8 and 16 bits (left and right columns), first using only the Farneback algorithm (top), then adding solenoidal correction (middle), and finally filtering the temperature images (bottom) as described in Sec. II B.

To quantify the error for each of these inversion schemes, the NRMSE is calculated between the reconstructed and the true flow. A second metric is the spatial correlation coefficient, which quantifies the spatial similarity of the 2D flow fields, as

$$\rho_U = \frac{\text{cov}(\mathbf{U}_{\text{OF}}, \mathbf{U}_{\text{True}})}{\sigma_{\mathbf{U}_{\text{OF}}} \sigma_{\mathbf{U}_{\text{True}}}}, \quad (21)$$

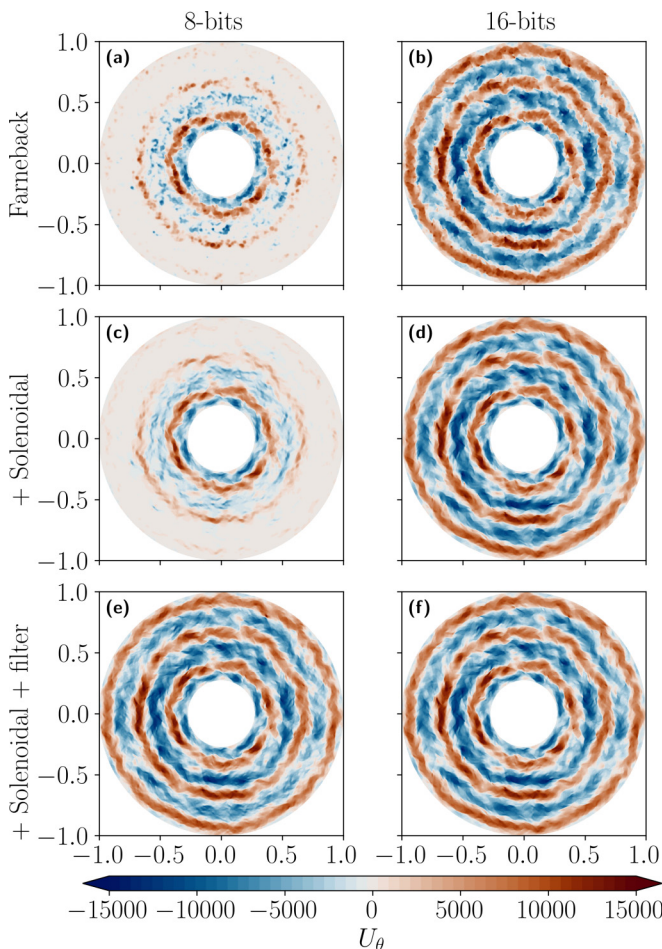


FIG. 6. TIV flow inversion for filtered and unfiltered images (image only rescaled and quantized), with and without solenoidal correction, for 8 and 16 bit color ranges.

where $\text{cov}(I, J) = \overline{(I - \bar{I})(J - \bar{J})}$ is the covariance function (with the overline denoting the mean value of a I, J two 2D fields) and $\sigma_I = \sqrt{\text{cov}(I, I)}$ is the standard deviation. This provides a value between -1 and 1 , with 1 being the value for identical fields and -1 being for anticorrelated fields. The values obtained are given, as percentages, in Table I.

We observe that in all cases the solenoidal correction improves the reconstructed flow. Filtering only significantly improves the results for 8-bit images, while the flow reconstructed for 16-bit images with solenoidal correction with (d), (f) and without (b) does not show significant differences.

TABLE I. Normalized RMS error and average correlation coefficient between the reconstructed TIV velocity fields of Fig. 6 and the true one. Each letter corresponds to one method of calculation.

	a	b	c	d	e	f
NRMSE (%)	82.1	34.9	80.8	30.4	32.0	30.4
$\overline{\rho_U}$ (%)	59.7	93.8	63.8	95.3	94.8	95.3

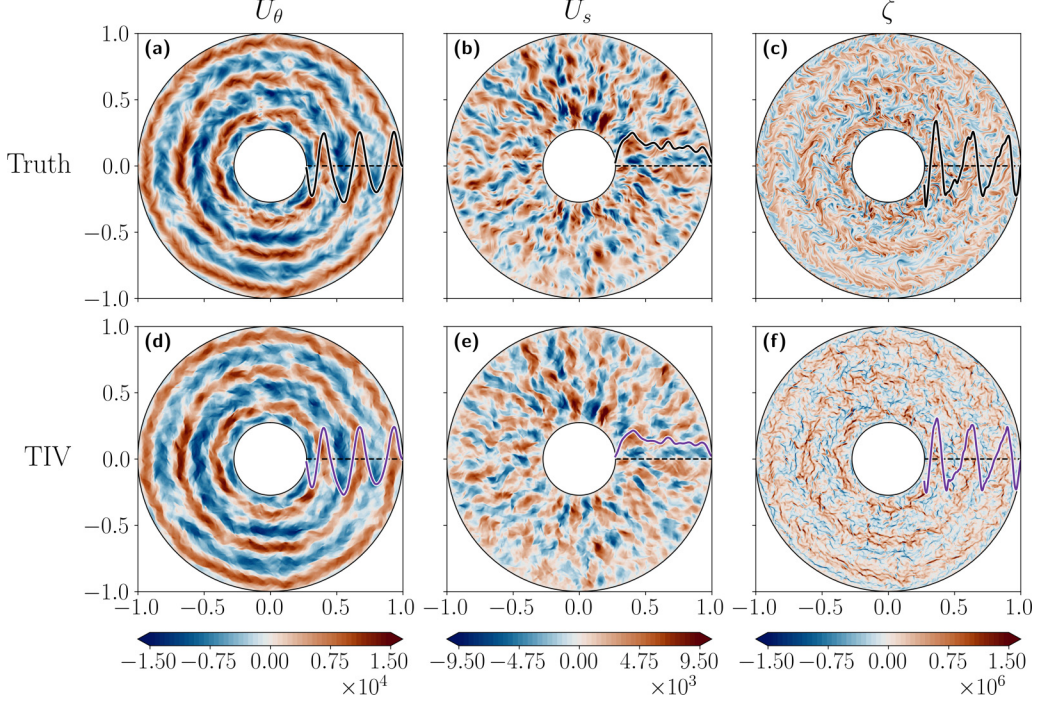


FIG. 7. Snapshots of the velocity field and vorticity. From left to right: Azimuthal velocity U_θ , radial velocity U_s , and vorticity ζ . Top: True fields, bottom: optical flow results (16-bit images, Farneback + filter, and solenoidal correction). The azimuthal average of U_θ and ζ , and the RMS of U_s are superimposed on the x axis (not at scale).

The histogram equalization filter allows information to be recovered when the image color quantization is insufficient to cover the entire image adequately, particularly in areas with low gradients (as is the case here for the outer part of the cylinder). This point is important for experimental images if they have a low effective quantization of the temperature field. The following contains only calculations performed with 16-bit images, solenoidal correction, and filters.

Figure 7 compares instantaneous velocity fields from the rapidly rotating QG convection simulation of the Coreaboloid setup (top) with those derived via TIV reconstruction (bottom). The figure shows, from left to right, the azimuthal velocity U_θ , the radial velocity U_s , and the axial vorticity ζ . The azimuthally averaged profiles of U_θ and ζ , and the RMS profile of U_s are superimposed on the x axis. The observed flow is a pattern of six alternating azimuthal jets. Our optical flow method reproduces the overall flow structure with good fidelity. The velocity is smooth and continuous and captures both large structures and small fluctuations.

The reconstructed velocity field shows a high spatial correlation coefficient with the reference field $\rho_U > 90\%$ for both components of the velocity, with a better correlation for the azimuthal velocity compared to the radial velocity. This indicates that the main flow structures and spatial patterns are well reproduced. The NRMSE remains relatively high ($\approx 30\%$), this metric being particularly severe for velocity fields that contain low-velocity regions and localized extrema.

The vorticity field requires a high degree of accuracy for the velocity gradients to be correct. However, the TIV ζ fields are of adequate quality ($\rho_\zeta \approx 44\%$ and $\text{NRMSE} \approx 102\%$). These values are summarized in Table II.

Figure 8 presents the azimuthal mean velocity and vorticity field, obtained from both the TIV inversion and the reference simulation, as well as the RMS U_s velocity. The TIV-derived profile

TABLE II. Normalized RMS error and correlation coefficient for instantaneous and time-averaged velocity and vorticity field, presented in Figs. 7 and 9. For instantaneous snapshots, values represent the mean $\pm$ one standard deviation computed over all instantaneous snapshots spanning 20 rotation periods.

	NRMSE (%)	$\rho(\%)$
$\mathbf{U} = [U_s, U_\theta]$	30.8 ± 0.6	$[89.1 \pm 0.8, 96.5 \pm 0.2]$
ζ	102.1 ± 1.8	44 ± 1
$\langle \mathbf{U} \rangle_t = [\langle U_s \rangle_t, \langle U_\theta \rangle_t]$	9.4	$[92.4, 99.7]$
$\langle \zeta \rangle_t$	42.3	90.8

reproduces both the jet amplitudes and their radial positions. A slight smoothing of the extrema is observed in the azimuthal mean TIV result. Nevertheless, the zero crossings of $\langle U_\theta \rangle_\theta(s)$ coincide almost perfectly with those of the simulation (0.3% error on average), indicating that the method captures the correct number and the correct position of the jets. The azimuthal mean of the radial velocity remains close to zero across radius, confirming that there is no artificial mean flow. This shows that the mass conservation, not ensured by the optical flow, is well enforced by the solenoidal projection. As for the azimuthal velocity, the RMS of the radial flow is weaker for TIV. Finally, the average vorticity agrees well, despite a poorer full field reconstruction than the velocity fields.

Figure 9 investigates the consistency of the TIV reconstructed flow upon time average over 20 rotation periods. The flow is inverted at each time step and then averaged. The figure is structured identically to Fig. 7. The time-averaged azimuthal velocity maintains the same alternating jet structure, while the small-scale turbulence is smoothed out. We could also mention that the prograde jets have sharp peaks, while retrograde jets are broader, with less curvature. This structural difference between prograde and retrograde jets is found in many studies of geophysical jets and is often attributed to potential vorticity mixing (e.g., [48]). This structural difference is evident in both the QG $\langle U_\theta \rangle_\theta$ profile as well as the TIV reconstructed profile. The amplitude and spatial distribution

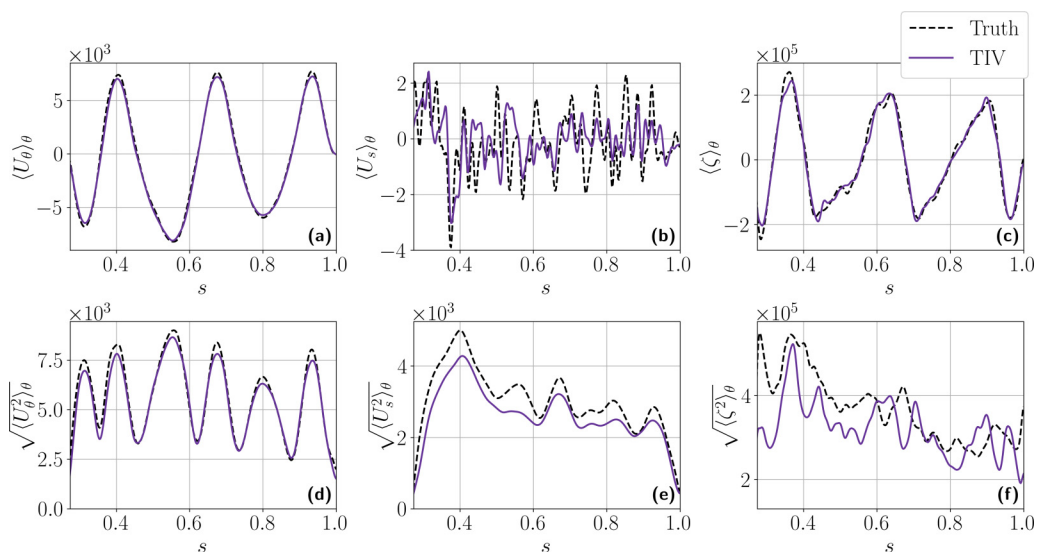


FIG. 8. Azimuthal average (top) and RMS average (bottom) of velocity field and vorticity. From left to right: Azimuthal velocity, radial velocity, and axial vorticity. Black dashed lines are the true field and purple solid lines are the ones reconstructed by optical flow (16-bit images, Farneback + filter and solenoidal correction).

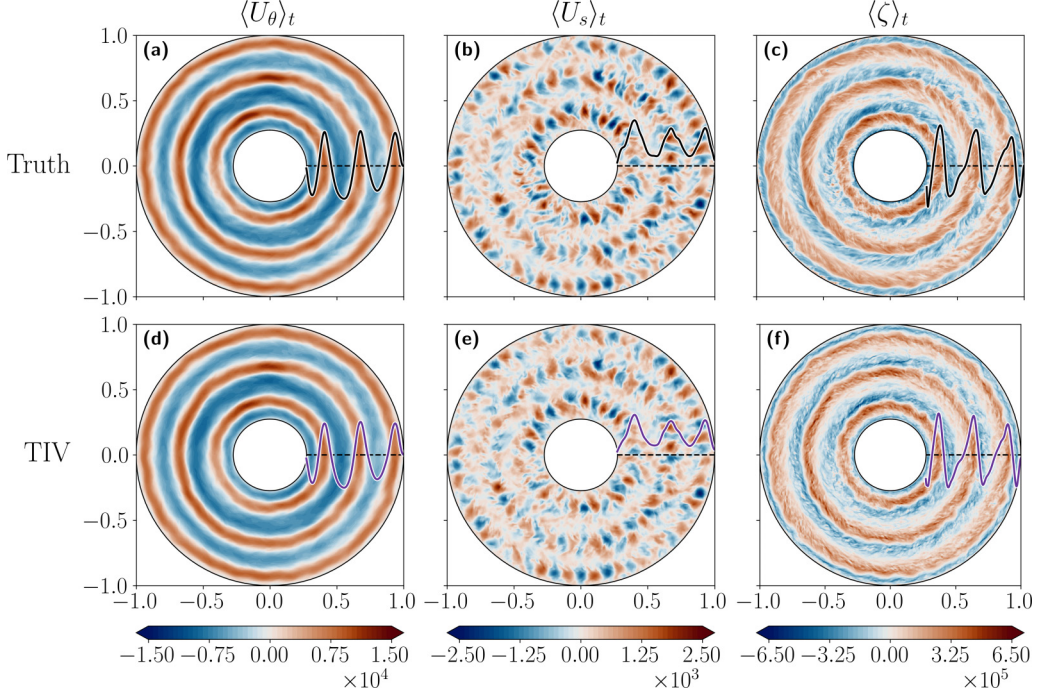


FIG. 9. Average over 20 rotation periods of velocity field and vorticity. From left to right: Azimuthal velocity $\langle U_\theta \rangle_t$, radial velocity $\langle U_s \rangle_t$, and vorticity $\langle \zeta \rangle_t$. Top: True data, bottom: optical flow results (16-bit images, Farneback + filter, and solenoidal correction). The azimuthal average of $\langle U_\theta \rangle_t$ and $\langle \zeta \rangle_t$, and the RMS of $\langle U_s \rangle_t$ are superimposed on the x axis (not at scale).

of the mean jets from the OF reconstruction agree with the reference field, confirming that the algorithm captures long-term motion. When averaged over time, the error between the true and reconstructed flow is greatly reduced (see Table II). For averaged radial velocity, we also observe quasisteady waves, emerging as expected only on prograde jets. These are characteristic of steady Rossby waves [49,50]. These waves are also well retrieved by TIV.

Figure 10 shows the Fourier ring correlation (FRC [51]) between the amplitude of the true velocity and the TIV reconstruction for an instantaneous velocity field and a time average over 20 rotation periods to quantify the quality of the spatial spectrum reconstruction. The Fourier ring correlation is computed as

$$\text{FRC}(k) = \frac{\sum_{\mathbf{q} \in \mathcal{R}(k)} f_1(\mathbf{q}) f_2(\mathbf{q})^*}{\sqrt{\sum_{\mathbf{q} \in \mathcal{R}(k)} |f_1(\mathbf{q})|^2 \sum_{\mathbf{q} \in \mathcal{R}(k)} |f_2(\mathbf{q})|^2}}, \quad (22)$$

where $f_1(\mathbf{q})$ and $f_2(\mathbf{q})$ are the Fourier transforms of $|U_{\text{True}}|$ and $|U_{\text{OF}}|$, $\mathcal{R}(k)$ denotes the annular ring of 2D spatial frequency vectors $\mathbf{q}$ with magnitude $\|\mathbf{q}\| \approx k$, and $(\cdot)^*$ is the complex conjugate. The FRC yields a correlation coefficient as a function of spatial frequency k , with values close to one indicating good agreement between the two fields. In Fig. 10, the FRC is shown as a function of local wave-number-dependent $\text{Pe}_k = |U_k| \text{Pr}/k$. This spectrally local Péclet number is calculated for each individual k value using the k -dependent amplitude $|U_k|$ of the Fourier transform spectrum of $|U_{\text{True}}|$ (and $\langle |U_{\text{True}}| \rangle_t$). For the instantaneous field, we observe that the correlation is greatly increased (over 75%) for $\text{Pe}_k > 1$ and has almost zero correlation for $\text{Pe}_k < 0.1$. This extends the observation of the idealized simulations of Sec. III to a fully nonlinear turbulent convective flow. This is improved for the time averaged fields, with higher FRC values for lower Pe_k .

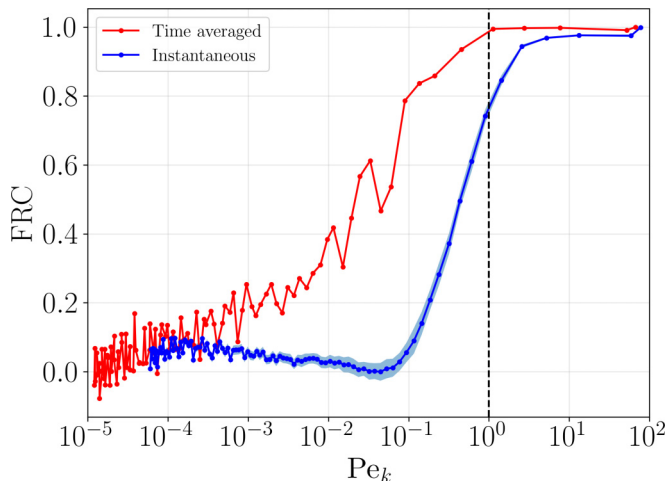


FIG. 10. Fourier ring correlation of the RMS velocity amplitude between true and TIV fields as a function of the local wave-number-dependent Péclet number, Pe_k , for an instantaneous snapshot and a time average over 20 rotation periods. For instantaneous FRC, values represent the mean $\pm$ one standard deviation computed over all instantaneous snapshots spanning the 20 rotation periods.

Our TIV results confirm that the combination of proper filtering, Farneback optical flow, and solenoidal projection methods provides an accurate reconstruction of the velocity field from thermal field images. Thus, our TIV method preserves the essential jet shapes, reproduces long-term averages, and remains computationally efficient (approximately 30 frames per second on a standard laptop).

V. DISCUSSION AND CONCLUSION

The primary goal of this study is to evaluate the performance of thermal image velocimetric reconstructions made using optical flow techniques on idealized thermal fields. This work adapted proven optical flow techniques to extract velocity fields from 2D temperature measurements. Although optical flow methods have proven their effectiveness, their application to thermal imagery in rapidly rotating fluid systems required careful consideration of temperature evolution dynamics, thermal diffusion effects, and the specific constraints of infrared measurements in rotating tank configurations. By testing TIV on simulated data where the true velocity field is known, this study provides a controlled evaluation of the accuracy and limitations of the method.

The results of the advection-diffusion simulations indicate that the method performs well for small-scale temperature fields and $Pe \gtrsim 1$. The use of optical flow method induces a null-space in which the velocity cannot be accurately reconstructed. However, this remains marginal in cases where the temperature structures are smaller in spatial scale than the flow to be resolved. The scale separation between temperature and velocity fields is governed by the Prandtl number Pr . Thus, we hypothesize that TIV will work optimally when $Pr \gtrsim 1$, as turbulence theory shows that temperature structures are at smaller scales than velocity structures in this Pr range [52]. Large Pe is a separate but also necessary condition, ensuring that small scales are maintained against diffusion. The method is therefore valid at all time steps for large Pr and Pe . However, for moderate Pe , the null-space grows progressively as diffusion smooths the small-scale temperature structures over a diffusive timescale (e.g., Fig. 5). The quasi-2D Coreaboloid simulations highlight the effectiveness of the TIV method for realistic and turbulent cases. TIV introduces some errors at small scales due to low local Péclet number values (e.g., $Pe \lesssim 1$) and the finite resolution of the optical flow

algorithm. Although TIV smooths small-scale features, it captures the primary flow structures and dynamics extremely well in all of our tests.

It is important to note that we conducted this study entirely using numerical simulations. The quasigeostrophic approximation, while well suited for rapidly rotating flows, inherently assumes negligible vertical velocity. This means that the results presented here are most applicable to experiments operating in the strongly rotating regime where two-dimensional dynamics dominate [53]. Real laboratory experimental flows deviate from purely 2D motions. For instance, vertical flow can introduce errors when moving from a 2D horizontal surface to a three-dimensional (3D) paraboloid. It is then more difficult to accurately project this 3D surface onto a 2D plane. Although our preliminary results from the comparison between midheight planar PIV and paraboloidal surface TIV appear to be in very good quantitative agreement, this complication needs to be taken into account in experimental applications.

In particular, the upper layer of the fluid, on which the temperature field is measured, is in contact with the air. This implies the generation of a boundary layer that may not be perfectly representative of the interior of the fluid. Further, any airflow above the surface can generate an Ekman boundary layer, involving shear and vertical pumping [11]. In the event of a temperature difference or excessive evaporation, vertical convection movements might also be observed [54], which may cause bias in the reconstructed optical flow.

Finally, even though thermal cameras now have very high spatial and thermal resolutions, experimental measurements inevitably introduce additional challenges compared to idealized simulations. The camera precision, while excellent, remains lower than that of perfect synthetic images. The camera also introduces thermal and electronic noise in the images. Although we designed our method with practical applications in mind, in particular by incorporating noise filtering, this will undoubtedly degrade the quality of the TIV. Moreover, practical experimental considerations, such as camera vibrations or surface waves, can distort the thermal images and may affect the optical flow reconstruction quality.

In summary, the current study demonstrates that TIV methods are suitable and highly promising for rapidly rotating free surface laboratory experimental applications. This will allow us to perform laboratory measurements in setups that are not suitable for PIV. It also opens up possibilities for studying rotating turbulence with only thermal imaging or coupled with PIV to obtain surface and internal fluid motions simultaneously. Our next step will be to devise a robust experimental implementation of our TIV algorithm on laboratory thermal images, such as those generated in the Coreaboloid [Fig. 2(a)] and other comparable experiments [55–58].

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R.M. developed the QG form of TIV employed here; he developed and conducted the advection-diffusion simulations and also analyzed all data products. C.G. developed and conducted the QG simulations. R.M. wrote the original manuscript. R.M., C.G., H.C., and J.M.A. revised the manuscript. J.M.A. proposed the study.

DATA AVAILABILITY

Some of the data that support the findings of this article are openly available (TIV package including an example notebook [38], advection-diffusion code [46]). Quasigeostrophic and advection-diffusion code raw output are not publicly available upon publication because it is

not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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